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ON BI-POLARIZATION AND THE MIDDLE CLASS IN LATIN AMERICA: A LOOK AT THE FIRST DECADE OF THE TWENTY-FIRST CENTURY

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This paper proposes new measures and graphical representations of the change in bi-polarization and in the relative importance of the middle class that took place in a given country during a given period. These tools extend, in fact, the concepts of inter-distribution income inequality and Lorenz curves by making, first, a distinction between overall, "pure growth based" and "shape related" distributional changes; and then between a "first-order" and a "second-order" change in "shape related" distributional change. The empirical illustration is based on data covering 17 Latin American countries in 2000 and 2009, obtained from the Latinobarómetro surveys for these years. The standard of living of individuals was derived on the basis of correspondence analysis. These new tools help us to understand the changes that took place in the distribution of standards of living during the period analyzed.

JEL Codes: D31, O15, O54

Keywords: bipolarization, generalized Lorenz dominance, inter-distributional income inequality, Latin America, Latinobarómetro

1. Introduction

In a recent paper, Kenny (2011) writes that "it is hard to find a set of characteristics or values that are consistently and uniquely middle class across countries and time." There is indeed in the literature a long list of traits that are supposed to help identify individuals who belong to the middle class. The literature

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seems to have stressed the following characteristics of individuals belonging to the middle class: they are supposed to be middle-aged people, who invest in their education, have a relatively small number of children (two or three?), spend more on health care, pay taxes, tend to own their own house or apartment, as well as one or two cars, and, as a consequence, have a significant amount of debt. They are also supposed to have some entrepreneurial spirit, to work in specific occupations, to have stable jobs, and the means to avoid poverty even when facing an unexpected shock such as becoming unemployed. They are also expected to hold common values such as a belief in the virtue of democracy (free elections and free speech) and of tolerance (e.g., toward minorities). Finally, it is also said that they are optimistic about the future, believe that they are doing better than their parents, belong to the middle spectrum of the distribution of incomes, no matter how such a middle range is defined, and are supposed to be one of the main engines of economic growth.

Needless to say, including all these characteristics (assuming they are all relevant) to determine who belongs to the middle class is an impossible task; among other reasons, because of the scarcity of data sources that would encompass all the potentially relevant variables mentioned previously. In addition, there is no agreement, among those who have attempted to define the middle class, regarding its most important features, although disposable income is almost always mentioned.

But even when the main focus is on income, there is no consensus concerning the critical thresholds, those that distinguish the middle class from the poor and from the rich. Hence there is a need to be careful when attempting to assess the size of the middle class in a given country, to find out whether its importance grew over time, to detect its main characteristics, or to determine whether the identity of those belonging to the middle class does not change or varies substantially over time.

The importance of the middle class is clearly related to the concept of bipolarization. Foster and Wolfson (2010) recommended making a distinction between four stages when attempting to measure the relative importance of the middle class: (1) choose a "space" (individual/family/household income, salary, expenditure in an income- or people-space); (2) define the "middle" (e.g., the median or the mean income); (3) fix a range around the middle (identify the middle class by determining a percentage interval above and below the median or the mean); and (4) aggregate the data.

Various definitions based on the "income space" have been proposed. Thurow (1984) assumed that the middle class includes the households whose income ranges from 75 to 125 percent of the median household income. Blackburn and Bloom (1985) recommended using a wider range (60–225 percent of the median). Other ranges have been proposed: 50–150 percent (Davis and Huston, 1992) and two-thirds to four-thirds of men's median weekly earnings (Lawrence, 1984). Birdsall (2007) suggested including in the middle class those individuals above the equivalent of $10 per day in 2005, and at or below the 90th percentile of the income distribution in their own country. In all these cases one computes which *share of the total population* the middle class includes.

Others have preferred to use definitions based on the people space. For Levy (1987), for example, the middle class ranges from the 20th to the 80th percentile. Whatever the definition adopted when using such an approach, one computes here the *share in total income* of those belonging to the selected population percentiles. Graphical representations for both the income and the people space have been proposed by Foster and Wolfson (2010).

There are however many cases where income data are not available to measure the standard of living. Filmer and Pritchett (2001) proposed solving this issue by using data on asset ownership and housing characteristics to derive for each household, via principal components analysis, what they called an "asset index." They concluded that "the asset index, as a proxy of economic status for use in predicting enrollments, is at least as reliable as conventionally measured consumption expenditures, and sometimes more so." Filmer and Scott (2011) reached similar conclusions, stating that "when per capita expenditure data are missing, the use of an asset index can clearly provide useful guidance to the order of magnitude of rich–poor differentials, however analysts should be aware that there are settings where the two approaches are likely to yield similar results, but others where they are more likely to differ."

In this paper we take a similar approach in so far as we construct a measure of living standards based on household data relative to asset ownership and access to services. However, given that all the variables are qualitative we prefer to use correspondence rather than principal components analysis.

The focus of the present paper is actually on the analysis of variations in the degree of bipolarization of the standard of living that may have occurred in several Latin American countries between 2000 and 2009. Our databases are the Latinobarómetro surveys which do not give information on the income of individuals, but on the assets available in the households and their access to various services. We first propose (Section 2) a graphical representation of the change in bi-polarization which is derived from the concept of inter-distributional change, making a distinction between a distributional change related to pure growth and one reflecting variations in the shape of the distribution. The latter effect is used to derive "first-order" and "second-order" definitions of the change in bi-polarization. Section 3 applies our approach to various countries in Latin America during the period 2000–09.

This paper makes several contributions to the literature. First, in cases where income, a cardinal variable, is not available, we propose an indirect way of measuring the bi-polarization of the distribution of standard of living, by asking which proportion of the population at two time periods has a standard of living smaller than some value. Such an approach should certainly be useful when measuring the standard of living via principal components or correspondence analysis since the "aggregate value" derived to measure the standard of living of an individual is a number whose magnitude has no intuitive interpretation. Moreover such an "aggregate value" turns out to be negative for many individuals. Second, our graphical approach is related to the concept of distributional change curves which, under specific assumptions, may allow one to draw implications concerning the change in welfare. Moreover the same graphical tool may also be used to understand what happened to the bipolarization of the standard of living. Third, the graphical device presented here allows one to reduce the number of curves by half when comparing two or more situations of change. Finally, the analysis that we present has important policy implications because we provide a tool that allows one to check in a very simple way whether during a given period the changes in the standard of living that have been observed were in favor of the middle class. We are also able to conclude whether such changes were only a consequence of growth, or were also related to variations in the shape of the distribution of standards of living.

