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WELL-BEING, DEPRIVATION, AND THE GREAT RECESSION IN THE U.S.: A STUDY IN A MULTIDIMENSIONAL FRAMEWORK

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We study changes in social well-being and deprivation in the U.S. during the Great Recession and the subsequent recovery. We outline an analytical framework for measuring well-being and deprivation in a multidimensional fashion when data on achievement in each dimension is assumed to be ordinal and binary in nature. We use data from the American Community Survey between 2008 and 2015 and find that there was a decline in social well-being and a rise in social deprivation in the U.S. during the recession followed by a reversal of trends during the recovery. Despite low deprivation levels among the White population, this population experienced the largest increase in deprivation during the recession and the least decline in deprivation in the recovery period. These results underscore the fact that the impact of recession and the subsequent recovery varied significantly across population groups.

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1. Introduction

The Great Recession at the turn of the last decade was a searing experience for the American society. The official poverty measure based on the head-count ratio increased from about 13 percent in 2008 to almost 15 percent in 2011.[1] In addition to affecting income levels, the recession had far reaching consequences on other indicators of quality of life such as health and housing. How the Great Recession affected the society's overall well-being and deprivation and what happened during

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[1]https://www.census.gov/topics/income-poverty/poverty.html.

the recovery which followed the recession are important issues which deserve serious study. The United Nations Human Development Report annually publishes a multidimensional poverty index for almost 100 developing countries. Several countries officially release their own estimates of multidimensional poverty measures. For example, Bhutan, Chile, Colombia, Costa Rica, Ecuador, El Salvador Mexico, and the Philippines release official estimates of multidimensional poverty. In addition to the official statistics, there has been a rapid expansion of literature estimating multidimensional measures among countries (Alkire *et al.*, 2015). One might expect that there would be similar studies measuring the impact of the Great Recession on multidimensional well-being and deprivation in the U.S. It is rather surprising that this has indeed not been the case. Dhongde and Haveman (2017) is perhaps the only comprehensive paper which systematically measures multidimensional deprivation in the U.S. during the Great Recession. As far as we know, ours is the first paper to use an axiomatic approach to measure multidimensional well-being and deprivation in the U.S. over a period of 8 years spanning the Great Recession and the subsequent recovery. We study changes in overall well-being and deprivation, as well as changes in these measures among several population groups.[2]

Our study uses a conceptual and analytical framework that draws on the contributions of Dhongde *et al.* (2016). While the contributions of Dhongde *et al.* (2016) focus on measures of social deprivation, we first study axiomatically a class of measures of social well-being in the context of ordinal and binary data. Our measures of social well-being can be constructed in three steps: (i) for each individual, we compute the individual's overall achievement defined as a weighted sum of her achievements in all dimensions (the weight attached to the achievement in any given dimension being the same for all individuals), (ii) each individual's overall achievement is then transformed to that individual's well-being, and (iii) the measure of social well-being is taken to be the sum of all individuals' well-beings. It may be of interest to note that the construction of our measures of social well-being resembles that of a prioritarian (or generalized utilitarian) social welfare function discussed in the social choice literature (Blackorby *et al.*, 2002; Parfit, 1997). In our framework of binary data, an individual's achievement in a dimension is either 0 or 1. An achievement of 0 in a dimension can therefore be regarded as non-achievement or deprivation in that dimension, and an achievement of 1 in a dimension can be regarded as an absence of deprivation in that dimension. We then take advantage of this close connection between an individual's dimensional achievement and dimensional deprivation and develop a new and intuitive way to measure an individual's overall deprivation from the individual's overall achievement. Since we accept the intuition that, in our multidimensional framework, an individual who has some dimensional deprivations, and, hence, some overall deprivation, may not necessarily be considered to be deprived and since we want to

[2]Though often referred to as multidimensional "poverty" in the literature, we choose to use the term multidimensional deprivation, for two reasons. First, we define deprivation as lack of certain achievements; a shortfall in these achievements may not necessarily translate into poverty. Second, the notion of poverty in policy debates in the U.S. seems to be closely associated with either income poverty or material hardship. We follow two of the recent empirical studies in the U.S., namely Dhongde and Haveman (2017) and Mitra and Brucker (2017) who refer to their indices as multidimensional deprivation measures rather than multidimensional poverty measures.

focus on the overall deprivation of the deprived individuals only, we introduce a benchmark level t ($t > 0$) such that an individual is considered deprived if and only if her overall achievement falls short of t. Our measure of social deprivation is then computed as the sum of overall deprivations of all individuals who are classified as deprived.

Our measures are related to some existing measures introduced in the literature on measuring multi-dimensional well-being and deprivation (see, for example, Aaberge and Brandolini (2015), for an extensive survey on related studies). For example, if the transformation function that is used to transform an individual's overall achievement to well-being is linear, then our measure is equivalent to the counting measure (Atkinson, 2003) widely used in the literature. However, if the transformation function is not linear, then the family of our measures behaves very differently from counting measures, and can avoid many pitfalls suffered by various counting measures. By appropriately choosing a transformation function to transform an individual's overall achievement to her well-being (see our discussions in Section 3), we ensure that our measure satisfies certain attractive properties.

We estimate the proposed indices using data from the American Community Survey (ACS), which is the largest household level surveys in the U.S. Our sample comprises more than 2 million individuals each year from ACS rounds: 2008 to 2015. We use the recommendations of the Commission on the Measurement of Economic Performance and Social Progress (Stiglitz *et al.*, 2009) as a guide in choosing the different dimensions or dimensions in terms of which we assess an individual's well-being. We choose 9 variables from the ACS which broadly capture the well-being dimensions mentioned in the Commission's report. We estimate trends in overall well-being and deprivation over time and test their sensitivity to multiple thresholds and weights. We also estimate these indices for population groups based on age, gender, nativity, race and ethnicity and find some interesting differences among the different population groups with respect to the level and change in deprivation during and after the recession.

The rest of the paper is organized as follows. In Section 2 we formulate the analytical framework and propose a class of measures of social well-being. In Section 3 we specify related measures of social deprivation. In Section 4 we use our framework to measure well-being and deprivation in the U.S. during the Great Recession and the following recovery period. In Section 5 we test the sensitivity of the estimates. The analysis is summarized in Section 6.

2. Measurement of Multidimensional Social Well-Being

2.1. *The Basic Notation*

Let $F = \{f_1, \ldots, f_m\}$ be a given set of dimensions or dimensions in terms of which individual as well as social well-being and deprivation are to be assessed, $M = \{1, \ldots, m\}$. Suppose the measurement of each individual's achievement in terms of each dimension is binary and ordinal, so that, for each dimension, there are only two levels of achievement: 1 (the individual is not deprived in terms of the dimension) and 0 (the individual is deprived in terms of the dimension). Let there be n individuals in the society. The society is denoted by $N = \{1, \ldots, n\}$. Let

$\mathcal{A}$ be the set of all $n \times m$ matrices with 0 or 1 entries. Each $A \in \mathcal{A}$ is interpreted as the society's achievement matrix: the (ij)th entry of the matrix records individual i's achievement in dimension f_j. For each individual $i \in N$ in the society and an achievement matrix $A \in \mathcal{A}$, let $M_1^i(A)$ denote the set of all $j \in M$, such that i's achievement in terms of f_j is 1.

2.2. *Measurement of Social Well-Being*

A measure of social well-being is a function g from $\mathcal{A}$ to $[0, 1]$. For all $A, B \in \mathcal{A}$, $g(A) \geq g(B)$ is interpreted as indicating that the society's well-being level under A is at least as high as the society's well-being level under B, with obvious corresponding interpretations for $g(A) > g(B)$ and $g(A) = g(B)$. We consider the following properties of well-being measures for the society.

Normalization

For all $A = (a_{ij}) \in \mathcal{A}$ and for all $\delta \in \{0, 1\}$, if $[a_{ij} = \delta$ for all $i \in N$ and all $j \in M]$, then $g(A) = \delta$.

