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MULTIDIMENSIONAL INEQUALITY ACROSS THREE DEVELOPED COUNTRIES

BY NICHOLAS ROHDE* AND ROSS GUEST

Griffith University, Gold Coast, Australia

This paper produces comparable estimates of multidimensional inequality for the U.S., Germany, and Australia. Two alternative approaches with differing interpretations are employed. The first method projects all facets of welfare onto a single variable which is then analyzed using standard univariate techniques. The second approach establishes equivalent-income distributions that *would* lead to an equalization of welfare, such that the difference between this counterfactual and the true income distribution can be measured. This difference is then interpreted as the degree of income redistribution required to offset welfare inequality. Using data on permanent incomes, health scores, years of education, and leisure times, we observe much higher levels of inequality in the U.S. than in Germany or Australia. Our results are highly statistically significant and hold over a large variety of weighting specifications.

JEL Codes: D31, D63

Keywords: distribution, equivalent income, multidimensional indices, welfare

1. INTRODUCTION

Economic inequality is a field that has attracted considerable research in recent years. Much of this work has focused on the distributional characteristics of income or some other monetary variable such as earnings, consumption, or wealth. However, a broad sense of agreement is emerging that inequality is a multidimensional phenomenon (Stiglitz *et al.*, 2009) which may be only tangentially related to the distribution of monetary resources. Other factors such as health and education are also important determinants of welfare, and their inclusion in a study of inequality may produce estimates that conflict with results from unidimensional analysis. Consequently, it is important to establish whether or not previously well-accepted empirical results hold when multiple facets of welfare are considered simultaneously.

The most general approach for this type of analysis involves searching for stochastic dominance rankings over multivariate distributions. Considerable

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*Correspondence to: Nicholas Rohde, Department of Accounting, Finance and Economics, Griffith University, Gold Coast, 170 Kessels Road, Nathan, QLD 4111, Australia (n.rohde@griffith.edu.au).

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effort has been expended in developing these techniques (see works by Tsui, 1999; Duclos *et al.*, 2006, 2011; Anderson, 2008; Muller and Trannoy, 2011, 2012; Gravel and Moyes, 2012; Pinar *et al.*, 2013; Yalonetzky, 2013; Sonne-Schmidt *et al.*, 2016), which has built upon classic earlier papers by Kolm (1977) and Atkinson and Bourguignon (1982). Dominance results are powerful when they occur and are motivated by the attractive welfare properties they imply. If distribution X dominates Y, then the ranking coincides with the social welfare orderings of the distributions subject to various minimally restrictive assumptions (such as being increasing and concave) on the form of the utility function. A second complementary approach is the use of summary measures, which provide cardinal rankings of countries regardless of whether or not a strict stochastic ordering is present. These indices are especially useful for policy analysis and the measurement of social progress, although they require making explicit further assumptions about the ways in which welfare emerges from the underlying variables. In this paper, we focus on the aggregative approach pioneered by Maasoumi and Jeong (1983), Maasoumi (1986), and Maasoumi and Nickelsburg (1988), which involves specifying a welfare function to map from multiple dimensions to a single variable that may then be analyzed using traditional univariate inequality measures.[1]

Empirically, the objective of the paper is to provide a comparison of multidimensional inequality in the U.S., Germany, and Australia. Motivation comes from the common perception that measures of cross-sectional *income* concentration tend to exaggerate U.S. inequality relative to other developed countries. This critique typically cites: (i) a failure to consider mobility over time (Burkhauser and Poupore, 1997; Leigh, 2009); and (ii) the omission of other important determinants of welfare, such as health, education, or leisure. By explicitly including these three other dimensions, and by basing our analysis on "permanent" variables (i.e. seven-year longitudinal averages), we are able to produce measurements that correspond closely to inequalities in ingrained socioeconomic disparities, which in turn addresses the drawbacks of static univariate analysis.

In order to produce a definitive inequality ranking across these three countries, our econometric method would ideally focus on establishing a stochastic dominance ordering, such that our results would be robust to choices within families of inequality statistics. However, as our data consist of four separate dimensions, this makes a complete set of dominance rankings implausible (a shadow price majorization is used to rule out this form of ordering in Appendix A.2, in the online supporting information). Instead, we employ the aforementioned aggregative approach to summarize the scores for each

[1]Aside from the aggregative class of measure, there are a number of single-step multivariate indices. Most of these are based upon some normative foundation, including multivariate generalizations of Atkinson, Kolm, and Kolm–Pollack indices. A survey is given in Weymark (2004). Alternatives come from extensions of the generalized entropy (GE) class of index (Tsui, 1999) and the Gini coefficient (Gajdos and Weymark, 2005; Decancq and Lugo, 2012). For a survey of this literature, see Lugo (2007). There is also a closely related body of work that measures multidimensional deprivation (see, e.g. Alkire and Foster, 2011; Bossert *et al.*, 2013).

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individual across the four dimensions. By studying the distribution of this index over our three countries, we are able to draw conclusions about their relative states of welfare inequality.[2]

In addition to providing these standard multidimensional estimates, we also analyze the distribution of welfare using an equivalent-income approach along the lines advocated by Fleurbaey (2011) and Fleurbaey and Gaulier (2009).[3] Money metrics are used to calculate the income that each unit "should" receive in order to offset differentials in welfare. By comparing the true income of the individual with their equivalent (or "compensating") income, we develop an indicator of relative socioeconomic advantage or disadvantage for each person. When an individual has an income that lies above (below) her equivalent, her welfare exceeds (falls short of) the sample average, and this advantage is valued as the excess of the true income over the equivalent. Thus the difference between the two income variables represents the amount of money that must be redistributed to remove the welfare effects of inequality. By averaging these discrepancies across individuals, we can gain an intuitive appreciation for the degree of income redistribution required to offset welfare inequality.

