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# IMPACT OF TAXES AND BENEFITS ON INEQUALITY AMONG GROUPS OF INCOME UNITS

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This paper analyzes the redistributive impact of the fiscal system and simultaneously explains how each tax and benefit instrument satisfies the principles of vertical and horizontal equity within and across different groups of income units. The decompositions of the redistributive effect are based on new axioms concerning the vertical and horizontal equity of the overall fiscal system, including taxes and benefits. The method is based on pairwise comparisons of income units and the "micro" concepts of income supremacy change, deprivation from reranking, and income distance change. The decomposition results provide more detailed insights into the income redistribution process than is typical in the literature. This is illustrated by an empirical application of the method to the Croatian scheme of personal income taxes and non-pension social benefits, in which households are divided into two groups, those with and those without children.

**JEL Codes**: D63, H22, H23

**Keywords**: decomposition by groups, horizontal effect, taxes and benefits, vertical effect

## 1. INTRODUCTION

The redistributive impact of a fiscal system can be analyzed along several dimensions. First, there is the question of satisfying the principles of vertical and horizontal equity (Lambert, 2001; Duclos and Araar, 2006). The principle of vertical equity requires that income distances between higher and lower income units should be decreased. The vertical effect measures the extent to which this principle is satisfied. The "classical" principle of horizontal equity stipulates that income units with equal pre-fiscal incomes should have equal post-fiscal incomes, whereas the "no reranking" principle of horizontal equity mandates that each income unit's ranking is preserved in the transition from pre- to post-fiscal income. Horizontal effects measure the extent to which these principles are violated (Duclos *et al*., 2003). Second, once the vertical and horizontal effects of the fiscal system are assessed, researchers seek to determine the relative contributions of individual tax and benefit instruments to these effects. Third, it is important to

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understand how the fiscal system redistributes income both within and across different socio-economic groups, which leads to further analysis of vertical and horizontal effects.

There are several papers that simultaneously address the *first* and the *second* matters discussed above, by decomposing the redistributive effect of the overall fiscal system into the contributions of taxes and benefits to vertical and horizontal effects (for a detailed review of the methods in this area, see Urban, 2014). Monti *et al.* (2013) capture the *first* and the *third* dimensions by measuring the vertical and horizontal aspects of tax systems. The present paper creates a comprehensive model that enables the measurement of *all three* of the aforementioned aspects of income redistribution. In the remainder of this section, we discuss the foundations of the method developed in subsequent sections.

Kakwani (1984) decomposes the redistributive effect into a vertical effect and a horizontal effect. Lambert (1985, 2001) decomposes the vertical effect into the contributions of taxes and benefits. The "relative deprivation" framework was introduced by Yitzhaki (1979) and Hey and Lambert (1980). In the analysis of income redistribution, the concept of relative deprivation was first employed by Duclos (2000), who defines the terms "fiscal harshness," "fiscal looseness," and "ill-fortune" and then relates them to the standard measures of vertical and horizontal effects. The relative deprivation framework employs the "mean difference" approach in the computation of the Gini coefficient.

Following Duclos (2000), Urban (2010) uses the relative deprivation framework to develop the concepts and measures of "income supremacy," "income distance," and "deprivation from reranking." When these micro measures are aggregated to the population level (using the mean difference approach), we obtain measures that are identical to those in Kakwani (1984). Urban (2010) also decomposes redistributive, vertical, and horizontal effects into contributions of taxes and benefits, and the resulting decomposition of vertical effects leads to contribution terms that are equivalent to those found in Lambert (1985, 2001).

Another foundation of the new method is the research of Pellegrino and Vernizzi (2013) and Monti *et al.* (2013), who dissect the redistributive effect of personal income tax into terms that quantify violations of the equity axioms presented by Kakwani and Lambert (1998). Monti *et al.* (2013) further decompose the new indices into contributions within and across groups, also using the mean difference approach to compute Gini-based indices.

Urban (2014) recognizes two different approaches that are widely used in the literature to determine vertical equity of separate tax and benefit instruments. According to the "prevalent" view, a tax (a benefit) is vertically equitable if the ratio between the tax (the benefit) and the pre-fiscal income is increasing (decreasing) in pre-fiscal income. This corresponds to the standard definition of tax progressivity (benefit regressivity). Conversely, the "alternative" view requires only that the absolute amount of a vertically equitable tax (benefit) be increasing (decreasing) in pre-fiscal income. These two views bring quite different judgments on whether a certain tax or benefit instrument reduces income inequality.[1]

This paper derives new axioms of vertical and horizontal equity for pairs of income units. The axioms are derived for two different normative frameworks, which correspond to the above-mentioned prevalent and alternative views. The basic axioms are related to the overall fiscal system, while the additional axioms are related to individual tax and benefit instruments. The derivation of new axioms is followed by a redefinition of the micro concepts "income supremacy," "income distance," and "deprivation from reranking." By aggregating these micro concepts at the population level, we obtain the indices of vertical, horizontal, and redistributive effects from Kakwani (1984), whereby these effects obtain new axiom-based interpretations. The equity axioms based on the prevalent view are then used to decompose the vertical, horizontal, and redistributive effects into the contributions of individual tax and benefit instruments. Finally, the new effects are further decomposed to expose income redistribution within and across groups.

The empirical application to the Croatian fiscal subsystem illustrates that the new method has the potential to provide detailed insight into the income redistribution process. We analyze how personal income taxes and non-pension social benefits redistribute income within and between two distinctive groups of households—those with children and those without children.

The remainder of this paper has four parts. Section 2 begins with data preparation and the definition of variables, followed by descriptions and discussion of the axioms of vertical and horizontal equity. The micro concepts of income supremacy change, deprivation from reranking, and income distance are then derived, and their relationships with the equity axioms are established. In Section 3, the micro measures are first decomposed into contributions of taxes and benefits and then decomposed further within and across groups of income units. Section 4 discusses the application of the new method to an empirical sample of Croatian households taken from the SILC database. Section 5 concludes.

## 2. The Concepts of Vertical and Horizontal Equity

### 2.1. *Data Preparation and Variables*

A typical dataset is a representative sample of $n$ households, and the total aggregated population consists of $N_r = \sum_{i=1}^{n} f_i$ households, where $f_i$ is a frequency weight of household $i$. Each household from the sample is categorized into one of $\Psi$ exclusive groups according to the selected exogenous characteristic. Thus, household $i$ belongs to group $\Theta_i$, $\Theta_i \in \{1, \ldots, \Psi\}$.[2]

[1]Essential to the Lambert (1985, 2001) decomposition is the prevalent view. The alternative view permeates the classical decomposition of income inequality into contributions of source components, proposed by Rao (1969) and others. The prevalent and alternative views are related to the concepts of "relative" and "absolute" inequality, envisaged by Dalton (1920) and developed thoroughly by Kolm (1976) and others. For subsequent developments in the field of taxation, see Pfingsten (1986) and Ebert (2010a).

[2]For example, the selected characteristic can be the "place of living" or the "household type." In the "place of living" example, $\Theta_i = 1$ may denote "the capital city," while $\Theta_i = 2$ and $\Theta_i = 3$ might stand for "other urban areas" and "rural areas," respectively. In case of "household type," $\Theta_i = 1$ could denote "a couple without children," while $\Theta_i = 2$ could represent "a couple with one child," etc.

Assume that the fiscal system consists of only one tax and one benefit, whose amounts are equal to $\tilde{T}_i$ and $\tilde{B}_i$, respectively. $\tilde{X}_i$ and $\tilde{Y}_i$ denote pre- and post-fiscal income, respectively, where $\tilde{Y}_i$ is obtained as

$$\tilde{Y}_i = \tilde{X}_i - \tilde{T}_i + \tilde{B}_i. \tag{1}$$

The equivalence scale factor, $e_i$, is typically a function of the number of adult household members, $a_i$, and children, $c_i$. The equivalized pre-fiscal income is obtained as $X_i = \tilde{X}_i / e_i$; $T_i$, $B_i$ and $Y_i$ are obtained analogously. In the process of equivalization, a new income unit is obtained, the so-called equivalent adult. Following Ebert (1999), the assumption here is that household $i$ has $e_i$ income units. The sample thus represents $N = \sum_{i=1}^{n} \varphi_i$ income units, where $\varphi_i = e_i f_i$.

