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HOUSEHOLD CHARACTERISTICS AND THE DISTRIBUTION OF INCOME IN ITALY: AN APPLICATION OF SOCIAL DISTANCE MEASURES

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The Lorenz criterion of preferable distributions fails to distinguish adequately between convergence to the global mean and clustering around local means. This concern has motivated independent work by Wolfson, and Esteban and Ray on the notion of polarization. In this paper I build on this recent work by providing a new method that characterizes changes in the entire distribution, rather than focussing only on dispersion. In particular, the approach proposed offers a new decomposition method of within- and between-group components that differs from the classical method of additively decomposable inequality indices. The new method can monitor which factors modified the entire distribution, where precisely on the distribution these factors had an effect, and what determined the variation in the level of social distance between groups or geographic areas. Summary statistics of the observed movements and of distance between and divergence among the estimated and the counterfactual distributions are provided as well as a new index of social distance. The new method is then applied to Italian data on income distribution between 1987 and 1995.

1. INTRODUCTION

The distribution of welfare has always been one of the main issues in economics. On the one hand, theorists have focussed on developing measures that satisfy appealing properties; on the other, applied researchers have used these measures to analyze the welfare of different societies. In his seminal work, Lorenz (1905) proposed a criterion to rank distributions. According to this, one distribution is preferred to another if the first can be obtained by a Pigou–Dalton transfer from the second (Atkinson, 1970). The worst possible distribution is that where there is only one individual who possesses everything; the best is that where the total amount of resources is shared equally among the members of a given society.

The Lorenz criterion has two major drawbacks. First, it offers a measure of inequality from the perspective of an impartial observer, an objective measure, which fails to take into account how people of a society perceive the level of inequality. Second, it fails to distinguish adequately between convergence to the global mean and clustering around local means.

The latter concern has motivated independent work by Wolfson (1994) and Esteban and Ray (1994) who have conceptualized the notion of polarization. The goal of this paper is to build on this recent work by providing a new method that characterizes changes in the entire distribution, rather than focus only on

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dispersion. In particular, I propose a new decomposition technique of *within*- and *between*-group components and introduce several indices for summarizing the results.

In keeping with the purposes of this paper, my approach to the decomposition analysis differs from the classical approach of additively decomposable inequality indices. If the purpose of the analysis is to understand what determines the variation in relative inequality, then the decomposition of indices that belong to the generalized entropy family is the most appropriate. If, on the other hand, the aim is to understand which factors modified the entire distribution, on which portion of the distribution these factors had an effect, and what determined the observed variations in social distance between groups in a given society, then the non-classical method described here is far better suited to the task.

The proposed technique is non-parametric. I use kernel density estimation methods to obtain an estimate of the income distribution and of its evolution through time, for the whole population and for its subgroups, without imposing any assumption on the distribution of the observed data. I then estimate *counterfactual* densities, i.e. what the density of income would have been in one year if household characteristics (between-group component) or the distribution of income among households with the same characteristics (within-group component) had remained at the level of the previous year by applying the same kernel method. Finally, I summarize the observed movements in the distribution with two kinds of indices. The first type of index (coefficients of distance and divergence) calculates the changes to the density of a given group over time, i.e. it measures the difference between any two given densities. The second index (the polarization index) calculates the existing "distance" between given groups within a society at one point in time; this index can then be used to track the moving apart of specific groups as classified according to certain household characteristics. For this purpose, I both use the Esteban–Ray polarization index and develop a new index that is better able to measure the actual social distance present in a society.

This paper contains an application of the new method to Italian data on the distribution of income.[1] The Italian case represents an interesting context for applying the decomposition. The early 1990s were a period of significant changes for Italy: from 1989 to 1993, the Italian economy was in recession and these years were accompanied by politics aimed at decreasing public spending and improving the economy's performance. Some important reforms took place: in the labor market, in 1992–93, unions, government and industrial employers reached agreements that affected bargaining and wage determination; in the financial and goods markets, the 1993 Single European Market established the free movement of goods, persons, services and capitals. In addition, these years experienced a boom of a particular kind of "work and train" contract for young workers, a contract that implied that young workers, generally more qualified, were paid relatively less. The effects of these changes have been analyzed in several studies but the main focus has been on the synthetic measures of dispersion of rather

[1] I will assume income as a proxy for welfare and not consumption, as I believe that it is important to distinguish, for example, between consumption of income and consumption of wealth, and between those who are starving and those who are fasting.

than the changes to the entire distribution of income. Furthermore, Italy can also be characterized by a "natural" model of social distance based on the geographic location of individuals. The great disparities existing in Italy between its geographic areas are, indeed, very noticeable.[2]

The study on Italian equivalent income among persons reveals that the social distance between individuals residing in different regions is pronounced and has increased a great deal over time. The distribution of income throughout the country became more dispersed due to a different behavior of the geographic areas. What caused these movements? Can we measure the increase in the social distance between regional groups in the Italian society and discover what determined it? This paper offers a tentative answer to these questions.

