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PRO-POORNESS ORDERINGS

By Satya R. Chakravarty and Nachiketa Chattopadhyay
Indian Statistical Institute

AND

Conchita D'Ambrosio
Université du Luxembourg

An indicator of pro-poorness of a growth profile associated with a distribution of income is a measure of the extent to which growth is biased towards the poor. This paper proposes a general approach to pro-poorness, called the *progressive sequential averaging principle* (PSA), relaxing the requirement of rank preservation due to growth. An endogenous benchmark for evaluating the growth of poor comes out naturally from this principle. A dominance relation on the basis of the above approach for a class of growth profiles is introduced through a simple device, called the PSA curve and its properties are examined in relation to the standard dominances in terms of the generalized Lorenz curve and the inverse generalized Lorenz curve. The paper concludes with an application to evaluate growth profiles experienced by the United States between 2001–07 and 2007–13.

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1. Introduction

The question of linking economic growth and poverty reduction has attracted wide interest from social scientists and policy makers in recent years. Loosely speaking, pro-poor growth refers to economic growth which is *favorable* to the poor in some unambiguous sense. The United Nations sustainable development agenda has been adopted by the world leaders in 2015 and, even though not legally binding, governments are expected to take ownership and establish national frameworks for the achievement of the 17 Goals by 2030. The first target of the 10th Goal—to reduce inequality within and among countries—is to progressively achieve and sustain income growth of the bottom 40 percent of the population at a rate higher than the national average. This indicator, introduced by the World Bank, is known as *shared prosperity*. Improvement in the latter requires growth to be inclusive of the less well-off. These facts will give further impetus to academic

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Correspondence: Satya Chakravarty, Indian Statistical Institute, Kolkata, India (satya@isical.ac.in, nachiketa@isical.ac.in)

research on the development of appropriate methods for the measurement of growth favorable for the poor.

There have been different approaches in the literature to the measurement of pro-poor growth which can be broadly classified in two groups. The first group of contributions focus more on the term “growth,” evaluating the gains or losses accruing to the poor and to the rest of the population. The second group of contributions capture more the term “poor” and evaluate pro-poorness if growth reduces poverty measured with some poverty indices without considering the changes in incomes of the rest of the population. We briefly review them in Section 3.

In this paper, we build on the definitions of pro-poorness of Kakwani and Pernia (2000) and later on by Ravallion and Chen (2003), Son (2004), Duclos (2009) and others to propose a general approach for ordering growth profiles in terms of pro-poorness. Our approach does not assume rank preservation of post growth incomes of individuals and introduces a notion of pro-poorness which we call Progressive Sequential Averaging Principle (PSA). PSA says that a growth pattern can be taken as pro-poor if the absolute average benefit of growth is more for the poor than that for the non-poor, given that the poverty threshold varies. The conditions to evaluate this fact are developed independently of the sizes of the poor and non-poor, hence allowing us to compare pro-poorness of the same country over time or of two countries with different population. Further the benchmarks for comparing growth of a group of poor is endogenously determined in this set up. This is because with variations in poverty threshold the number of poor in the given distribution is likely to vary. Hence our formulation of a partial ordering of pro-poorness, as Essama-Nssah and Lambert (2009) pointed out, “will be most suitable for international and inter-temporal pro-poorness comparisons” (p. 763). Consequently, calculation of any pro-poorness indicator is not required to judge whether one growth pattern is more pro-poor than another in terms of the partial ordering. In addition, we do not choose any a priori fixed standard of growth as benchmark, which is a major deviation of our framework from Duclos’ (2009) formulation. Our results apply to any endogenous growth pattern that is observed under the transformation of an initial distribution to a posterior distribution following the growth process. We also demonstrate that this pro-poorness ordering can be checked by seeking dominance in terms of the PSA curve we introduce. We apply our proposal to evaluate growth patterns in the U.S. in 2001–07 and 2007–13. We show that growth not always favor the poor, in the sense that we will formalize in the following sections.

The paper is organized as follows. The next section is concerned with the definitions and notation. Section 3 presents the review of the literature and motivations of the PSA approach. Section 4 analyses, with illustrations, the properties of the PSA approach to the measurement of pro-poorness. This section also presents a dominance relation for absolute pro-poor growth with a sufficient condition and its relation with standard dominance criteria. The empirical illustration to the U.S. is contained in Section 5. Finally, Section 6 concludes.

2. Preliminaries: Notation and Definitions

Let $x_1, x_2, \dots, x_m$ be incomes of m-persons in the society, where each $x_i > 0$. The set of income distributions in this society is D^m, the strictly positive part of the m-dimensional Euclidean space $\mathfrak{R}^m$. The set of all income distributions is $D = \bigcup_{m \in N} D^m$, where N is the set of positive integers. For any $x \in \mathfrak{R}^m$, we denote the ill-fare ranked permutation of x, by $\hat{x}$ that is, $\hat{x}_1 \leq \hat{x}_2 \leq \dots, \leq \hat{x}_m$. For any two ill-fare ranked distributions $\hat{x}$ and $\hat{x}'$ over a given population size m, $\hat{x} > \hat{x}'$ means that $\hat{x}_i \geq \hat{x}'_i$ for all $1 \leq i \leq m$, with strict inequality for at least one i. For any $x \in \mathfrak{R}^m$, we denote the welfare ranked permutation of x, by $\tilde{x}$ that is, $\tilde{x}_1 \geq \tilde{x}_2 \geq \dots, \geq \tilde{x}_m$. For any $1 \leq i \leq m$, we write 1^i for the i–coordinated vector of ones $(1,1,...,1)$.

Let $z > 0$ be the arbitrarily given poverty line. It is assumed that z can take values in the interval $[z_-, z_+]$. A person is called poor if his income falls below the poverty line, otherwise he is called non-poor. For any income distribution x, we denote the set of poor persons by $\Pi(x)$ and let $|\Pi(x)|$ (or, simply $|\Pi|$) be the number of poor in x. Likewise, the set of non-poor and the number of non-poor persons are denoted respectively by $R(x)$ and $|R(x)|$ (or, simply $|R|$). Let x^{Π} and x^R be the income distributions of the poor and non-poor respectively. Thus, $\hat{x} = (\hat{x}^{\Pi}, \hat{x}^R)$.

