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STOCHASTIC ANALYSIS OF INPUT–OUTPUT MULTIPLIERS ON THE BASIS OF USE AND MAKE TABLES

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Although technical coefficients are estimated on the basis of flow data (use and make matrices), they are rarely treated as random variables. If this is done, an error term is added to the coefficients, rather than derived from the distribution of the data. Even so, the calculation of multipliers, by means of the Leontief inverse, is difficult. Due to the nonlinearity of this operation, the multiplier estimates are biased. By going back to the flow data, this paper provides unbiased and consistent employment and output multipliers estimates for the Andalusian economy. Rectangular use and make matrices are accommodated and problems associated with the construction and estimation of technical coefficients and the Leontief inverse are circumvented.

1. INTRODUCTION

The prime use of input and output accounts is the estimation of multiplier effects, such as the employment and output effects of increases in alternative final demand components. The multipliers are "given" by the Leontief inverse of the matrix of input–output coefficients. The practice of interrelating accounts and input–output multipliers can be decomposed into three steps (see Figure 1).

Step 1 consists of filling data gaps, imputing values to non-observed establishments, and summation over firms within industries. These operations are straightforward and produce the so-called use and make tables, U and V, which display the commodity inputs and outputs of the industries. The off-diagonal elements of the make table are the so-called secondary products, which must be treated one way or another in Step 2. The System of National Accounts (UN, 1993) advocates the so-called commodity technology, which involves inversion of the make table. Anyway, the result is a matrix of input–output coefficients, A. The third and last step is Leontief inversion, $(I - A)^{-1} = I + A + A^2 + \dots$. In multiplier analysis, the first term represents the direct effect, the second term the

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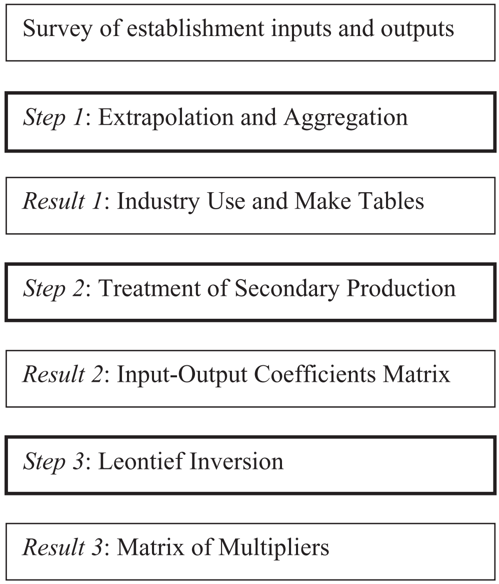

Figure 1. From Establishment Data to Input–Output Multipliers

direct input requirement, and the third and further terms the indirect input requirements.

The literature is as piecemeal as Figure 1 suggests. The theory of input–output coefficients addresses Step 2 and analyzes alternative models for their construction. Results are partial and problems persist, such as the problem of negative coefficients. Basically, input–output coefficients measure inputs per units of outputs and the division by outputs involves the inversion of the make table, a complicated, nonlinear operation. The stochastic input–output literature focuses on Step 3, analyzing the transmission of errors under Leontief inversion. Here the problem is also nonlinearity, but not one associated with the presence of secondary products. In fact, the problem is already there in a one-sector economy, where matrix A reduces to a scalar a. Because the Leontief inverse, $(I - a)^{-1} = 1 + a + a^2 + \ldots$, is a convex function of input–output coefficient a, Young's Theorem yields that the expectation of the Leontief inverse exceeds the Leontief inverse of the expectation of the input–output coefficient. Since the latter constitutes the standard Leontief inverse, it follows that the standard Leontief inverse underestimates its true value. Simonovits (1975) and Kop Jansen (1994) extend this result, albeit under rather restrictive assumptions, such as the independence of technical coefficients. Dietzenbacher (1995) and Roland-Holst (1989) find more overestimation than underestimation.

In this paper we make two interrelated contributions to the literature. First, we derive information on the precision of multipliers not from stochastic assumptions on the input–output coefficients, but from the variability of the underlying input and output statistics across establishments. In other words, we go back to square 1 in Figure 1. Second, we integrate the steps of Figure 1, by reducing the formulas for multipliers to the establishment use and make tables. To our delight, the nonlinearities, which plague the construction of input–output coefficients and the transmission of errors in the Leontief inverse, neutralize each other. In this way we are able to present consistent linear unbiased estimates of multipliers. We contrast our results with the official ones of the Institute of Statistics of Andalusia (IEA).