The paper is structured as follows. Section 2 previews the data, while Section 3 introduces the concept of aggregative indices and presents baseline estimates of welfare inequality. Section 4 calculates equivalent incomes for each individual and measures the degree of income distribution required to eliminate welfare inequality. Section 5 presents a number of further empirical results and Section 6 offers some concluding comments. Further supplementary results (univariate analysis and dominance tests) are presented in the appendices (in the online supporting information).

2. Data

Data come from the Cross National Equivalence File (CNEF), which is an agglomeration of harmonized panels from a number of developed countries. Observations are taken from 2001 to 2007, which consists of four waves of U.S. data and seven waves for Germany and Australia, sourced respectively from PSID, SOEP, and HILDA. The fewer waves for U.S. data is due to the PSID switching from being an annual survey to biennial in 1997. For the U.S., we require observations for all four periods, while for Germany and Australia we require that at least six from seven waves contain non-missing data. It should be

[2]Examples of cross-national studies of this sort include: Gravel *et al.* (2005), who search for dominance between 12 OECD countries over both income and access to public goods; Brandolini (2008), who compares income and health inequalities across France, Germany, Italy, and the United Kingdom; and Duclos *et al.* (2011), who study India and Mexico. Decancq *et al.* (2009) take the concept further by studying the world distribution of multidimensional well-being. Other notable applied multidimensional papers include Decancq and Lugo (2012), Decancq and Ooghe (2010), Yalonetzky (2012), and Nilsson (2010).

[3]These authors define a utility function for individual $u_i(x_i, z_i)$, where x_i is the income of individual i and z_i is a vector of living conditions including prices, access to public goods, social connections, and so forth. By fixing a benchmark of conditions z^*, one can obtain an equivalent income by solving $u_i(x_i, z_i)=u_i(x_i^*, z^*)$ for x_i^*. This is the income level at which the individual is indifferent between circumstances z_i and z^*, and hence the approach allows for individual heterogeneity in circumstances.

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noted that due to the differing frequencies of the surveys, it is impossible to achieve a consistent set of waves without ignoring large quantities of data, while insisting on a balanced panel for each country will introduce an asymmetrical attrition effect on the U.S. due to the smaller number of waves.

Our unit of analysis is the individual; however, as observations for non-income characteristics are often not present for all members, we assume that the characteristics of the head are representative of the entire household. This enables us to increase our sample size but has the effect of ignoring intra-household inequality. We focus only on working-aged heads, as life-cycle factors tend to unduly influence all four variables prior to the age of 25 and after 65. Non-conforming data are dropped, along with negative observations, and each household is weighted by the product of its size and a cross-sectional weight.

Inequality is assessed over (i) household income, (ii) years of education, (iii) self-reported health status, and (iv) weekly leisure time. These variables are all common correlates of subjective life satisfaction (see Kahneman and Krueger, 2006, and references therein) and are the most accessible of the set recommended by Stiglitz *et al*. (2009).[4] Here, income is used as a proxy for claims on resources, education levels are used to capture amorphous characteristics such as job satisfaction and status, and health is seen as a fundamental human functioning. Similarly, leisure (play) is supported as a key variable by Nussbaum (2000) and also represents a price for having acquired higher scores on the income and education criteria.[5] It is noted that this choice of variables is by no means exhaustive and it is not always clear what roles they should play in the analysis. For example, education can be seen as a means to an end or an end in itself, and is complicated by the fact that it is a predictor of future earnings (Trannoy, 2005). Further, access to public goods is sometimes used (Gravel *et al*., 2005), but is not included here.

Once chosen, the variables are defined as follows. Household income applies to the head, partner, and other family members and consists of labor earnings, asset flows, private transfers, private pensions, public transfers, and social security pensions less total household taxes. There are some slight differences in definitions occurring between the countries, which can be found by consulting the relevant codebooks. To account for fixed costs within the household, each income is then divided by the square root of the household size. As more recent waves of the U.S. data do not record post-government incomes, we use instead the TAXSIM series generated by Feenberg and Coutts (1993), which simulates the role of household taxes. Our education variable is measured in years of formal schooling of the household head (again see the codebooks for definitions) and is topcoded at 17 years for the U.S., 18 years for Germany, and 18.5 years for Australia. This difference in topcoding is likely to lead to a small understatement of inequality in U.S. education relative to the other two countries. Subjective health satisfaction is then measured in terms of an ordinal variable coded from one to five, where five indicates the greatest level of satisfaction and one the lowest. As this variable is

[4]Recommended dimensions omitted from our study are (i) political voice and governance, (ii) social connections and relationships, (iii) the environment, and (iv) a sense of security.

[5]This variable is also rarely employed in multidimensional analysis, Merz and Rathjen (2014) being a notable exception. Working hours have also been included in related contexts (e.g. Fleurbaey and Gaulier, 2009; Pistolesi, 2009; Niehaus and Peichl, 2011).