2. Measuring Changes in Bi-Polarization

2.1. *The Concept of Overall Distributional Change and Inter-Distributional Inequality and Lorenz Curves*

The concepts of inter-distributional inequality and Lorenz curves were introduced in the literature by Butler and McDonald (1987) and may be summarized as follows. Assume two different density functions, $f(x)$ and $h(x)$, describing the distribution of income x in a given country at two different periods 0 and 1. Let $F(x)$ and $H(x)$ be the two distribution functions corresponding to $f(x)$ and $h(x)$. These two distribution functions, $F(x)$ and $H(x)$, will now be plotted respectively on the horizontal and vertical axes of a one-by-one square. In other words, for each income x we plot the percentage of individuals with an income lower than or equal to x observed in the distributions $F(x)$ and $H(x)$. If the "distributional change curve" obtained happens to be completely below the diagonal, we can certainly conclude that the distribution $H(x)$ first-order stochastically dominates the distribution $F(x)$. More generally, if most of the curve lies below the diagonal we can conclude that the population with the income distribution $h(x)$ has an economic advantage over the population with an income distribution $f(x)$ (see Bishop *et al.*, 2003). However, if most of the curve lies above the diagonal we would conclude that the population with the income distribution $f(x)$ has an economic advantage over the population with an income distribution $h(x)$.

2.2. *Distributional Change in the Case of Pure Growth*

Let us now call m_x^f and m_x^h the median incomes corresponding to the distributions $f(x)$ and $h(x)$ and let us, for example, assume that $m_x^h > m_x^f$. Let now $k(x)$ be the density function obtained when the density function $f(x)$ is horizontally translated by an amount $\left(m_x^h - m_x^f\right)$. Finally, let $K(x)$ be the distribution functions corresponding to the density function $k(x)$. A plot of $K(x)$ on the vertical axis versus that of $F(x)$ on the horizontal axis would then give us a "distributional change curve" that would only be affected by growth (assuming that $F(x)$ refers to time t and $K(x)$ to time $t + 1$) since $K(x)$ was derived from $F(x)$ by a translation. Assuming there was positive growth (since we postulated that $m_x^h > m_x^f$), the distribution change curve obtained will then start at some point A on the horizontal axis (see Figure 1). The segment OA would then represent the proportion of individuals who at time t (corresponding to distribution $F(x)$) had an income lower than the lowest income at time $t + 1$ (corresponding to distribution $K(x)$). Such a distributional curve would also end at point B and the segment BC would represent the share of the population who at time $t + 1$ had an income higher than the highest income at time t (see Figure 1). In the particular and exceptional case where the lowest income at time $t + 1$ would be higher than the highest income at

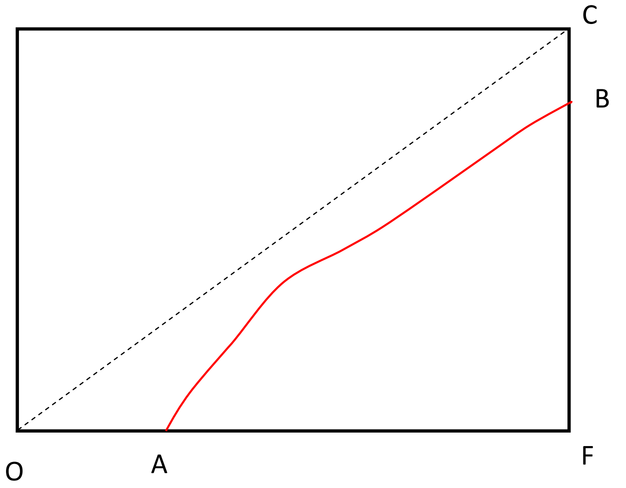


Figure 1. Distributional Change Curve in the Case of Pure Positive Growth

time t, the distributional change curve would become identical to the broken curve OFC. In such a case we know that there would be no overlap between the distributions $f(x)$ and $k(x)$, and the index of distributional change $DC(x)$, defined as being equal to twice the area between the distributional curve and the diagonal (OC), would evidently be equal to 1. The complement to one of the distributional change index may hence be considered as a measure of the degree of overlap between the distributions $h(x)$ and $k(x)$ in the case of positive growth over time. A similar graphical representation can be proposed for the case where there was negative growth, that is, when the median of the distribution $K(x)$ is smaller than that of the distribution $F(x)$.

2.3. *Distributional Change: The "Pure Shape Effect"*

Assume now that we compare the cumulative distributions $H(x)$ (on the vertical axis) and $K(x)$ (on the horizontal axis). By construction (see Section 2.2) these two distributions have the same median incomes. If for each income x we now plot the percentage of individuals with an income lower than or equal to x observed in the distributions $H(x)$ and $K(x)$, we will find that, as expected, the distributional change curve obtained will pass through the point (0.5, 0.5) since these two distributions have the same median. We also know that, by definition, the slope of this curve is positive. As a consequence, up to the point (0.5, 0.5) the curve will be located in the lower left square of size 0.5 by 0.5 while beyond the point (0.5, 0.5) the curve will be located in the upper right square of size 0.5 by 0.5.

The signed sum of the areas lying between such a "pure shape-related" distributional change curve and the diagonal would hence be a good measure of such a distributional change. Note however that here again, in computing this sum, we have to give a positive sign to any area below the diagonal and a negative sign to any area lying above the diagonal.

2.4. *The "Pure Shape" Effect and a "First-Order" Measure of the Change in Bi-Polarization*

We can also derive from the plot of $H(x)$ (on the vertical axis) versus $K(x)$ (on the horizontal axis) a measure of change in bi-polarization, as will now be shown.

If for any income x below the median we know that the proportion of individuals with an income lower than or equal to x is higher for distribution $H(x)$ than for distribution $K(x)$, then, up to the median, the curve obtained will not only be in the lower left square of size 0.5 by 0.5 but it will also lie above the diagonal. Conversely, if for any income x above the median the proportion of individuals with an income higher than or equal to x is lower for distribution $H(x)$ than for distribution $K(x)$, then, beyond the median, the curve obtained will not only be in the upper right square of size 0.5 by 0.5 but it will also lie below the diagonal. In fact the more distant the curve obtained is from the diagonal in these two 0.5 by 0.5 squares, the more bi-polarization there is in the distribution $H(x)$ in comparison to the distribution $K(x)$.

We can therefore measure the relative bi-polarization of the distribution $H(x)$ in comparison to the distribution $K(x)$ by the sum of the areas lying between the curve and the diagonal, in the two 0.5 by 0.5 squares previously defined. However, here these areas will be given a positive value if the curve lies above the diagonal in the lower left square, and if it lies below the diagonal in the upper right square.

Since the curve may cross the diagonal at least once, we will more generally define the sign given to the areas lying between the diagonal and the curve, as follows. In the lower left square of size 0.5 by 0.5, any area lying between the curve and the diagonal, which is located below the diagonal, will be given a negative sign, while any area lying between the diagonal and the curve, which is located above the diagonal, will be given a positive sign. Conversely, in the upper right square of size 0.5 by 0.5, any area lying between the curve and the diagonal, which is located below the diagonal, will be given a positive sign, while any area lying between the curve and the diagonal, which is located above the diagonal, will be given a negative sign. The sum of all these areas will then be considered as a measure of the relative bi-polarization of the distribution $H(x)$ when compared to the distribution $K(x)$.

Since the higher this relative bipolarization the lower the relative importance of the middle class at time 1 when compared to time 0, we have here a measure of the change in the relative importance of the middle class that took place between two time periods (ignoring the impact of economic growth).