Assume that the economy experiences some income growth and the distribution x becomes y. The underlying individual absolute growth function g can be defined as $g: D^1 \to \mathfrak{R}^1$. Note that the notion of poor and the growth profile is with respect to the pre-growth income distribution $\hat{x}$. The growth levels are denoted by $b = (b_1, b_2, \dots, b_m)$, where $b_i = g(\hat{x}_i) = \hat{y}_i - \hat{x}_i$. For any $b \in \mathfrak{R}^m$, the growths accruing to poor and non-poor are denoted respectively by b^{Π} and b^R. Since growth can as well be negative, $b \in \mathfrak{R}^m$. For any growth vector $b \in \mathfrak{R}^m$, $\bar{b}$ stands for the mean $\frac{1}{m}\sum_{j=1}^{m} b_j$ of the growth vector b. We denote the set of all growth vectors by $\mathfrak{R} = \bigcup_{m \in N} \mathfrak{R}^m$.

The generalized Lorenz curve, $GL\left(u, \frac{i}{m}\right)$ of $u \in \mathfrak{R}^m$, is the plot of cumulative sums $\frac{1}{m}\sum_{j=1}^{i} \hat{u}_j$ against cumulative population proportions $\frac{i}{m}$, $i = 0, 1, \cdots, m$, where $GL(u, 0) = 0$. When divided by the mean of u, the generalized Lorenz curve becomes the Lorenz curve.

Definition 1: For any $u, v \in \mathfrak{R}^m$, we say that u generalized Lorenz dominates v (written as $u \geq_{GL} v$) if $\frac{1}{m}\sum_{j=1}^{i} \hat{u}_j \geq \frac{1}{m}\sum_{j=1}^{i} \hat{v}_j$, $1 \leq i \leq m-1$, with $>$ for at least one i.

The relation $u \geq_{GL} v$ means that the generalized Lorenz curve of u is never below that of v and above at some places (at least). $\geq_{GL}$ is a quasi-ordering in the sense that it is transitive but not complete. It is a population replication invariant relation. This enables us to compare generalized Lorenz curves over different population sizes. If the means of the distributions u and v are the same, $u \geq_{GL} v$ becomes $u \geq_L v$, the Lorenz domination of u over v.

Another well-known dominance relation is the inverse generalized Lorenz dominance criterion.[1] The inverse Lorenz curve $IL\left(u, \frac{i}{m}\right)$ of a vector $u \in \mathfrak{R}^m$ is the

[1]It may be noted that the inverse Lorenz curve is the original version of the Lorenz curve that was proposed in Lorenz (1905). We thank a referee for bringing this to our attention.

plot of cumulative proportions $\frac{1}{m\bar{u}}\sum_{j=0}^{i}\tilde{u}_j$ against cumulative population proportions $\frac{i}{m}$, $i=1,\cdots,m$, where $IL(u,0)=0$. This curve is the reflection of the Lorenz curve of the vector u, when the values are non-decreasingly ordered, around the line of equality (see Jenkins and Lambert, 1997). The inverse generalized Lorenz curve $IGL\left(u,\frac{i}{m}\right)$ of a profile u is obtained by scaling up its inverse Lorenz curve by its mean, that is, $IGL\left(u,\frac{i}{m}\right)=\bar{u}IL\left(u,\frac{i}{m}\right)$.

Definition 2: For any two vectors $u,u'\in\mathfrak{R}^m$, we say that u dominates u' by the inverse generalized Lorenz criterion, what we write $u\geq_{IGL}u'$, if $\frac{1}{m}\sum_{j=1}^{i}\tilde{u}_j\geq\frac{1}{m}\sum_{j=1}^{i}\tilde{u}'_j$, $i=0,1,\cdots,m$, with $>$ for at least one i. Note that the inverse generalized Lorenz curve is also population replication invariant, that is, $IGL\left(u,\frac{i}{m}\right)$ coincides with $IGL\left(u^k,\frac{i}{km}\right)$, where u^k is the k–fold replication of u.

3. Review of the Literature and Motivations

In order to make our exposition clear, we divide this section into two sub-sections, where the former is concerned with the literature review and latter deals with motivations and contributions of our research.

3.1. *Review of Literature*

Kakwani and Pernia (2000) and Pernia (2003) raised the basic issue of how to check that a growth profile experienced by a country at certain point of time has benefitted the poor more than the non-poor. One way of observing whether the poor have enjoyed the benefits of growth proportionally more than the non-poor is to see if relative inequality or absolute inequality is reduced in the post growth income profile. This gave rise to the concept of relative and absolute pro-poor growths (Kakwani and Pernia, 2000; Kakwani and Son, 2008). Grosse et al. (2008) and Zheng (2011) have also stated that absolute growth is pro-poor if the benefits of growth enjoyed by the poor are higher than that enjoyed by the non-poor. Ravallion and Chen (2003) interpreted pro-poor growth as the growth which reduces poverty with respect to some poverty index, without considering the changes in incomes of the rest of the population.

As pointed out by Duclos (2009) and Araar et al. (2009) the main issues on pro-poor growth are (i) whether we look at relative or absolute reduction of inequality/poverty, (ii) how to separate the poor from non-poor, i.e. choice of poverty line and (iii) how to evaluate the benefit. They provide a unifying normative framework using stochastic dominance.

Ravallion and Chen (2003) introduced the notion of a growth incidence curve (GIC). If $u(p)=F^{-1}(p)$ denotes the p-th quantile function, where F^{-1} is the inverse of the cumulative distribution function (cdf) $F(u)$, the GIC plots the change in $u(p)$ between pre and post growth distributions minus one, for all $p\in[0,1]$. It turns out that the area under the GIC up to the headcount index is identical to the change in the Watts (Watts, 1968) index times minus one. This method is equivalent to seeking first order stochastic dominance of the post-growth profile over the pre-growth one. First order dominance of a distribution u over that of v is the requirement that the cdf of u never lies above that of v. A generalization of this using other poverty indices was considered by Kraay (2006). Dhongde and Silber (2016), using a variant of the relative concentration curve, a plot of cumulative income shares of post growth incomes against that of pre growth incomes, where shares are ranked according to some unambiguous criterion, proposed a unified framework for the measurement of distributional change. They use a Gini-related index with weights based on individual income shares in the GIC framework of Ravallion and Chen (2003) to propose measures of pro-poor growth.