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qualitative it is subjected to a linear scale, such that a health score of four is twice as good as a health score of two. Although one may reasonably object to this or any other scaling, it is felt that the problem is not too serious in practice, as it is well accepted that inequality measurements regularly employ weights or distance functions that are chosen out of simplicity or convenience. There are also some interpretation issues associated with subjective health scores (such as whether scores are interpersonally comparable, or if respondents implicitly adjust for factors such as their own age); however, we gain confidence from Currie and Madrian (1999), who observe that such scores are highly correlated with medically determined health status. Lastly, leisure time is measured by weekly non-working hours outside of non-paid labor less 56 hours for sleep. The figure of 56 hours of sleep is the product of eight hours per night over seven nights, a value which is approximately in line with historical averages and medical recommendations (Alvarez and Ayas, 2004). Plots of the marginal distributions of each variable are provided in Appendix A.1 (in the online supporting information).

3. Defining the Welfare Function

The data are denoted $X \in \mathbb{R}_{++}^{n \times k \times t}$, where X_{ijt} is the score of the ith individual on welfare dimension j in year t. To obtain welfare scores for each individual, we require an aggregation function and select a variation of the CES (Maasoumi and Jeong, 1985; Maasoumi, 1986) functional form

$$w_{it} \propto \left[\sum_{j=1}^{k} \alpha_j \left(\frac{1}{n} \sum_{t=1}^{T} X_{ijt}^{\gamma} \right)^{\beta} \right]^{\frac{1}{\beta}}, \tag{1}$$

which has the property of minimizing a multivariate β-entropy measure of pairwise divergences between w and X. To simplify, we set $\gamma = 1$, which imposes perfect intertemporal substitutability for each dimension such that individuals are indifferent about the order in which incomes and other facets of welfare arrive. Thus we are making a simplifying assumption by measuring inequality in "permanent" well-being along the lines of Burkhauser and Poupore (1997), rather than inequality in a specific income stream that accounts for substitutability, as in Maasoumi and Trede (2001) and Maasoumi and Jeong (1985). In addition, for computational ease we standardize each attribute relative to its mean, which makes inequality a purely relative concept, such that proportional changes to the distributions of our variables will have no effect. Having averaged out the intertemporal dimension, a row vector of mean-standardized individual specific scores is then written as $x_i \in \mathbb{R}_{++}^{k}$ and a column vector of corresponding attribute scores is $x_j \in \mathbb{R}_{++}^{n}$. Column means will thus have the property $\bar{x}_j = \frac{1}{n} \sum_{i=1}^{n} x_{ij} = 1 \, \forall j$, while row means will not.

The welfare function in equation (1) is further characterized by $\alpha_j \in [0, 1]$, which is a dimension weight normalized such that $\sum_{j=1}^{k} \alpha_j = 1$, and by $\beta \in (-\infty, \infty)$, which is a secondary weighting parameter that dictates the degree of substitutability. It is well known that this functional form has the elasticity of

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substitution $\sigma=1/(1-\beta)$, where various special cases exist for a number of specifications for β.[6] For the sake of practicality, α and β are held constant over the dimensions, across individuals, and across countries.[7] This represents a simplification, as it is likely that in practice the true weights will vary over all three sets of criteria; however, it is pragmatic to assume away these sources heterogeneity in the same manner implicit in most measures of economic inequality (Weymark, 2004). As a consequence, the welfare function is best interpreted as a paternalistic set of value judgments from the analyst's point of view, rather than a set of personal utilities. A second challenge involves selecting appropriate values for these parameters. Decancq and Lugo (2013) give some techniques for selecting α weights, including the use of normative and data-driven schemes, and we consider these below.

A simple approach to α weight setting is to select values on the basis of subjective judgments of each variable's relative importance. Although normative weights can be selected on the basis of opinion, a typical method is to apply them equally using $\alpha_j=1/k$. This is usually justified with an appeal to agnosticism; however, Chowdhury and Squire (2006) show using survey data that expert opinion is highly consistent with agnostic weights, which will be used as one variant in this paper, denoted $\alpha_E \in \mathbb{R}^k_+$. A second set of α weights is derived from the data by recognizing that under agnostic weights, the potential for double counting a latent variable exists if two dimensions are equally weighted and highly correlated.[8] Should the overlapping dimensions be independently relevant, this double counting is appropriate; however, if the same variable appears twice, the phenomenon can be accounted for by choosing $\alpha_j \propto^{-1} \sum_{j=1}^{k} |\rho_{j,j^*}| \forall j \neq j^*$. Here, ρ_{j,j^*} is the correlation between dimensions such that α_j is inversely proportional to the strength of the relationship between x_j and the other variables. In practice, we find that these weights are not too dissimilar across countries (although they do change over quantiles) and for this reason we employ an average for use over all three countries. Again this disallows heterogeneity in weights, which would be preserved if unique values were employed for each country; however, maintaining a consistent set of weights aids comparability, which we see as a more desirable feature. The data-driven weights employed are denoted $\alpha_D \in \mathbb{R}^k_+$.[9]

Less guidance is available for the substitution parameter β. Values less than or equal to unity are typical, which imply a preference for equality of attribute scores within each individual; however, beyond this point there is little theoretical rationale for selecting any particular value. We use a broader range of values,

[6]When $\beta \to 1$, the elasticity is infinite and the index becomes a weighted linear sum of the attributes. For $\beta \to 0$, $\sigma = 1$, which gives unit elasticity of substitution. Further, as $\beta \to -\infty$, $\sigma \to 0$ and the functional form collapses to the Leontief specification, where no substitution between the dimensions is allowed. In this case, w_i is simply equal to the minimum of the attribute scores.

[7]For the importance of homothetic preferences in the use of money-metric utilities, see Blackorby and Donaldson (1988).

[8]A further method that allows for individual heterogeneity is to use *most favorable* weights (Cherchye *et al.*, 2007). These weights allow α to vary over i to give maximal welfare to each unit. Some restrictions need to be imposed such that all the weight does not gravitate toward the single most favorable criterion.