Normalization requires that, if every individual's achievement is 0 (resp. 1) in every dimension, then the society's well-being is 0 (resp. 1); it is a convenient, though dispensable, assumption.

Anonymity

Let σ be a bijection from N to N. Then, for all $A, B \in \mathcal{A}$, if $[a_{ij} = b_{\sigma(i)j}$ for all $i \in N$ and all $j \in M]$, then $g(A) = g(B)$.

Anonymity, which requires that a permutation of achievement vectors of the individuals should leave social well-being unchanged, essentially implies that individuals are symmetrically treated when we aggregate their achievement vectors to reach a social well-being index.

Monotonicity

For all $A, B \in \mathcal{A}$, if $[a_{ij} \geq b_{ij}$ for all $i \in N$ and all $j \in M$ and $A \neq B]$, then $g(A) > g(B)$.

Monotonicity is a compelling axiom which requires that the society's well-being must increase when, other things remaining the same, the achievements of some individuals in some dimensions increase.

Independence

For all $A, B, A', B' \in \mathcal{A}$, and for all $i' \in N$, if [(for all $i \in N \setminus \{i'\}$: $a_{ij} = b_{ij}$ and $a'_{ij} = b'_{ij}$ for all $j \in M$), and $(a_{i'j} = a'_{i'j}$ and $b_{i'j} = b'_{i'j}$ for all $j \in M)$], then $g(A) - g(B) = g(A') - g(B')$.

Independence requires that, when, starting with a given achievement matrix, the achievement vector of one individual changes without any change in the other individuals' achievement vectors, the resulting change in the society's well-being does not depend on what those other individuals' initial achievement vectors may be.

Normalization, Anonymity, Monotonicity, and Independence are fairly straightforward axioms and some version or other of each one of these axioms is often used in the literature. Our next axiom, Additivity, is the counterpart of the Additivity axiom used by Dhongde *et al.* (2016) in measuring multidimensional deprivation (a similar property for measures of multidimensional deprivation had also been introduced earlier by Bossert *et al.*, 2013) and plays a crucial role in determining the form of what emerges as an individual's well-being function in our conceptual framework. Additivity requires that, for any given achievement matrix, the (individual) well-being measure is additively separable among the dimensions.

Additivity

For each $j \in M$, there exists a function φ_j such that, for all $A, B \in \mathcal{A}$ with, for some $i' \in N$, $[a_{ij} = b_{ij} = 0$ for all $j \in M$ and all $i \in N \backslash \{i'\}]$, we have $g(A) \geq g(B) \Longleftrightarrow \sum_{j=1}^{m} \varphi_j \left(a_{i'j}\right) \geq \sum_{j=1}^{m} \varphi_j (b_{i'j})$.

The implications of the above properties to be imposed on a well-being measure are summarized in the following proposition.

Proposition 1. A well-being measure, g, for the society satisfies Normalization, Anonymity, Monotonicity, Independence and Additivity if and only if

(1) for some increasing function $\theta: [0, 1] \to [0, 1]$ with $\theta(0) = 0, \theta(1) = 1$, and some positive constants $w_1, \ldots, w_m$ with $w_1 + \cdots + w_m = 1$, we have $g(A) = \frac{1}{n} \sum_{i=1}^{n} \theta(\sum_{j=1}^{m} w_j a_{ij})$ for all $A \in \mathcal{A}$.

The proof of Proposition 1 is similar to that of Proposition 2 of Dhongde *et al.* (2016), and we omit it. It is worth noting that, implicit in the proof of Proposition 1, for each dimension $f_j \in F$, the weight $w_j > 0$ attached to f_j is the contribution of the dimension f_j, when an individual's achievement in this dimension is 1, to the individual's overall achievement. Then, the expression $\sum_{j=1}^{m} w_j a_{ij}$ figuring in (1) can be regarded as the overall achievement of individual i given the achievement vector $(a_{i1}, \ldots, a_{im})$ of individual i. We interpret θ in (1) as the well-being function of an individual, which specifies the individual's well-being as a function of her overall achievement (note that, by Anonymity, all the individuals have the same well-being function θ). The exact form of the well-being function θ is essentially a matter of value judgment and it is possible for reasonable evaluators to differ about the specification of the function θ.[3] In our empirical applications, we shall take θ as a power function so that

[3]This, of course, depends on the interpretation of the concept of individual well-being. If individual well-being is interpreted as having a descriptive content independent of dimensions, then the form of the well-being function can be considered to be a testable empirical assumption; for instance, this is the case when well-being is interpreted as happiness or desire fulfillment. Conforming to the spirit of the functioning and capability approach to individual well-being, we are, however, interpreting individual well-being as the value that the evaluator of an individual's well-being attaches to the individual's achievement vector (for a discussion of this specific point, see Sen, 1987, pp. 36–7). Given our interpretation, the form of the individual well-being function is a matter of value judgment.

(2) for some $\alpha\,(\alpha>0)$, $\theta\left(\sum_{j=1}^{m} w_j a_{ij}\right)=(\sum_{j=1}^{m} w_j a_{ij})^{\alpha}$ for all $i\in N$, where a_{ij} is the achievement of individual i in dimension j.

Several remarks concerning the family of measures of well-being defined in (1) and in (2) are in order. First, the family of well-being measures defined in (1) has been studied axiomatically by Chakravarty and D'Ambrosio (2006) in the context of measuring social exclusion. Their axioms restrict the corresponding θ function to have a "convexity" property: θ is increasing with a non-decreasing rate. Second, in our framework where achievement levels are either 0 or 1, the family of well-being measures defined in (2) coincides with a commonly used family, $\sum_{i=1}^{n}(w_1(a_{i1})^{\frac{1}{\beta}}+\cdots+w_m(a_{im})^{\frac{1}{\beta}})^{\beta}$ where $\beta>0$, of well-being measures in the literature (see, among others, Maasoumi, 1986; Bourguignon, 1999; and Decancq and Lugo, 2013). This is because, for any $i\in N$ and any $j\in M$, when $a_{ij}=0$, we have $(a_{ij})^{\frac{1}{\beta}}=0$, and when $a_{ij}=1$, we have $(a_{ij})^{\frac{1}{\beta}}=1$. Consequently, in our framework, $\sum_{i=1}^{n}(w_1(a_{i1})^{\frac{1}{\beta}}+\cdots+w_m(a_{im})^{\frac{1}{\beta}})^{\beta}=\sum_{i=1}^{n}(w_1a_{i1}+\cdots+w_1a_{i1})^{\beta}$. Third, when the θ function in (1) is linear (or α in (2) is equal to 1), the measure in (1) and (2) is equivalent to the simple counting measure (head-count method). The simple counting measure is a special case of more elaborate counting measures that have attracted much attention and have been extensively studied in the literature (see Aaberge and Brandolini, 2015 for an extensive survey on related studies). However, when the θ function in (1) is not linear, the family of measures defined in (1) behaves very differently from counting measures, and these measures can avoid many pitfalls suffered by various counting measures (Pattanaik and Xu, 2018).

3. Measurement of Multidimensional Social Deprivation

3.1. *From Measures of Social Well-Being to Measures of Social Deprivation*

Instead of developing a measure of social deprivation in an axiomatic fashion, we develop this measure intuitively. In our framework where data on each achievement is binary, the following statements seem reasonable: (i) for any two achievement vectors, say x and y in $\{0,1\}^m$, the increase (resp. decrease) in an individual's overall achievement when she goes from x to y is equal to the decrease (resp. increase) in the individual's deprivation when she goes from x to y; and (ii) a zero achievement vector gives a maximum deprivation (normalized to 1) for an individual and the achievement vector with an achievement level of 1 for every dimension gives a minimum deprivation (normalized to 0) for the individual. With these two intuitions, it can be checked that, for every individual $i\in N$ and for every achievement vector a_i of individual i, 1-$\theta(\sum_{j=1}^{m} w_j a_{ij})$ measures i's deprivation when i's overall achievement is measured by $\theta(\sum_{j=1}^{m} w_j a_{ij})$. Therefore, if individual i's well-being function is given by (for some $\alpha>0$) $(\sum_{j=1}^{m} w_j x_{ij})^{\alpha}$ for all $x_i\in\{0,1\}^m$, then 1-$(\sum_{j=1}^{m} w_j x_{ij})^{\alpha}$ measures i's deprivation and is treated as individual i's deprivation function.