2. From Data to Coefficients and from Coefficients to Multipliers

Modern input–output accounting distinguishes commodities $i = 1, \ldots, n$ and activities $j = 1, \ldots, m$. (We have traced this approach to Edmonston, 1952.) At the most disaggregated level, an activity represents a plant. Plant j uses inputs (both factor services and commodities) to make products (commodities). For reasons of national accounting, it is customary to list the inputs in the j-th column of *use* matrix $U = (u_{ij})$ but the outputs in the j-th row of *make* matrix $V = (v_{ji})$. The requirements of input i by industry j are proportional to its products v_{jk}. If we assume that the proportions, a_{ik}, are independent of the industry (the so-called commodity technology assumption), we obtain for the technical coefficients:

$$(1)\qquad u_{ij} = \sum_{k=1}^{n} a_{ik} v_{jk} \quad \text{for all } i = 1, \ldots, n \text{ and } j = 1, \ldots, m.$$

According to Konijn and Steenge (1995), an input–output matrix A has to fulfill the commodity technology assumption to achieve full consistency with fundamentals of input–output analysis. Kop Jansen and ten Raa (1990) arrived at the same conclusion on axiomatic grounds. Moreover, Avonds (2005) demonstrated that the commodity technology assumption need not break the economic circuit, as de Mesnard (2004) argued.

If there are more activities than commodities ($m > n$), the system of equation (1) is overdetermined, an error term must be attached, and the input–output coefficients become regression coefficients. Several studies have attempted to estimate technical coefficients from econometric models with cross-section data on firms' inputs and outputs. Miernyk *et al.* (1970) quantify the level of uncertainty in measured technical coefficients. Mattey and ten Raa (1997) support the commodity technology hypothesis for United States manufacturing.

In the literature on stochastic input–output analysis, technical coefficients are the point of departure for the analysis of the probabilistic properties of the Leontief inverse, $B = (I - A)^{-1}$. Kop Jansen (1994) reviews how stochastics affect the multipliers, i.e. the distributional properties of the Leontief inverse. Basically, Young's Theorem extends (assuming independence and symmetry) and the expected value of the Leontief inverse is underestimated by the Leontief inverse of the expected value of the A-matrix. This bias of the standard Leontief inverse will be revealed by our results. For purists the situation is as follows. Denote the true value of the input–output matrix by α and of its Leontief inverse by β. Denote the estimate of α by A and of β by B. The standard estimator of β is $B = (I - A)^{-1}$. The expectation of B is $\mathrm{E}(B) = \beta +$ bias. Denote the estimator we will develop in the next section by $\boldsymbol{B}$. Since it will be unbiased, $\mathrm{E}(\boldsymbol{B}) = \beta$. It follows that the bias of the standard Leontief inverse is $\mathrm{E}(B) - \mathrm{E}(\boldsymbol{B})$. We will estimate this bias by $B - \boldsymbol{B}$. If the assumptions of the extension of Young's Theorem are valid, we expect $B < \boldsymbol{B}$, or that standard multipliers *under*estimate our estimates of the true values.

An output multiplier is given by the total value of production needed to satisfy a euro worth of a particular component of final demand and a multiplier measures the associated number of workers. We will adopt the commodity technology hypothesis for intermediate and labor inputs. Use and make transactions exclude imports and are valued at basic prices. Net commodity taxes and non-deductible Value Added Tax (VAT) are excluded, as are trade and transport margins. The latter are assigned to the trade and transport services industry. The measurement in basic prices accommodates the treatment of net exports as part of final demand.

Employment multipliers are derived from labor coefficients. Commodity technology labor coefficients are determined by the following expression:

$$L = lV^{\mathrm{T}}, \tag{2}$$

where L represents a row vector of labor employment (of order m), l is the row vector of labor coefficients and V^{T} the transposed make matrix. Inflation by the Leontief inverse yields the employment multipliers (λ):

$$\lambda = l(I - A)^{-1}. \tag{3}$$

Multipliers (3) measure the employment generated by a monetary unit expended on alternative commodities. It is no workers per worker figure, but a kind of return-on-investment measure.

In traditional input–output analysis all matrices are square ($m = n$) and equations (1) and (2) imply the well-known commodity technology coefficients $A = U(V^{\mathrm{T}})^{-1}$ and $l = L(V^{\mathrm{T}})^{-1}$ (Kop Jansen and ten Raa, 1990). In this case, the employment multipliers (3) reduce to:

$$\lambda = L(V^{\mathrm{T}})^{-1}\left[I - U(V^{\mathrm{T}})^{-1}\right]^{-1} = L\left\{\left[I - U(V^{\mathrm{T}})^{-1}\right]V^{\mathrm{T}}\right\}^{-1} = L(V^{\mathrm{T}} - U)^{-1}. \tag{4}$$

or

$$L = \lambda(V^{\mathrm{T}} - U). \tag{5}$$

If there are more activities than commodities ($m > n$), the system of equations (5) is overdetermined, an error term must be attached, and the employment multipliers become regression coefficients:

$$L = \lambda(V^{\mathrm{T}} - U) + \varepsilon. \tag{6}$$

In (6) L is a row vector of order m with labor employment, λ is a row vector of order n with employment multipliers, V is a make matrix of order $m \times n$, U is a use matrix of order $n \times m$ and ε is a row vector of independently normally random disturbance errors with zero mean and constant variance, with order m. Notice that m is the number of establishments or observations and that net outputs by commodities would therefore constitute the independent variables of the resulting model. The estimation of employment multipliers becomes a matter of multiple linear regression analysis, with linear, unbiased and consistent multipliers estimates. In Section 3, we estimate (6) for the Andalusian economy in the year 1995.