Having defined all the necessary variables, let **M** be the matrix of column vectors:

$$\mathbf{M} = \begin{pmatrix} X_1 & Y_1 & T_1 & B_1 & \varphi_1 & \Theta_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ X_i & Y_i & T_i & B_i & \varphi_i & \Theta_i \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ X_n & Y_n & T_n & B_n & \varphi_n & \Theta_n \end{pmatrix}. \tag{2}$$

Each row of the matrix **M** represents one household from the sample. The order of these rows is determined by the data collector. However, for purposes of the analysis presented below, we must order the household units according to pre-fiscal income. Therefore, the rows in **M** are sorted in increasing order of the values from the first column:[3]

$$\mathbf{M}^x = \begin{pmatrix} X_1^x & Y_1^x & T_1^x & B_1^x & \varphi_1^x & \Theta_1^x \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ X_i^x & Y_i^x & T_i^x & B_i^x & \varphi_i^x & \Theta_i^x \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ X_n^x & Y_n^x & T_n^x & B_n^x & \varphi_n^x & \Theta_n^x \end{pmatrix}. \tag{3}$$

From $\mathbf{M}^x$ we extract the values of pre-fiscal income ($X_i^x$), tax ($T_i^x$), benefit ($B_i^x$), post-fiscal income ($Y_i^x$), weight ($\varphi_i^x$), and group ($\Theta_i^x$). The superscript $x$ denotes that income units are sorted in increasing order of pre-fiscal income.

Total and mean pre-fiscal income are equal to $\Sigma_X = \sum_{i=1}^{n} X_i^x \varphi_i^x$ and $\mu_X = \Sigma_X/N$, respectively. $\Sigma_T$, $\Sigma_B$ and $\Sigma_Y$ ($\mu_T$, $\mu_B$, and $\mu_Y$) are total amounts (means) of the tax, the benefit, and post-fiscal income, respectively, and are obtained

[3]If the sample includes a large subgroup of income units with identical incomes, the units within this subgroup should be sorted in random order, to minimize the bias in calculation of index $D_{Y;x}$ from equation (7). For more details on this issue, see Urban (2013).

analogously to $\Sigma_X$ ($\mu_X$). The average rates of the tax and the benefit are $\tau = \Sigma_T/\Sigma_X$ and $\beta = \Sigma_B/\Sigma_X$, respectively. The average rate of post-fiscal income is $\eta = \Sigma_Y/\Sigma_X = 1 - \tau + \beta$.

Equation (1) describes the relationship among pre-fiscal income, post-fiscal income, the tax, and the benefit. The following equations restate this relationship for equivalized income units, as follows:

(4) $$Y_i^x = X_i^x - T_i^x + B_i^x;$$

(5) $$X_i^x - Y_i^x = T_i^x - B_i^x.$$

Equation (5) demonstrates that the wedge between pre- and post-fiscal income is created by the tax and the benefit. This wedge is called the *net tax* and is expressed as follows:

(6) $$U_i^x = T_i^x - B_i^x = X_i^x - Y_i^x.$$

2.2. *Standard Decompositions of Redistributive Effect*

The redistributive effect of the fiscal system is equal to $P = G_X - G_Y$, where $G_X$ and $G_Y$ are the Gini coefficients of pre- and post-fiscal income. Kakwani (1984) decomposes $P$ into parts that explain the satisfaction of the vertical and horizontal equity principles, namely the vertical effect, $V$, and the horizontal effect, $H$, as follows:

(7) $$P = V - H = (G_X - D_{Y;x}) - (G_Y - D_{Y;x}),$$

where $D_{Y;x}$ is the concentration coefficient of post-fiscal income with respect to pre-fiscal income rankings. In $D_{Y;x}$, the letter $x$ in the subscript denotes that income units are sorted in increasing order of pre-fiscal income. $H$ is conceived by Atkinson (1980) and Plotnick (1981), whereas $V$ is introduced by Kakwani (1977). Equation (7) shows that $H$, which is by construction always positive, represents a subtraction from $V$, leading to the net effect $P$. Thus, $V$ can be interpreted as a potential redistributive effect that would be achieved in the absence of horizontal inequity, i.e. if $H = 0$ (Urban, 2013).

Lambert (1985, 2001) decomposed $V$ into contributions of taxes and benefits, as follows:

(8) $$V = \frac{\tau}{\eta}(D_{T;x} - G_X) + \frac{\beta}{\eta}(G_X - D_{B;x}),$$

where $D_{T;x}$ and $D_{B;x}$ are the concentration coefficients of the tax and the benefit, respectively, with respect to pre-fiscal income rankings. If the tax (the benefit) is proportional with pre-fiscal income, we would have that $D_{T;x} = G_X$ ($D_{B;x} = G_X$), and the contribution of the tax (the benefit) would be zero.

In the mean difference approach, $G_X$, $D_{Y;x}$ and $G_Y$ are obtained as follows:

(9)
$$G_X = (N\Sigma_X)^{-1}\sum_{j=1}^{n}\sum_{i=j+1}^{n}\varphi_i^x\varphi_j^x(X_i^x - X_j^x);$$
$$D_{Y;x} = (N\Sigma_Y)^{-1}\sum_{j=1}^{n}\sum_{i=j+1}^{n}\varphi_i^x\varphi_j^x(Y_i^x - Y_j^x);$$
$$G_Y = (N\Sigma_Y)^{-1}\sum_{j=1}^{n}\sum_{i=j+1}^{n}\varphi_i^x\varphi_j^x\left|Y_i^x - Y_j^x\right|.$$

Thus, the coefficients from equation (9) are obtained for all pairs of units $\{i, j\}$ such that $i > j$. $D_{T;x}$ and $D_{B;x}$ are obtained analogously to $D_{Y;x}$. The calculation approach taken in equations (9) is somewhat different from the standard use of the mean difference method, which accounts for *all* pairs of units, and not only those pairs for which $i > j$.

### 2.3. *Axioms of Vertical and Horizontal Equity of the Overall Fiscal System*

Following the approach of Kakwani and Lambert (1998), we propose several axioms for an equitable fiscal system, for pairs of income units. The first two axioms, VSR and VSA, represent two alternative definitions of a vertically equitable fiscal system. The third axiom, HS, defines a horizontally equitable fiscal system. These are our basic three axioms and are used to derive additional axioms of vertically and horizontally equitable taxes and benefits, as presented in the following section. In the nomenclature of axioms, *V* and *H* represent vertical and horizontal, respectively; *S*, *T* and *B* represent overall fiscal system, taxes, and benefits, respectively; and *R* and *A* designate "relative differences" and "absolute differences" approaches, respectively.

Henceforth, I and J are income units whose pre-fiscal incomes are $X_i^x$ and $X_j^x$, respectively, such that $X_i^x > X_j^x$. Thus, unit I ranks above unit J on the pre-fiscal income ladder, and from the construction of $\mathbf{M}^x$, it follows that $i > j$.

The shares of unit I's and J's pre-fiscal incomes in total pre-fiscal income are $X_i^x/\Sigma_X$ and $X_j^x/\Sigma_X$, respectively, whereas their shares in total post-fiscal income are $Y_i^x/\Sigma_Y$ and $Y_j^x/\Sigma_Y$, respectively. For unit I (unit J), in the transition from pre- to post-fiscal income, the share in total income thus changes from $X_i^x/\Sigma_X$ to $Y_i^x/\Sigma_Y$ (from $X_j^x/\Sigma_X$ to $Y_j^x/\Sigma_Y$).

*Axiom VSR. For units I and J, a fiscal system is vertically equitable if, in the transition from pre- to post-fiscal income, the difference between I's and J's shares in total income is reduced.* Axiom VSR is expressed by the following equation:

(10)
$$\frac{Y_i^x}{\Sigma_Y} - \frac{Y_j^x}{\Sigma_Y} < \frac{X_i^x}{\Sigma_X} - \frac{X_j^x}{\Sigma_X}.$$

To simplify equation (10), we first convert $\Sigma_X$ into $\Sigma_Y$ by using the property $\Sigma_X = \Sigma_Y/\eta$, and then we multiply all values by $\Sigma_Y$ and thereby obtain the following inequality:

(11) $$Y_i^x - Y_j^x < \eta(X_i^x - X_j^x).$$

Equation (11) represents another way of expressing axiom VSR. According to equation (11), the fiscal system is *vertically equitable* for units I and J, if $Y_i^x - Y_j^x < \eta(X_i^x - X_j^x)$. Conversely, if $Y_i^x - Y_j^x > \eta(X_i^x - X_j^x)$, the fiscal system is *vertically inequitable*. When $Y_i^x - Y_j^x = \eta(X_i^x - X_j^x)$, the fiscal system is *vertically neutral* for units I and J. The concept of vertical neutrality is here introduced to theoretically delineate the borderline between vertically equitable and inequitable fiscal systems. A vertically neutral system is neither equitable nor inequitable.

*Axiom VSA. For units I and J, a fiscal system is vertically equitable if, in the transition from pre- to post-fiscal income, the difference between I's and J's incomes is reduced.* Axiom VSA is expressed by the following equation:

(12) $$Y_i^x - Y_j^x < X_i^x - X_j^x.$$

According to equation (12), the fiscal system is *vertically equitable* for units I and J, if $Y_i^x - Y_j^x < X_i^x - X_j^x$. Conversely, if $Y_i^x - Y_j^x > X_i^x - X_j^x$ ($Y_i^x - Y_j^x = X_i^x - X_j^x$), the fiscal system is *vertically inequitable* (*vertically neutral*) for units I and J.