[9]Using this method, health receives the greatest relative weighting of 0.34, income is weighted at 0.17, education at 0.21, and leisure at 0.28.

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TABLE 1
WELFARE INEQUALITY ESTIMATES: THEIL'S L AND T MEASURES

Weighting	U.S.		Germany		Australia	
	$\hat{L}$	$\hat{T}$	$\hat{L}$	$\hat{T}$	$\hat{L}$	$\hat{T}$
S1	0.0634	0.0799	0.0227	0.0242	0.0239	0.0259
	(0.00589)	(0.01090)	(0.00135)	(0.00158)	(0.00154)	(0.00198)
S2	0.0225	0.0235	0.0079	0.0080	0.0091	0.0093
	(0.00136)	(0.00176)	(0.00036)	(0.00038)	(0.00046)	(0.00050)
S3	0.0148	0.0150	0.0057	0.0057	0.0067	0.0068
	(0.00076)	(0.00089)	(0.00024)	(0.00024)	(0.00028)	(0.00030)
S4	0.0222	0.0216	0.0079	0.0079	0.0089	0.0090
	(0.00096)	(0.00093)	(0.00036)	(0.00036)	(0.00040)	(0.00041)
S5	0.0150	0.0145	0.0058	0.0058	0.0066	0.0067
	(0.00064)	(0.00060)	(0.00026)	(0.00025)	(0.00027)	(0.00027)

Notes: The first row (S1) contains estimates based upon the weighting scheme $\beta=1$, $\alpha_k=1/2$ and $\alpha_j=1/(2k-2)$, and the second row (S2) uses the scheme $\beta=0.5;\alpha_E$. Row three (S3) uses $\beta=0.5;\alpha_D$, while rows four (S4) and five (S5) use $\beta\rightarrow 0;\alpha_E$ and $\beta\rightarrow 0;\alpha_D$. Standard errors are given below each estimate in parentheses.

including (i) $\beta=1$, (ii) $\beta=0.5$ and (iii) $\beta\rightarrow 0$. Full specifications are determined by combining the various α and β weights; however, for $\beta=1$ we employ $\alpha_k=1/2$ and $\alpha_j=1/(2k-2)$. This results in five sets of weights which will be referred to in shorthand as specifications S1–S5.[10]

3.1. *Estimates of Welfare Inequality*

Once the functional forms for w are established, welfare scores are generated for each individual in each country. The distributions of w can then be analyzed using any univariate technique, and we employ Theil's (1967) two entropy-based inequality measures:

$$L(w)=\frac{1}{n}\sum_{i=1}^{n}-\ln\left(\frac{w_i}{\bar{w}}\right) \qquad \text{and} \qquad T(w)=\frac{1}{n}\sum_{i=1}^{n}\frac{w_i}{\bar{w}}\ln\left(\frac{w_i}{\bar{w}}\right).$$

Both measures are widely used in empirical analysis and can be interpreted as the information content of an indirect message converting either $w\rightarrow\bar{w}$ (L) or $\bar{w}\rightarrow w$ (T). When welfare is perfectly equal (i.e. $w_i=\bar{w}$ for $i=1\ldots n$), then $L(w)=0$ and $T(w)=0$, while increasingly unequal distributions will yield greater positive values in both cases. Of the two measures, the L index is more sensitive to the low end of the distribution, while the T index is more affected by transfers closer to the mean.

Results for L and T are presented in Table 1 for the permanent welfare inequality in our three countries. In addition to each estimate, bootstrap standard errors are provided in parentheses, to illustrate the degree of uncertainty attributable to sampling variation.

[10]S(1) is $\alpha_k=1/2$ and $\alpha_j=1/(2k-2)$, while combining weights gives S(2) $\beta=0.5;;\alpha_E$, S(3) $\beta=0.5;;\alpha_D$, S(4) $\beta\rightarrow 0;;\alpha_E$, and S(5) $\beta\rightarrow 0;;\alpha_D$.

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Estimates of welfare inequality from Table 1 show that over both measures and all weighting specifications, ingrained welfare inequality was substantially higher in the U.S. than in either Germany or Australia. Starting with the linear aggregation function (S1), we see that estimates for L and T were around 0.06–0.08 for the U.S., while for Germany and Australia they were less than half of the U.S. levels, ranging from 0.023 to 0.036 for both measures. For S2 and S4 (which used equal dimension weights but differing substitution weights), the U.S. estimates were around 0.22 for both the T and L measures, whereas the German and Australian estimates ranged from around 0.008 to 0.009, with the Australian estimates typically a little higher. A similar pattern is evident for S3 and S5, with high U.S. estimates, while the Australian and German estimates were again similar and much lower. Given the breadth of this result, we conclude that the standard inequality ordering of these countries (i.e. high in the U.S., low in Germany) is not overturned when the additional dimensions are included in the analysis, and when results are based upon longitudinal averages rather than annual observations.