Output multipliers, μ are given by the column totals of the Leontief inverse:

$$\mu = e(I - A)^{-1} \tag{7}$$

The only difference with equation (4) is the replacement of the row vector of labor coefficients l by the unit vector $e = (1 \dots 1)$. In standard input–output analysis the output multipliers (7) are represented by:

$$\mu = e\left[I - U\left(V^{\mathrm{T}}\right)^{-1}\right]^{-1} = eV^{\mathrm{T}}\left(V^{\mathrm{T}} - U\right)^{-1} \tag{8}$$

or

$$eV^{\mathrm{T}} = \mu\left(V^{\mathrm{T}} - U\right). \tag{9}$$

Analogous to the system of equations (5), the output multipliers become regression coefficients when there are more activities than commodities ($m > n$). That is, net outputs would be considered again as exogenous variables.

$$eV^{\mathrm{T}} = \mu\left(V^{\mathrm{T}} - U\right) + \varepsilon. \tag{10}$$

In (10), eV^{T} is a row vector of total outputs of establishments (of order m) and μ, a row vector of output multipliers (of order n), having V, U and ε the same meaning as for employment multipliers. In the next section, equation (10) will also be estimated for the Andalusian economy in 1995.

For both the employment and the output multipliers the huge sample size justifies our normality assumption, by the Central Limit Theorem.

3. Results

The sample used by IEA (1999) to construct published use and make tables, covered nearly 45 percent of total regional outputs and more than a third of total employment. The IEA completed the initial survey-based information on industries' disaggregated turnovers and purchases with other statistical sources from the National Statistical Institute (INE), the Central Balance Sheet Office and public institutions (health services, government budget data, education, agriculture, etc) to achieve such large sample coverage. However, while most observations were establishments, some industries' data had to be consolidated (see Table 1). Since no establishments can be identified, Public Administration is treated as a single observation. For data availability reasons agriculture, public hospitals, public education services and public social services are also consolidated, among others. Thus, the sample size of our analysis is 18,084 observations.

The official input–output table (IEA, 1999) was compiled using the commodity technology model as a first estimation. Next, changes were made

TABLE 1

Employment Multipliers (number of workers per 600,000 Euros)

	Description	Sample Size	Estimated Multiplier	Leontief Inverse Multiplier	p Value	Lower Bound	Upper Bound	Industry Share* (%)	Commodity Share** (%)
1	Fruit and vegetables	1	23.9	24.1	0	23.3	24.6	3.0	0.2
2	Olive and vine	1	21	21.2	0	20.5	21.5	5.1	0.1
3	Other agriculture and related services	1	14.6	14.8	0	14.2	15.1	2.9	15.9
4	Livestock and hunting	1	21.1	21.3	0	20.7	21.5	1.4	10.2
5	Forestry and related services	1	30	30.3	0	29.5	30.5	6.0	7.7
6	Fish and fishing products	173	11	18.3	0	9.7	12.4	24.7	0.0
7	Coal mining	4	7.2	13.7	0	5	9.3	3.6	0.0
10	Metallic minerals	7	7	8.8	0	4.2	9.8	28.2	0.0
11	Non-metallic and non-energetic minerals	1	11	10.9	0	10.1	12	3.4	2.2
12	Meat and meat products	239	6.4	9.8	0.0001	3.2	9.7	8.0	2.4
13	Canned and preserved fish, fruit and vegetables	148	15	17.5	0	12.1	17.9	3.8	12.5
14	Fats and oils	252	3.4	14.5	0	2.2	4.7	3.7	1.0
15	Milk and dairy products	34	11.5	11.6	0	6.5	16.4	5.6	21.7
16	Grain mills, bakery, sugar mills, etc	469	4.6	15.6	0	2.7	6.5	3.3	4.0
17	Miscellaneous food products	139	6	6.8	0.001	2.4	9.5	12.6	4.8
18	Wine and alcoholic beverages	105	7.7	11.7	0	6.2	9.3	7.8	0.9
19	Beer and soft drinks	23	5.5	7.2	0	4.7	6.3	0.7	3.3
20	Tobacco products	3	5.4	7	0.0478	0.1	10.7	0.0	0.0
21	Textile mill products	1	10.6	10.5	0	10.2	11	4.1	1.4
22	Clothing products	483	4.4	18.4	0.0186	0.7	8	1.4	0.2
23	Leather tanning, leather products and footwear	135	7.2	18.5	0.0013	2.8	11.5	2.9	0.5
24	Cork and wood products	299	8	22.6	0.0413	0.3	15.7	2.8	1.0
25	Paper and allied products	51	3.6	7.1	0	2	5.1	2.5	2.3
26	Printing, publishing and editing services	447	33.9	10.7	0.0868	–4.9	72.7	28.2	1.7
27	Petroleum refining products	9	2	1.6	0	1.5	2.6	2.9	0.5
28	Basic chemical products	52	2.2	5.1	0.0079	0.6	3.7	2.2	7.2
29	Other chemical products	96	5	8.7	0.0185	0.8	9.2	15.4	3.6
30	Rubber and plastic products	122	3.3	9.1	0.0001	1.6	4.9	3.2	2.4
31	Cement, lime and allied products	200	4.3	10.2	0.0018	1.6	7	2.1	1.1
32	Ceramics, clay, bricks and other products for building	126	11.7	17.7	0	8.6	14.7	1.5	2.6
33	Stone and glass products	282	7.1	15.5	0	5.2	9.1	1.2	1.4