What is the difference between axioms VSR and VSA? Both axioms focus on reductions in the difference between I's and J's incomes. However, while VSR concerns the *relative* differences (expressed as shares of total income), VSA concerns the *absolute* difference in the incomes of units I and J.

Equations (11) and (12) indicate that the difference between axioms VSR and VSA is manifested by use of the term $\eta$. For example, suppose that $\Sigma_X = 10{,}000$, $\Sigma_Y = 5000$ (consequently, $\eta = 0.5$), $X_i^x = 100$, $X_j^x = 80$, $Y_i^x = 96$ and $Y_i^x = 84$. The system is clearly vertically equitable for VSA, as the absolute difference in incomes between I and J falls from 20 to 12. However, inequality (11) is not satisfied, and according to VSR, the system is vertically inequitable because the pre-fiscal (post-fiscal) income share of unit I is 0.20 percent (0.24 percent) higher than unit J's share. Thus, the difference between I's and J's shares in total income increases in the transition from pre- to post-fiscal income.

In conclusion, VSR focuses on the relative shares of income units in total income, whereas VSA addresses absolute differences in income. Therefore, we can say that VSR belongs to the "*relative* differences framework" (henceforth RDF), whereas VSA shapes the "*absolute* differences framework" (henceforth ADF). The comparison of equations (11) and (12) shows that when $\eta = 1$, the two frameworks (axioms) for judging vertical equity lead to identical characterizations.

*Axiom HS. For units I and J, a fiscal system is horizontally equitable if, in the transition from pre- to post-fiscal income, the unit I remains higher ranked than unit J on the income ladder.* Axiom HS is expressed as follows:

(13) $$Y_i^x - Y_j^x \geq 0.$$

Axiom HS embeds the "no reranking" principle of horizontal equity. If $Y_i^x \geq Y_j^x$, then there is no reranking of income units and the fiscal system is *horizontally equitable*. If $Y_i^x < Y_j^x$, then income unit I, which ranks higher than unit J on the pre-fiscal income ladder, ranks lower than unit J on the post-fiscal income ladder. In that case, a violation of horizontal equity principles has occurred in the form of reranking; the fiscal system is *horizontally inequitable* for units I and J.

Taking axioms VSR and HS together, as expressed by equations (11) and (12), respectively, we obtain the following relationship, which shapes the vertically and horizontally equitable fiscal system for units I and J in the RDF:

(14) $$\eta(X_i^x - X_j^x) > Y_i^x - Y_j^x \geq 0.$$

A similar expression is obtained for ADF, as follows:

(15) $$X_i^x - X_j^x > Y_i^x - Y_j^x \geq 0.$$

Axioms VSR, VSA, and HS are derived for income units I and J, whose pre-fiscal incomes are $X_i^x$ and $X_j^x$, respectively, such that $X_i^x > X_j^x$. In a special case, where the pre-fiscal incomes of units K and L are exactly equal, i.e. $X_k^x = X_l^x$, the axioms require amendments, which are presented in Appendix 1.

### 2.4. *Equity of Taxes and Benefits*

Once we have determined whether the fiscal system is equitable or inequitable, we consider the additional issue of whether its particular elements—taxes and benefits—are equitable. As observed below, the impact of one tax (benefit) on the vertical equity of the overall fiscal system can only be established in *relative* terms, that is, through comparison with a benchmark. This benchmark is a counterfactual vertically and horizontally equitable fiscal system (henceforth CEFS) in which each tax (benefit) is equal-yield to its counterpart actual tax (benefit). Furthermore, each tax (benefit) in CEFS is distributed according to a pattern determined by the analyst's normative view.

Urban (2014) describes two standard normative views of the roles of taxes and benefits in achieving vertical equity in a fiscal system. Under the "prevalent view," the benchmark is the CEFS in which each tax (benefit) is *proportional to pre-fiscal income* (Lambert, 2001). In the "alternative view," the benchmark is the CEFS in which each tax (benefit) is *equal in amount for all income units*.

The tax, benefit, and post-fiscal income given by the CEFS under the prevalent view are, respectively, equal to:

$$\dot{T}_i^x = \tau X_i^x; \quad \dot{B}_i^x = \beta X_i^x; \quad \dot{Y}_i^x = \eta X_i^x. \tag{16}$$

The CEFS under the alternative view is described by the following equations:

$$\ddot{T}_i^x = \tau \mu_X; \quad \ddot{B}_i^x = \beta \mu_X;. \quad \ddot{Y}_i^x = \eta \mu_X. \tag{17}$$

We can easily check that the averages of the counterfactual taxes and benefits are equal to their actual counterparts, that is, $\dot{\tau} = \ddot{\tau} = \tau$, and $\dot{\beta} = \ddot{\beta} = \beta$. Equations (16) state that, given the first CEFS, the tax (benefit; post-fiscal income) is proportional to pre-fiscal income. As observed below, the prevalent view is compatible with the RDF. Therefore, the system shown by equations (16) is called the "relative CEFS" (henceforth RCEFS). The RCEFS is quite different from the alternative view CEFS, defined by equations (17), in which all income units have equal *absolute* amounts of tax and benefit. Because of its compatibility with the ADF, the system presented by equations (17) is called the "absolute CEFS" (henceforth ACEFS).

### 2.5. *Axioms of Vertical and Horizontal Equity of Taxes and Benefits for the Prevalent View*

The deviations of actual tax, benefit, post-fiscal and pre-fiscal income from those of the RCEFS, as presented in equation (16), are, respectively:

$$\begin{aligned} \widehat{T}_i^x &= T_i^x - \dot{T}_i^x = T_i^x - \tau X_i^x; \\ \widehat{B}_i^x &= B_i^x - \dot{B}_i^x = B_i^x - \beta X_i^x; \\ \widehat{Y}_i^x &= Y_i^x - \dot{Y}_i^x = Y_i^x - \eta X_i^x = X_i^x - \widehat{T}_i^x + \widehat{B}_i^x; \\ \widehat{X}_i^x &= X_i^x - 1 \cdot X_i^x = 0. \end{aligned} \tag{18}$$

The terms $\widehat{T}_i^x$, $\widehat{B}_i^x$ and $\widehat{Y}_i^x$ tell us how far are the actual values of the tax, benefit and post-fiscal income from their RCEFS values. Their signs indicate the direction of the deviation from RCEFS. For example, $\widehat{T}_i^x < 0$ implies that unit I pays less tax under the actual system than under the proportional system.

Using the properties $\widehat{Y}_i^x = Y_i^x - \eta X_i^x$ and $Y_i^x = \widehat{Y}_i^x + \eta X_i^x$ from equation (18), equations (11) and (13) can be rewritten as follows:

$$\widehat{Y}_j^x - \widehat{Y}_i^x > 0; \tag{19}$$

(20) $$\widehat{Y}_j^x - \widehat{Y}_i^x \leq \eta(X_i^x - X_j^x).$$

Combining equations (19) and (20), which present new ways of expressing axioms VSR and HS, respectively, we determine the boundaries of the vertically and horizontally equitable fiscal system for units I and J, previously defined in equation (14), as follows:

(21) $$\eta(X_i^x - X_j^x) \geq \widehat{Y}_j^x - \widehat{Y}_i^x > 0.$$

As shown in Appendix 2, $\widehat{Y}_j^x - \widehat{Y}_i^x$ is equal to the following:

(22) $$\widehat{Y}_j^x - \widehat{Y}_i^x = (\widehat{T}_i^x - \widehat{T}_j^x) + (\widehat{B}_j^x - \widehat{B}_i^x).$$

The axioms VSR and HS can now be rewritten, respectively, as follows:

(23) $$(\widehat{T}_i^x - \widehat{T}_j^x) + (\widehat{B}_j^x - \widehat{B}_i^x) > 0;$$

(24) $$(\widehat{T}_i^x - \widehat{T}_j^x) + (\widehat{B}_j^x - \widehat{B}_i^x) \leq \eta(X_i^x - X_j^x).$$

Recall equation (16), which says that in the RCEFS, the tax equals $\dot{T}_i^x = \tau X_i^x$, and the benefit equals $\dot{B}_i^x = \beta X_i^x$. Therefore, in the RCEFS, we have $\widehat{T}_i^x = \widehat{T}_j^x = \widehat{B}_i^x = \widehat{B}_j^x = 0$ and $(\widehat{T}_i^x - \widehat{T}_j^x) + (\widehat{B}_j^x - \widehat{B}_i^x) = 0$. By substituting these values into equations (23) and (24), we observe that the RCEFS is vertically neutral and horizontally equitable in the RDF.

Based on axioms VSR and HS, as presented in equations (23) and (24), respectively, we can derive further axioms of vertical and horizontal equity, for the tax and the benefit. In deriving these axioms for the *tax* (the benefit), we assume that the *benefit* (the tax) belongs to the RCEFS.