These approaches are based on absolute changes in poverty levels only. Osmani (2005) adopted an intermediate framework and consequently there have been different summary measures of pro-poorness. Essama-Nssah (2005); Essama-Nssah and Lambert (2009) identify a growth pattern with the elasticity function of individual income with respect to total income and note its importance as a vehicle in the measurement of pro-poorness. This elasticity function q is defined as $q=\frac{d\ln h}{d\ln H}$, where h and H denote individual income and total income respectively. When q is multiplied by the growth rate of mean income (that is, the ratio of change in mean income and the mean income), we get the growth-incidence function defined by Ravallion and Chen (2003). On the other hand, the Kakwani et al., 2006; Kakwani and Son (2008) pro-poorness measure of poverty equivalent growth rate is given by the product of the growth rate of mean income and a pro-poorness measure. The pro-poorness measure is defined as the ratio between the elasticity of a poverty index with respect to mean income for any growth pattern and the corresponding elasticity for a distributionally neutral pattern of growth (that is, $q=1$). A new family of measures emerges from the general structure which is consistent with Osmani's (2005) framework. Measures stemming from the general structure have the convenient property of being decomposable across income sources. Such decomposition can be applied to look at the percentage contributions of different income sources to overall pro-poorness. A policy maker may be interested in looking at such contributions for some policy recommendation to make the growth pattern more pro-poor for a major source of income. Zheng (2011) introduces a consistency property on measures based on elasticities. According to this property, suppose a given income distribution x can grow into two alternative distributions w and v with the same mean. Further, assume that the growth from x to w is regarded as more pro-poor than the growth from x to v. If now w and v grow further into $\hat{w}$ and $\hat{v}$ respectively with the same mean along an (relative or absolute) inequality-neutral growth path, then the growth from x to $\hat{w}$ should also be treated as more pro-poor than the growth from x to $\hat{v}$. In other words, if one growth pattern is regarded as more pro-poor than another growth pattern at a given growth rate, then pro-poor ranking between the growth patterns should remain unchanged if the growth rate becomes higher. Zheng (2011) demonstrates that there is no poverty-growth-elasticity measure that satisfies the growth-rate consistency axiom along the two inequality neutral paths. See also Klasen, 2008; Klasen and Misselhorn (2008) for a related discussion. Zheng (2011) also investigates the conditions to be imposed on a poverty index under which a poverty-growth elasticity measure can be used in a consistent way. Jenkins and Lambert's (1997) TIP curve (Three "I"s of Poverty: Incidence, Intensity and Inequality) dominance turns out to be a sufficient condition for relative growth rate consistency. Assuming that the incomes are arranged in a non-decreasing order, the TIP curve of an income distribution is a plot of cumulative relative shortfalls of the incomes from the poverty line in the corresponding censored distribution. The censored income distribution associated with a distribution of incomes is obtained by replacing all the incomes above the poverty line by the poverty line itself.

Second order stochastic dominance, which is equivalent to seeking whether the generalized Lorenz curve shifts upward, has also been studied by Duclos (2003) and Son (2004). Using this dominance one can equivalently make pro-poor judgements in terms of poverty reduction for a general class of poverty indices. For example, one can use the theorem of Atkinson (1987) to conclude that if the generalized Lorenz curve shifts upward, poverty will be analogously reduced for a general class of additive poverty measures for all poverty lines. Thus Lorenz domination or generalized Lorenz domination is a useful artifice for judging pro-poor growth.

But generalized Lorenz domination of a distribution over another is also equivalent to the condition that the former can be obtained from the latter by a finite sequence of rank preserving income increments and a finite sequence of rank preserving progressive transfers or simply by finitely many rank preserving income increments (see Foster and Shorrocks, 1988, Lemma 2) and Chakravarty (2009, Theorem 2.1). A similar remark concerning rank preserving progressive transfers holds in the context of Lorenz domination. Thus, both Lorenz and generalized Lorenz relations require rank-preservation of incomes for the relations to be well-defined.

3.2. *Motivations and Contributions*

It is evident from the above discussion that the GIC, the TIP curve and the poverty growth curve (Son, 2004) are intimately linked to the Lorenz or generalized Lorenz curve. Therefore, all such approaches used to judge pro-poorness and those directly based on the Lorenz or generalized Lorenz domination presuppose rank preservation of post growth incomes. In this paper, we wish to withdraw this assumption since post growth income ranks of individuals do often change. Thus, we only identify individuals by their ill-fare ranks in the initial distribution and allow re-ranking of post growth incomes. As a result, the set of poor or non-poor may or may not be same in the two distributions. Relaxation of rank preservation assumption may be criticized from the point of view that for a fixed poverty line, the re-ranking may generate a widely different poor and non-poor sets. To address this, we vary our poverty line so that our criteria of pro-poorness consider all possible sets of poor and non-poor. Hence, any individual whose rank is changed in the post growth profile would be present in some subgroup of poor. Obviously, as the set of poor is enlarged by taking a higher poverty line, the effect of re-ranking on the two sets will become insignificant. This indicates that retaining the basic philosophy of pro-poorness of Kakwani and Pernia (2000) and Pernia (2003), we look at the issue from a more general perspective. First, we look at the growths experienced by any set of poor vis-a-vis the complementary set of non-poor. Next, we evaluate the growth profiles of a given set of poor and the corresponding set of non-poor by a suitable evaluation function. Finally, we regard the growth profile as pro-poor if all such evaluations are higher for all sets of poor than for the corresponding sets of non-poor. This may be considered as the first contribution of our paper.

Duclos (2009) and Araar et al. (2009) explain that the basic framework of judging pro-poorness, which is consistent with the Kakwani and Pernia (2000) and Son (2004) approach, is to see whether the distributive change makes the proportional change in the income of the poor no less than some norm or benchmark, like the growth rate of mean income or some quantiles like the median. We clearly show how this recommendation of Araar et al. (2009) and Duclos (2009) and can be incorporated in our quite general framework. Although Ravallion and Chen (2003) consider pro-poorness from another perspective, Araar et al. (2009) and Duclos (2009) show that their result can also be included in the benchmark framework stated above. This, in a sense, indicates a unifying character of our approach.

We show that in our method, if the evaluation of growth is taken as the mean growth, the norm of comparison is endogenously obtained as the overall mean of the growth profile. Thus, in our framework, starting with the comparison of every set of poor with the set of corresponding non-poor, we end up at the requirement of pro-poorness with the condition where, average growth of any set of poor should not be less that the overall mean growth. Therefore, our approach explicitly incorporates the idea of pro-poor growth as advocated by Kakwani and Pernia (2000) and Pernia (2003) and that of Duclos (2009) in a more general way. We refer to this as the *Progressive Sequential Averaging Principle* (PSA, for short) condition for pro-poor growth.

Note that if the evaluation function changes, the requirement for pro-poorness will accordingly change. For instance, if the evaluation is performed using the median instead of the mean, and we demand that under pro-poorness, any poor subgroup median should not be less than the median of the corresponding non-poor subgroup, we get the benchmark as the overall median to which every poor subgroup median is compared. Growth will be pro-poor if the median of any subgroup of poor is not less than the overall median. Hence, our criterion turns out to be quite general. This means that we can change the norm and evaluate pro-poorness analogously; this is another contribution of the paper to the existing literature. Note however, we may not get benchmarks for every evaluation function. For example, if we evaluate the poor subgroup by the minimum growth in the subgroup and apply our method of comparison, all profiles where the minimum growth is experienced by the richest satisfy pro-poorness. However, this is not a necessary condition and hence no benchmark can be given.