Given the above discussion, we proceed to the measure of social deprivation. Suppose there exists a positive real number $t \in [0, 1]$ such that, for an achievement matrix $A \in \mathcal{A}$, a person i is *deprived* if and only if $\sum_{j=1}^{m} w_j x_{ij} < t$. It is clear that this criterion for identifying a deprived person is equivalent to saying that, for an $A \in \mathcal{A}$, a person i is deprived iff her deprivation exceeds 1- t^{α}. Thus, in the framework of 0 or 1 achievement levels, our approach to identification is formally equivalent to the counting approach (Atkinson, 2003). Depending on the value of t, it is possible that a person may have inadequate levels of achievement in some dimensions without being called deprived under our definition. If $t = 1$ then any individual whose achievement is inadequate in at least one dimension is said to be deprived—this is known as the union approach; if $0 < t < \min\{w_1, \ldots, w_m\}$, then an individual is deprived if her achievement in each dimension is inadequate—this is the so-called intersection approach (Atkinson, 2003). For any achievement matrix $A \in \mathcal{A}$ and for a given $t \in [0, 1]$, let $N^d(A, t)$ denote the set of individuals who are deprived. Let $h: \mathcal{A} \to [0, 1]$ be a measure of the society's overall deprivation. For any given achievement matrix $A \in \mathcal{A}$, $h(A)$ measures the overall deprivation of the society under A: for any achievement matrices $A, B \in \mathcal{A}$, $h(A) \geq h(B)$ is to be interpreted as indicating that the society's deprivation level under A is at least as high as the society's deprivation level under B. Our measure of social deprivation will then be given as:

(3) for some $\alpha > 0$, $h(A) = \frac{1}{n} \sum_{i \in N^d(A, t)} [1 - (\sum_{j=1}^{m} w_j a_{ij})^{\alpha}]$ for all $A \in \mathcal{A}$.

Note that, in the assessment of social well-being, the entire population is considered, while in the evaluation of social deprivation, only those that are classified as deprived are considered. This asymmetric treatment of social well-being and social deprivation is reflected in the Weak Focus axiom in the literature (see Dutta, *et al.*, 2003; Permanyer, 2014; and Pattanaik and Xu, 2018). This axiom basically states that a measure of social deprivation should be insensitive to changes of a non-deprived individual's achievements (as long as this non-deprived individual stays non-deprived). If one seeks to focus on the urgent problem of measuring and relieving the deprivation of those individuals who are classified as deprived, it makes sense to ignore the deprivation of the individuals who have some deprivations in some dimensions but whose overall deprivations are not severe enough for them to be considered deprived.

3.2. *Further Restrictions on Measures of Well-Being and Deprivation*

Recall that (2) gives us a class of social well-being measures; for each value of α, we get one social well-being measure belonging to this class. Similarly, (3) gives us a corresponding class of social deprivation measures. One can think of plausible restrictions on the values of α figuring in (2) and (3). Let $\rho: \{0, 1\}^m \to [0, 1]$ be an individual's well-being function and $\sigma: \{0, 1\}^m \to [0, 1]$ be an individual's deprivation function. Consider the following two properties.

Clustered Dimensional Deteriorations and Well-being (CDDW)

Let $v, x, y, z \in \{0,1\}^m$ be such that, for some distinct $k, k' \in M$, $[(v_k = v_{k'} = 1)$ and $(x_k = 1$ and $x_{k'} = 0)$ and $(y_k = 0$ and $y_{k'} = 1)$ and $(z_k = z_{k'} = 0)]$ and for all $j \in M \setminus \{k, k'\}$, $v_j = x_j = y_j = z_j$. Then $\rho(v) - \rho(z) > [\rho(v) - \rho(x)] + [\rho(v) - \rho(y)]$.

Clustered Dimensional Deteriorations and Deprivation (CDDD)

Let $v, x, y, z \in \{0,1\}^m$ be such that, for some distinct $k, k' \in M$, $[(v_k = v_{k'} = 1)$ and $(x_k = 1$ and $x_{k'} = 0)$ and $(y_k = 0$ and $y_{k'} = 1)$ and $(z_k = z_{k'} = 0)]$ and for all $j \in M \setminus \{k, k'\}$, $v_j = x_j = y_j = z_j$. Then $\sigma(z) - \sigma(v) > [\sigma(x) - \sigma(v)] + [\sigma(y) - \sigma(v)]$.

The two properties are essentially equivalent given our postulated relation between an individual's well-being function and her deprivation function (see Section 3.1). They are based on the intuition underlying a restriction proposed by Stiglitz *et al.* (2009, p. 15) for measures of individual deprivation: *"… the consequences for quality of life of having multiple disadvantages far exceed the sum of their individual effects*".

To see the intuition of CDDD, consider four individual achievement vectors with two dimensions: $v = (1,1)$, $x = (1,0)$, $y = (0,1)$, and $z = (0,0)$. The increase in deprivation resulting from a switch from v to z is given by $\sigma(z) - \sigma(v)$. The increases in individual deprivation resulting from a switch from v to x and from a switch from $v$to y are given, respectively, by $\sigma(x) - \sigma(v)$ and $\sigma(y) - \sigma(v)$. The change from v to z involves simultaneous changes from 1 to 0 in achievements in two different dimensions, namely, f_1 and f_2, while the switch from v to x involves a change from 1 to 0 in the achievement in dimension f_2 only and the switch from v to y involves a change from 1 to 0 in the achievement along dimension f_1 only. Thus, what CDDD says is that the total harm caused by two different dimensional deprivations occurring simultaneously is greater than the sum of the separate harms caused by those two dimensional deprivations occurring one at a time. Intuitively, we find CDDD (resp. CDDW) captures a very plausible property of an individual deprivation function (resp. an individual well-being function). One can prove the following proposition. Its proof is elementary and we shall omit it.

Proposition 2 An individual well-being function $\rho(x) = (\sum_{j=1}^{m} w_j x_j)^{\alpha}$ $(\alpha > 0)$ satisfies CDDW if and only if $0 < \alpha < 1$; and an individual deprivation function $\sigma(x) = 1 - (\sum_{j=1}^{m} w_j a_{ij})^{\alpha}$ $(\alpha > 0)$ satisfies CDDD if and only if $0 < \alpha < 1$.

Before concluding this section, we would like to note the connections between our CDDW and two other well-known concepts in the literature on multidimensional approach to well-being and deprivation. First, there is a close link between CDDW and an intuitive notion of two dimensions being substitutes of each other. A possible formulation of the notions of two dimensions being substitutes or complements can be as follows. Suppose the well-being measure for an individual is given by $\rho(x)$ for all $x \in \{0,1\}^m$. Then for all distinct $f_k, f_{k'} \in F$, f_k and $f_{k'}$ are said to be *substitutes* for the individual if and only if

(4) for all $v, x, y, z \in \{0,1\}^m$ such that $[(v_j = x_j = y_j = z_j$ for all $j \in M - \{k, k'\})$ and $(v_k = v_{k'} = 1)$ and $(x_k = 1$ and $x_{k'} = 0)$ and $(y_k = 0$ and $y_{k'} = 1)$ and $(z_k = z_{k'} = 0)]$, we have $\rho(x) - \rho(z) > \rho(v) - \rho(y)$.

Replacing "<" in (4) by ">" and "=", one gets, respectively, the definition of two dimensions being *complements* for an individual and the definition of two dimensions being *neutrally related* for an individual.