Such a measure of the change in bi-polarization should however be called a "first-order" measure because it is directly linked to what Foster and Wolfson (2010) have called the "First-Order Polarization Curve" as will now be shown.[1]

Let A and B be two distributions of a continuous variable Y, and we define $F_A(y) \equiv \Pr[Y \leq y]$ for distribution A and, likewise, $y(q_A) \equiv F_A^{-1}(q)$ for $0 \leq q \leq 1$. If m stands for the median, we can standardize the distributions by *dividing* each value by the median. Such division yields the variable z and we will have, for example,

[1]The following presentation draws from Foster and Wolfson (2010).

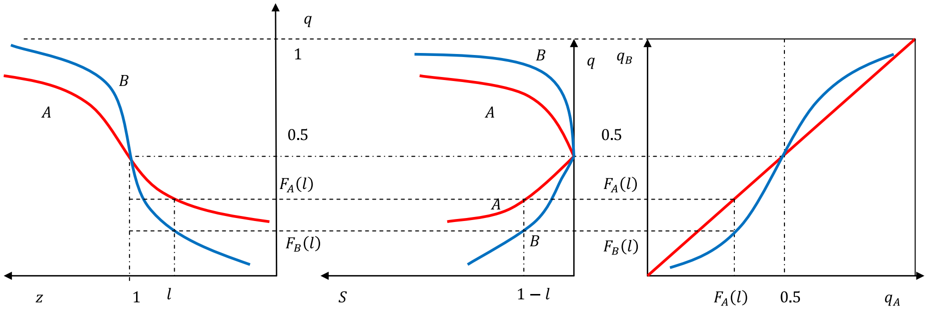


Figure 2. First-Order Polarization Curves and Cumulative Relative Distributions

$z(0.5) = 1$, $z(q) < 1 \forall 0 \le q < 0.5$ and $z(q) > 1 \forall 0.5 < q \le 1$. Define now the spread from the median: $S_A(q) \equiv |z(q_A) - 1|$. Compare the following two (first-order) change-in-polarization indices:

The one based on the first-order polarization curve (middle graph in Figure 2) is:

$$P_1 = \int_0^1 [S_B(q) - S_A(q)]dq = \int_0^{\max\{S_A(0),S_B(0)\}} [q_B(S) - q_A(S)]\mathbb{I}(q_A \le 0.5 \wedge q_B \le 0.5)dS + \int_0^{\max\{S_A(1),S_B(1)\}} [q_A(S) - q_B(S)]\mathbb{I}(q_A \ge 0.5 \wedge q_B \ge 0.5)dS.$$

The one based on the cumulative relative distribution (right graph in Figure 2) is:

$$R_1 = 2\int_0^{0.5} [q_B(q_A) - q_A]dq_A + 2\int_{0.5}^1 [q_A - q_B(q_A)]dq_A.$$

Net positive values mean that B exhibits relatively more polarization than A (although there may be compensation effects operating at different percentiles of the distributions). Net negative values mean that A exhibits relatively more polarization.

Note the proportionality relationship between P_1 and R_1. When one spread difference, $[S_A(q) - S_B(q)]$, increases, the right-hand side of P_1 has to increase as well and that can only be accomplished by the widening of some of the percentile gaps, $q_A(S) - q_B(S)$, below the median and/or $q_B(S) - q_A(S)$ above the median. Hence R_1, which is a function of both sets of gaps, also increases when a spread difference increases. Both P_1 and R_1 can be expressed as functions of the percentile gaps, but while the first index is a weighted sum of these gaps in which the weights are dS, the second index is a weighted sum of percentile gaps in which the weights are dq.

Note also that a Pigou–Dalton transfer *across the median* should reduce polarization and increase P_1 and R_1, if A represents the pre-transfer distribution and B represents the post-transfer distribution.

Finally, it should be stressed (see Appendix A) that measuring first-order change in bipolarization via the plot of $H(x)$ (on the vertical axis) versus $K(x)$ (on the horizontal axis) gives exactly the same result as the one that would be obtained when working with the standardized distances $S_A(q)$ and $S_B(q)$ defined previously and introduced by Foster and Wolfson (2010).

2.5. *The "Pure Shape" Effect and a "Second-Order" Measure of the Change in Bi-Polarization*

Foster and Wolfson (2010) have also discussed what happens if a Pigou–Dalton transfer takes place *on one side of the median*. If the transfer preserves ranks, then it is easy to show that P_1 is insensitive to it, as it is only measuring polarization with respect to the median. However such transfer should increase bi-polarization as it concentrates the distribution on the side of the median where the transfer took place. An index that is sensitive to these transfers and thus measures changes in bipolarization can be constructed using second-order polarization curves.

We define the cumulative spread from the median as:

$$C_A(q) \equiv \left| \int_q^{0.5} S_A(q)\,dq \right| = \left| \int_q^{0.5} |z(q_A) - 1|\,dq \right|.$$

We can then compare the following two change-in-polarization indices:

The one based on the second-order polarization curve is:

$$P_2 = \int_0^1 [C_B(q) - C_A(q)]\,dq = \int_0^{\max\{C_A(0),C_B(0)\}} [q_B(C) - q_A(C)]\mathbb{I}(q_A \leq 0.5 \wedge q_B \leq 0.5)\,dC + \int_0^{\max\{C_A(1),C_B(1)\}} [q_A(C) - q_B(C)]\mathbb{I}(q_A \geq 0.5 \wedge q_B \geq 0.5)\,dC.$$

The one based on the cumulative relative distribution is:

$$R_2 = 2\int_0^{0.5} [q_B(q_A) - q_A]\,dq_A + 2\int_{0.5}^{1} [q_A - q_B(q_A)]\,dq_A.$$

Note again the proportionality relationship between P_2 and R_2. Also note that the indices attach more weight to spreads closer to the median. Therefore if a Pigou–Dalton transfer occurs on one side of the median, A represents the pre-transfer distribution and B represents the post-transfer distribution, then the indices take a positive value, thereby showing an increase in bipolarization.[2]

One may wonder whether the graphical approach proposed here is not a simple application of the methodology originally developed by Foster and Wolfson (2010). As was already stressed in the introduction, our approach is in fact well suited for cases where, in the absence of income data, one has to estimate the standard of living via multivariate analysis, such as principal components or

[2]Graphs similar to those given in Figure 2 are available upon request from the authors.

correspondence analysis, and it turns out that the estimated individual standard of living has no clear interpretation and is moreover often negative.

3. An Empirical Illustration: Changes in the Bi-Polarization of Living Standards in Latin America between 2000 and 2009

3.1. *Estimating the Standards of Living in Latin American Countries*

As mentioned previously, the 2000 and 2009 Latinobarómetro surveys do not provide any data on income. But these surveys provide information on individual possession of durable goods and access to certain services. Eleven durables goods or basic services were taken into account: television, refrigerator, home, personal computer, washing machine, phone, car, second home, access to drinking water, access to hot water, and sewage facilities. To estimate individual standards of living we use correspondence analysis.[3] This correspondence analysis was implemented *separately* for each country and we used the first component (axis) to estimate the standard of living of the individuals. It should hence be clear that we cannot compare different countries, but we can check for each country what happened to the degree of bi-polarization of the distribution of standards of living.