As already noted, the existing literature on pro-poor growth is intimately related to reduction of inequality or poverty and hence consideration of shifting up of the Lorenz curve or the generalized Lorenz curve at all points (that is, without any reference to a particular poverty line) are of crucial importance. These dominations have well-known equivalent formulations in terms of welfare functions like S-concave functions. Therefore, it may be worthwhile to examine the scope of these dominations in our proposed scheme of pro-poorness. As these orderings are replication invariant, we consider a pro-poorness definition, as an alternative to our PSA, by requiring generalized or inverse generalized domination of any non-poor subgroup growth profile by its complementary poor subgroup growth profile. We observe that these criteria are quite strong in nature limiting the class of pro-poor growth profiles. However, PSA is much weaker and is satisfied whenever the above criteria are satisfied. Of course, the converse is not true.

This further motivates us to base the judgement solely on the direct benefits that accrue to the poor. What we gain by this is to have a wider class of growth profiles for which we can have a conclusive result about pro-poorness since the requirement of pro-poorness is weaker. To illustrate, let us suppose for a positive average growth and rank preservation in post growth profile, the post growth profile $x+b \geq_L x$, where, $\geq_L$ denotes Lorenz domination. Note that, the growth values are not ranked but correspond to the ill-fare ranks, $\hat{x}$, of the initial profile (see Section 1 for the formal definition of ill-fare). One can easily verify that this is equivalent to the sets of conditions $\frac{\sum_{j=1}^{i} b_j}{\bar{b}} \geq \frac{\sum_{j=1}^{i} \hat{x}_j}{\bar{x}}$, $i=1,..,n$, where $\bar{b}$ and $\bar{x}$ are the means of the respective vectors. A similar result is developed in Son (2004) using a continuous version of the Lorenz curve which presupposes rank preservation. Our result is therefore more general in nature. This generality represents one more enrichment of the literature.

Analogously, using PSA, rank preservation and assuming a positive average growth, one can easily show that the post growth profile necessarily generalized Lorenz dominates the initial profile. Again, this result does not hold if rank preservation is given up. Thus, although the concept of pro-poorness in the existing literature is closely connected with the assumption of rank preservation, the issue is not a pre-requisite in our framework. This is because the b_j values give rise to the ordered n-tuple b and hence this coordinate type ordering helps us to clearly demarcate the benefits between poor and non-poor giving us the justification of basing the pro-poorness condition solely on the vector b.

It may be worthwhile to note that we use the notion of "relative" in case of PSA pro-poorness in the sense that it is comparing the growth of a poor class with its complement non-poor class. (This contrasts with the "absolute" concepts where unilateral improvements/worsening of the poor are considered for judging pro-poorness, where improvements/worsening of the non-poor are not considered). On the other hand, improvements/worsening can be evaluated as an "absolute" difference or a "relative" ratio. Since in deriving the PSA condition (see below), we are cumulating growth values, this implicitly reflect "absolute" changes. Further, the growth vector is only linked to the ill-fare ranked pre-growth incomes and is taken as given in the setup.

In the following sections of the paper we build up on the above discussion by formalizing the PSA principle. This principle generates a partial ordering of pro-poorness and demonstrates that this pro-poorness ordering can be checked by seeking dominance in terms of the PSA curve we introduce.

4. Formal Definitions and Propositions

Definition 3: For any income distribution $x \in \mathfrak{R}^m$, we say that the growth vector b is pro-poor according to the PSA condition, if for any subgroup $\{1,2,\cdots,i\}$ of poor persons with income distribution $\hat{x}'^{\Pi}$ and $(m-i)$ non-poor persons with income distribution $\hat{x}'^{R}$, $\frac{1}{i}\sum_{j=1}^{i} b_j \geq \frac{1}{m-i}\sum_{j=i+1}^{m} b_j$, $1 \leq i \leq m-1$, with $>$ for at least one i, where $m \in N$ is arbitrary.

That is, the PSA condition demands that the mean growth of the first *i* poor persons is not lower than the mean growth of the remaining persons and for at least one sub-group it is higher. For the comparison to be meaningful we need to assume that there is at least one poor person and at least one non-poor person. This approach parallels the stochastic dominance analysis employed in the context of poverty ordering (see Foster and Shorrocks, 1988 and Foster et al. 2013).[2]

As stated earlier, note that the PSA condition stated above gives rise to a natural benchmark for the evaluation of the growth of any subgroup. To see this, we can rewrite the inequalities in Definition 3 as $(m-i)\sum_{j=1}^{i} b_j \geq i\sum_{j=i+1}^{m} b_j$ leading to $m\sum_{j=1}^{i} b_j \geq i\sum_{j=1}^{m} b_j$ which in turn gives us $\frac{1}{i}\sum_{j=1}^{i} b_j \geq \bar{b}$, $1 \leq i \leq m-1$.

Thus if we evaluate the poor subgroup growth by its mean, the benchmark or standard is the overall mean. The same result holds if we change the evaluation by the median growth. The comparable benchmark will then be overall median for any poor subgroup.

We now illustrate some sufficient but not necessary conditions for PSA to be satisfied. One can easily observe that the above inequalities in Definition 3 are satisfied if the b_i's are welfare ranked, that is $(b_1 \geq b_2 \geq \ldots \geq b_n)$. However, this is a very strong sufficient condition for the growth vector to be pro-poor.

A weaker sufficient condition is the requirement that the sequence of partial means of the growth vector is decreasing. To see this consider the relation between the (*k*-th) and (*k+1*-th) partial means

$$\frac{b_1 + \ldots + b_k}{k} \geq \frac{b_1 + \ldots + b_{k+1}}{k+1}. \tag{1}$$

For $k = n-1$, we have $\frac{b_1+\ldots+b_{n-1}}{n-1} \geq \frac{b_1+\ldots+b_n}{n} = \bar{b}$. Since these means are decreasing in k, we have $\frac{b_1+\ldots+b_k}{k} \geq \bar{b}$ for all k, which in turn can be rewritten as the (*k*-th) pro-poor condition described above. Thus if the sequence of partial means of the growth vector decreases as the set of poor increases then the PSA condition is implied. However, the converse is not true and it is only a sufficient condition. This type of decreasingness of partial means is used in reliability theory involving distributions with decreasing mean residual life (see Shaked and Shanthikumar, 2007).

It may be worthwhile to note that this weaker sufficient condition (1) can be rewritten as an equivalent condition $\frac{b_1+\ldots+b_k}{k} \geq b_{k+1}$. Thus a sufficient condition for pro-poorness is that the growth for a marginally non-poor person should not be above the average growth enjoyed by the persons poorer than her. This condition is quite intuitive and appealing.