TABLE 1 (*continued*)

	Description	Sample Size	Estimated Multiplier	Leontief Inverse Multiplier	p Value	Lower Bound	Upper Bound	Industry Share* (%)	Commodity Share** (%)
34	Primary metal products	58	0.8	2.3	0.005	0.2	1.3	2.1	0.2
35	Fabricated metal products	1,087	6.6	15.6	0.0001	3.3	9.9	1.2	3.6
36	Machinery and mechanic equipment	1	14.1	14	0	13.4	14.8	4.2	1.9
37	Computers and office equipment	1	5.5	5.9	0	4.7	6.3	0.2	5.0
38	Electrical and electronic machinery	88	3.6	8.6	0.0166	0.7	6.6	0.8	3.1
39	Electronic materials, radio and television equipment	1	7.7	7.5	0	6.9	8.5	3.1	0.0
40	Professional and scientific instruments	60	3.6	9.2	0.0664	–0.2	7.4	1.1	1.8
41	Motor vehicles transportation equipment	45	11.3	8.6	0	6.4	16.2	1.4	0.4
42	Naval transportation and repairing services	65	2.3	15.4	0.1319	–0.7	5.4	2.3	0.2
43	Miscellaneous transportation equipment	22	1.5	10.5	0.2899	–1.2	4.2	2.6	0.2
44	Furniture	601	9.7	23.1	0.0017	3.7	15.8	0.5	2.7
45	Miscellaneous manufactured products	1	14.6	14.6	0	13.9	15.3	1.6	1.5
46	Recycling products	1	9.8	11.1	0	8	11.5	0.0	0.0
47	Electricity and irrigation services	51	2.8	3.4	0	2.6	2.9	1.8	0.6
48	Gas and water steam and irrigation services	1	3.5	3.8	0	2.1	5	1.1	0.2
49	Water and sewerage services	67	10.8	11.8	0	8.1	13.5	6.5	18.2
50	Construction	1,136	7	12.2	0	5.2	8.7	1.8	3.4
51	Preparing, installation and finishing construction services	1	18.3	18.4	0	17.7	18.8	9.4	4.1
52	Petrol and motor vehicles trade services	380	18.1	17.6	0.1137	–4.3	40.4	14.8	0.0
53	Repair motor vehicles services	1	15.9	16.3	0	15.5	16.3	2.0	6.4
54	Wholesale trade	4,535	3.4	12.4	0.3184	–3.3	10.2	1.2	3.9
55	Retail trade and repair domestic and personal effects	4,113	42.9	22.1	0.1186	–11	96.8	4.5	0.6
56	Hotel services	20	19.4	14.1	0	12.8	26	26.3	3.3
57	Bar and restaurant services	352	31.1	18.8	0	23.2	39.1	1.2	7.5
58	Railway transportation services	1	10.6	11.1	0	9.9	11.2	2.6	0.0
59	Other earthbound transportation services	1	17	17.1	0	16.1	17.8	1.7	15.6
60	Sea and river transportation services	2	7.2	13.7	0	5	9.4	0.0	0.0
61	Air transportation services	1	8.4	7.6	0	4.7	12.1	19.9	0.0
62	Allied transportation services	142	8.1	13.6	0.0007	3.4	12.8	4.4	3.2

63	Post and communication services	205	5.2	10.4	0	4.7	5.8	1.1	5.2
64	Finances	1	8.7	9.1	0	7.9	9.5	0.0	0.0
65	Insurance	1	12.1	15.9	0	10.3	13.9	0.0	0.0
66	Allied financial services	1	6.4	15.4	0.0021	2.3	10.5	0.1	0.7
67	Real estate	1	2.1	2	0	1.6	2.6	0.9	4.7
68	Machinery and equipment rental	9	8.8	11	0.0001	4.5	13.2	3.0	2.1
69	Computer services	84	15.1	12.7	0.3185	–14.6	44.9	12.8	40.3
70	Research and development	1	20.1	19.9	0	19.5	20.6	0.0	0.0
71	Accounting and law activity services	235	72	18.1	0.106	–15.3	159.3	12.8	19.9
72	Engineering and architectural technical services	31	23.2	16.5	0.0203	3.6	42.8	0.5	8.9
73	Marketing services	8	1.5	21.5	0.3509	–1.7	4.7	3.2	81.9
74	Security services	48	23.5	31.6	0.0004	10.5	36.4	1.6	16.7
75	Cleaning services	1	32.7	32.1	0	31.6	33.7	1.4	0.3
76	Other business services	1	4.3	19.7	0.7309	–20.3	29	42.7	41.5
77	Public administration	1	19	18.6	0	18.2	19.8	7.6	0.0
78	Public education services	2	18.3	17.5	0	17.4	19.2	0.2	0.0
79	Private education services	150	31.8	28.1	0	29.8	33.8	10.5	1.2
80	Public medical and hospital services	1	16.6	16.8	0	16.4	16.8	0.0	0.0
81	Private medical and hospital services	4	4.9	7.6	0.0008	2	7.8	0.3	0.1
82	Public social services	1	25.6	25.7	0	25.4	25.9	0.0	0.0
83	Private social services	1	40.2	40.6	0	39.2	41.2	2.8	1.5
84	Public drainage and sewerage services	15	23.8	16.5	0	20.5	27.1	1.7	36.4
85	Social services	1	22.2	18.8	0	14.2	30.2	25.8	0.0
86	Cinema, video, radio and television services	1	29.1	5.9	0	25.2	33	44.0	3.6
87	Other amusement, cultural, sport and recreation services	538	9.6	12.4	0	8.4	10.9	35.3	4.1
88	Personal services	1	31.9	32.6	0	31.6	32.3	0.4	4.0
89	Household employers services	1	116.2	116.2	0	116.2	116.2	0.0	0.0