To determine whether the observed differentials in inequality are statistically significant, we also conduct bootstrap hypothesis tests. Consider two generic countries, A and B, and the GE inequality measure I. We are interested in testing the null hypothesis $H_0 : I_A = I_B$ against the alternative $H_1 : I_A \neq I_B$ using test statistic $\tau_{AB} = (I_A - I_B)/\sqrt{\sigma_A^2 + \sigma_B^2}$, where σ_A^2 and σ_B^2 are the asymptotic variances of GE measures given by Cowell (1989). To simulate the case where the null hypothesis holds, w_A and w_B are pooled and $S = 1,000$ samples of size n_A and n_B are drawn with replication. In each case, we employ the indicator function

$$\phi = \begin{cases} 0 & \text{if}; \tau_{AB} \leq \hat{\tau}_{AB}, \\ 1 & \text{if}; \tau_{AB} > \hat{\tau}_{AB}, \end{cases}$$

such that P-values can calculated as

$$P = 2 \times \min\left(\frac{1}{S} \sum_{s=1}^{S} \phi_s, \frac{1}{S} \sum_{s=1}^{S} 1 - \phi_s \right).$$

The null hypothesis is then rejected if P is less than the level of significance (taken to be 5 percent).

TABLE 2

Hypothesis Tests on Inequality Measures

Specification	$L_{US,GR}$	$T_{US,GR}$	$L_{US,AU}$	$T_{US,AU}$	$L_{AU,GR}$	$T_{AU,GR}$
S1	0.000	0.000	0.000	0.000	0.392	0.449
S2	0.000	0.000	0.000	0.000	0.016	0.028
S3	0.000	0.000	0.000	0.000	0.001	0.005
S4	0.000	0.000	0.000	0.000	0.009	0.008
S5	0.000	0.000	0.000	0.000	0.024	0.014

Notes: The P-values are based upon two-tailed tests of equality of T_A and T_B or L_A and L_B across each pair of countries

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The results from Table 2 indicate that U.S. inequality was significantly higher than both German and Australian inequality over all weighting specifications. The first two columns show rejections of the null of equal L (and T) statistics for the U.S. and Germany at all levels, while the next two columns show the same result for the U.S. and Australia. The comparison of Germany and Australia in the final two columns is more ambiguous. For specifications S2–S5, we found significant differences in Australian and German inequality at 5 percent significance for both the L and T measures. Given that these rejections occur when the point estimate for Australia exceeds that for Germany, they imply a slightly higher level of Australian inequality. It is observed that these results rely upon weighting specifications that impose imperfect substitution between the dimensions. When specification S1 is used, the null of significant differences is not rejected for either measure, implying that the result of higher Australian inequality depends upon either this lack of substitutability or the increased weighting on income used in this welfare function.

4. Calculating Equivalent Incomes

The results presented in Section 3 show that subject to our assumptions, permanent welfare inequality is much larger in the U.S. than in Germany or Australia. However, as the inequality metrics have abstract interpretations, it is difficult to conceptualize the degree of difference between the countries. In this section, we attempt to give an intuitive guide to the size of the observed inequalities with the use of equivalent incomes. We assume that for each individual, the determinants of well-being are exogenous, such that changes in income, education, health, or leisure act causally to affect welfare as defined by the relevant utility function. We then calculate a hypothetical income level that would compensate for any shortfalls across non-monetary dimensions by restoring each person's welfare to the sample average. This allows differentials in welfare to be measured in purely monetary terms. By averaging the gap between an individual's true income and their equivalent, we are able to obtain a measure of the degree of redistribution required to fully offset welfare inequality.

To calculate the equivalent income, let x_{ik} denote the observed mean-relative income of individual i and define x_{ik}^* as the equivalent. The welfare function becomes

$$(2) \qquad w_i^*\left(\alpha,\beta,x_{ij},x_{ik}^*\right)=\begin{cases}\left[\left(\sum_{j=1}^{k-1}\alpha_j x_{ij}^{\beta}\right)+\alpha_k x_{ik}^{*\beta}\right]^{\frac{1}{\beta}}, & \beta\neq 0,\\ \left[\prod_{j=1}^{k-1} x_{ij}^{\alpha_j}\right]\times x_{ik}^{*\alpha_k}, & \beta\to 0,\\ \left[\sum_{j=1}^{k-1}\alpha_j x_{ij}\right]+\alpha_k x_{ij}^{*}, & \beta=1,\end{cases}$$

where distribution x_k^* can be found in accordance with the following conditions.

C 1. Equality: Income x_{ik}^* is chosen such that equality in welfare is achieved, that is, we effectively solve the system of n simultaneous equations[11]

[11]The n equations correspond to the n observations, while the $(n+1)$th equation is required to specify an average welfare level specified in *C2*.

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$$
(3)\quad \begin{cases}
w_1^*\left(\alpha,\beta,x_{11},x_{12},\ldots x_{1k-1},x_{1k}^*\right) &= \dfrac{1}{n-1}\sum_{i\neq 1}^{n}\left[\left(\sum_{j=1}^{k-1}\alpha_j x_{ij}^{\beta}\right)+\alpha_k x_{ik}^{*\beta}\right]^{\frac{1}{\beta}},\\
w_2^*\left(\alpha,\beta,x_{21},x_{22},\ldots x_{2k-1},x_{2k}^*\right) &= \dfrac{1}{n-1}\sum_{i\neq 2}^{n}\left[\left(\sum_{j=1}^{k-1}\alpha_j x_{ij}^{\beta}\right)+\alpha_k x_{ik}^{*\beta}\right]^{\frac{1}{\beta}},\\
\vdots & \qquad\qquad \vdots\\
w_n^*\left(\alpha,\beta,x_{n1},x_{n2},\ldots x_{nk-1},x_{nk}^*\right) &= \dfrac{1}{n-1}\sum_{i=1}^{n-1}\left[\left(\sum_{j=1}^{k-1}\alpha_j x_{ij}^{\beta}\right)+\alpha_k x_{ik}^{*\beta}\right]^{\frac{1}{\beta}}.
\end{cases}
$$

As there is insufficient information to yield a unique solution for x_k^*, we also require:

C 2. Non-negativity and Normalization: The counterfactual income is also non-negative ($x_{ik}^* \geq 0$) and has the same mean as the true distribution of income ($\bar{x}_{ik}^* = 1$).