Assuming that the benefit belongs to the RCEFS, we have that $\widehat{B}_j^x - \widehat{B}_i^x = 0$. For equations (23) and (24) to continue to hold, we must have:

(25) $$\widehat{T}_i^x - \widehat{T}_j^x > 0;$$

(26) $$\widehat{T}_i^x - \widehat{T}_j^x \leq \eta(X_i^x - X_j^x).$$

Similarly, assuming that the tax belongs to the RCEFS, we have $\widehat{T}_i^x - \widehat{T}_j^x = 0$. For equations (23) and (24) to continue to hold, we must have:

(27) $$\widehat{B}_j^x - \widehat{B}_i^x > 0;$$

(28) $$\hat{B}_j^x - \hat{B}_i^x \le \eta(X_i^x - X_j^x).$$

Equations (25) through (28) represent four additional axioms in the RDF. Equation (25) represents axiom VTR of the vertical equity of the tax. Equation (27) shows axiom VBR of the vertical equity of the benefit. Furthermore, equations (26) and (28) express axioms HTR and HBR, respectively, with regard to the horizontal equity of the tax and the benefit, respectively.

Axioms VTR and VBR from equations (25) and (27) can be rewritten as follows, respectively:

(29) $$\frac{T_i^x - T_j^x}{X_i^x - X_j^x} > \tau;$$

(30) $$\frac{B_j^x - B_i^x}{X_i^x - X_j^x} > \beta.$$

The left-hand side of equation (29) describes the rate, or "speed" of a tax increase in pre-fiscal income. For units I and J, the tax is vertically equitable when this rate is higher than the average rate, $\tau$. Similarly, but conversely, the benefit is vertically equitable when the rate of its increase in pre-fiscal income is lower than $\beta$, as shown by equation (30). The definition of vertical equity of a tax from equation (29) differs from the standard concept of "tax progressivity," according to which, the ratio between the tax and pre-fiscal income $(T_i^x / X_i^x)$ must be increasing. For a comparison of VTR and Kakwani and Lambert's (1998) axiom 2, which embodies the standard tax progressivity concept, see the discussion in Appendix 4.

Furthermore, axioms HTR and HBR from equations (26) and (28), respectively, can be rewritten as follows:

(31) $$T_i^x - T_j^x \le (\eta + \tau)(X_i^x - X_j^x);$$

(32) $$B_j^x - B_i^x \le (\eta - \beta)(X_i^x - X_j^x).$$

### 2.6. *Axioms of Vertical and Horizontal Equity of Taxes and Benefits for the Alternative View*

The axioms for the alternative view are derived in Appendix 3, and are expressed by the following equations:

(33) $$T_i^x - T_j^x > 0;$$

(34) $$B_j^x - B_i^x > 0;$$

(35) $$T_i^x - T_j^x \le X_i^x - X_j^x;$$

(36) $$B_j^x - B_i^x \le X_i^x - X_j^x.$$

Equation (33) represents axiom VTA of the vertical equity of the tax. Equation (34) shows axiom VBA of the vertical equity of the benefit. Furthermore, equations (35) and (36) express axioms HTA and HBA, with regard to the horizontal equity of the tax and the benefit, respectively. For units I and J, the tax is vertically equitable when $T_i^x > T_j^x$. Similarly, but conversely, the benefit is vertically equitable if $B_i^x < B_j^x$.

### 2.7. *Discussion of the New Axioms*

The equitable fiscal systems described by equations (14) and (15) both accord with the classical Pigou–Dalton principle of transfers, which states that a money transfer from the richer to the poorer income unit, where the transfer is sufficiently small that the ranks of two income units do not change, is inequality-reducing (welfare-enhancing). Suppose that in the first step of the redistribution process, €$h$ is taken from unit I and given to unit J. Unit I has $Y_i^x = X_i^x - h$ and unit J has $Y_j^x = X_j^x + h$. Consequently, $\Sigma_Y = \Sigma_X$, and $\eta = 1$. Clearly, the transfer satisfies axioms VSR and VSA, as $Y_i^x - Y_j^x = X_i^x - X_j^x - 2h < X_i^x - X_j^x$. Furthermore, if $h \le (X_i^x - X_j^x)/2$, axiom HS is also satisfied, as $Y_i^x - Y_j^x = X_i^x - X_i^x - 2h > 0$.

Axioms VSR, VSA, and HS are also compatible with the concepts of "minimal progressiveness" and "incentive preservation" proposed by Fei (1981). According to Fei (1981), in the framework of a "balanced budget" ($\tau = \beta \Rightarrow \eta = 1$), a "fiscal program" is "minimally progressive" for units I and J if $U_i^x > U_j^x$. From equation (6) the following relationship is derived:

(37) $$U_i^x - U_j^x = (X_i^x - Y_i^x) - (X_j^x - Y_j^x) = (X_i^x - X_j^x) - (Y_i^x - Y_j^x).$$

From equation (37) it follows that axioms VSR and VSA from equations (11) and (12), respectively, are satisfied if $U_i^x - U_j^x > 0$, which is identical to Fei's (1981) condition of minimal progressivity. Additionally, according to Fei (1981), a fiscal program is incentive preserving for income units I and J if $Y_i^x > Y_j^x$, which is in line with axiom HS. A fiscal program that is both minimally progressive and incentive preserving is "rational," according to Fei (1981). Hence, the fiscal systems described by equations (14) and (15) can be characterized as "rational," assuming $\eta = 1$.

Thus, according to the concept of minimal progressivity, a sufficient condition for vertical equity is a monotonic increase in net tax, that is, $U_i^x - U_j^x > 0$. We are tempted to conjecture that a sufficient condition for a tax to be vertically equitable is that it also be monotonically increasing. Indeed, $T_i^x - T_j^x > 0$ is the condition set by axiom VTA in equation (33). However, axiom VTR in equation (29) is much more demanding than Fei's rule because, for a tax to be vertically equitable, $T_i^x - T_j^x$ must exceed $\tau(X_i^x - X_j^x)$. We arrive at a somewhat paradoxical situation that gives rise to several questions: Why is VTR so strict compared with Fei's concept? Are VTR and Fei's concepts compatible? Is Fei's concept of minimal progressivity still applicable when we analyze single tax and benefit instruments?

At the outset, it is crucial to remember that Fei's concept of minimal progressivity works within the framework of a closed fiscal system that consists of both taxes and benefits and where a balanced budget is assumed. Imagine that the money collected by $T_i^x$ is distributed back to income units by means of a counterfactual equal-yield and vertically neutral benefit. In the relative (absolute) differences framework, this benefit is defined by RCEFS (ACEFS) and is equal to $B_i^{R;x} = \tau X_i^x$ $(B_i^{A;x} = \tau\mu_X)$. This is the major point: by choosing the normative framework, we set a benchmark distribution for evaluation of $T_i^x$. The benchmark in RCEFS is *more rigorous* than that in ACEFS.

We focus on the fiscal subsystem comprising only the actual tax and the counterfactual benefit in RCEFS. The net tax is obtained as follows:

$$(38)\qquad U_i^{R;x} = T_i^x - B_i^{R;x} = (\widehat{T}_i^x + \tau X_i^x) - \tau X_i^x = \widehat{T}_i^x.$$

As mentioned above, a sufficient condition for a vertically equitable tax system is $U_i^x - U_j^x > 0$. From equation (38), in the RDF, we have that:

$$(39)\qquad U_i^{R;x} - U_j^{R;x} = \widehat{T}_i^x - \widehat{T}_j^x > 0.$$

Thus, Fei's (1981) condition of minimal progressivity of a net tax in the RDF is $\widehat{T}_i^x - \widehat{T}_j^x > 0$, which is identical to axiom VTR. Therefore, VTR is compatible with the concept of minimal progressivity. The strong condition $(\widehat{T}_i^x - \widehat{T}_j^x > 0)$ is implied by the benchmark used in the RDF, which requires that for a tax to be vertically neutral, it must be proportional to pre-fiscal income. Appendix 6 provides a numerical example to guide this discussion.

### 2.8. *Income Supremacy Change, Income Distance Change, and Deprivation from Reranking*

In the remainder of the paper, we work only in RDF. However, all of the concepts and decompositions presented below could be analogously derived in ADF. The "pre-fiscal income supremacy" of unit I over unit J is equal to $(X_i^x - X_j^x)/\Sigma_X$ and measures unit I's contentment with having a larger share of pre-fiscal income than unit J. The "pre-fiscal income distance" between units I and J is equal to $|X_i^x - X_j^x|/\Sigma_X$ and is identical to $(X_i^x - X_j^x)/\Sigma_X$ because $X_i^x > X_j^x$.