Source: Own elaboration and IEA (1999).
*Secondary product share of industry outputs. **Secondary product share of commodity outputs.

manually depending on the resulting negatives and their possible causes, i.e. errors in use and make tables, heterogeneity of the industry classification and vertical integration, where sometimes the industry technology assumption was preferable. Nevertheless, for comparison purposes, the Leontief inverse based input–output multipliers shown in Table 1 were not constructed on the basis of the official A matrix but on a pure commodity technology basis. This means that equations (3) and (7) were computed using published use and make tables and $A = U(V^{\mathrm{T}})^{-1}$.

Labor data are expressed in full-time equivalent numbers of workers. This is survey based for each industry and has been checked by IEA using other regional and national labor statistics.

Employment Multipliers

The employment multipliers estimates are presented in Table 1. For comparison, the second column displays the employment multipliers based on published use and make tables under the commodity technology assumption.

The model has been estimated for 87 commodities by means of ordinary least squares. The resulting R-squared is 0.9948, which is quite satisfactory. Due to the presence of certain forms of unknown heteroskedasticity, the White estimate (White, 1980) of the covariance matrix of estimated coefficients was used to provide consistent and robust standard errors. We find that problems of autocorrelation (not expected in cross-sectional data) and multicollinearity do not plague our analysis. Only 12 of the 7,482 (87×86) possible off-diagonal elements of a correlations matrix with 87 different explanatory variables were higher than 0.5, with only one higher than 0.75. Eventually, 76 estimated multipliers are significant at the 95 percent confidence level. All remaining estimators are assumed to be zero (no impact) at the same confidence level, since the null hypothesis is accepted in each one of the cases. Negative values of multipliers are insignificant.

Three major contributions are provided by the results presented in Table 1:

(1) In most cases, the Leontief inverse based employment multipliers *over*-estimate the true values. Indeed, 57 of 87 commodities have lower employment multipliers than those calculated with the published use and make matrices. On the other hand, 19 commodities have higher employment multipliers. Our findings contradict the underestimation of the Leontief inverse found in Simonovits (1975), or rather its restrictive assumptions, such as the independence of technical coefficients, and firm up the conclusions of Dietzenbacher (1995) and Roland-Holst (1989).

(2) Input–output estimates are unbiased and consistent, providing confidence intervals for employment multipliers. These intervals may be seen as a measure of the true estimates of the accuracy of multipliers. About 56 percent of the published based input–output multipliers values are included.

(3) The estimated bias of employment multipliers is generally positively related with secondary production. Commodities that are the primary output of industries with sizeable secondary production and commodities

of which a large share is produced as secondary output have employment multipliers with larger estimated bias, as measured by the difference between the published data based and our estimated employment multipliers. Estimated multipliers not significant at the 95 percent confidence level are set zero, and therefore the estimated bias equals the official data based multiplier.

The Spearman's rank correlation between estimated and official data based multipliers is 0.65. Additionally, four out of the six most prominent rank reversals are insignificant at the 95 percent confidence level, i.e. other business services have large proportions of secondary activities (42.74 percent), their primary products are produced elsewhere in sizeable amounts (41.45 percent) and nearly 82 percent of marketing services are produced by other industries. Also, just one out of the seven commodities with highest estimated bias (in absolute values) are significant at a 5 percent significance level, i.e. cinema, video, radio and television services, where the secondary activities of the corresponding industry represent almost 44 percent of their total production.

From a theoretical view, when some industries with no secondary activities produce commodities for which other industries provide sizeable amounts, it is reasonable to assume that the technologies used by the rest of the economy for making such commodities should not match that of the industries for which they are primary products (the latter industries' technology can be considered as commodities technology since no secondary products are involved). This could explain the sizeable estimated bias of marketing services, computer services and public drainage and sewerage services.

On the contrary, when some commodities are produced by a single industry with large proportions of secondary outputs, it is reasonable to assume that these primary commodities are not produced according to a commodity technology hypothesis if the estimated bias is sizeable. Such is the cases for cinema, video, radio and television services and printing, publishing and editing services.

Output Multipliers

Maintaining the number of observations of the last section, the output multipliers are as presented in Table 2.