Note also that in correspondence like in principal components analysis, the weights given to the various durable goods and access to services are endogenously determined (they are derived from the first component). As stressed by Asselin (2009), the weights obtained in correspondence analysis have, however, two advantages. First, correspondence analysis gives more weight to indicators with a smaller number of "hits." In other words if, for example, for a dimension of the standard of living like having a refrigerator, we observe that only a few individuals do not have a refrigerator, then these individuals will be given a higher weight. The second property is reciprocal bi-additivity. This property states that the composite "standard of living" score of an individual is the simple average of the factorial weights of the "standard of living" categories for this individual and that the weight of a given dimension of "standard of living" is the simple average of the composite "standard of living" scores of the population units that belong to the given dimension.

It should be emphasized however that the type of analysis we conduct, like that of increased spread or bipolarity, has to be conducted in terms of the aggregate standard of living. It does not seem possible to go back to the observed variables (goods and services) from these latent variables in terms of these movements.[4] As stressed by Filmer and Scott (2011), "While principal components analysis is easy to implement, it remains a black box." The same is evidently true of correspondence analysis. Filmer and Scott (2011) review alternative approaches (e.g., factor analysis, counting approaches, item response theory), all of which have their advantages as well as their shortcomings.

[3]See Benzécri and Benzécri (1980), Asselin and Vu Tuan Anh (2008), and Asselin (2009) for more details on correspondence analysis and on when the latter is preferable to principal components analysis.

[4]We thank an anonymous referee for having drawn our attention to this point.

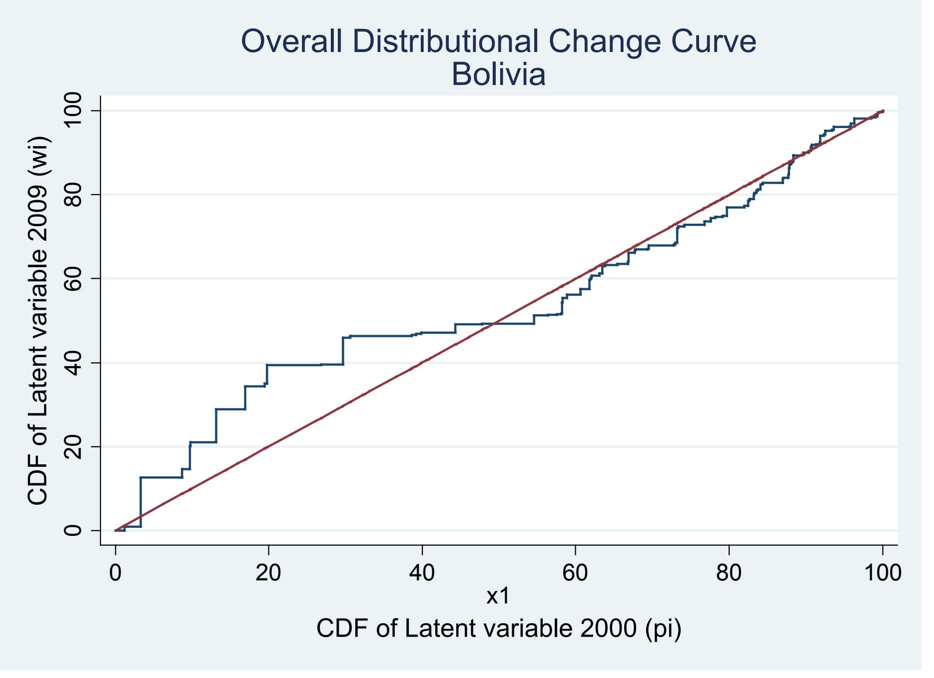


Figure 3. Overall Distributional Change for Bolivia

3.2. *Deriving the Distributional Change Curves*

By comparing the standards of living in 2000 and 2009, we can now derive the various distributional change curves.

Overall Distributional Change

The results of the computation of the measures of the overall, "pure growth related," and "shape related" distributional change are summarized in Table 1.

It appears that the overall distributional change was positive only in Venezuela, Uruguay, El Salvador, Argentina, and Costa Rica, the change being very small in the two last countries.

The highest negative values were observed in Honduras, Guatemala, Ecuador, Panama, Paraguay, Bolivia, Nicaragua, and Peru. These negative values imply that in these countries, as a whole, the standard of living was lower in 2009 than in 2000, although this was not necessarily true for all the segments of the population.

As graphical illustration we present in Figure 3 the case of Bolivia.[5] We observe that below the median most of the curve is above the diagonal, which implies that the living conditions of most of the "poor" (those with living standards below the median) worsened between 2000 and 2009. We also see that,

[5]The graphs for the other countries are given in Appendix B.

TABLE 1

Values of the Overall, "Pure Growth Based" and "Shape Related" Distributional Change Measures for the Period 2000–09 in Various Latin American Countries

Country	Overall Distributional Change	"Pure Growth Based" Distributional Change	(First-Order) "Shape Related" Distributional Change for Standards of Living Below the Median	(First-Order) "Shape Related" Distributional Change for Standards of Living Above the Median	Total (First-Order) "Shape Related" Distributional Change
Argentina	0.0083	0.1515	–0.0626	–0.0485	–0.1111
Bolivia	–0.0360	0.0110	–0.0487	0.0097	–0.0390
Brazil	–0.0169	0.0648	–0.0432	–0.0061	–0.0493
Colombia	–0.0254	0.0435	–0.0374	–0.0098	–0.0472
Costa Rica	0.0077	0.0761	–0.0404	–0.0250	–0.0654
Chile	–0.0076	0.1597	–0.0747	–0.0269	–0.1016
Ecuador	–0.0575	0.0739	–0.0891	–0.0084	–0.0975
El Salvador	0.0116	0.1091	–0.0747	0.0042	–0.0705
Guatemala	–0.0616	–0.2177	0.0554	0.0758	0.1312
Honduras	–0.0715	–0.0852	0.0037	0.0122	0.0159
Mexico	–0.0007	0.0509	–0.0307	–0.0017	0.0324
Nicaragua	–0.0308	–0.0706	0.0277	0.0237	0.0514
Panama	–0.0491	0.0545	–0.0757	0.0007	–0.0750
Paraguay	–0.0397	–0.0301	–0.0382	0.0159	–0.0223
Peru	–0.0306	0.0416	–0.0519	0.0000	–0.0519
Uruguay	0.0161	0.1043	–0.0560	–0.0229	0.0789
Venezuela	0.0260	–0.0366	0.0494	0.0009	0.0503

above the median, most of the curve is below the diagonal so that most of the "rich" (those with living standards higher than the median) improved their living conditions.