The "post-fiscal income distance" between units I and J is defined as $|Y_i^x - Y_j^x|/\Sigma_Y$. The "post-fiscal income supremacy" between units I and J is equal to $(Y_i^x - Y_j^x)/\Sigma_Y$, and can be either positive or negative. If $Y_i^x - Y_j^x > 0$, unit I enjoys pleasure from having a larger share of post-fiscal income than unit J. However, if $Y_i^x - Y_j^x < 0$, unit I feels "negative pleasure," or dissatisfaction, from having a lower share of post-fiscal income than unit J, which ranks below unit I on the pre-fiscal income ladder. The situation in which $Y_i^x - Y_j^x < 0$ has previously been denoted as a violation of the horizontal equity principle that is defined by axiom HS. To measure the violation of the "no reranking" horizontal equity principle, the "deprivation from reranking" is defined as:

$$\rho_{i,j}^x = \frac{|Y_i^x - Y_j^x| - (Y_i^x - Y_j^x)}{\Sigma_Y}. \tag{40}$$

When $Y_i^x - Y_j^x > 0$ (no reranking), $\rho_{i,j}^x = 0$. However, when $Y_i^x - Y_j^x < 0$ (reranking occurs), $\rho_{i,j}^x = 2(Y_j^x - Y_i^x)/\Sigma_Y$.

The "income supremacy change" is the difference between pre-fiscal and post-fiscal income supremacy:

$$\sigma_{i,j}^x = \frac{X_i^x - X_j^x}{\Sigma_X} - \frac{Y_i^x - Y_j^x}{\Sigma_Y} = \frac{\eta(X_i^x - X_j^x) - (Y_i^x - Y_j^x)}{\Sigma_Y}; \tag{41}$$

$$\sigma_{i,j}^x = \frac{\widehat{Y}_j^x - \widehat{Y}_i^x}{\Sigma_Y}. \tag{42}$$

$\sigma_{i,j}^x$ can be positive or negative. When $\sigma_{i,j}^x > 0$, it indicates that unit I's supremacy has decreased in the transition from pre- to post-fiscal income. Comparing equations (42) and (11), we conclude that $\sigma_{i,j}^x > 0$ satisfies axiom VSR. Conversely, $\sigma_{i,j}^x < 0$ indicates that I's supremacy has increased, which indicates a violation of axiom VSR.

Thus, $\sigma_{i,j}^x$ properly signals violations of the vertical equity principle, but is insensitive to violations of the horizontal equity principle. Indeed, reranking (when $Y_i^x - Y_j^x < 0$) inflates $\sigma_{i,j}^x$, which thereby indicates enhanced satisfaction of axiom VSR, whereas axiom HS is simultaneously violated. Therefore, we introduce a somewhat different measure, the "income distance change," which is defined as follows:

$$\delta_{i,j}^x = \sigma_{i,j}^x - \rho_{i,j}^x = \frac{\eta(X_i^x - X_j^x) - (Y_i^x - Y_j^x)}{\Sigma_Y} - \frac{|Y_i^x - Y_j^x| - (Y_i^x - Y_j^x)}{\Sigma_Y}; \tag{43}$$

$$\delta_{i,j}^x = \frac{\eta(X_i^x - X_j^x) - |Y_i^x - Y_j^x|}{\Sigma_Y}.$$

As the difference between $\sigma_{i,j}^x$ and $\rho_{i,j}^x$, the measure $\delta_{i,j}^x$ is constructed in a manner that accounts for the satisfaction of axiom VSR, but cancels the deprivation caused by the violation of axiom HS. It rewards what is desired, that is, the decrease of the difference between unit I's and unit J's respective shares of total income, but penalizes the excessive part of that decrease, which leads to reranking. Appendix 8 provides a numerical illustration of the concepts derived in this section.

## 3. Decompositions of Vertical and Horizontal Effects

### 3.1. *Contributions of Taxes and Benefits to Income Supremacy Change*

Equation (22) suggests that income supremacy change, $\sigma_{i,j}^{x}$, from equation (42) can be decomposed into contributions of the tax, $(\widehat{T}_i^x - \widehat{T}_j^x)/\Sigma_Y$, and the benefit, $(\widehat{B}_j^x - \widehat{B}_i^x)/\Sigma_Y$. Furthermore, equation (25) states that the tax is vertically equitable if $\widehat{T}_i^x - \widehat{T}_j^x > 0$. Similarly, equation (27) indicates that benefit is vertically equitable if $\widehat{B}_j^x - \widehat{B}_i^x > 0$. Therefore, $(\widehat{T}_i^x - \widehat{T}_j^x)/\Sigma_Y$ and $(\widehat{B}_j^x - \widehat{B}_i^x)/\Sigma_Y$ naturally explain how a fiscal system satisfies the principle of vertical equity in RDF.

For units I and J, the income supremacy change induced by the tax and the benefit is equal to $\sigma_{i,j}^{x} = [(\widehat{T}_i^x - \widehat{T}_j^x) + (\widehat{B}_j^x - \widehat{B}_i^x)]/\Sigma_Y$. The contributions of the tax and the benefit to $\sigma_{i,j}^{x}$ can be expressed, respectively, simply as:

(44)
$$\sigma_{i,j}^{T} = \frac{\widehat{T}_i^x - \widehat{T}_j^x}{\Sigma_Y};$$
$$\sigma_{i,j}^{B} = \frac{\widehat{B}_j^x - \widehat{B}_i^x}{\Sigma_Y}.$$

Summing up $\sigma_{i,j}^{x}$, $\sigma_{i,j}^{T}$, and $\sigma_{i,j}^{B}$ for all pairs of units $\{i, j\}$ such that $i > j$, and dividing by $N$, we obtain:

(45)
$$N^{-1}\sum_{j=1}^{n}\sum_{i=j+1}^{n}\varphi_i^x\varphi_j^x\sigma_{i,j}^{x} = N^{-1}\sum_{j=1}^{n}\sum_{i=j+1}^{n}\varphi_i^x\varphi_j^x\sigma_{i,j}^{T} + N^{-1}\sum_{j=1}^{n}\sum_{i=j+1}^{n}\varphi_i^x\varphi_j^x\sigma_{i,j}^{B};$$
$$V = V_T + V_B.$$

In equation (45), the term on the left-hand side is the average income supremacy change, and is identical to $V = G_X - D_{Y;x}$ from equation (7). We observed above that $\sigma_{i,j}^{x}$ measures both satisfaction of and violations of axiom VSR. Therefore, $V$, as the aggregate sum of all $\sigma_{i,j}^{x}$, is the measure of the *net aggregate satisfaction* of axiom VSR.[4] The terms on the right-hand side of equation (45), $V_T$ and $V_B$, are the contributions of the tax and the benefit to the average

[4]This is somewhat different from the approach of Monti *et al.* (2013), whose reranking indexes consider *only* the pairs of units $\{i, i > j\}$, for which $\sigma_{i,j}^{x} < 0$. The framework presented in this paper could be extended in such a way that $V$ is decomposed into two parts: a first part (call it $V^e$), that sums the values of $\sigma_{i,j}^{x}$ for all the pairs for which $\sigma_{i,j}^{x} \geq 0$ (i.e., when the vertical equity principle is not violated), and a second part (call it $V^n$), that sums the values of $\sigma_{i,j}^{x}$ for all pairs for which $\sigma_{i,j}^{x} < 0$ (i.e., when the vertical equity principle is violated). Consequently, $V^e \geq 0$, $V^n < 0$ and $V^e + V^p = V$. However, this matter is left for further research.

income supremacy change. These contributions fully coincide with the contributions obtained by Lambert (1985, 2001) from equation (8).

The *relative* contributions of the tax and the benefit to $\sigma^x_{i,j}$ can be expressed, respectively, as:

(46)
$$s^T_{i,j} = \frac{\widehat{T}^x_i - \widehat{T}^x_j}{\sigma^x_{i,j}\Sigma_Y};$$
$$s^B_{i,j} = \frac{\widehat{B}^x_j - \widehat{B}^x_i}{\sigma^x_{i,j}\Sigma_Y}.$$

Decomposition terms from equation (44) can be rewritten as:

(47)
$$\sigma^T_{i,j} = \sigma^x_{i,j}s^T_{i,j} = \sigma^x_{i,j}\frac{\widehat{T}^x_i - \widehat{T}^x_j}{(\widehat{T}^x_i - \widehat{T}^x_j) + (\widehat{B}^x_j - \widehat{B}^x_i)};$$
$$\sigma^B_{i,j} = \sigma^x_{i,j}s^B_{i,j} = \sigma^x_{i,j}\frac{\widehat{B}^x_j - \widehat{B}^x_i}{(\widehat{T}^x_i - \widehat{T}^x_j) + (\widehat{B}^x_j - \widehat{B}^x_i)}.$$

for all $\sigma^x_{i,j} \neq 0$ (see numerical illustration of these concepts in Appendix 9).

### 3.2. *Contributions of Taxes and Benefits to Deprivation from Reranking*

How can deprivation from reranking, $\rho^x_{i,j}$, from equation (40), be decomposed into contributions of taxes and benefits? We have observed that the natural decomposability of $\sigma^x_{i,j}$ arises from the fact that $X^x_i - Y^x_i = T^x_i - B^x_i$ for all *i*. Conversely, as we recall from equation (40), $\rho^x_{i,j}$ features only the values of $Y^x_i$. Nonetheless, we can again employ the terms $\widehat{T}^x_i - \widehat{T}^x_j$ and $\widehat{B}^x_j - \widehat{B}^x_i$. These elements reflect the roles of the tax and the benefit in removing a fiscal system from the horizontally equitable RCEFS, and are used to construct the axioms HTR and HBR for the horizontal equity of taxes and benefits, in equations (26) and (28).