The proposed model has been estimated for 87 commodities by means of ordinary least squares and, as before, with quite satisfactory goodness of fit (R-squared equal to 0.9993). The White (1980) estimated covariance matrix of estimated regression coefficients was used to obtain consistent standard errors. The model is free from serial correlation (as expected in cross-sectional data) and multicollinearity issues. This time, 84 estimated multipliers are significant at a 5 percent significance level. The same as in employment multipliers holds for non-significant estimates and negative values.

The output multipliers provide similar results as the employment multipliers presented above. These are:

(1) Mostly, official data based output multipliers are *over*estimated and not underestimated. It is remarkable that 73 of 87 commodities have lower output multipliers than published data based multipliers, whilst 11

TABLE 2

Output Multipliers

	Description	Estimated Multiplier	Leontief Inverse Multiplier	p Value	Lower Bound	Upper Bound
1	Fruit and vegetables	1.266	1.323	0	1.3	1.3
2	Olive and vine	1.142	1.179	0	1.1	1.2
3	Other agriculture and related services	1.226	1.264	0	1.2	1.2
4	Livestock and hunting	1.325	1.357	0	1.3	1.3
5	Forestry and related services	1.224	1.262	0	1.2	1.2
6	Fish and fishing products	1.163	1.23	0	1.0	1.3
7	Coal mining	0.724	1.799	0.0325	0.1	1.4
10	Metallic minerals	1.337	1.365	0	1.3	1.4
11	Non-metallic and non-energetic minerals	1.523	1.562	0	1.5	1.6
12	Meat and meat products	1.447	1.664	0	1.2	1.7
13	Canned and preserved fish, fruit and vegetables	1.755	1.872	0	1.6	2.0
14	Fats and oils	1.439	1.949	0	1.3	1.5
15	Milk and dairy products	1.654	1.603	0	1.4	1.9
16	Grain mills, bakery, sugar mills, etc	1.201	1.534	0	1.1	1.3
17	Miscellaneous food products	1.3	1.402	0	1.1	1.5
18	Wine and alcoholic beverages	1.433	1.696	0	1.3	1.5
19	Beer and soft drinks	1.309	1.417	0	1.2	1.4
20	Tobacco products	1.117	1.159	0	1.0	1.3
21	Textile mill products	1.273	1.289	0	1.3	1.3
22	Clothing products	1.165	1.336	0	1.0	1.4
23	Leather tanning, leather products and footwear	1.177	1.366	0	1.1	1.3
24	Cork and wood products	0.679	1.431	0.0069	0.2	1.2
25	Paper and allied products	1.117	1.314	0	1.0	1.2
26	Printing, publishing and editing services	0.507	1.243	0.1699	–0.2	1.2
27	Petroleum refining products	1.271	1.245	0	1.2	1.3
28	Basic chemical products	1.206	1.621	0	1.0	1.4
29	Other chemical products	1.031	1.287	0	1.0	1.1
30	Rubber and plastic products	0.921	1.29	0	0.8	1.1
31	Cement, lime and allied products	1.183	1.739	0	1.0	1.4
32	Ceramics, clay, bricks and other products for building	1.182	1.415	0	1.1	1.3
33	Stone and glass products	1.299	1.546	0	1.2	1.4
34	Primary metal products	1.058	1.2	0	1.0	1.1
35	Fabricated metal products	1.04	1.272	0	0.9	1.2

36	Machinery and mechanic equipment	1.266	1.284	0	1.2	1.3
37	Computers and office equipment	1.296	1.352	0	1.3	1.3
38	Electrical and electronic machinery	1.045	1.156	0	1.0	1.1
39	Electronic materials, radio and television equipment	1.17	1.202	0	1.2	1.2
40	Professional and scientific instruments	1.043	1.155	0	1.0	1.1
41	Motor vehicles transportation equipment	1.318	1.243	0	1.1	1.5
42	Naval transportation and repairing services	1.024	1.502	0	1.0	1.0
43	Miscellaneous transportation equipment	1.032	1.232	0	1.0	1.1
44	Furniture	1.21	1.464	0	1.1	1.4
45	Miscellaneous manufactured products	1.34	1.367	0	1.3	1.4
46	Recycling products	1.346	1.493	0	1.2	1.5
47	Electricity and irrigation services	1.072	1.434	0	1.1	1.1
48	Gas and water steam and irrigation services	1.05	1.147	0	0.9	1.2
49	Water and sewerage services	1.293	1.448	0	1.1	1.5
50	Construction	1.353	1.67	0	1.2	1.5
51	Preparing, installation and finishing construction services	1.352	1.374	0	1.3	1.4
52	Petrol and motor vehicles trade services	–0.168	1.295	0.5752	–0.8	0.4
53	Repair motor vehicles services	1.197	1.232	0	1.2	1.2
54	Wholesale trade	0.593	1.253	0	0.3	0.8
55	Retail trade and repair domestic and personal effects	0.917	1.391	0	0.8	1.0
56	Hotel services	1.264	1.329	0	1.1	1.4
57	Bar and restaurant services	1.355	1.523	0	1.2	1.5
58	Railway transportation services	1.22	1.252	0	1.2	1.2
59	Other earthbound transportation services	1.422	1.465	0	1.4	1.4
60	Sea and river transportation services	1.876	2.256	0	1.7	2.0
61	Air transportation services	1.321	1.304	0	1.2	1.5
62	Allied transportation services	1.106	1.607	0	1.0	1.2
63	Post and communication services	1.069	1.145	0	1.1	1.1
64	Finances	1.183	1.193	0	1.2	1.2
65	Insurance	1.711	1.74	0	1.7	1.8
66	Allied financial services	1.421	1.431	0	1.3	1.5
67	Real estate	1.082	1.088	0	1.1	1.1
68	Machinery and equipment rental	1.151	1.236	0	1.1	1.3