"Pure Growth Related" Distributional Change

The results concerning the "pure growth related" distributional change are also given in Table 1. It appears that, as a whole, growth was highest in Chile, Argentina, El Salvador, Uruguay, Costa Rica, and Brazil. On the other side the countries where, as a whole, growth was negative were Guatemala, Honduras, Nicaragua, Venezuela, and Paraguay. Figure 4 illustrates the case of positive growth (Argentina) and negative growth (Honduras).[6]

"Shape Related" Distributional Change

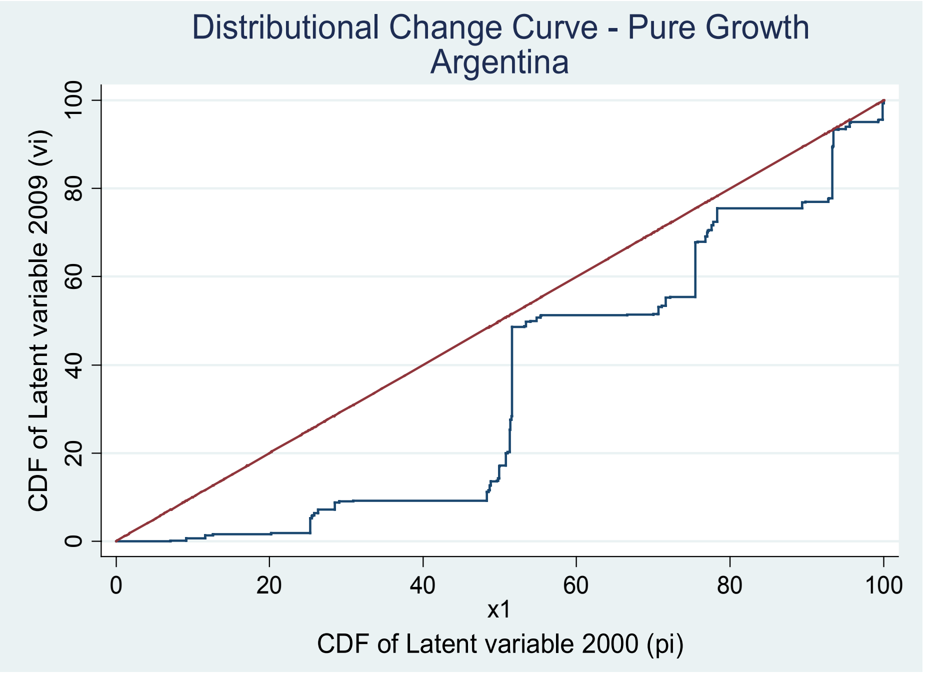
We now turn to the analysis of "shape-related" distributional change. We refer first to what was called previously "first-order" change in the bi-polarization curve. Table 1 indicates that this type of distributional change was highest (and positive) for Guatemala, Uruguay, Nicaragua, and Venezuela and lowest (and negative) for Argentina, Chile, Ecuador, Panama, El Salvador, and Costa Rica.

Figure 5 presents as an illustration these "shape related" first-order distributional change curves for Ecuador, Guatemala, and Venezuela.[7] In the case of Guatemala, we observe that from a pure change-in-shape point of view, almost everyone in 2009 would have had a higher standard of living than in 2000. Thus there was a shift of the observations toward the median among the "poor" between 2000 and 2009. We also observe that, among the "rich," there was a rightward shift of the observations toward higher standards of living. For Ecuador the "shape related" distributional change was almost nil for those having a standard of living above the median, whereas for those with a standard of living below the median, there was a downward shift towards the lowest levels of standard of living.

Variations in "First-Order" Bi-Polarization

The observations that were just made concerning the "(first-order) shape-related distributional change" have evidently implications concerning the variation between 2000 and 2009 in the degree of bi-polarization of the distribution of standards of living. A new measure of change in bi-polarization was previously proposed; Table 2 gives the value of this index for the various Latin American countries. We recall that a distinction has to be made between the "poor," those whose standard of living is below the median, and the "rich," those with a standard of living higher than the median. If the curve for the poor is mostly above the diagonal, then the poor have become "poorer" (assuming the two distributions compared have the same median standard of living) so that bipolarization increases. On the other hand, bipolarization will increase if the "shape-related"distributional curve corresponding to the rich is mostly below the diagonal, since this implies that the rich have become richer.

[6]The graphs for the other countries may be obtained upon request from the authors.
[7]The graphs for the other countries are given in Appendix B.



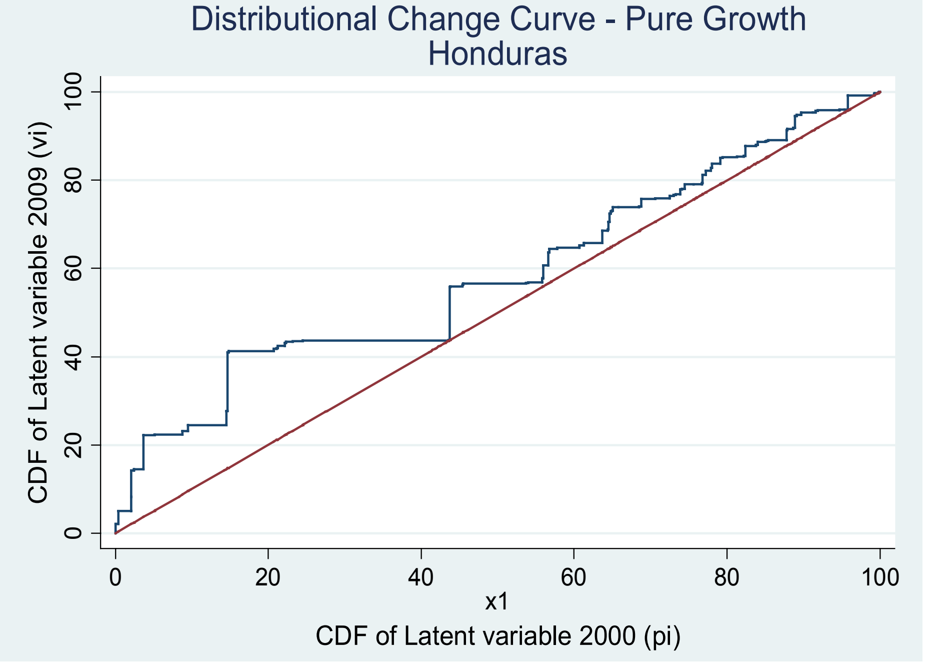
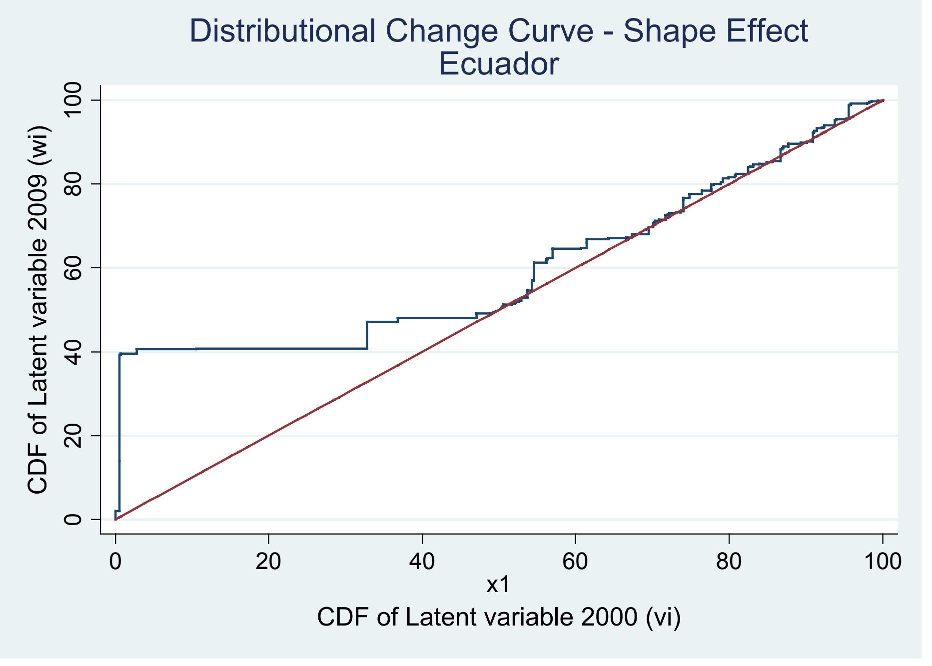