$C1$ ensures that $n\times 1$ dimensional *compensating vector* x_k^* is sufficient to equalize welfare subject to the weighting specification employed. $C2$ requires that equalization is obtainable by a process of mean-preserving transfers on the actual income variable x_k.

It is apparent that under certain circumstances x_k^* will not exist, subject to $C1$ and $C2$. Should an individual be highly disadvantaged on some non-monetary characteristic, it may be that the income level required to compensate is greater than the sum of the entire distribution, and therefore that no process of mean-preserving spreads could equalize welfare. This problem places some restrictions on the weights that can be employed in equation (1). Although this sort of imposition is undesirable, it can be seen that the requirement that x_k^* exists does not rule out too many sensible sets of weights. Indeed, we have always been able to obtain x_k^* using numerical methods for any α where $\beta < 0$ and many cases when $\beta > 0$. Situations where x_{ik}^* does not exist occur when the welfare levels of two individuals will not cross with *any* redistribution of income. Intuitively, it can be seen that this is not a problem when $\beta < 0$, as this implies $\lim_{x_{ik^*}\to 0} w_i^* = 0$. As all individuals start at this lower bound and w_i^* is continuous and monotonic in x_{ik}^*, it becomes clear that x_k^* will exist in these instances.[12] Further, the monotonicity of w in x_{ik}^* implies that solutions will be unique.

To obtain x_k^*, an iterative method is used. We start with an egalitarian distribution of income (i.e. $x_{ik}^* = 1$ for $i = 1\ldots k$) and determine the welfare distribution w_i^* with average $\bar{w}$. Each individual then has income transferred to them if $w_i^* < \bar{w}$ and away from them if $w_i^* > \bar{w}$ in proportion to $w_i^*/\bar{w}$. The repetition of this process over a sufficient number of iterations sees the process converge, yielding equality in w_i^* while satisfying $C2$.

5. The Joint Distribution of x_K and x_k^*

Having completed this process, we now examine the relationship between the true income distribution x_k and the equivalent distribution x_k^*. Diagrammatic depictions of

[12]Despite the existence of solutions for $\beta < 0$, strongly negative values are not always desirable, as the low degree of substitutability often necessitates severe redistribution of income to equalize welfare. For example, cases where $\beta = -0.5$ have resulted in the most disadvantaged 1 percent receiving over 30 percent of all income.

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x_k and x_k^* are of particular interest as they allow a geometric interpretation of welfare gaps, and provide a contrasting analogue with univariate income inequality measures. Consider that if $x_k = x_k^*$, then each person has exactly their welfare-equalizing income, and as such no redistribution is required to offset welfare inequality. Conversely, deviations from this relationship imply disparities in welfare which may be eliminated with an appropriate set of transfers. In this sense, $x_k = x_k^*$ forms a *line of no redistribution*, which becomes the reference against which the empirical distribution is measured.

Ordinarily, a scatter plot would be sufficient for depicting this relationship; however, the interpretation is complicated by the uneven weights that apply to the observations (e.g. due to differing household sizes). To get around this problem, a bivariate kernel is used to model the joint distribution such that the weights can be included in the representation. A two-stage process is used where a fixed-bandwidth kernel is employed as a calibration device for a secondary adaptive estimator. The general form of the pilot kernel is

$$\hat{f}_H(x_k; x_k^*) = \frac{1}{n}\sum_{i=1}^{n} K_H(x - x_d), \tag{4}$$

where x_d contains observations on x_k and x_k^*, H is a 2 × 2 bandwidth matrix, $K_H(x) = |H|^{-\frac{1}{2}} K\left(H^{-\frac{1}{2}}x\right)$, and K is a Gaussian kernel (Wand and Jones, 1993). The choice of bandwidth is fundamental to the performance of the kernel and it is common to use either

$$H = \begin{bmatrix} h^2 & 0 \\ 0 & h^2 \end{bmatrix} \quad \text{or} \quad H = \begin{bmatrix} h_1^2 & 0 \\ 0 & h_2^2 \end{bmatrix},$$

where the former uses radially symmetric kernels while the latter gives elliptical kernels that are parallel to axes x_k and x_k^*. The latter is more flexible and the choice of elements is usually made on the basis of minimizing a quadratic loss function. A number of techniques for selecting bandwidths exist, including cross-validation methods and plug-in estimators. If univariate x is Gaussian, then it can be seen that $h = 1.06n^{-\frac{1}{5}}\hat{\sigma}_x$ minimizes the mean integrated squared error (MISE), while replacing $\hat{\sigma}_x$ with interquartile range $R(x)$ works well if data are heavy tailed. Applying this rule to a diagonal matrix gives

$$H = \begin{bmatrix} \left(1.06n^{-\frac{1}{5}} \times \min\left(\hat{\sigma}_{x_k}, \frac{\mathrm{R}(x_k)}{1.34}\right)\right)^2 & 0 \\ 0 & \left(1.06n^{-\frac{1}{5}} \times \min\left(\hat{\sigma}_{x_k^*}, \frac{\mathrm{R}(x_k^*)}{1.34}\right)\right)^2 \end{bmatrix},$$

where $\hat{\sigma}_{x_k}$ and $\hat{\sigma}_{x_k^*}$ are the estimated standard deviations. This density is then used to calibrate an adaptive bandwidth $\hat{f}_{H_i}(x_k; x_k^*) = \frac{1}{n}\sum_{i=1}^{n} K_{H_i}(x - x_r)$, where $H_i = \lambda_i \times H$, in which $\lambda_i = \left(G/\hat{f}_H(x_k; x_k^*)\right)^{\frac{1}{2}}$ and $G = \left[\prod_{i=1}^{n} \hat{f}_H(x_k; x_k^*)\right]^{\frac{1}{n}}$. This is the estimator of Abramson (1982), which employs a low bandwidth where observations are clustered together such that important details are not smoothed away, while still allowing for a high degree of smoothing when observations are sparse.