[2]Blackorby and Donaldson (1980) suggested a generalization of the Sen (1976) index using a social welfare function and showed that the welfare function must be completely strictly recursive in the sense that welfare of every subgroup of poor must be separable from that of the non-poor. Evidently this also assumes variability of the poverty line.

The Lorenz curve or the generalized Lorenz curve are well known tools for comparing distributions in the inequality literature. Such dominations are also linked to the summary evaluation of the distributions in terms of S-concave functions. Therefore, it is natural that we seek to invoke such dominances in our set up and try to get a measure of pro-poorness. As already noted, the basic set up of demarcating the poor and non-poor remains as before. We only need to change the dominance or the corresponding evaluation function.

We apply the above definitions to our pro-poorness set up to get to the following definition of pro-poorness.

Definition 4: For any income distribution $x \in \mathfrak{R}^m$, we say that the growth vector b is pro-poor by the generalized Lorenz criterion if for any subgroup $\{1,2,\cdots,i\}$ of poor persons with income distribution $\hat{x}^{\Pi}$ and $(m-i)$ non-poor persons with income distribution $\hat{x}^R$, $(b_1,b_2,\ldots,b_i) \geq_{GL} (b_{i+1},b_{i+2},\ldots,b_m)$ $1 \leq i \leq m-1$, where $m \in N$ is arbitrary.

Likewise, we can define pro-poorness using the inverse generalized Lorenz criterion.

We next show that the PSA condition drops out as an implication of the generalized Lorenz criterion or the inverse generalized Lorenz criterion. First of all, note that the PSA conditions are all relative order invariant with respect to the subgroups. That is, once the set of poor and non-poor are defined, rearranging the values within the group does not change the conditions.

For the growth vector $(b_1,b_2,\ldots,b_n)$ corresponding to the ill-fare ranked distribution $(\hat{x}_1,\hat{x}_2,\ldots,\hat{x}_n)$, the PSA inequalities are:

$$b_1 \geq \frac{b_2+b_3+\ldots+b_n}{n-1}, \tag{2}$$

$$\frac{b_1+b_2}{2} \geq \frac{b_3+\ldots+b_n}{n-2} \tag{3}$$

$$\ldots$$

$$\frac{b_1+b_2+\ldots+b_{n-1}}{n-1} \geq b_n, \tag{4}$$

with strict inequality in at least one case.

Further, consider the (k-th) equation

$$\frac{b_1+\ldots+b_k}{k} \geq \frac{b_{k+1}+\ldots+b_n}{n-k}$$

where the first k persons are poor and the other $(n-k)$ persons are non-poor. Consider the dominance $(b_1,b_2,\ldots b_k) \geq_{GL} (b_{k+1},b_{k+2},\ldots,b_n)$. We replicate the ill-fare ranked permutations of the vectors $(b_1,b_2,\ldots b_k)$ and $(b_{k+1},b_{k+2},\ldots,b_n)$, $(n-k)$ times and k times respectively, to get two vectors $b_\pi = (\widehat{b}_1,\ldots\widehat{b}_1,\widehat{b}_2,..\widehat{b}_2,\ldots.\widehat{b}_k,\ldots\widehat{b}_k)$ and $b_r = (\widehat{b}_{k+1},\ldots\widehat{b}_{k+1},\widehat{b}_{k+2},..\widehat{b}_{k+2},\ldots.\widehat{b}_n,\ldots\widehat{b}_n)$. The dominance conditions $(b_1,b_2,\ldots b_k) \geq_{GL}$

$\left(b_{k+1}, b_{k+2}, ..., b_n\right)$ are equivalent to $b_\pi \geq_{GL} b_r$ which now are obtained as a series of inequalities starting with $\widehat{b}_1 \geq \widehat{b}_{k+1}$ to the last one $(n-k)(\widehat{b}_1 + ... + \widehat{b}_k) \geq k(\widehat{b}_{k+1} + ... + \widehat{b}_n)$. This last inequality can be rewritten as $(n-k)\left(b_1 + ... + b_k\right) \geq k\left(b_{k+1} + ... + b_n\right)$ which is the k-th PSA inequality. Likewise, the inverse generalized Lorenz condition leads as well to the PSA inequality.

The above discussion enables us to state the following.

Proposition 1: For any income distribution $x \in \mathfrak{R}^m$, and for any subgroup $\left\{1,2,\cdots,i\right\}$ of poor persons with income distribution $\hat{x}^\Pi$ and $(m-i)$ non-poor persons with income distribution $\hat{x}^R$, each of the two dominances $\left(b_1, b_2, \ldots, b_i\right) \geq_{GL} \left(b_{i+1}, b_{i+2}, \ldots, b_m\right)$ and $\left(b_1, b_2, \ldots, b_i\right) \geq_{IGL} \left(b_{i+1}, b_{i+2}, \ldots, b_m\right)$ ensures that the PSA condition holds, where $1 \leq i \leq m-1$, where $m \in N$ is arbitrary.

The terminal condition of the GL and the IGL orderings is our PSA condition. In fact, it is the only common condition between the GL and the IGL orderings. In view of this we can state the following corollary.

Corollary 1: For any income distribution $x \in \mathfrak{R}^m$, and for any subgroup $\left\{1,2,\cdots,i\right\}$ of poor persons with income distribution $\hat{x}'^\Pi$ and $(m-i)$ non-poor persons with income distribution $\hat{x}'^R$, the only common condition which holds for each of the two dominances $\left(b_1, b_2, \ldots, b_i\right) \geq_{GL} \left(b_{i+1}, b_{i+2}, \ldots, b_m\right)$ and $\left(b_1, b_2, \ldots, b_i\right) \geq_{IGL} \left(b_{i+1}, b_{i+2}, \ldots, b_m\right)$ is the PSA condition, where $1 \leq i \leq m-1$, where $m \in N$ is arbitrary.

It is clear from the above discussions that the requirement of pro-poorness in terms of generalized Lorenz dominance or equivalently in terms of strict S-concave evaluation of the growth vector of any poor or non-poor subgroup is quite strong which may rule out many growth vectors to be not pro-poor which are termed pro-poor by PSA. This is because under generalized Lorenz dominance, for each i-th partitioning of the population into poor and non-poor subgroups, the sum of benefits enjoyed by the j-poorest persons in the poor subgroup is at least as high as the corresponding sum for in the non-poor subgroup, where, $1 \leq j \leq i$, $1 \leq i \leq m-1$, with at least one strict inequality. In contrast, the PSA is a much weaker condition as it does not require the additional restrictions of dominance given a partition, which are required for the pro-poorness in terms of generalized Lorenz dominance. The same comments apply to inverse generalized Lorenz dominance as well.