Figure 4. "Pure Growth Based" Distributional Change Curves



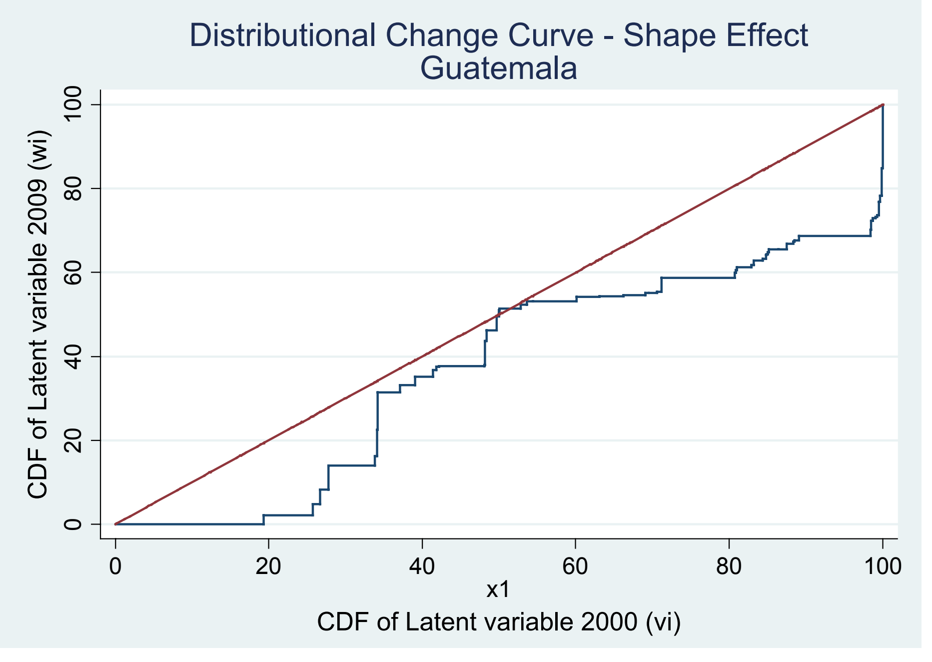


Figure 5. "(First Order) Shape Effect" Distributional Change

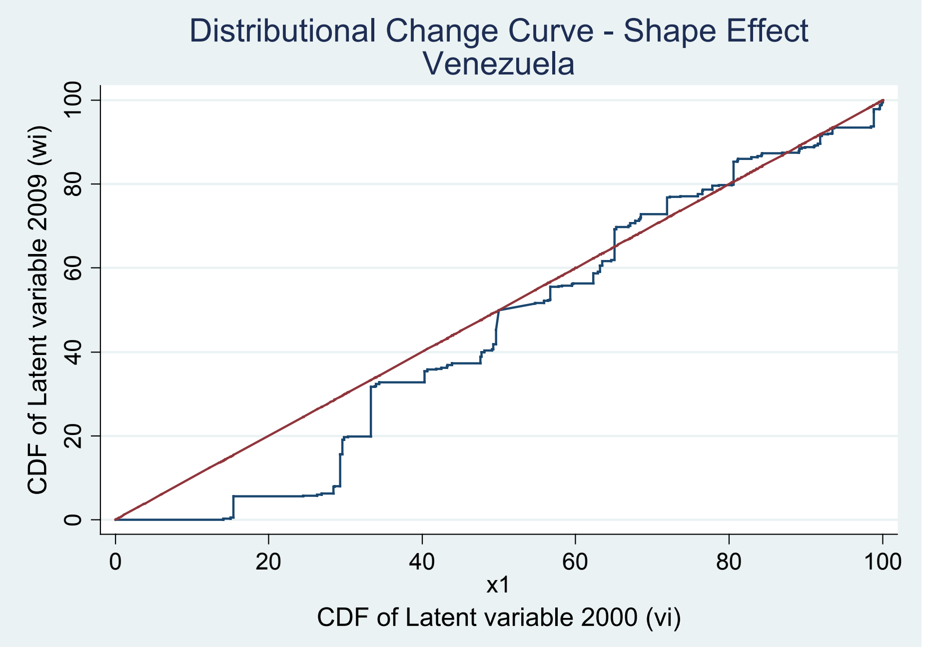


Figure 5. Continued

TABLE 2

Value of the "(First-Order) Change in Bi-Polarization" Index for Various Latin American Countries

	"Change in Bi-Polarization" Index	Rank
Argentina	0.0142	4
Bolivia	0.0584	14
Brazil	0.0370	10
Colombia	0.0276	7
Costa Rica	0.0155	5
Chile	0.0371	11
Ecuador	0.0806	17
El Salvador	0.0801	16
Guatemala	0.0204	6
Honduras	0.0080	3
Mexico	0.0288	8
Nicaragua	–0.0042	2
Panama	0.0746	15
Paraguay	0.0541	13
Peru	0.0509	12
Uruguay	0.0332	9
Venezuela	–0.0473	1

Table 2 then indicates that bipolarization increased the most in Ecuador and El Salvador whereas it decreased only in Venezuela and Nicaragua. Figure 5 shows, for example, that in the case of Venezuela, what happened was essentially an improvement in the standards of living of the poor.

It is important to understand the difference between the numbers that appear in the last column of Table 1 and the second column of Table 2. The last column of Table 1 gives the value of the first-order shape related distributional change. So up to the median, if the curve is below the diagonal there is a smaller percentage at time 0 than at time 1 of individuals with a standard of living smaller than, say, some value x, and hence the area below the diagonal should be given a positive sign. This is also true when this value x is greater than the median. For Argentina, for example, we have negative values before (–0.0626) and after (–0.0485) the median so that the shape related distributional curve should be above the diagonal (see Figure 6a). The total value of the first-order shape related distributional change shape is then (–0.0626) + (–0.0485) = –0.1111.