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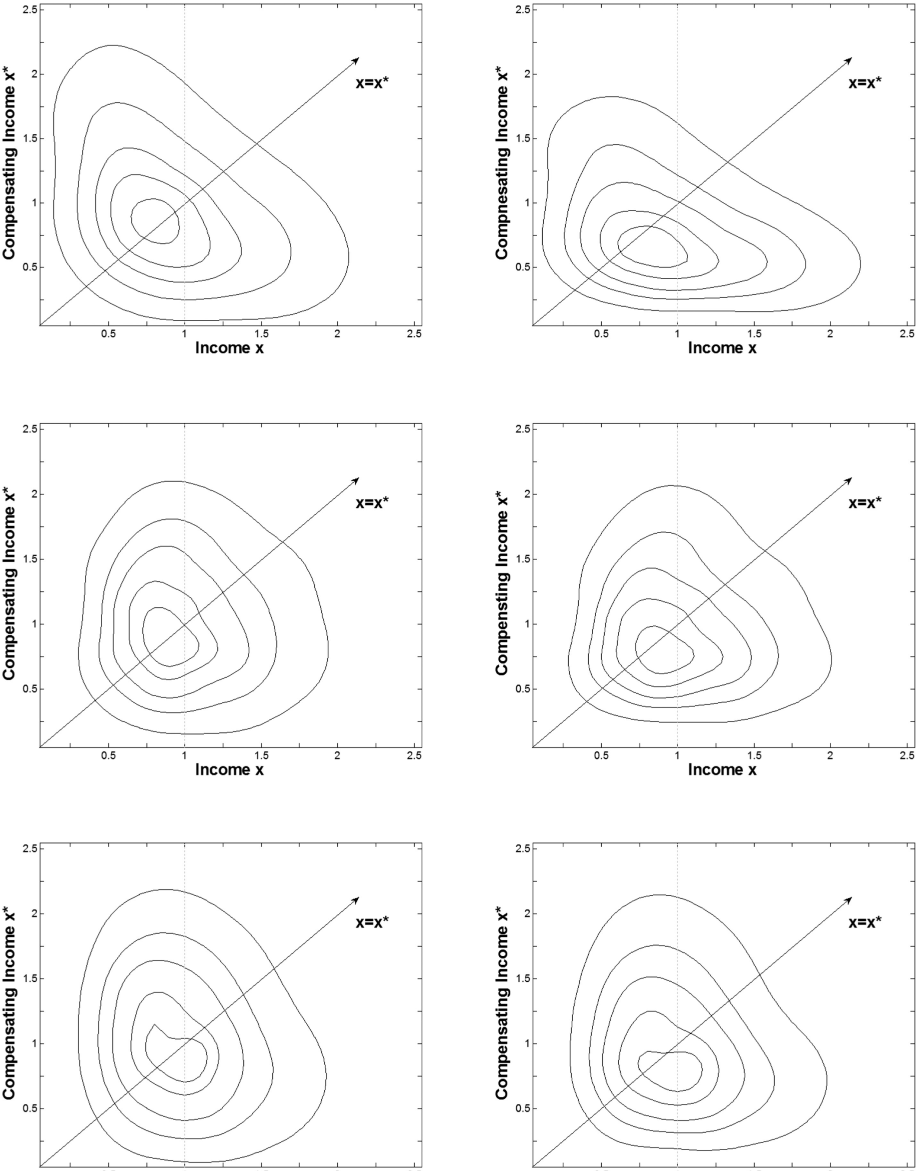

Figure 1. The Contour Densities of x_k and x_k^* for the U.S., Germany, and Australia

Notes: The boxes on the left give level curves of $\hat{f}_{H_i}(x_k; x_k^*)=c$, where x_k^* is determined using weighting specification S3 (i.e. $\beta=0.5; \alpha_D$), while the boxes on the right give the level curves using weighting specification S5 (i.e. $\beta \rightarrow 0, \alpha=\alpha_D$). The top two panels are for the U.S. and the level curves are at 0.065, 0.2, 0.4, 0.6, and 0.85, and 0.065, 0.2, 0.4, 0.75, and 1.1. The second two panels are for Germany and both sets of level curves are at 0.065, 0.2, 0.6, and 1.2. The final two boxes are for Australia and this same set of values is used.

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Such adaptive estimators are generally found to strongly outperform their fixed-bandwidth counterparts in terms of MISE (Sain, 2002).

Once the kernel is fitted, the distributions are depicted with contour plots consisting of level curves of the fitted density. That is, $\hat{f}_{H_i}(x_k; x_k^*) = c$ produces a curve joining all values of x_k and x_k^* that share the same estimated frequency. The process thus characterizes a three-dimensional "landscape" portrait of the joint distribution.

The estimator is applied for the three countries where two selected specifications (S3 and S5) are used, varying β over the fixed set of data-driven weights. In addition, the lines $x_k = x_k^*$ and $x_k = 1$ are included in each plot such that the dispersion of density away from these benchmarks can be observed, where the latter corresponds to the case of perfect income equality.