TABLE 2 (*continued*)

	Description	Estimated Multiplier	Leontief Inverse Multiplier	p Value	Lower Bound	Upper Bound
69	Computer services	1.597	1.231	0.0001	0.8	2.4
70	Research and development	1.076	1.086	0	1.0	1.1
71	Accounting and law activity services	2.465	1.477	0.069	–0.2	5.1
72	Engineering and architectural technical services	1.156	1.409	0	0.8	1.5
73	Marketing services	1.046	1.68	0	0.9	1.2
74	Security services	1.227	1.094	0	0.8	1.7
75	Cleaning services	1.104	1.11	0	1.1	1.1
76	Other business services	1.002	1.427	0.03	0.1	1.9
77	Public administration	1.309	1.317	0	1.3	1.3
78	Public education services	1.027	1.051	0	1.0	1.0
79	Private education services	1.193	1.284	0	1.1	1.2
80	Public medical and hospital services	1.13	1.142	0	1.1	1.1
81	Private medical and hospital services	1.214	1.208	0	1.2	1.3
82	Public social services	1.178	1.195	0	1.2	1.2
83	Private social services	1.471	1.524	0	1.4	1.5
84	Public drainage and sewerage services	1.165	1.346	0	1.1	1.2
85	Social services	2.136	2.124	0	1.9	2.4
86	Cinema, video, radio and television services	2.097	1.411	0	1.9	2.3
87	Other amusement, cultural, sport and recreation services	1.689	1.457	0	1.7	1.7
88	Personal services	1.385	1.443	0	1.4	1.4
89	Household employers services	1	1	0	1.0	1.0

Source: Own elaboration and IEA (1999).

commodities have higher output multipliers. Again, most of the output multipliers obtained by using published data are overestimated and not underestimated, confirming Dietzenbacher (1995) and Roland-Holst (1989).

(2) Unbiasedness and consistency of estimated input–output multipliers, jointly with derived confidence intervals. In this case, 34.5 percent of published data based multipliers are included within them.

(3) Once again, the estimated bias of output multipliers has a positive relationship with secondary production. The Spearman's rank correlation between official data based and estimated multipliers is 0.51. Moreover, we find that the three products with the highest estimated bias correspond exactly with the three commodities which are insignificant at the 95 percent confidence level, namely accounting and law activity services (12.8 percent of secondary productions and 19.9 percent of services produced elsewhere), printing, publishing and editing services (28.2 percent of secondary outputs) and petrol and motor vehicles trade services (14.8 percent of secondary activities).

It is surprising that the output multipliers are more accurate than the employment multipliers. With only three exceptions, the p-values in Table 2 are smaller than the corresponding ones in Table 1.

Some macro checks have been carried out to test the robustness and coherence of the results by using equations (5) and (9) with our estimated input–output multipliers and the published net outputs matrix. Consequently, the estimated total employment requirements reach 1,871,800 people, which is only 2.38 percent higher than the official value (1,828,400); furthermore, the estimated total outputs, which yield €105,799 million, is just 3.65 percent higher than published total productions (€102,070 million).

4. Conclusions

Technical coefficients are the subject of two disjoint bodies of literature. The construction of technical coefficients is linked to flow data (use and make matrices), but stochastics are imposed on the coefficients when multipliers are calculated, by means of the Leontief inverse. Due the nonlinearity of this operation, the multiplier estimates are biased as it is generally argued that the Leontief inverse underestimates input–output multipliers.

In this paper, we let the flow data tell the stochastics and take them all the way to confidence intervals for multipliers. We focus on the use and make matrices instead of the *A*-matrix to obtain unbiased and consistent multipliers estimates. Our output and employment multipliers are normally distributed and do not suffer from over- or underestimation. Our results for the Andalusian economy indicate that the Leontief inverse is not underestimated but overestimated in most cases.

Statistical offices combine use and make flow data (including inversion of the make matrix) to construct input–output coefficients, and economists invert the Leontief matrix to determine the output and cost multipliers of the economy. The construction and the inversion are nonlinear operations with complicated errors transmission and have been studied in relative isolation. This paper shows, however, that an integrated analysis, from the use and make data directly to the multipliers, provides simple, unbiased and consistent estimates.

Appendix: Data

The Andalusian Input–Output Framework 1995 (MIOAN95) is one of the first Spanish input–output tables based on the new European System of Accounts (ESA-95) published by EUROSTAT (1996). The IEA provided the cross-section inputs and outputs establishment data. These data were used for the elaboration of the Input–Output Andalusian Framework 1995 (IEA, 1999).