But for the measurement of (first-order) bipolarization (Table 2), as was mentioned previously, "in the lower left square of size 0.5 by 0.5, any area lying between the curve and the diagonal, which is located below the diagonal, will be given a negative sign while any area lying between the diagonal and the curve, which is located above the diagonal, will be given a positive sign," while "in the upper right square of size 0.5 by 0.5, any area lying between the curve and the diagonal, which is located below the diagonal, will be given a positive sign while any area lying between the curve and the diagonal, which is located above the diagonal, will be given a negative sign." So the measure of change in bipolarization should be 0.0626 (positive sign) + (–0.0485) = 0.0142, which is the value which appears for Argentina in the first row of Table 2.

Variations in "Second-Order" Bi-Polarization

If one draws what we called previously "second-order" shape-related distributional change curves, it is certainly possible to find results which will be quite different from those based on the concept of "first-order" shape-related distributional change curves. Such differences may occur because the hypotheses underlying these two sets of curves are not the same. The curves derived on the basis of a "first-order" shape-related effect make only the assumption of "increased spread," i.e. that bipolarization increases only when a rich person (whose income is above the median income) becomes richer (ceteris paribus) or a poor person (whose income is lower than the median income) becomes poorer. In other words, bipolarization increases (the index of bipolarization change is higher) if you transfer income from a poor person (income below the median) to a rich person (income above the median) so that the average gap between the rich and the poor increases.

The curves derived on the basis of a "second-order" shape-related effect add to the assumption of "increased spread," that of "increased bipolarity," which means that if you transfer income from a rich person to a less rich person (with both individuals having an income higher than the median) or from a poor person to a poorer person (with both individuals having an income below the median), bipolarization increases. "Increased bipolarity" means therefore that the dispersion of income among the rich, and/or among the poor, becomes smaller.

In fact the two sets of curves turn out to be quite different.[8] Let us take a closer look at one of the "second-order" curves, that for Argentina, and compare it with the "first-order" curve for the same country. For the "first-order' change in bi-polarization curve we observe (see Figure 6a) for Argentina that for the poor, most of the area is above the diagonal, which means that on average the poor are in 2009 further away from the median (they became poorer between 2000 and 2009). For the rich, most of the area is above the diagonal, which means that on average the rich are now closer to the median, and hence on average have become less rich.

Now let us take a look at the "second-order" change in bi-polarization curve for Argentina (see Figure 6b). We then observe that for the poor the curve is below the diagonal. We may then conclude, *in addition* to what we inferred previously on the basis of the "first-order" change in bi-polarization curve, that the dispersion of the incomes of the poor has increased. This in itself decreases bipolarization, and this effect is stronger than the one described on the basis of the "first-order" change in bi-polarization curve, which implied that the poor have become poorer and are on average located further away from the median.

For the rich the "second-order" change in bipolarization curve is below the diagonal, so that bipolarization has increased. This means that, in addition to what was mentioned previously on the basis of the "first-order" change in bipolarization curve (i.e., the "rich" have become less rich and are located closer to the median), the dispersion of the incomes of the rich has decreased (which raises bipolarization, ceteris paribus) and this effect is stronger than the former effect (the rich are on average closer to the median which implies a lower bipolarization).

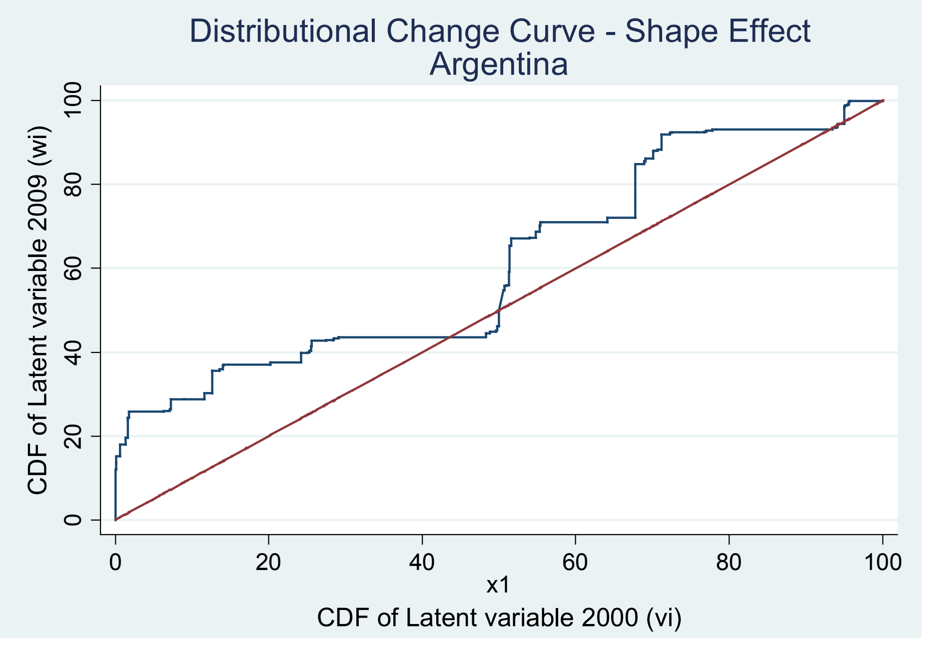
From these "second-order" distributional change curves we can derive an index of change in "second-order" bipolarization which will be defined in the same way as we defined previously an index of "first-order" change in bipolarization. In other words, when the curve for the poor is mostly above the diagonal, then the poor have become "poorer" (assuming the two distributions compared have the same standard of living) so that bipolarization increases. On the contrary, bipolarization will increase if the "shape-related" distributional curve corresponding to the rich is mostly below the diagonal, because this would imply that the rich have become richer.

Table 3 gives the value of such an index of "second-order" change in bipolarization for all the countries examined previously. In addition it shows the breakdown of this index into two components, one which shows the impact on bipolarization of the distributional change among the "poor," and another one which shows the effect of distributional change among the "rich."

It appears that the "second-order" increase in bipolarization was highest for Paraguay, Bolivia, Chile, Peru, and El Salvador. But there was an important

[8]The graphs for these "second order" shape related distributional change curves are also given in Appendix B.

(a)



(b)

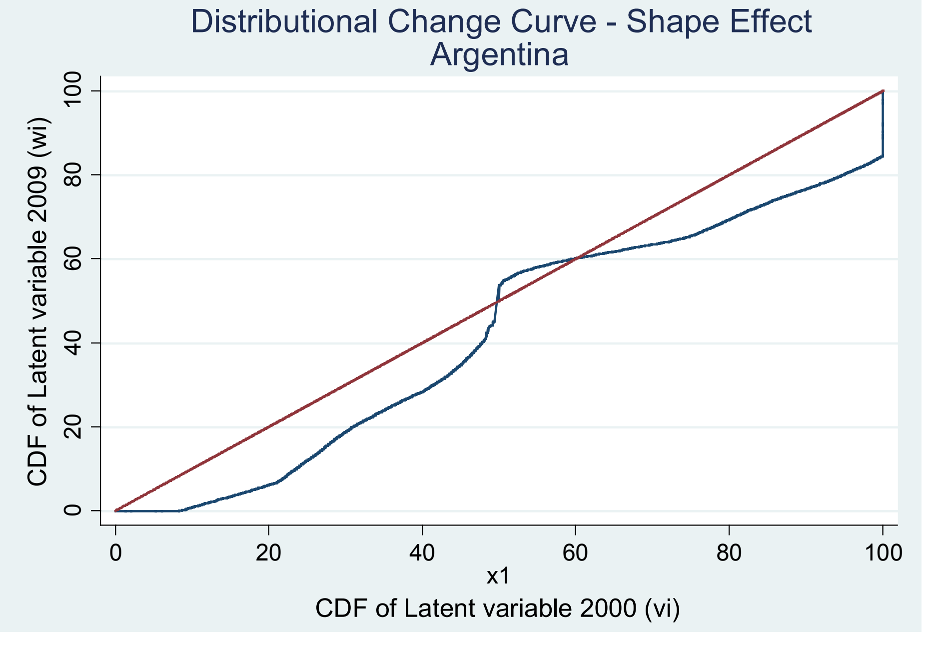


Figure 6. Comparing (a) "First Order" and (b) "Second Order" Shape Related Distributional Change Curves: The Case of Argentina