Accordingly, for pairs of income units $\{i, j\}$ for which reranking occurs, that is, $Y^x_i < Y^x_j$, we calculate the contributions to $\rho^x_{i,j}$ with the following procedure:

(48)
$$\rho^T_{i,j} = \rho^x_{i,j}s^T_{i,j} = \rho^x_{i,j}\frac{\widehat{T}^x_i - \widehat{T}^x_j}{(\widehat{T}^x_i - \widehat{T}^x_j) + (\widehat{B}^x_j - \widehat{B}^x_i)};$$
$$\rho^B_{i,j} = \rho^x_{i,j}s^B_{i,j} = \rho^x_{i,j}\frac{\widehat{B}^x_j - \widehat{B}^x_i}{(\widehat{T}^x_i - \widehat{T}^x_j) + (\widehat{B}^x_j - \widehat{B}^x_i)},$$

for all $\sigma^x_{i,j} \neq 0$, where $s^T_{i,j}$ and $s^B_{i,j}$ are the relative contribution terms defined in equation (46). The two equations in (48) are undefined if $\sigma^x_{i,j} = 0$. Additionally, if $\sigma^x_{i,j}$ is close to zero, biases in the calculation of contributions to deprivation from reranking may occur. Appendix 5 explains how to technically solve these issues in empirical research.

Analogously to equation (45), summing up $\rho^x_{i,j}$, $\rho^T_{i,j}$ and $\rho^B_{i,j}$ for all pairs of units $\{i, j\}$ such that $i > j$, and dividing by $N$, we obtain:

(49)
$$N^{-1}\sum_{j=1}^{n}\sum_{i=j+1}^{n}\varphi^x_i\varphi^x_j\rho^x_{i,j} = N^{-1}\sum_{j=1}^{n}\sum_{i=j+1}^{n}\varphi^x_i\varphi^x_j\rho^T_{i,j} + N^{-1}\sum_{j=1}^{n}\sum_{i=j+1}^{n}\varphi^x_i\varphi^x_j\rho^B_{i,j};$$
$$H = H_T + H_B.$$

The term on the left-hand side of equation (49) is the average deprivation from reranking and is identical to $H$ from equation (7). Because $\rho^x_{i,j}$ measures violations of axiom HS, $H$ is the measure of the average violation of the horizontal equity principle. The terms on the right-hand side of equation (49), $H_T$ and $H_B$, are the contributions of the tax and the benefit to average deprivation from reranking, respectively.

### 3.3. *Contributions of Taxes and Benefits to Income Distance Change*

Equation (43) states that $\delta^x_{i,j} = \sigma^x_{i,j} - \rho^x_{i,j}$, which means that we can decompose $\delta^x_{i,j}$ into contributions of taxes and benefits with equations (47) and (75) (see Appendix), as follows:

(50)
$$\delta^x_{i,j} = \delta^T_{i,j} + \delta^B_{i,j} = (\sigma^T_{i,j} - \rho^T_{i,j}) + (\sigma^B_{i,j} - \rho^B_{i,j}).$$

Analogously to equations (45) and (49), by summing up $\delta^x_{i,j}$, $\delta^T_{i,j}$ and $\delta^B_{i,j}$ for all pairs of units $\{i, j\}$ such that $i > j$, and dividing by $N$, we obtain:

(51)
$$N^{-1}\sum_{j=1}^{n}\sum_{i=j+1}^{n}\varphi^x_i\varphi^x_j\delta^x_{i,j} = N^{-1}\sum_{j=1}^{n}\sum_{i=j+1}^{n}\varphi^x_i\varphi^x_j\delta^T_{i,j} + N^{-1}\sum_{j=1}^{n}\sum_{i=j+1}^{n}\varphi^x_i\varphi^x_j\delta^B_{i,j};$$
$$P = P_T + P_B.$$

The term on the left-hand side of equation (51) is the average income distance change, which fully corresponds to $P$ from equation (7). The terms on the right-hand side of equation (51), $P_T$ and $P_B$, are the contributions of the tax and the benefit to the average income distance change, respectively.

### 3.4. *Decomposition by Groups*

Having decomposed the redistributive effect ($P$) first into vertical ($V$) and horizontal ($H$) effects and then into contributions of taxes and benefits, in equations (45), (49), and (51), in this section, we present the final methodical endeavor of this research—the decomposition of the aforementioned indices into parts that represent the contributions within and across groups of income units. As explained earlier, income units from a pair $\{i, j\}$ belong to groups $\Theta^x_i$ and $\Theta^x_j$, respectively. We now define the following indicator function for pre-fiscal income ordering:

(52)
$$I^x_{i,j,k,l} = \begin{cases} 1 & \text{if} \quad \Theta^x_i = k \quad \text{and} \quad \Theta^x_j = l \\ 0 & \text{otherwise} \end{cases},$$

where $i = 1, \ldots, n$, $j = 1, \ldots, n$, $k = 1, \ldots, \Psi$ and $l = 1, \ldots, \Psi$.[5]

$G_X$, $D_{Y;x}$ and $G_Y$ from equation (9) can now be decomposed into the parts associated with different groups of income units, as follows:

(53)
$$G^{k,l}_X = (N\Sigma_X)^{-1}\sum_{j=1}^{n}\sum_{i=j+1}^{n} I^x_{i,j,k,l}\varphi^x_i\varphi^x_j(X^x_i - X^x_j);$$
$$D^{k,l}_{Y;x} = (N\Sigma_Y)^{-1}\sum_{j=1}^{n}\sum_{i=j+1}^{n} I^x_{i,j,k,l}\varphi^x_i\varphi^x_j(Y^x_i - Y^x_j);$$
$$G^{k,l}_Y = (N\Sigma_Y)^{-1}\sum_{j=1}^{n}\sum_{i=j+1}^{n} I^x_{i,j,k,l}\varphi^x_i\varphi^x_j\left|Y^x_i - Y^x_j\right|.$$

In equation (53), the component $G^{k,l}_X$ represents the part of average income distance ($G_X$) obtained only for pairs of units in which the pre-fiscally dominant unit (the unit with rank $i$, i.e., unit I) belongs to group $k$, whereas the pre-fiscally inferior unit (the unit with rank $j$, i.e., unit J) belongs to group $l$. When $k = l$, the contribution to $G_X$ is "within group," whereas, when $k \neq l$, the contribution to $G_X$ is "across groups." The same principle applies analogously to all of the terms that contain "$k$, $l$" in their superscripts.

$G_X$, $D_{Y;x}$ and $G_Y$ are sums of their components, that is, $G_X = \sum_k^{\Psi}\sum_l^{\Psi} G^{k,l}_X$, $D_{Y;x} = \sum_k^{\Psi}\sum_l^{\Psi} D^{k,l}_{Y;x}$ and $G_Y = \sum_k^{\Psi}\sum_l^{\Psi} G^{k,l}_Y$. Kakwani's (1984) decomposition of $P$ into $V$ and $H$ from equation (7), is decomposed further within and across groups, by combining the elements from equation (53), in the following manner:

(54)
$$\sum_k^{\Psi}\sum_l^{\Psi}(G^{k,l}_X - G^{k,l}_Y) = \sum_k^{\Psi}\sum_l^{\Psi}(G^{k,l}_X - D^{k,l}_{Y;x}) - \sum_k^{\Psi}\sum_l^{\Psi}(G^{k,l}_Y - D^{k,l}_{Y;x});$$
$$\sum_k^{\Psi}\sum_l^{\Psi} P^{k,l} = \sum_k^{\Psi}\sum_l^{\Psi} V^{k,l} - \sum_k^{\Psi}\sum_l^{\Psi} H^{k,l}.$$

where $P^{k,l} = G^{k,l}_X - G^{k,l}_Y$, $V^{k,l} = G^{k,l}_X - D^{k,l}_{Y;x}$ and $H^{k,l} = G^{k,l}_Y - D^{k,l}_{Y;x}$. $P^{k,l}$ ($V^{k,l}$; $H^{k,l}$) represents the contribution to the overall distance change (income supremacy change; deprivation from reranking) of the pairs of units $\{i, j\}$ in which unit $i$ belongs to group $k$, while unit $j < i$ belongs to group $l$.

Similarly, we adapt decompositions (45), (49) and (51), which become:

(55)
$$N^{-1}\sum_{j,i} h^x_{i,j,k,l}\sigma^x_{i,j} = N^{-1}\sum_{j,i} h^x_{i,j,k,l}\sigma^T_{i,j} + N^{-1}\sum_{j,i} h^x_{i,j,k,l}\sigma^B_{i,j};$$
$$V^{k,l} = V^{k,l}_T + V^{k,l}_B.$$

[5]For example, assume that $\Psi = 2$. We have two units, $i$ and $j$, that belong to groups 2 and 1, respectively (thus, $\Theta_i = 2$ and $\Theta_j = 1$). Therefore, the values of the indicator function will be equal to 1 for $I^x_{i,j,k=2,l=1}$ and equal to 0 for all other combinations, i.e., $I^x_{i,j,1,1}$, $I^x_{i,j,1,2}$ and $I^x_{i,j,2,2}$.