IEA publishes two use tables, which differ by valuation. One is valued at purchasers' prices and the other at basic prices, which is the same as the former but excluding trade and transport margins and net commodity taxes (see Viet, 1994, p. 28). (Trade and transport margins are simply reallocated from the commodities where they are included, at purchasers' values, to the use matrix rows of trade and transport services.) The make table is published exclusively at basic prices. The United Nations System of National Accounts (SNA) recommends basic values; production costs of good and services are measured before they are conveyed to the market for consumption so that the effects of tax and subsidy policies as well as of differences in types of economic transactions are isolated.

IEA transforms use data at purchasers' prices into basic prices, as described below. The use and make tables at basic prices are balanced to obtain the final official accounts for the input–output framework.

Since all input and output data provided by IEA were valued at purchasers' prices and at basic prices, respectively, we subtracted trade and transport margins and also net commodity taxes from establishment inputs in order to have the same valuation (basic prices) for inputs and outputs and to estimate equations (6) and (9). As detailed below, we applied the same formula, formalized here for the first time, as IEA used for the elaboration of the use matrix at basic prices and assumed equality of margins and net commodity taxes between establishments in industry *j*, which consume some commodity *k*. We will focus now on the procedure.

According to the ESA-95, the intermediate uses at basic values are equal to the intermediate uses at purchasers' prices minus trade and transport margins and minus net commodity taxes. Let u_{kj}^{b} and u_{kj}^{p} be the total inputs of commodity *k* by industry *j* (excluding imports) at basic and at purchasers' prices, respectively. Then, we can write out that:

$$u_{kj}^{b} = u_{kj}^{p} - T_{kj}^{d} - T_{kj} - N_{kj} - H_{kj}, \tag{A1}$$

where, for each use of commodity *k* by industry *j*, T_{kj}^{d} and T_{kj} are the total amount of trade and transport margins, respectively, N_{kj} is the total amount of net commodity taxes (excluding non-deductible VAT) and H_{kj} is the total amount of non-deductible VAT.

We will assume that the trade margins are proportional to the use data at purchasers' prices. The proportions are defined by:

$$T_{kj}^{d} = t_{kj}^{d} u_{kj}^{p}, \quad 0 < t_{kj}^{d} < 1. \tag{A2}$$

Next, we will assume that net commodity taxes (excluding non-deductible VAT) and transport margins are proportional to the use data at basic prices:

$$N_{kj} = n_{kj} u_{kj}^{b}, \quad 0 < n_{kj} < 1; \tag{A3}$$

$$T_{kj} = t_{kj} u_{kj}^{b}, \quad 0 < t_{kj} < 1. \tag{A4}$$

With respect to VAT, the hypothesis is as follows:

$$H_{kj} = h_{kj}\left(\frac{u_{kj}^{p}}{1+h_{kj}}\right), \quad 0 < h_{kj} < 1. \tag{A5}$$

Then, by substituting (A2), (A3), (A4) and (A5) in (A1), we obtain:

$$u_{kj}^{b} = u_{kj}^{p}\left(\frac{1 - t_{kj}^{d} - \dfrac{h_{kj}}{1+h_{kj}}}{1 + t_{kj} + n_{kj}}\right). \tag{A6}$$

Yet, this formula would be used to transform use data from basic values to purchasers' values when dealing with industries but not with establishments. However, our purpose is to estimate the unknown u_{kji}^{b}, that is, the total use of commodity k by an establishment i from industry j at basic prices. Then, since survey available data is based on establishments of a particular industry and not on products, we denote u_{kji}^{p} and u_{kji}^{b} as purchasers' and basic prices use data, respectively. Based on (A6), our objective would be to apply the following formula for each establishment, i:

$$u_{kji}^{b} = u_{kji}^{p}\left(\frac{1 - t_{kji}^{d} - \dfrac{h_{kji}}{1+h_{kji}}}{1 + t_{kji} + n_{kji}}\right). \tag{A7}$$

A problem arises when available information does not enable us to value establishment specific t_{kji}^{d}, h_{kji}, t_{kji} and n_{kji}. In this case, we will assume equality of margins and net commodity taxes across firms in industry j, which consumes some commodity k. We consequently use (A7) with:

$$t^d_{kji} = t^d_{kj} \quad \textit{for all } i,$$
$$t_{kji} = t_{kj} \quad \textit{for all } i,$$
$$n_{kji} = n_{kj} \quad \textit{for all } i,$$
$$h_{kji} = h_{kj} \quad \textit{for all } i,$$

so that the formula becomes:

(A8) $$u^b_{kji} = u^p_{kji}\left(\frac{1 - t^d_{kj} - \dfrac{h_{kj}}{1 + h_{kj}}}{1 + t_{kj} + n_{kj}}\right).$$

Once trade and transport margins and net commodity taxes have been subtracted from use flow data, the last step would be to reallocate the subtracted total trade and domestic transport margins to trade and transport industries, respectively. This was done with the help and technical support of IEA.

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