Our definition here is based on Pattanaik *et al.* (2012). It is worth noting that if we treat the well-being measure $\rho(x)$ as a differentiable function of continuous variables, then our notion of substitutability is the so-called Auspitz-Lieben-Edgeworth-Pareto (ALEP) notion of substitutes: two dimensions f_j and f_k are substitutes if $\frac{\partial^2 \rho}{\partial j \partial k} < 0$ (Samuelson, 1974). Applying the ALEP notion of substitutes to the well-being measure $\rho(x) = (\sum_{j=1}^{m} w_j x_j)^{\alpha}$ (assuming that dimensions are continuous variables), we note that $\frac{\partial^2 \rho}{\partial j \partial k} = \alpha(\alpha - 1) w_j w_k (\sum_{j=1}^{m} w_j x_j)^{\alpha - 2}$. For this cross partial to be negative, noting that $\alpha > 0, w_j > 0, w_k > 0$, we must have $\alpha < 1$ and vice versa.[4] Note that the ALEP definitions of substitutes and complements, as well as our definitions, implicitly assume that the well-being function for an individual is cardinal. Also, at the risk of laboring the obvious, it may be worth noting that, in our present context, the question of whether certain attributes are substitutes or complements is not a question about empirical facts but a question about the value judgments that one wants to incorporate in the assessment of an individual's well-being.

Intuitively, under our definitions, f_k and $f_{k'}$ are substitutes (resp. complements) if the increase in the well-being of an individual caused by a switch of f_k from inadequate (0) to adequate (1), when the individual's achievement in terms of every other dimension is held fixed, is larger (resp. smaller) when the achievement in terms of $f_{k'}$ is held fixed at inadequate (0) rather than adequate (1). It is easy to check that, if an individual's well-being function satisfies CDDW, then every dimension is a substitute (in the sense specified above) of every other dimension.

Secondly, CDDW is related to the "association-decreasing rearrangement" principle (also known as correlation decreasing switches) in the literature on multidimensional inequality and deprivation.[5] To see this, we first note that the expression $\rho(v) - \rho(z) > [\rho(v) - \rho(x)] + [\rho(v) - \rho(y)]$ in CDDW can be equivalently written as $\rho(x) + \rho(y) > \rho(v) + \rho(z)$.[6] Note that the achievement vector v dominates the achievement vector z. For illustrative purpose, suppose there are two individuals and two dimensions. Suppose individual 1's achievement vector is $v = (1,1)$, and individual 2's achievement vector is $z = (0,0)$. Now, rearrange their achievements so that individual 1's achievement is given by $x = (1,0)$ and individual 2's achievement is given by $y = (0, 1)$. Note that there is no vector-dominance relation between x and y (after the rearrangement) so that neither individual has unambiguously more of everything than the other. Also the overall achievement levels in society remain the same, but the correlation between the two achievement vectors has been reduced. The association-decreasing rearrangement principle stipulates that social

[4]We thank one of the referees for pointing out the conceptual similarity between the two notions.

[5]See, for example, Atkinson and Bourguignon (1982), Bourguignon and Chakravarty (2018), and Tsui (2002); see also Pattanaik *et al.* (2012) for a related axiom that has the same spirit as the association-decreasing rearrangement principle.

[6]The idea underlying CDDW corresponds to the association decreasing principle in the literature on multidimensional risk (see, Richard, 1975), and is linked to submodularity/supermodularity in mathematics (see, Topkis, 1998; Muller and Scarsini, 2012).

well-being should increase. Given $\rho(x)+\rho(y)>\rho(v)+\rho(z)$ and Proposition 1, CDDW and the association-decreasing rearrangement principle are equivalent in our context.

4. Well-Being, Deprivation and the Great Recession in the U.S

In this section, we use the proposed framework to estimate trends in well-being and deprivation among the population in the U.S. We find several studies in the literature which compare well-being measures globally between countries or among nations within the European Union.[7] However few studies have measured well-being in the U.S. Typically these studies have used either a "dashboard" approach or a "composite index approach"; none has used an axiomatic approach to measure well-being in the U.S. The dashboard approach provides a marginal distribution of achievement. For instance, the Federal Reserve Bank's Report on the Economic Well-Being of U.S. Households (2016) publishes annual report on the percent of population with banking or credit access, the percent of population with automobile loans, the percent of population with retirement savings, and so on. The dashboard approach thus does not estimate the overall social well-being or the trends there in. In composite index approach the proportions of individuals with achievement in each dimension is computed first and then these proportions are aggregated into some type of composite index. For example, the Gallup-Healthways annual report on the State of American Well-being (2016) ranks states, communities and congressional districts by a composite index based on telephone interviews about health related dimensions such as life evaluation, emotional and physical health and so on. Other studies, such as Oswald and Wu (2011) estimated life satisfaction equations by controlling for people's personal characteristics and found no correlation between states' regression-adjusted well-being and their GDP per capita. Deaton (2012) analyzed daily recorded data on self-reported subjective well-being and found that Americans reported sharp declines in their life evaluation and sharp increases in worry and stress, during the financial crisis. As far as we are aware, this is the first study which uses an axiomatic approach to measure well-being in the U.S. The social well-being measure is based on certain intuitively attractive properties such as the CDDW property discussed in the previous section. We estimate the well-being level for each individual by aggregating achievements across dimensions and then aggregate individual well-being levels to estimate overall social well-being. Thus unlike the dashboard or composite index approach, we estimate a social well-being index by taking into account the joint distribution of each individual's achievements.

4.1. *Data*

The ACS collects information on demographic, social, economic, and housing characteristics of the sample population. It randomly selects samples in all

[7]See Berenger and Verdier-Couchane (2002); Deaton (2012); and UNDP Human Development Report (2016) for comparisons between countries and Pittau *et al*. (2013); Marlier *et al.* (2012); Whelan *et al*. (2014), and the OECD Better Life Index http://www.oecdbetterlifeindex.org/ for comparisons within the European Union.

counties across the nation (and all municipios in Puerto Rico) every month. We use ACS records on adults, aged 18 and above, from the Public Use Microdata Sample (PUMS) files.[8] We do not include children in our sample since many of the achievements we choose are appropriate for adults and not for children. We compile data starting from 2008 (previous rounds data is not considered consistent) till 2015.

Although the choice of dimensions is largely dictated by the availability of relevant data in the ACS, we use as guidance, the recommendations made by the Commission on the Measurement of Economic Performance and Social Progress (Stiglitz *et al.*, 2009). Previous studies measuring multidimensional deprivation in the U.S. also have used the Commission's recommendations listed in its report (pp. 14–5) (Dhongde and Haveman, 2017; Mitra and Brucker, 2017). Table 1 lists the different variables in ACS that we choose to reflect an individual's achievement and deprivation in a particular dimension. Detailed explanation of each dimension is provided in the Appendix to the paper.

As seen in Table 1, different dimensions are measured in different units making it difficult to aggregate an individual's achievements across dimensions. For instance, in Table 1, income, and housing costs are continuous variables, the number of disabilities, rooms per household or kitchen and plumbing facilities are discrete variables. Education and ability to speak English fluently are categorical (ordinal and discrete) variables whereas being employed or having health insurance are binary variables. Typically, indices based on the counting approach use dichotomous or binary variables (Aaberge and Brandolini, 2015); we convert data on all 9 indicators to a binary 0-1 form.

4.2. *Parameter Values, Thresholds and Weights*

We choose a power function so that the social well-being index is estimated as $g(A) = \frac{1}{n}\sum_{i=1}^{n}(\sum_{j=1}^{m} w_j a_{ij})^{\alpha}$ for all $A \in \mathcal{A}$ where $\alpha > 0$, and a_{ij} is the achievement of individual i in dimension j. The social deprivation index is estimated as $h(A) = \frac{1}{n}\sum_{i \in N^d(A,t)}[1 - (\sum_{j=1}^{m} w_j a_{ij})^{\alpha}]$ for all $A \in \mathcal{A}$, where $\alpha > 0$ and $N^d(A, t)$ is the set of individuals who are classified as deprived overall. We assume that $\alpha = \frac{1}{2}$, which implies that g satisfies CDDW and h satisfies CDDD. In order to estimate the social deprivation index, we also need to choose a value for t, since an individual i is considered deprived in our framework if and only if $\sum_{j \in M_1^i(A)} w_j < t$. We assume $t = \frac{2}{3}$, so that if we assign equal weights to all dimensions ($w_1 = \cdots = w_9 = \frac{1}{9}$), then an individual with fewer than 6 out of 9 achievements (that is, deprived in 4 or more of the 9 dimensions) is regarded as being deprived overall. This threshold aligns with the threshold used in the European Union, where "severally materially deprived" are all persons who cannot afford at least 4 out of 9 amenities. In the next section, we change each of these values and re-estimate the indices.