(56) $$N^{-1}\sum_{j,i} h^x_{i,j,k,l}\rho^x_{i,j} = N^{-1}\sum_{j,i} h^x_{i,j,k,l}\rho^T_{i,j} + N^{-1}\sum_{j,i} h^x_{i,j,k,l}\rho^B_{i,j};$$
$$H^{k,l} = H^{k,l}_T + H^{k,l}_B.$$

(57) $$N^{-1}\sum_{j,i} h^x_{i,j,k,l}\delta^x_{i,j} = N^{-1}\sum_{j,i} h^x_{i,j,k,l}\delta^T_{i,j} + N^{-1}\sum_{j,i} h^x_{i,j,k,l}\delta^B_{i,j};$$
$$P^{k,l} = P^{k,l}_T + P^{k,l}_B.$$

where $h^x_{i,j,k,l} = I^x_{i,j,k,l}\varphi^x_i\varphi^x_j$ and $\sum_{j,i}$ is the abbreviation for operators $\sum_{j=1}\sum_{i=j+1}(\cdot)$. In equations (55), (56), and (57), the vertical, horizontal, and redistributive effects, respectively, are decomposed in terms of both taxes and benefits, and within and across groups. The terms $V^{k,l}_T$, $H^{k,l}_T$ and $P^{k,l}_T$ ($V^{k,l}_B$, $H^{k,l}_B$ and $P^{k,l}_B$) represent the within- and across-group contributions of the tax (the benefit) to the income supremacy change ($V$), the deprivation from reranking ($H$), and the average distance change ($P$). Finally, we obtain the following equation:

(58) $$\sum_k^{\Psi}\sum_l^{\Psi}(P^{k,l}_T + P^{k,l}_B) = \sum_k^{\Psi}\sum_l^{\Psi}(V^{k,l}_T + V^{k,l}_B) - \sum_k^{\Psi}\sum_l^{\Psi}(H^{k,l}_T + H^{k,l}_B),$$

which represents the ultimate decomposition of the overall distance change ($P$), addressing all three "dimensions" mentioned in the Introduction—vertical and horizontal equity, taxes and benefits, and within and across groups.

Before turning to the application of the newly derived decompositions, an important note must be made concerning the interpretation of the within-group and across-group contribution terms presented in equations (53), (54), and (58). All of these contributions depend on the number of units in groups $k$ and $l$ (Ebert, 2010b). In empirical applications, the sizes of groups can vary significantly. Thus, for example, we may observe relatively small values of some index $M^{k,l}$, for pairs of groups $\{k = p,\ l \neq p\}$ and $\{k = p,\ l \neq p\}$. However, this may be observed simply because group $p$ is small relative to the other groups. To avoid misinterpreting the role of group $p$, Monti *et al.* (2013) "normalize" their contribution terms, that is, divide each term $M^{k,l}$ by the sum of all terms $M^{k,l}$ in which group $p$ is included. Such normalization was not applied in the following exercise, as we exclusively consider two household groups, the sizes of which are quite similar. However, if there are several groups involved, or in cross-country comparisons, a procedure should be applied to equalize the contribution terms.

## 4. Application

Appendix 10 applies the analytical tools developed above to an imaginary population of twelve units. In the following empirical exercise, we analyze the impact of one tax and one benefit within and across two income groups in Croatia in 2010. *Taxes* ($T^x_i$) include personal income tax (henceforth PIT), the surtax and the "crisis tax," whereas *benefits* ($B^x_i$) consist of basic social

TABLE 1
Summary Data for Empirical Population

| | Percentage of Equivalent Units | Mean Equivalent Pre-Fiscal Income (EUR) | Average Tax Rate | Average Benefit Rate | Average Post-Fiscal Income Rate |
|---|---|---|---|---|---|
| Group 1 | 48.2 | 7642 | 11.1 | 1.6 | 90.5 |
| Group 2 | 51.8 | 6025 | 8.9 | 5.5 | 96.6 |
| Overall | 100.0 | 6804 | 10.1 | 3.4 | 93.3 |

*Source*: Author's calculations.

assistance, child benefit, maternity and baby allowance, unemployment benefits, and sickness benefits.[6]

The research question is how taxes and benefits redistribute income within and across households without children and households with children. We use the 2011 EU-SILC data for Croatia. From the sample of 6,403 households, we exclude 2,128 households in which all members are older than 60. The remaining households are divided into two groups: (1) households without children, and (2) households with one or more children. A child is defined as any person up to the age of 18. Thus, $\Theta_i^x = 1$ for group 1, and $\Theta_i^x = 2$ for group 2. Household pre-fiscal incomes, taxes, benefits, and post-fiscal incomes are adjusted using the "modified OECD scale."

Table 1 shows basic information for the analyzed system. Group 1 has 48.2 percent of all equivalent units, compared to the somewhat larger group 2. Thus, the two groups represent relatively similar shares of the population. Group 1 has a significantly higher mean pre-fiscal income; precisely, it is 27 percent higher than group 2's mean. The average tax rate is relatively low, 10.1 percent, and is 2.2 percentage points higher for group 1 (11.1 percent) than for group 2 (8.9 percent). Although they pay taxes at a lower rate, the members of group 2 on average receive benefits at a rate of 5.5 percent, which is nearly 4 percentage points higher than the rate at which members of group 1 receive benefits, that is, 1.6 percent. As a combined result of taxes and benefits, average income falls by 9.5 percent for group 1 but only 3.4 percent for group 2, as shown in the last column of Table 1.

The fiscal subsystem has reduced income inequality from $G_X = 0.3631$ to $G_Y = 0.3065$, or by $P = 0.0566$, as shown in Table 2. Thus, the average income distance decreased by 15.6 percent. The fall of average income supremacy was $V = 0.0602$, which is higher than $P$ by the amount of $H = 0.0036$, which indicates that the excess loss of income supremacy equaled 1 percent of $G_X$.

A more detailed examination of inequality and the process of change is summarized in Table 3, which demonstrates that nearly one-half of pre-fiscal inequality originates *within* groups 1 and 2. Namely, $G_X^{1,1}$ ($G_X^{2,2}$) is equal to 26.1 percent (23.0 percent) of $G_X$. The other half of $G_X$ is attributable to across-group

[6]The PIT is progressive (in the standard sense), given the relatively high basic personal allowance and the 2010 schedule with four rates (15–45 percent). The tax base of the surtax is the amount of PIT, and the rates vary from 0 to 18 percent. The "crisis tax" is a type of PIT that existed during 2009 and 2010. The tax base is post-PIT income, and the rates are 2 and 4 percent. Basic social assistance and child benefit are means tested.

TABLE 2

Basic Results for the Empirical Population

| | Index | % $G_X$ | % $P$ |
|---|---|---|---|
| $G_X$ | 0.3631 | 100.0 | |
| $G_Y$ | 0.3065 | 84.4 | |
| $D_{Y;x}$ | 0.3029 | 83.4 | |
| $P$ | 0.0566 | 15.6 | 100.0 |
| $V$ | 0.0602 | 16.6 | 106.4 |
| $H$ | 0.0036 | 1.0 | 6.4 |

*Source*: Author's calculations.

TABLE 3

Decomposition Within and Across Groups for the Empirical Population

| | $G_X^{k,l}$ | | $G_X^{k,l}$ (% $G_X$) | |
|---|---|---|---|---|
| | $l=1$ | $l=2$ | $l=1$ | $l=2$ |
| $k=1$ | 0.0948 | 0.1227 | 26.1 | 33.8 |
| $k=2$ | 0.0621 | 0.0835 | 17.1 | 23.0 |

| | $G_Y^{k,l}$ | | $G_Y^{k,l}$ (% $G_Y$) | |
|---|---|---|---|---|
| | $l=1$ | $l=2$ | $l=1$ | $l=2$ |
| $k=1$ | 0.0798 | 0.1004 | 26.0 | 32.8 |
| $k=2$ | 0.0552 | 0.0710 | 18.0 | 23.2 |

| | $P^{k,l}$ | | $P^{k,l}$ (% $P$) | | $P^{k,l}$ (% $G_X^{k,l}$) | |
|---|---|---|---|---|---|---|
| | $l=1$ | $l=2$ | $l=1$ | $l=2$ | $l=1$ | $l=2$ |
| $k=1$ | 0.0150 | 0.0223 | 26.4 | 39.4 | 15.8 | 18.2 |
| $k=2$ | 0.0069 | 0.0125 | 12.1 | 22.1 | 11.0 | 15.0 |

| | $V^{k,l}$ | | $V^{k,l}$ (% $V$) | | $V^{k,l}$ (% $P^{k,l}$) | |
|---|---|---|---|---|---|---|
| | $l=1$ | $l=2$ | $l=1$ | $l=2$ | $l=1$ | $l=2$ |
| $k=1$ | 0.0155 | 0.0236 | 25.8 | 39.1 | 103.9 | 105.7 |
| $k=2$ | 0.0074 | 0.0137 | 12.3 | 22.8 | 108.2 | 109.8 |

| | $H^{k,l}$ | | $H^{k,l}$ (% $H$) | | $H^{k,l}$ (% $P^{k,l}$) | |
|---|---|---|---|---|---|---|
| | $l=1$ | $l=2$ | $l=1$ | $l=2$ | $l=1$ | $l=2$ |
| $k=1$ | 0.0006 | 0.0013 | 15.9 | 34.9 | 3.9 | 5.7 |
| $k=2$ | 0.0006 | 0.0012 | 15.4 | 33.8 | 8.2 | 9.8 |