In the following, we formulate the PSA principle in the form of a graphical device that enables us to compare growth profiles. This is similar in spirit to the Lorenz quasi-ordering. For this, note that the PSA condition can also be interpreted in terms of non-negativity of the net average excess growth vector of the poor from the mean growth. To see this formally, consider the growth vector b which satisfies the PSA condition. Hence we have $\frac{1}{i}\sum_{j=1}^{i} b_j \geq \frac{1}{m-i}\sum_{j=i+1}^{m} b_j$, for $i = 1,..,m-1$. Consider now the vector $d_i = \frac{1}{i}\sum_{j=1}^{i} b_j - \frac{1}{m-i}\sum_{j=i+1}^{m} b_j \geq 0, \quad i = 1,..,m-1.$

This vector represents the distance of the average growth of the poorer subgroup from that of the non-poorer subgroup as the poverty line varies. One can rewrite $d_i \geq 0,\ i=1,..,m-1$ as $\frac{1}{i}\sum_{j=1}^{i}\left(b_j-\bar{b}\right)\geq 0,\ i=1,...,m-1$, where $\bar{b}$ is the average growth. Thus, the PSA condition can be alternatively expressed as the requirement that the net average excess growth from the mean growth level for any subgroup of poor be nonnegative. Further, $\frac{1}{i}\sum_{j=1}^{i}\left(b_j-\bar{b}\right)\geq 0,\quad i=1,..,m-1$ can be also written as $\sum_{j=1}^{i} b_j \geq i\bar{b},\ i=1,..m-1$. Since $\sum_{j=1}^{m} b_j = m\bar{b}$, we can write this condition as

(3a) $$\frac{1}{m\bar{b}}\sum_{j=1}^{i} b_j \geq \frac{i}{m}\ ,\ i=1,...,m,\ \text{if}\ \bar{b}>0,$$

(3b) $$\text{or}\ \frac{1}{m\bar{b}}\sum_{j=1}^{i} b_j \leq \frac{i}{m}\ ,\ i=1,...,m,\ \text{if}\ \bar{b}<0$$

If we now successively plot $\frac{1}{m\bar{b}}\sum_{j=1}^{i} b_j$ against $\frac{i}{m}, i=1,..,m$ we get a curve. We may call it the PSA curve (see Figure 1). When there is overall positive growth, from (3a) it follows that if this curve is above the 45^{o} line, we have a pro-poor growth profile. From equation 3(b), we observe that in case of negative overall growth, the pro-poor profile is below this line: the poor are sharing less than the non-poor the negative outcomes. Consequently, the 45° line can be interpreted as the neutral line where all absolute (negative or positive) levels of growths are equal. The first of these curves have a structure similar to the Industrial Concentration Curve which plots cumulative non-negative output shares of firms in an industry against rank ordered number of firms, where the firms are ranked from the highest

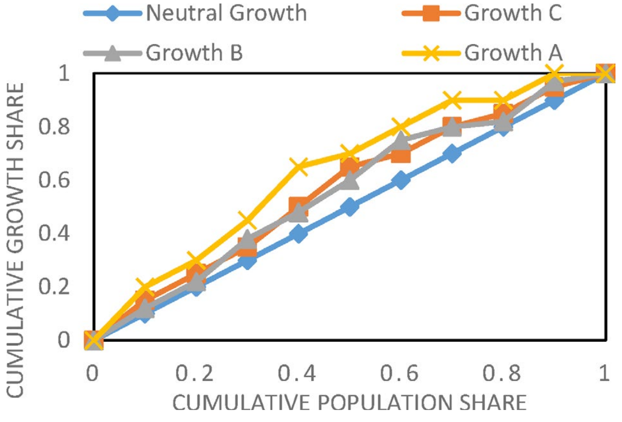

Figure 1. Examples of PSA curves in case of positive growth rates

to the lowest in terms of output levels connecting (0,0) to (1,1), where at least one voutput share is positive. However, since b_i can have any sign, the cumulative shares can be numerically greater than one and hence there is no upper (lower) bound of the pro-poor curves with positive (negative) average levels of growth.

We now illustrate the concept of pro-poorness in the PSA sense using both positive and negative average growths with the following example. Let $m=5$ and the $b^1=(4,5,-1,0,2)$ and $b^2=(0,-3,-1,1,-7)$ with $\bar{b}^1=2>0, \bar{b}^2=-2<0$. The cumulation is $(4,9,8,8,10)\geq(2,4,6,8,10)$, where the latter is the neutral growth line, and, similarly $(0,-3,-4,-3,-10)\geq(-2,-4,-6,-8,-10)$. Hence both the growth vectors are PSA pro-poor. The cumulants show that the poor are having a larger share of a positive overall growth than the corresponding non-poor and lesser (negative) burden of an overall negative growth. If we divide the cumulants by the respective $\bar{b}^1=2$, $\bar{b}^2$ and $m=5$ (10 and -10) we have the equations (4a) and (4b) as $(0.4,0.9,0.8,0.8,1)\geq(0.2,0.4,0.6,0.8,1)$ and $(0,0.3,0.4,0.3,1)\leq(0.2,0.4,0.6,0.8,1)$ respectively. Thus in case of positive growth the PSA curves are above the line of equality and below this line in case of overall negative growth, when growth is pro-poor.

Some properties of this PSA curve can be now stated. One can easily see that this curve is population replication invariant. Hence we can compare the curves across different populations with varying sizes. The more the curve is away from the neutral line, the more we have pro-poorness, irrespective of whether the curve is above or below the diagonal line. When all the growth values are non-negative (respectively non-positive), with at least one being positive (respectively negative), it ensures that the average growth is positive (respectively negative). In this special case, when growth values are all non-negative (respectively non-positive) with at least one being positive (respectively negative), the highest pro-poorness is reached when all the growths accrue to the poorest (respectively non-poorest) person with others experiencing zero growth. While in the former situation, the curve is represented by the left and upper boundaries of a unit square, in the latter situation it coincides with the lower and right boundaries of the unit square.

Using the above interpretation we can now define a dominance relation between two growth vectors by requiring that the PSA curve of one is above (below) the PSA curve of the other where both are above (below) the neutral line. The ordering involved in the PSA curve is an incomplete, transitive ordering.

In order to study the dominance relation analytically, we consider the following.

We first note that the growths are linked to the pre-growth ordered distribution and hence are independent of the ordering of the distribution in the post growth scenario. In the inequality literature we talk of progressive transfers by considering a transfer of income from a person to a poorer person such that the post transfer income rankings of the affected persons are not altered. Such transfer generates a Lorenz dominating distribution. The result is that if we start with two distributions of fixed total of which one Lorenz dominates the other we can get a finite series of Lorenz dominating distributions which starts and ends with the given distributions.