[8]PUMS files provide data from areas with population of 65,000 or more. Individual records are replicated using person weights; data on individual's household characteristics are used as well. Individuals living in group quarters such as college residence halls, residential treatment centers, skilled nursing facilities, group homes, military barracks, correctional facilities, and workers' dormitories are not included.

TABLE 1
MULTIPLE DIMENSIONS OF WELL-BEING

Dimensions Recommended by the Commission	Relevant Data Available in the ACS
Std. of living	1. Individual's family income is above poverty threshold
	2. Individual's housing costs are less than 50% of household income
Health	3. Individual has fewer than 2 of 6 disabilities
Education	4. Individual has at least a high-school diploma
Personal activities including work	5. Individual is employed
Economic security	6. Individual has health insurance coverage
Social connections and relationships	7. Individual belongs to a household where at least one person speaks English
Environment	8. Individual belongs to a household which has less than one occupant per room
	9. Individual belongs to a household which has kitchen and plumbing facilities

Source: Stiglitz *et al.* (2009) and the American Community Survey User Guide.

4.3. *Multidimensional Well-being and Deprivation Over Time*

Trends in the Well-being and Deprivation Index

Table 2 shows values of indices for both well-being and deprivation between 2008 and 2015. With the onset of the Great Recession, the well-being index declined by 0.27 percent between 2008 and 2009, and further by 0.18 percent in 2010. Well-being improved in the post-recession period, slower at first and then at a more rapid rate. Between 2011 and 2015, well-being index increased on average by 0.2 percent.[9]

A reverse trend is observed in estimates of the deprivation index, which increased during the recession (2008 to 2010) and declined from 2011 onwards. By the end of the recovery period, deprivation levels (in 2015) were lower than deprivation levels at the start of the recession (in 2008). Note that the deprivation index is much smaller in value than the well-being index since the deprivation index is calculated only for those individuals who are overall deprived; in this case, individuals deprived in 4 or more of the 9 dimensions. The largest increase in social deprivation was seen between 2008 and 2009 (13.33 percent) whereas the largest decrease occurred between 2014 and 2015 (16.39 percent).

Social Deprivation Index and the Official Poverty Measure

The trend in the social deprivation index differs significantly from the trend in the official income poverty measure (Table 2). During the recession, income poverty among the adult population increased from 11.4 percent in 2008 to 12.8 in 2010. However even after the end of the recession, during the recovery period,

[9]The National Bureau of Economic Research (NBER) dates the beginning of the recession as December 2007; officially the recession lasted through June 2009.

TABLE 2
MULTIDIMENSIONAL WELL-BEING AND DEPRIVATION INDICES OVER TIME

Years	Well-Being Index	Percent Change	Deprivation Index	Percent Change	Poverty Index	Percent Change
2008	0.954		0.009		11.35	
2009	0.952	−0.27	0.010	13.33	12.27	8.07
2010	0.950	−0.18	0.010	5.53	12.88	5.01
2011	0.950	0.04	0.010	−1.41	12.83	−0.40
2012	0.952	0.21	0.009	−10.32	12.84	0.09
2013	0.953	0.10	0.009	−6.02	12.81	−0.24
2014	0.956	0.32	0.007	−12.64	12.85	0.27
2015	0.959	0.31	0.006	−16.39	11.69	−9.00

Source: Well-being and deprivation index are based on authors' calculations using ACS data. $\alpha = \frac{1}{2}, t = \frac{2}{3}$, all dimensions weighted equally. Values of well-being and deprivation index have been rounded to three decimal places; percent changes are rounded to two decimal places and are based on index values extending to eight decimal places. Poverty Index is compiled from U.S. Census Bureau Poverty Status for Adults aged 18 and above.

income poverty levels were stubbornly stagnant at around 12.8 percent between 2010 and 2014. Only after 2014, did the income poverty rates decline significantly. These trends suggest that during the recovery period, income levels did not recover rapidly. However, other indicators of well-being such as availability of health insurance, housing costs, high-school completion rates improved during this period (see Section 4.4 below for details). These improvements, though not captured by income poverty, were reflected in the multidimensional deprivation index. Dhongde and Haveman (2017) also find a similar result— unlike the official poverty measure which did not show any decline, the multidimensional deprivation index better reflects the economic recovery since the recession (pp. 485–6).

Trends in the Components of the Well-being Index

Note that the social well-being index is decomposable and can be expressed as the weighted average of the well-being of non-deprived and that of the deprived, where the weights measure the shares of the two groups in the population. In Table 3 we summarize estimates of these different components of the well-being index.

As expected, we see in Table 3 that the well-being among the non-deprived was higher than that among the deprived. Overall, about 97 percent of the adult population was not deprived and about 4 percent was deprived (individuals who were deprived in at least 4 of the 9 dimensions). During the recession, the share of the deprived population increased from 3 to 4 percent. There was not much decline in the well-being among the deprived; it remained constant at around 0.719 from 2009 to 2011. Thus during the recession, there was a rise in the incidence of the deprived and not much change in the average intensity of deprivation. However in the post-recession recovery period (2012–2015), both the incidence of the deprived decreased as well as their well-being improved. Compared to 2008, by 2015, the percentage of deprived population was lower and their overall well-being index was higher suggesting an overall improvement in the incidence and average intensity of well-being.

4.4. *Trends in Deprivation along each Dimension*

In order to further understand the trends in these indices, we analyze the trend in each dimension separately. In Figure 1, we plot the percentage of deprived individuals in each dimension between 2008 and 2015. Observing the trends in Figure 1, we can broadly classify the 9 dimensions in two groups. The first group comprises of 4 dimensions with greater proportion (more than 10 percent) of deprived population. These include health insurance coverage, income poverty, housing costs and schooling. The percentage of individuals without any health insurance increased from 16.8 percent in 2008 to 18 percent in 2010 and, since then, it declined gradually back to 16.8 percent in 2014. Recall that most of the major provisions of the Affordable Care Act (ACA) were enacted in January 2014. As a result of the ACA, the percentage of individuals deprived of health insurance declined to 13.3 percent in 2014 and further to 10.7 percent in 2015. The percentage of individuals with high housing costs and the percentage of high

TABLE 3

Decomposing the Well-Being Index Between the Non-Deprived and the Deprived

	WB Among Non-Deprived	Population Share of Non-Deprived	WB Among Deprived	Population Share of Deprived	Total WB
2008	0.962	0.97	0.720	0.03	0.954
2009	0.960	0.97	0.719	0.03	0.952
2010	0.959	0.96	0.719	0.04	0.950
2011	0.959	0.96	0.719	0.04	0.950
2012	0.960	0.97	0.721	0.03	0.952
2013	0.961	0.97	0.722	0.03	0.953
2014	0.963	0.97	0.722	0.03	0.956
2015	0.965	0.98	0.723	0.02	0.959

Source: Authors' calculations based on ACS data. $\alpha = \frac{1}{2}$, $t = \frac{2}{3}$, all dimensions weighted equally. Values of well-being index have been rounded to three decimal places.