TABLE 3
"Second-Order" Change in Bipolarization

Country	Shape (before median)	Shape (after median)	Total Change in Bi-Polarization
Argentina	–0.0496	0.0360	–0.0136
Bolivia	0.0216	0.0490	0.0707
Brazil	0.0214	0.0258	0.0473
Colombia	–0.0176	0.0085	–0.0092
Costa Rica	–0.0094	0.0358	0.0264
Chile	–0.0147	0.0775	0.0628
Ecuador	–0.0105	0.0506	0.0402
El Salvador	0.0148	0.0368	0.0516
Guatemala	0.0512	–0.0268	0.0244
Honduras	–0.0024	–0.0018	–0.0041
Mexico	0.0013	0.0240	0.0254
Nicaragua	0.0240	–0.0038	0.0202
Panama	0.0004	0.0453	0.0457
Paraguay	0.0388	0.0848	0.1237
Peru	0.0200	0.0360	0.0560
Uruguay	–0.0335	0.0394	0.0059
Venezuela	–0.0045	–0.0487	–0.0532

decrease in this "second-order" bipolarization in Venezuela, although this decrease was more a consequence of what happened to the "rich" than of what happened to the "poor."

4. Concluding Comments

This paper proposed a new index and graphical representation of the change in bi-polarization and in the relative importance of the middle class that took place in a given country during a given period. These tools extend the concepts of inter-distribution income inequality and Lorenz curves by making a distinction between overall, "pure growth based" and "shape related" distributional changes.

The empirical illustration was based on data covering 17 Latin American countries in 2000 and 2009, obtained from the Latinobarómetro surveys for these years. The standard of living of individuals was derived on the basis of correspondence analysis. The new tools proposed in this paper help us to understand the changes that took place in the distribution of standards of living in Latin America during the period analyzed. They also suggest a new way of determining what happened to the middle class between 2000 and 2009.

This empirical analysis examined the case of a "first-order" as well as that of a "second-order" change in bi-polarization.

Appendix A

Call x the vector of actual standards of living at time 0 and y the vector of actual standards of living at time 1.

Call m_0 and m_1 the median values of the standards of living at times 0 and 1.

Define now a hypothetical set of standards of living z where z is defined as $z = x + (m_1 - m_0)$, assuming m_1 is greater than m_0.

In defining what we called the "shape related distributional change" we are comparing

Prob$\{z < A\}$ with Prob$\{y < A\}$ for all possible values of A.

We have now to make a distinction between the cases where A is greater or smaller than the common median m_1 of z and y.

<u>First case: A is greater than the median m_1 (assume that $m_1 > 0$).</u>

We then compare Prob$\{z < A\}$ with Prob$\{y < A\}$

which is equivalent to comparing Prob$\{(z - m1) < (A - m1)\}$

with Prob$\{(y - m1) < (A - m1)\}$

which is equivalent to comparing $\text{Prob}\left\{\frac{(z-m1)}{m1} < \frac{(A-m1)}{m1}\right\}$

with $\text{Prob}\left\{\frac{(y-m1)}{m1} < \frac{(A-m1)}{m1}\right\}$

which is equivalent to comparing $\text{Prob}\left\{\left|\frac{(z-m1)}{m1}\right| < \left|\frac{(A-m1)}{m1}\right|\right\}$

with $\text{Prob}\left\{\left|\frac{(y-m1)}{m1}\right| < \left|\frac{(A-m1)}{m1}\right|\right\}$

In short if, say, Prob$\{z < A\}$ < Prob$\{y < A\}$, we should also find in this case that

$$\text{Prob}\left\{\left|\frac{(z-m1)}{m1}\right| < \left|\frac{(A-m1)}{m1}\right|\right\} < \text{Prob}\left\{\left|\frac{(y-m1)}{m1}\right| < \left|\frac{(A-m1)}{m1}\right|\right\}.$$

<u>Second case: A is smaller than the median m_1 ($m_1 > 0$).</u>

We then compare Prob$\{z < A\}$ with Prob$\{y < A\}$

which is equivalent to comparing Prob$\{-A < -z\}$ with Prob$\{-A < -y\}$

which is equivalent to comparing Prob$\{(m1 - A) < (m1 - z)\}$

with Prob$\{(m1 - A) < (m1 - y)\}$

which is equivalent to comparing $\text{Prob}\left\{\frac{(m1-A)}{m1} < \frac{(m1-z)}{m1}\right\}$

with $\text{Prob}\left\{\frac{(m1-A)}{m1} < \frac{(m1-y)}{m1}\right\}$

But since $(m1 - A)$, $(m1 - z)$ and $(m1 - y)$ are positive while m1 is also positive, the ratios $\frac{(m1-A)}{m1}$, $\frac{(m1-z)}{m1}$ and $\frac{(m1-y)}{m1}$ are all positive and hence

this is equivalent to comparing $\text{Prob}\left\{\left|\frac{(m1-A)}{m1}\right| < \left|\frac{(m1-z)}{m1}\right|\right\}$

with $\text{Prob}\left\{\left|\frac{(m1-A)}{m1}\right| < \left|\frac{(m1-y)}{m1}\right|\right\}$

To simplify, call α the positive ratio $\left|\frac{(m1-A)}{m1}\right|$.

We are therefore comparing Prob $\left\{\left|\frac{(m1-z)}{m1}\right|>\alpha\right\}$ with Prob $\left\{\left|\frac{(m1-y)}{m1}\right|>\alpha\right\}$

and if this is true for every possible value of A as long as A < z and A < y, we have the equivalence between having [Prob$\{z < A\}$ < Prob$\{y < A\}$] (which indicates an increase in bipolarization between times 0 and 1) and $\left[\text{Prob}\left\{\left|\frac{(m1-z)}{m1}\right|>\alpha\right\}<\text{Prob}\left\{\left|\frac{(m1-y)}{m1}\right|>\alpha\right\}\right]$.

However, using Foster and Wolfson's (2010) approach this also indicates that bipolarization increased between periods 0 and 1 (y is more bipolarized than z).

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Supporting Information

Additional Supporting Information may be found in the online version of this article at the publisher's web-site:

Appendix B: Overall Distributional Change, Growth Component, First Order and Second Order Shape Effects