*Source*: Author's calculations.

inequality. Thus, $G_X^{1,2}$ ($G_X^{2,1}$) indicates that 33.8 percent (17.1 percent) of the average pre-fiscal income distance derives from the income supremacy of group 1 (group 2) members over group 2 (group 1) members. This relationship between $G_X^{1,2}$'s and $G_X^{2,1}$'s shares in $G_X$ is in accordance with the results in Table 1, indicating that group 1 has a much greater average pre-fiscal income than group 2.

TABLE 4

Contributions of Taxes and Benefits for the Empirical Population

| | $P_T^{k,l}$ | | $P_B^{k,l}$ | | $P_T^{k,l}$ (% $P^{k,l}$) | |
|---|---|---|---|---|---|---|
| | $l=1$ | $l=2$ | $l=1$ | $l=2$ | $l=1$ | $l=2$ |
| $k=1$ | 0.0108 | 0.0119 | 0.0042 | 0.0104 | 72.2 | 53.4 |
| $k=2$ | 0.0061 | 0.0067 | 0.0008 | 0.0059 | 88.9 | 53.2 |
| | $V_T^{k,l}$ | | $V_B^{k,l}$ | | $V_T^{k,l}$ (% $V^{k,l}$) | |
| | $l=1$ | $l=2$ | $l=1$ | $l=2$ | $l=1$ | $l=2$ |
| $k=1$ | 0.0110 | 0.0120 | 0.0046 | 0.0115 | 70.5 | 51.1 |
| $k=2$ | 0.0062 | 0.0068 | 0.0012 | 0.0069 | 84.2 | 49.5 |
| | $H_T^{k,l}$ | | $H_B^{k,l}$ | | $H_T^{k,l}$ (% $H^{k,l}$) | |
| | $l=1$ | $l=2$ | $l=1$ | $l=2$ | $l=1$ | $l=2$ |
| $k=1$ | 0.0002 | 0.0001 | 0.0004 | 0.0011 | 26.0 | 9.7 |
| $k=2$ | 0.0002 | 0.0001 | 0.0004 | 0.0011 | 27.0 | 11.2 |

*Source*: Author's calculations.

Income distance is decreased for all within- and across-group combinations, as shown by a comparison of $G_Y^{k,l}$ and $G_X^{k,l}$. This decrease was essentially evenly distributed within- and across-groups (compare the values $P^{k,l}$ as percentages of $P$ with the corresponding values $G_X^{k,l}$ as percentages of $G_X$). Therefore, the composition of income inequality changed only slightly (compare the values $G_Y^{k,l}$ as percentages of $G_Y$ with the corresponding values $G_X^{k,l}$ as percentages of $G_X$).

Group 2 accounts for 7.6 percent more income units than group 1 (see Table 1), but the horizontal inequity within group 2 is twice higher than within group 1 (compare $H^{2,2}$ with $H^{1,1}$ in Table 3). When the values $H^{k,l}$ are compared to the corresponding $P^{k,l}$ terms, we observe that horizontal inequity is more pronounced between group 2 pre-fiscal income superiors and group 1 pre-fiscal inferiors ($H^{2,1}$ represents 8.2 percent of $P^{2,1}$), than between group 1 pre-fiscal income superiors and group 2 pre-fiscal income inferiors (the share of $H^{1,2}$ in $P^{1,2}$ is 5.7 percent). Two thirds of excess deprivation from reranking ($H$) emerges in the processes in which group 2 pre-fiscal income inferiors outrank: (a) the pre-fiscal income superiors from their own group (as shown by $H^{2,2}$), and (b) the group 1 pre-fiscal superiors (as shown by $H^{1,2}$).

The terms $H_T^{k,l}$ and $H_B^{k,l}$ in Table 4 demonstrate that benefits have much greater influence on the creation of horizontal inequity than taxes. The role of taxes in the creation of $H$ is comparatively strong when pre-fiscal income superiors from groups 1 and 2 are outranked by units from group 1 (as shown by $H_T^{1,1}$ and $H_T^{2,1}$). By contrast, the role of benefits is decisive when pre-fiscal income superiors from groups 1 and 2 are *outranked* by pre-fiscal income inferiors from group 2 (as shown by $H_B^{1,2}$ and $H_B^{2,2}$).

$V_T^{1,1}$ and $V_T^{2,1}$ represent more than two-thirds of $V^{1,1}$ and $V^{2,1}$, respectively, whereas the shares of taxes in $V^{1,2}$ and $V^{2,2}$ (represented by $V_T^{1,2}$ and $V_T^{2,2}$), respectively, are similar to the shares of benefits (Table 4). Thus, taxes are more influential than benefits in decreasing the average income supremacy between group 1 and group 2 pre-fiscal income superiors and group 1 pre-fiscal income inferiors. However, benefits are equally important as taxes in reducing the average income supremacy between group 1 and group 2 pre-fiscal income superiors and group 2 pre-fiscal inferiors.

## 5. Conclusion

This paper presents a new analysis of the redistributive impact of the fiscal system, simultaneously explaining how each tax and benefit instrument acts to satisfy the principles of vertical and horizontal equity both within and across different socio-demographic groups. The advantages of the method developed here are its comprehensiveness and simplicity of interpretation.

Income inequality is measured as the average income distance across all pairs of income units in society. The redistributive mechanism initiates the transition from pre-fiscal to post-fiscal income, whereby the average income distance is changed from $G_X$ (pre-fiscal) to $G_Y$ (post-fiscal), or by $P = G_X - G_Y$. The redistributive effect, $P$, is decomposed into a vertical effect, $V$, and a horizontal effect, $H$, as $P = V - H$. $V$ represents the average change in *income supremacy*, whereas $H$ represents the average *excessive* reduction in income supremacy caused by rerankings of income units. $V$ measures the potential average reduction in income distance that could be achieved in the absence of horizontal inequity.

Various tax and benefit instruments have different contributions to $V$ and $P$. These contributions depend on the average tax and benefit rates and the distributions of taxes and benefits. The determination of whether a certain tax (benefit) decreases inequality is always made in reference to some benchmark counterfactual distribution. The prevalent benchmark in the literature is based on proportionality with pre-fiscal income. Therefore, this paper developed decompositions for the "relative" perspective. However, other perspectives deserve attention in future research, such as the "absolute" and "intermediate" perspectives. The latter represents a combination of the relative and absolute perspectives, analogous to intermediate concepts of income inequality (Bossert and Pfingsten, 1990).

Typically, income units belong to different socio-demographic groups. Using the tax-benefit system, the government attempts to horizontally equalize the living standards of different families. For example, imagine two couples with same pre-fiscal (money) income. The couple with children receives a greater family tax allowance than the couple without children. The intention of policy makers is to equalize the two families' post-fiscal incomes according to differences in needs. Are the family tax allowance and child benefits sufficiently large to serve this purpose, or are they perhaps too high, and induce rerankings?[7]

The empirical analysis using the new decompositions indicated that Croatian personal income taxes and non-pension social benefits reduce the average income distance by 15.6 percent. The redistributive effect could be 6.4 percent higher if

[7]A comprehensive theoretical and empirical analysis of these questions is provided by Lambert and Yitzhaki (2013).

horizontal inequities were eliminated. Benefits have an overwhelming role in achieving horizontal inequity, while taxes have a higher contribution in achieving vertical equity. Rerankings of income units are relatively more pronounced in the case when households with children are outranked by households without children.[8] However, a considerable amount of reranking occurs in the opposite direction, and also within the households with children. More detailed analysis is necessary to reveal the precise outcomes, but this exercise clearly demonstrates one of possible uses of the new method, namely, identifying inequity patterns in the tax-benefit system.

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[8]Monti *et al.* (2013) obtain similar results for the Italian personal income tax. The authors find that families with one or more children are the most penalized by rerankings and conclude that "the system of tax credits for dependent relatives (children in particular) is not so generous as it should be in order to limit this unpleasant outcome."

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## Supporting Information

Additional Supporting Information may be found in the online version of this article at the publisher's web-site:

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