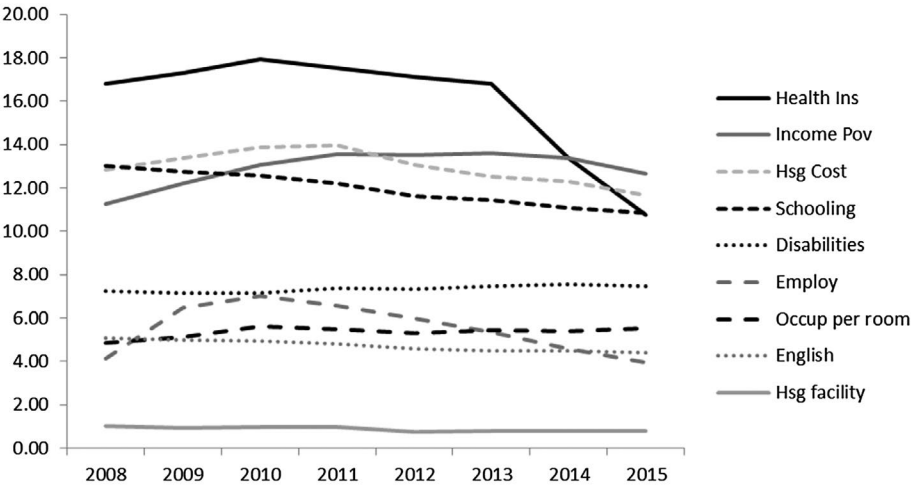

Figure 1. Trends in Deprivation along each Dimension.
Source: Authors' estimates of percent of population deprived in each dimension based on ACS data.

school drop outs continued to decline throughout the years. Our estimates of income poverty are very close to the official poverty estimates. Income poverty rates increased from 11.3 percent in 2008 to 13 percent in 2010. Between 2010 and 2014, poverty rates continued to stagnate at 13 percent or above and declined only slightly to 12.7 percent in 2015. As seen in Figure 1, the second group comprises of those dimensions where the proportion of deprived individuals is much lower (less than 10 percent). Of these, the percentage of individuals unemployed increased during the recession, it was as high as 7 percent in 2010 and then gradually declined to 4 percent in 2014–2015. There was not much change in deprivation incidence in the remaining 3 dimensions, namely, the percentage of individuals with two or more disabilities (7 percent), the percentage of individuals in living in households with limited English speaking ability (4.5 percent) and the percentage of individuals in households with no plumbing and/or kitchen facilities (0.9 percent). The lack of significant changes in the percentage of people having two or more disabilities and the percentage of people living in households without anyone speaking English is not surprising since the incidence of such deprivation is unlikely to change over a relatively short period. In Section 5, when we introduce hierarchical structures on dimensions, we categorize the first group of dimensions as basic and assign them greater weight and classify the second group of dimensions as non-basic, with lower weights.

Unemployment as an Indicator of Deprivation

Unemployment is an important indicator of deprivation, especially since we measure deprivation in the U.S. during the Great Recession and subsequent recovery. Although our sample includes adult individuals (18 and above), unemployment as an indicator of deprivation is relevant largely to the working-age

population (18–64 years). In Figure 2, we compare the trends in unemployment rates between the entire population and the working-age population. Unemployment rates peaked in 2010 for both groups and were slightly lower than the official unemployment rate at 10 percent (Bureau of Labor Statistics, 2012).[10] As expected, unemployment rate was higher among the working-age population but it followed a trend similar to the unemployment rate among the entire population.

We suspect that the incidence of unemployment may have been even higher among working-age adults than seen in Figure 2, since many discouraged workers quit searching for jobs during the recession. There is evidence showing that the number of marginally attached and discouraged workers during the recession rose substantially, and the labor force participation rate declined (Kalleberg and Von Wachter, 2017).[11] The ACS asks individuals whether they are unemployed or whether they are not in the labor force, among other options related to the employment status (given in the Appendix). There is no detailed information in the ACS regarding the number of working hours, full/part-time employment, voluntarily/involuntarily not in the labor force-all of which is important for well-being measurement.[12] Using the ACS data, we find that between 2008 and 2015, the percentage of working-age adults who reported not to be in the labor force steadily increased from about 20 percent to almost 23 percent (Figure A1 in the Appendix). In the next sub-section, we separately estimate well-being indices by taking into account these differences.

4.5. *Multidimensional Well-being and Deprivation among Population Groups*

Next, we compare changes in the well-being and the deprivation index by considering population groups by age, gender, race and ethnicity and whether a person is native or foreign-born. In Table 4, we compare percentage changes in well-being and deprivation, for instance, among Whites, Blacks, Asians and Hispanics. For a convenient analysis, we divide the 8 year time period in three intervals: 2008–2010 is the period of recession, 2010–2012 is the immediate period following the recession with short term recovery, and 2012–2015 is the period capturing relatively long-term recovery. Estimates of the indices for each year, for every population group are given in Appendix Table A1.

[10]To be consistent with other deprivation indicators, we estimate the unemployment rate as the number of unemployed persons as a percent of the relevant entire population. The Bureau of Labor Statistics, however, calculates the unemployment rate as the number of unemployed persons as a percent of the labor force. The labor force is the total number of employed and unemployed persons.

[11]The Bureau of Labor Statistics defines persons marginally attached to the labor force as those who currently are neither working nor looking for work but indicate that they want and are available for a job and have looked for work sometime in the past twelve months. Discouraged workers, a subset of the marginally attached, are those who give a job-market-related reason for not currently looking for work (Kalleberg and Von Wachter, 2017).

[12]Although the Bureau of Labor Statistics publishes six alternative measures of labor underutilization, similar detailed data on employment status is not compiled by the U.S. Census Bureau in its household surveys such as the ACS. Household surveys, however, are an important source of data for estimating multi-dimensional well-being and deprivation measures. Brandolini and Viviano (2012) underscore the need for statistical agencies to re-examine existing labor statistics by taking into account the diversity of labor market conditions. We invite the Census Bureau to provide more details on the employment status, in line with the Bureau of Labor Statistics.

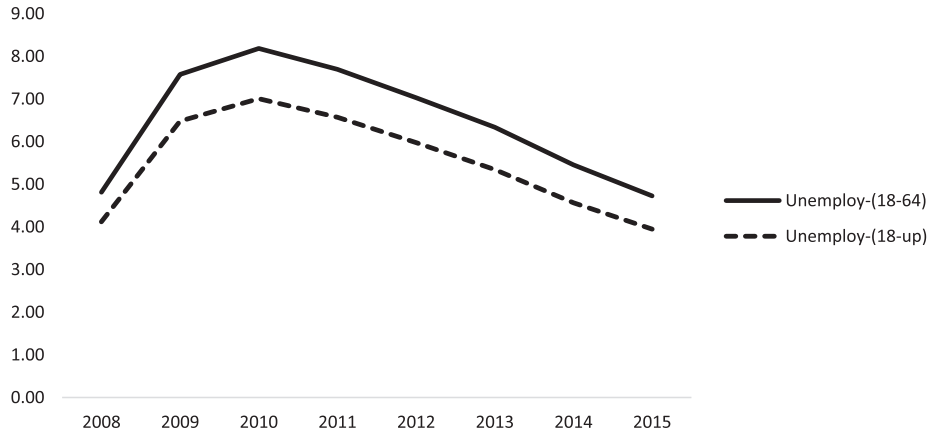

Figure 2. Trends in Unemployment among Working-age Population vs. the Entire Adult Population.
Source: Authors' estimates of percent of unemployed based on ACS data.

Compared to percentage changes in the deprivation index, percentage changes in the well-being index are smaller in magnitude as seen in Table 4. As expected, the sign of a change in the well-being index is opposite the sign of a change in the deprivation index. In the last row of Table 4, we provide percentage changes for the entire adult population (from Table 2), which serves as a benchmark to compare percentage changes among different groups. For all population groups except for the elderly adults (age 65 and above), deprivation increased and well-being decreased during the recession (2008–2010). The recession had a significant impact on the working-age adults, who experienced an increase in deprivation by 23 percent. The elderly adults, on the other hand, did not experience any rise in deprivation during the recession. This is largely explained by the fact that the recession had a significant impact on indicators such as having health insurance, completing high-school or being employed for the working-age population; these indicators did not change significantly for the elderly adults. Compared to the benchmark, we observe that the absolute magnitude of the change in both the well-being and the deprivation index was greater among men and lower among women. Similarly, we find that during the recovery period, the improvement in well-being among foreign-born individuals was greater compared to the improvement in well-being among general population.