The top two panels in Figure 1 show level curves of the estimated joint distribution $\hat{f}_{H_i}(x_k; x_k^*)$ for the U.S. under the two selected weighting specifications.[13] In both cases, there is evidence of a negative correlation between x_k and x_k^*, which implies that persons with high actual incomes tend to require lower incomes to bring them to average social welfare. This indicates that higher than average incomes correspond with higher performances on other welfare dimensions for the U.S. under these weighting specifications. The difference between the fitted density and the 45° line is notable in both cases and appears greater than from the vertical line, illustrating the effect of including the additional non-income variables in the analysis.

The results for Germany in the two central panels show some obvious differences from those for the U.S. In both cases, there is little difference between the bivariate distributions generated using the different β weights; however, there is much less evidence of a negative correlation existing between the observed income and the equivalent income. For this reason, the dispersion relative to the diagonal line appears similar to the dispersion around the gray vertical line, indicating a similarity between the degree of redistribution required and univariate (income) inequality. This suggests that the relationship between the non-monetary dimensions is less correlated than for the U.S., such that a person who was well off on one dimension is less likely to also be advantaged in other ways. The final two panels in Figure 1 for Australia look broadly similar to those for Germany; however, there is some evidence of a negative correlation between x_k and x_k^*, and both plots appear to have similar degrees of variation along the vertical and horizontal axes. A comparison of three countries suggests that the joint density for the U.S. diverges further from both the line $x_k = x_k^*$ and the line $x_k = 1$. The first comparison indicates that in monetary terms, the welfare gaps are greatest in the U.S., while the second indicates that U.S. permanent income inequality in isolation is also likely to be high relative to the other two countries.

The dispersion of density around the line $x_k = x_k^*$ can be explicitly measured using the direct distances between the income pairs. This is

$$R(x_k \parallel x_k^*) = \frac{1}{2n}\sum_{i=1}^{n} |x_{ik} - x_{ik}^*| \tag{5}$$

[13]Note that we have used different contours for the higher frequencies across countries in order to make the plots as informative as possible. This comes at the expense of some comparability across the countries for the innermost contours.

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TABLE 3
THE SIZE OF THE INEQUALITY-ELIMINATING REDISTRIBUTIONS

	Weighting Specification				
Measure	S1	S2	S3	S4	S5
U.S.	0.2607	0.3220	0.3829	0.3434	0.4340
	(0.007)	(0.007)	(0.008)	(0.009)	(0.014)
Germany	0.1649	0.1972	0.2437	0.2000	0.2590
	(0.004)	(0.004)	(0.005)	(0.004)	(0.007)
Australia	0.1682	0.2100	0.2639	0.2128	0.2716
	(0.004)	(0.005)	(0.006)	(0.005)	(0.006)

Notes: The first column (S1) contains estimates based upon the weighting scheme $\beta=1$, $\alpha_k=1/2$ and $\alpha_j=1/(2k-2)$, and the second column (S2) uses the scheme $\beta=0.5; \alpha_E$. Column three (S3) uses $\beta=0.5; \alpha_D$, column four (S4) uses $\beta \to 0; \alpha_E$, and column five (S5) uses $\beta \to 0; \alpha_D$.

and is interpreted as the degree of redistribution required to eliminate welfare inequality. Here, $R\left(x_k \parallel x_k^*\right)$ is analogous to the relative mean deviation (RMD) inequality index frequently applied to analyze inequality of incomes. The index lies between a value of zero (when $x_k=x_k^*$) and one (when all income goes to one individual and all the compensating income is required by another). Results based on weighting specifications S1–S5 are given in Table 3.

Estimates from Table 3 indicate that the U.S. would require much greater levels of income redistribution (relative to the current distribution) to offset inequalities in health, education, and leisure. Estimates range from around 0.26 to about 0.43 and generally congregate around 0.3–0.4, indicating that welfare could be equalized by transferring 30–40 percent of all U.S. income from high- to low-welfare individuals. For Germany and Australia these estimates are lower, and average around 20 and 25 percent of the total income distribution, respectively. Thus the relative degrees of required redistribution closely resemble the welfare inequality orderings presented in Section 3. It is of additional interest to compare these estimates to RMD estimates based purely on income. These values are 0.238 for the U.S., and 0.151 and 0.158 for Germany and Australia. As these figures are considerably less than the welfare-equalizing transfers, it follows that a process of transfers that equalizes incomes would still fall substantially short of equalizing welfare.

6. CONCLUSION

The paper has measured permanent multidimensional inequality in the U.S., Germany, and Australia using data on income, education, health, and leisure. Aggregative welfare-based techniques were employed and we found evidence of higher U.S. inequality relative to Germany or Australia across a variety of alternative parametric assumptions. Australia appeared to be slightly more unequal than Germany, although the result was confined to a limited number of weighting specifications. In addition, we employed an equivalent-income approach to quantify the degree of redistribution of income required to offset welfare inequality, and again found these estimates to be much larger in the U.S. The inequality orderings presented in the paper are thus consistent with many cross-sectional univariate studies of the distributions of factors such as income, wealth, and health. We therefore conclude that income mobility and the

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distribution of non-monetary resources do not reverse or even weaken the standard findings of high U.S. inequality relative to other developed countries.

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Supporting Information

Additional supporting information may be found in the online version of this article at the publisher's web-site

Appendix A.1 Marginal Distributions of Each Variable
Figure A.1: Marginal Distributions
Appendix A.2 Dominance Testing
Table A.1: Lorenz Dominance Tests—US, Germany and Australia

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