In case we have two growth vectors of which one PSA dominates the other, the following natural question arises: Is it possible to have a sequence of PSA dominating growth vectors so that we can have the two distributions as the initial and final distributions of the sequence? Here we have to keep in mind that the growth vector components can have any sign. Hence in this case we define progressivity as a transfer of growth from a richer person to a poorer person so that the average growth of the poor remains higher.

Definition 5 For $b,b' \in \mathfrak{R}^m$, b' is obtained from b by a progressive redistribution of growth, if there is a pair (i,j) such that $b'_i - b_i = b_j - b'_j = \eta > 0$, and $b_l = b'_l$ for all $l \neq i, j$, where $i(j)$ represents the suffix of a poorer(richer) person.

Note that the growth vector can have negative components and the components are not ordered. Hence the redistribution may involve a recipient (a relatively poor person with respect to pre growth income) whose existing growth is higher than the existing growth of a transferee (a relatively richer person with respect to the pre growth profile). Both the existing growth components may be negative. However, since the growth components can be ultimately linked to the ill-fare ranks of the pre growth distribution, we can still talk of a transfer which is progressive in the sense that it is taking place from a richer person to a poorer person. Such a transfer generates a more PSA pro-poor distribution of growth.

The following proposition, whose proof is easy, can now be stated.

Proposition 2: For $b,b' \in \mathfrak{R}^m$, if b' is obtained from b by a progressive redistribution of growth, then the PSA curve of b will be higher than that of b'.

Finally, we have

Proposition 3: For $b,b' \in \mathfrak{R}^m$, if the PSA curve of b lies higher than that of b', b' can be obtained from b by a successive progressive redistribution of growth.

Proof: We have $\sum_{j=1}^{i} b_j \geq \sum_{j=1}^{i} b'_j$ $i = 1,\ldots,m$ with strict inequality for at least one i and $\sum_{j=1}^{m} b_j = \sum_{j=1}^{m} b'_j$. Let k be the highest integer such that $\sum_{j=1}^{k} b_j > \sum_{j=1}^{k} b'_j$. This implies that $b'_{k+1} > b_{k+1}$. Hence we can transfer benefit from b'_{k+1} to b'_k, which, being a transfer from the richer $k+1$-th person to the poorer k-th person, is progressive so that $\sum_{j=1}^{k} b_j = \sum_{j=1}^{k} b'_j$. Repeated application of this procedure enables us to reach the lower curve from the higher curve.

It may so happen that two pro-poor curves may intersect and hence no ordering is possible between two pro-poor growth profiles. In that case, we may consider a summary measure which will enable us to compare any two pro-poor growth profiles. Using the condition of pro-poorness $d_i = \frac{1}{i} \sum_{j=1}^{i} \left(b_j - \bar{b} \right) \geq 0, i = 1,\ldots,m-1$, a simple measure of pro-poorness of a pro-poor growth profile could be:

$$\frac{1}{m}\sum_{i=1}^{m-1}\frac{1}{i}\sum_{j=1}^{i}\left(b_j-\bar{b}\right). \tag{4}$$

In case two pro-poor curves intersect, we can compare pro-poor growth by employing this index. Average of any increasing transformation of d_i values can as well be used for this purpose.

This index is simply the population average of excesses of average benefits of different poor subgroups over the grand average of benefits. Therefore, the index may be termed as the pro-poorness index of mean of sequential excess-benefit averages. More compactly, we can refer to the index as PSA index of pro-poorness. It may be noted that, while most of the pro-poorness indices (e.g. the one given by the difference between the Watts poverty indices for pre and post–growth distributions of incomes) assume rank preservation of individuals, we do not make any such assumption. In view of this, our index is not comparable with such indices.

In line with the existing literature, if the growth vector does not alter the rankings in the post growth distribution, the following proposition can be easily verified.

Proposition 4: If any income distribution experiences a rank preserving positive average growth $\bar{b}$ which is more pro-poor in the PSA sense than another positive average growth $\bar{c}$, then the resultant distribution for $\bar{b}$ generalized Lorenz dominates the corresponding distribution for $\bar{c}$ if $\bar{b} \geq \bar{c}$.

Thus a more pro-poor growth in the PSA sense will be inequality reducing as long as overall growth is not less than that for another pro-poor growth profile.

A similar proposition can also be stated for two pro-poor PSA distributions arising out of negative overall growths.

We can also observe that on the basis of the PSA curve, we can define pro-richness of a growth profile by requiring that the PSA curve of the growth profile always remains below (respectively above) the diagonal neutral line when the average growth is positive (respectively negative). Following the procedure discussed above, we can therefore define a partial ordering of pro-rich growth profiles. If all the growth values are non-negative (respectively non-positive) when the average growth is positive (respectively negative), the highest pro-richness will be achieved when all non-negative (respectively non-positive) growths accrue to the richest (respectively poorest) person only and consequently the PSA curve is given by the lower (respectively upper) horizontal and the right (respectively left) vertical boundaries of the unit square (Figure 1).

It may now be worthwhile to explore the converse of Proposition 4. The question, therefore, is whether Lorenz domination of one distribution over another has any implication on the pro-poor dominance, in the PSA sense, of the related growth profiles when the distributions experience growths. Towards this, we have the following proposition.

Proposition 5: If one post growth profile of a distribution Lorenz dominates another post growth profile associated with the same distribution, where both the growth profiles are pro-poor, rank preserving and have identical average, then the former growth profile PSA dominates the latter growth profile.

Note that, since here we are considering a relation involving PSA domination, we must consider pro-poor growth profiles only. Rank preservation helps us link the growth vector to the corresponding ill-fare ranking of the pre-growth distribution, which is a requirement of PSA dominance. Finally, since PSA dominance reflects the nearness of the growth profile from the constant (average) growth profile, we need to have the same average growth for comparison.

5. Empirical Illustration

We use the March supplement Current Population Survey (CPS) from 2002, 2008 and 2014 which is the source of official poverty estimates of the U.S. We assign to each individual an income level equal to the household total income divided by the square root of the number of household members. To allow for comparisons over time, we adjust incomes for inflation using the Consumer Price Index (CPI-U-RS), with base year 2007. We drop negative incomes (369 observations in the three years). A household total income refers to the previous calendar year, hence we evaluate growth profiles between 2001–07 and 2007–13. Sample weights are used in the analysis. Our sample in 2001/2007/2013 is composed of respectively 216,985/206,316/139,368 individuals. The mean/median equivalent real income of the sample increased from 37,823.11/28,982.98 in 2002 to 44,220.75/34,310.23 in 2008 and to 47,798.11/36,000 in 2014.