Consider the deprivation index among different racial/ethnic groups. Table A1 in the Appendix shows that deprivation "levels" were the highest among Hispanics, similar among Asians and Blacks and least among Whites. However as seen in Table 4, the "decrease" in deprivation between 2012 and 2015 was least among the Whites compared to the other groups. In general, deprivation declined significantly (about 30 to 40 percent) among all population groups between 2012 and 2015, owing to a decline in the number of individuals who were: (i) without health insurance, (ii) without high school diploma and (iii) with high housing costs as seen in Figure 1.

TABLE 4
VARIATION IN MULTIDIMENSIONAL WELL-BEING AND DEPRIVATION AMONG POPULATION GROUPS

			Well-Being		Deprivation	
% Change	2008-2010	2010-2012	2012-2015	2008-2010	2010-2012	2012-2015
Age						
18-64	−0.59	0.26	0.83	23.06	−11.54	−32.81
65-above	0.22	0.19	0.25	−9.41	−5.80	−9.49
Gender						
Male	−0.51	0.33	0.77	22.99	−14.40	−33.23
Female	−0.39	0.17	0.69	16.39	−8.76	−29.57
Nativity						
Native	−0.41	0.19	0.63	24.15	−9.94	−37.24
Foreign-born+	−0.49	0.64	1.45	13.10	−13.36	−28.45
Race/Ethnicity						
White	−0.39	0.21	0.60	20.29	−9.85	−30.88
Black	−0.56	0.24	1.19	19.04	−8.15	−39.73
Asian	−0.41	0.24	0.90	16.39	−0.90	−34.12
Hispanic++	−0.44	0.89	1.76	10.84	−17.32	−31.08
Overall	−0.45	0.25	0.73	19.60	−11.59	−31.35

[+]Includes naturalized citizens and non-citizens.
[++]Includes Hispanic, Spanish and Latinos.

5. Testing the Sensitivity of Estimates

The value of any index depends on the underlying assumptions made. Hence it is important to test how sensitive the trends observed in these estimates are by considering alternative values. We treat previous estimates in Table 2 as the benchmark, where we used the values $\alpha = \frac{1}{2}$, $t = \frac{2}{3}$ and weighted all dimensions equally. In the following sub-sections, we change each of these assumptions, one at a time and re-estimate the indices. We find that the trends in the indices observed above are robust to changes in these assumptions.

5.1. *Sensitivity to Different Power Functions*

Holding constant all other parameters, including same weights to all dimension, we estimate in Table 4, the social well-being index $\frac{1}{n}\sum_{i=1}^{n}(\sum_{j=1}^{m} w_j a_{ij})^{\alpha}$, for different values of the power function, namely when $\alpha = \frac{1}{2}$as in the benchmark case and when $\alpha = 1$. Recall that, for each achievement matrix A and each individual i, $\sum_{j=1}^{m} w_j a_{ij}$ is interpreted as i's overall achievement, and consequently, for a given α $(0 < \alpha)$, $\left(\sum_{j=1}^{m} w_j a_{ij}\right)^{\alpha}$ is interpreted as the individual's well-being. When $\alpha = 1$, we have a special case, when, (i) the individual i's well-being coincides with the individual i's overall achievement and (ii) the social well-being index no longer satisfies the CDDW property. Thus, when $\alpha = 1$,the society's well-being is simply the average of all the individuals' overall achievements. If we take the average of the well-being index, when $\alpha = 1$, it is equal to 0.91, implying that on average, most individuals had achievements in 8 out of 9 attributes. As seen in Table 5, a similar trend in the well-being index is observed for the two alternative values of α.

5.2. *Sensitivity to Different Weighting Structures*

So far we have assigned equal weights to all dimensions. Next, we discuss a weighting scheme that allows different weights for different dimensions. In Proposition (1), $w_1, \ldots, w_m$ are the relative weights attached to the different dimensions in calculating the overall achievement of an individual; thus, they are value-based parameters. When we have a relatively large number of dimensions, our intuition about the exact relative weight to be attached to each dimension may not be very precise. Also, with a large number of different weights for the

TABLE 5
Well-Being Estimates With Equal Weights for All Dimensions

Years	$\alpha = \frac{1}{2}$	$\alpha = 1$
2008	0.954	0.915
2009	0.952	0.911
2010	0.950	0.908
2011	0.950	0.908
2012	0.952	0.912
2013	0.953	0.913
2014	0.956	0.919
2015	0.959	0.924

Source: Authors' calculations based on ACS data; all dimensions weighted equally.

dimensions, checking the sensitivity of estimates of social well-being or deprivation to changes in these relative weights becomes a rather unwieldy exercise. In such cases, one possible procedure for making the task of assigning weights and checking the sensitivity of our estimates to changes in the weights more manageable may be to partition the class, F, of dimensions to a small number subclasses $F^1, \ldots, F^K$ $(K \geq 2)$ with the assumption that all dimensions in the same subclass have identical weights and, the weight attached to each dimension in the subclass F^1 is a where $a > 0$, and for each $t \in \{2, \ldots, K\}$, the weight attached to each dimension in the subclass F^t is $\gamma^{t-1}a$ where $\gamma \in (0, 1)$. For every $t \in \{1, \ldots, K\}$, let m_t denote the number of dimensions in F^t. Then,

$$am_1 + \gamma am_2 + \cdots + \gamma^{K-1} am_K = 1$$

so that

$$a = \frac{1}{m_1 + \gamma m_2 \cdots + \gamma^{t-1} m_t + \cdots + \gamma^{K-1} m_K}$$

Consequently, for each dimension in the partition F^t, the weight is given by

$$\frac{\gamma^{t-1}}{m_1 + \gamma m_2 \cdots + \gamma^{t-1} m_t + \cdots + \gamma^{K-1} m_K}$$

First, consider a relatively simple structure where we have exactly two tiers in the hierarchy $(K = 2)$: basic dimensions $\left(F^1\right)$ and non-basic dimensions $\left(F^2\right)$. Suppose F^1 contains the dimensions of health insurance, income poverty, housing cost, and high-school education $(m_1 = 4)$ and F^2 contains the remaining five dimensions $(m_2 = 5)$, namely, disabilities, employment, English fluency, occupancy per room, and housing facilities. Suppose equal weight is attached to each dimension in the same tier in the hierarchy. We calculate the weights using the formula $\frac{\gamma^{k-1}}{m_1 + \gamma m_2 \cdots + \gamma^{k-1} m_k + \cdots + \gamma^{K-1} m_K}$. Let $\gamma = \frac{1}{2}$, then the weight attached to each basic dimension is equal to 2/13 and that attached to each non-basic dimension is 1/13. We refer to this weighting structure as a 2-tier hierarchy.

Next, suppose we partition the dimensions into three categories as highly important $\left(F^1\right)$, moderately important $\left(F^2\right)$ and less important $\left(F^3\right)$. In this case, we have

(5) for every dimension $f_j \in F^1, w_j = a = 1/[m_1 + \gamma m_2 + \gamma^2 m_3]$; for every dimension $f_j \in F^2, w_j = \gamma/[m_1 + \gamma m_2 + \gamma^2 m_3]$; and for every dimension $f_j \in F^3, w_j = \gamma^2/[m_1 + \gamma m_2 + \gamma^2 m_3]$.

Given (5), checking the sensitivity of the estimates of social well-being and deprivation to changes in the relative weights of dimensions boils down to checking the sensitivity of the estimates of social well-being and deprivation to changes in only one parameter, γ. Note that our partition of F into subclasses, F^1, F^2, and

F^3, in the fashion described above does not necessarily involve the types of lexicographic principles which are discussed in several recent contributions.[13] For instance, in the case of our partition of F into subclasses F^1, F^2, and F^3, depending on the exact values of m_1, m_2, m_3, and γ, it is possible that the adverse effect that a switch from 1 to 0 in an individual's achievement along a single dimension in F^1 may have on her well-being may be outweighed by the combined favorable effects, on the same individual's well-being, of simultaneous switches from 0 to 1 in her achievements in several dimensions in $F^2 \cup F^3$.

Let the dimensions, income, housing costs, health insurance and high-school education, be highly important; let the dimensions, employment and disabilities, be moderately important; and, finally, let the dimensions, English fluency, occupancy per room, and housing facilities, be less important. Thus we have: $m_1 = 4, m_2 = 2$, $m_3 = 3$. Assuming $\gamma = \frac{1}{2}$, the weight attached to each highly important dimension is equal to 4/23, to each moderately important dimension is 2/23 and to each less important dimension is 1/23. We refer to this weighting structure as 3-tier hierarchy.

In Table 6, we list estimates of the deprivation index, when all dimensions are equally weighted (benchmark), and compare these with estimates based on 2-tier and 3-tier hierarchy. As seen in Table 6, for any given year, the deprivation index increases as the tiers in the hierarchy increase. This is because, in each partition, dimensions with greater percent of deprived individuals happen to figure in higher tiers in the hierarchy. However the trend in the deprivation index over time is evident even when the index is estimated using 2-tier and 3- tier hierarchy.

5.3. *Sensitivity to Different Threshold Values*

As a final sensitivity exercise, we alter the value of the threshold used in the social deprivation index and re-estimate the proportion of deprived population. Recall, that an individual i is deprived if and only if her weighted achievement score is less than some threshold value, $\sum_{j \in M_1^i(A)} w_j < t$. In the benchmark case, assuming all dimensions are equally weighted, a threshold value of $t = \frac{6}{9}$ implies that an individual who is deprived in 4 or more of the 9 dimensions is regarded as being deprived overall. Now consider a higher threshold, $t = \frac{4}{9}$, which means that an individual who is deprived in 6 or more of the 9 dimensions is regarded as being deprived overall and a lower threshold, $t = \frac{8}{9}$, which implies that an individual who is deprived in any 2 or more of the 9 dimensions is regarded as overall deprived.

In Table 7 we estimate the proportion of population identified as deprived for alternative threshold values. As expected, compared with the benchmark, the proportion of deprived increases if the threshold is lower and decreases for a higher threshold. On average, 21 percent adults were deprived in at least 2 of the 9 dimensions, 3 percent were deprived in at least 4 of the 9 dimensions whereas only about 0.15 percent were deprived in 6 or more dimensions.[14] Thus significantly lower

[13]See, among others, Esposito and Chiappero-Martinetti (2016) and Dhongde *et al.* (2017).
[14]Dhongde and Haveman (2017) find that on average 15 percent of working age adults (18 to 64) were deprived in at least 2 out of 6 dimensions.

TABLE 6
Deprivation Estimates: Hierarchical Structures Among Dimensions

Years	Benchmark	2-Tier Hierarchy	3-Tier Hierarchy
2008	0.009	0.022	0.035
2009	0.010	0.024	0.037
2010	0.010	0.025	0.039
2011	0.010	0.025	0.039
2012	0.009	0.023	0.037
2013	0.009	0.022	0.036
2014	0.007	0.020	0.033
2015	0.006	0.017	0.029

Source: Authors' calculations based on ACS data. $\alpha = \frac{1}{2}$, $\gamma = \frac{1}{2}$, $t = \frac{2}{3}$.

proportion of the population experienced more than 4 overlapping deprivations, a finding also echoed in Dhongde and Haveman (2017).

6. Summary

This is the first study to undertake an axiomatic approach and empirically estimate changes in social well-being and deprivation in the U.S. during the Great Recession and the subsequent recovery spanning a period of about 8years (2008–2015). We measured well-being and deprivation using data on 9 dimensions for more than 2 million adults. We found that for the society as well as for different social groups, well-being decreased and deprivation increased during the recession (2008 to 2010) and that the trend was reversed in the recovery period (2010–2015). While these general trends, which are robust to a host of sensitivity tests, are not entirely unexpected, the magnitudes of the changes are interesting, especially when we compare the changes across different social groups. For instance, the Whites show the largest increase in deprivation between 2008 and 2010. At the same time, when we consider the decline in deprivation during the recovery period between 2012 and 2015, the Blacks show the greatest decline and the Whites show the least decline. The Whites had the least improvement in their well-being during the recovery period (2012–2015) among all population groups.

TABLE 7
Proportion of Deprived by Varying Threshold Values

	Benchmark At Least 4 Dimensions	Higher Threshold At Least 6 Dimensions	Lower Threshold At Least 2 Dimensions
2008	3.06	0.15	20.61
2009	3.45	0.19	21.92
2010	3.64	0.19	22.82
2011	3.59	0.18	22.64
2012	3.25	0.14	21.73
2013	3.06	0.12	21.24
2014	2.67	0.11	19.73
2015	2.25	0.08	18.24
Average	3.12	0.15	21.12

Source: Authors' calculations based on ACS data. $\alpha = \frac{1}{2}$, all dimensions weighted equally.

During recession, though well-being declined among working-age population, it did not decline significantly among elderly adults. Although ours is not a causal analysis, and we are not in a position to explain these differences, the measurement of well-being and the changes therein will be helpful in designing policy responses to economic shocks in the future.

Very few studies previously have estimated multidimensional well-being measures in the U.S. A state-wide economic well-being index published by the Federal Reserve Bank or the subjective well-being questions by Gallup-Healthways are some examples. Though a good start at compiling data, these reports use either a dashboard or composite index approach and do not take into account the joint distribution of each individual's achievements. Dhongde and Haveman (2017) estimate a multi-dimensional deprivation index similar to the UN-MPI during the recession in the U.S. However our study improves upon theirs since: (i) we propose a new framework to estimate social well-being as well as deprivation, (ii) our deprivation index satisfies certain desirable properties such as CDDD, (iii) we introduce a much more flexible weighting structure and (iv) we include more dimensions of well-being and cover the recession as well as the recovery period.

It seems to us that the application of multidimensional analysis of well-being and deprivation has been overwhelmingly in the context of developing countries. In particular, the multidimensional framework has been used much less to study well-being and deprivation in the U.S. We believe this is partly due to lack of data in the U.S. which can be used to measure quality of life indicators. A variety of data collected by the U.S. Census Bureau compiles comprehensive information on income and related variables. The Current Population Survey (CPS) Annual Social and Economic Supplement (ASEC) is used to measure official poverty measure as well as the supplemental poverty measure and the Survey of Income and Program Participation (SIPP) is a longitudinal data collecting information on program participation of individuals and households in the U.S.[15] Unlike the CPS-ASEC, the SIPP has information on housing conditions. We use the Census's American Community Survey, given its extensive coverage, although that data too has limitations in terms of information compiled on well-being dimensions. None of the above data sets (CPS-ASEC, SIPP, ACS) collect information on indicators such as discouraged workers, neighborhood quality, political voice, social connections and the environment.[16] In order to stimulate further research efforts on multidimensional well-being in the U.S., it is important to compile a household level dataset which has information on income alongside a wider set of well-being dimensions. We hope that our study will highlight the need for better data collection and will be a useful addition to the meager existing literature on the impact of the recession on the well-being and deprivation of the American society.

[15]The official poverty measure is based on cash resources, the supplemental poverty measure uses cash resources and also includes noncash benefits and subtracts necessary expenses such as taxes and medical expenses.

[16]Mitra and Brucker (2017) note that the CPS has data on variables such as political voice and governance as well social connections but this data is collected as part of supplements that cannot be linked to the data on income from the ASEC supplement and thus cannot be used in a multidimensional measure that includes income.

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Supporting Information

Additional supporting information may be found in the online version of this article at the publisher's web site:

Appendix

Figure A1: Trend in the percent of working-age adults not in the labor force

Table A1: Multi-dimensional Well-being and Deprivation by Population Groups