We follow the anonymous approach to pro-poor growth which demands to compute the growth levels experienced by percentiles of the income distributions. Hence we partition the income distributions in 100 groups and evaluate growth of mean and median income of these groups. The years of analysis are characterized by a recession in 2001, an economic expansion between 2002 and 2007, and the Great Recession which officially lasted from December 2007 to June 2009.

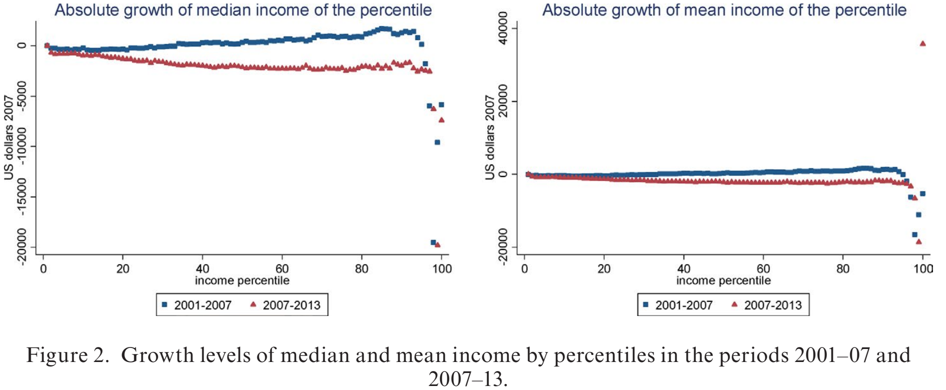

Figure 2. Growth levels of median and mean income by percentiles in the periods 2001–07 and 2007–13.

TABLE 1
Summary statistics of growth levels of the percentiles in the periods 2001–07 and 2007–13

	Mean	Std. Dev.	Min	Max
Mean Income Growth 2007-2013	−1668.67	4193.29	−18624.98	35703.02
Median Income Growth 2007-2013	−2096.91	1998.81	−19807.74	0
Mean Income Growth 2001-2007	−78.65	2268.96	−16518.64	1657.80
Median Income Growth 2001-2007	−91.04	2443.61	−19533.24	1708.05

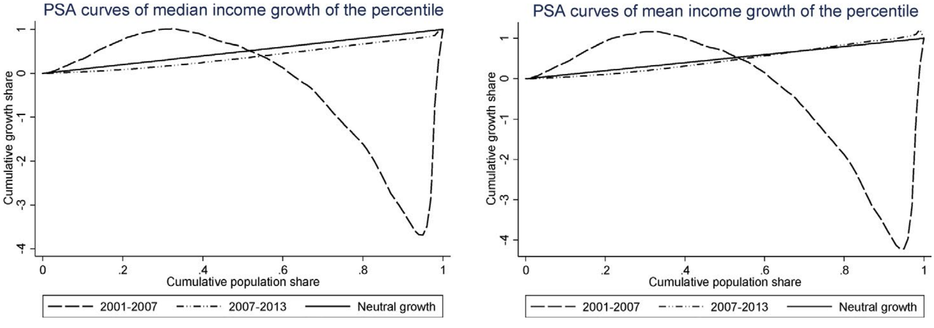

Figure 3. PSA Curves of growth profiles in the periods 2001–07 and 2007–13

Figure 2 contains the plot of the growth levels of both mean and median income of each percentile and Table 1 contains the summary statistics. The global mean of the growth levels of both mean and median income in the percentiles is always negative. The difference between the two periods is striking and is to be expected given the Great Recession hitting families hard in the second period. The values range from −79/−91 for mean/median growth in 2001–07 to very high figures of −1,669/−2097 in 2007-2013. The global values are the results of contrasting paths in the single percentiles, as clear from Figure 2: in 2001–07 the growth of both mean and median income is negative up to the 33rd percentile, it becomes then positive up to until the 96th percentile where negative values are observed once again. The mean and median results differ in 2007–13: while the growth of median income is always negative, for mean income we observe a positive value only for the last percentile. This value is as high as 35703.02, the maximum value observed. The PSA curves are plotted in Figure 3.

Have these changes been pro-poor?

To be pro-poor, given that the global mean income is negative in both periods, the PSA curve should be always below the neutral growth line. In this case, the poor are sharing less than the non-poor the negative outcomes. This is what we observe only for the growth of median income of the percentiles in 2007–13. The other PSA curves cross the neutral growth line, from above in the years 2001–07, independently of the use of mean or median income, from below in 2007–13 in case of growth in mean income of the percentiles. An interesting feature emerges from the shape of the curve for the cycle 2001–07. Above the single point of crossing with the line of neutrality, as a consequence of the recession higher negative burden of overall negative growth was borne by the poor and it increased at an increasing rate (as indicated by the increasing and concave curve) up to certain cumulative population fractions; after which the burden of the poor started decreasing at a decreasing rate. Below the point of intersection, the growth became pro-poor, probably, as a consequence of economic expansion between 2002 and 2007. This happened irrespective of whether the mean or the median was chosen as the benchmark. Hence in three out of the four cases we consider, we observe pro-poorness only once. If the U.S. aims at obtaining a path of growth which is pro-poor reforms are needed in order to allow the poor to benefit more than the non-poor in all cases.

Given that the pro-poor curves intersect and hence no ordering is possible between the pro-poor growth profiles, we have computed the PSA index of pro-poorness defined in (4). The values of the index are 743.9 (319.2) in 2007–13 and 22.5 (6.5) in 2001–07 for the growth levels of median (mean) income of the percentiles. According to this summary measure the second period, i.e. 2007–13, is always more pro-poor than the first.

6. Conclusions

We have built on the ideas of pro-poorness put forward by Kakwani and Pernia (2000) and later on by Ravallion and Chen (2003), Son (2004), Duclos (2009) and others to develop a general approach for a growth profile associated with a distribution of income to be called pro-poor, in terms of a curve: the PSA curve. We introduce a partial ordering of growth profiles on the basis of this curve. This ordering is implied by the two standard orderings, the generalized Lorenz ordering and the inverse generalized Lorenz ordering but not conversely.

We apply the proposed ordering to evaluate growth profiles experienced by U.S. individuals in two periods, 2001–07 and 2007–13, both characterized by negative average growth levels in mean and median incomes of the percentiles. Growth in the U.S. is pro-poor only in one case, 2007–13, when we consider median income of each group. In this case, the poor are sharing less than the non-poor the negative outcomes.

Our propositions establish the relation of the PSA with the generalized Lorenz criterion. A natural line of investigation is to look for a similar relation between the PSA and the relative concentration curve. As we have noted in Section 1 implicit under the formulation based on concentration curve is some notion of ordering. But in our set up no such assumption is made. Consequently, the two notions are non-comparable.

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