The main result of the decomposition method is that changes in household characteristics did not have a significant influence on the evolution of the Italian density of income within each geographic area during the period under examination. Most of the observed variation can be attributed to the within-group income schedule that underwent a dramatic change. The exceptions are in some areas changes in the employment status of heads of household and in the number of earners due, respectively, to a rise in the proportion of retired and unemployed heads, and to an increase in both the female participation rate and in the tendency of adult working children to leave the family later.

Within the densities of the different groups, we can see a common pattern across the different geographic areas, in that the groups whose densities underwent the largest changes in the years of analysis are the same. However, the densities of these groups changed in different ways in different areas.[3]

The most striking result is the effect that the pension system had on the densities of retired heads of the households. The retired heads in the north and to a slightly smaller extent in the center experienced an improvement of their relative position. In contrast, the pension system did not at all favor the households of the south.

Another effect that distinguishes the south from the north and the center is its large increase in the dispersion of the density of the two-earners group. Throughout Italy, female participation rate increased, but in the south the contribution of the additional earner to total household income was not high enough to bring these new two-earner households in line with those households that had already belonged to this category in 1987.[4]

The rest of the paper is organized as follows: in the next two sections I introduce the decomposition method and the non-parametric technique that will be used to estimate the densities. Section 4 discusses the indices used to summarize

[2]These are characterized by differences in industrial development and composition (less development accompanied by firms with lower technology, smaller dimensions, lower per-worker productivity in the south), in unemployment rate (higher in the south especially among the young), in the composition of the population in terms of number of family components (more children in the south), in the average age of the population (younger in the south), in the average number of earners (lower in the south and lower female participation rate in the south), implying that the effect of the reforms and the performance of the economy was probably not the same in all areas.

[3]The exception to this is the self-employed group whose density moved towards lower levels of income throughout Italy.

[4]This result is consistent with the findings of Nizzoli and Wolleb (1989).

the observed movements in the densities of income. The application of the method to Italy is contained in Section 5. Section 6 concludes the paper.

2. The Estimation Method

The estimation method I use here is optimally derived from a generalization of the kernel density estimator to take into account the sample weights attached to each observation, namely, from the *adaptive* or *variable kernel*.

The adaptive kernel is built with a two-stage procedure: a density is determined in the first stage in order to obtain the optimal bandwidth parameter; in the second stage, the final density is computed. The estimate of the density function, $\hat{f}(y)$, is determined directly from the data of the sample, $y_1, y_2, \ldots, y_N$, without assuming its functional form *a priori*. The only assumption made is that there exists a density function $f(y)$, from which the sample is extracted. In detail, the estimated density in the first stage is:

$$\tilde{f}(y_j) = \sum_{i=1}^{N} \frac{\theta_i}{h_N} K\left(\frac{y_j - y_i}{h_N}\right) \qquad \forall y_j \tag{1}$$

while the final estimate is:

$$\hat{f}(y_j) = \sum_{i=1}^{N} \frac{\theta_i}{h_N \lambda(y_i)} K\left(\frac{y_j - y_i}{h_N \lambda(y_i)}\right) \qquad \forall y_j \tag{2}$$

where N is the number of observations of the sample, h_N is the bandwidth parameter, $K(\cdot)$ is the kernel function,[5] $\lambda(y_i) = \{\tilde{f}(y_i)/g\}^{-1/2}$ where g is the geometric mean of $\tilde{f}(y_i)$. The sample weights are normalized to sum to one, $\Sigma_i \vartheta_i = 1$.

I estimate the density functions of the logarithm of income for two reasons:

- The kernel estimator has difficulties handling densities that have a high degree of asymmetry. It is possible to show that the smallest *MISE* depends on f through $R(f'') = \int f''(y)^2\, dy$, which is a measure of the total curvature of f. The magnitude of this quantity gives an indication of how well f can be estimated even when h_N is chosen optimally. Hence for a density with high skewness, kurtosis, several modes, $|f''(y)|$ will assume relatively high values implying a larger value $R(f'')$. It has been shown[6] that densities close to normality appear to be easiest for the kernel estimator to estimate. As the density of the level of income resembles a Log-normal its logarithm will, hence, be similar to a Normal.
- I am interested in the movements of the distributions over time. These can be detected more easily by shrinking the tails present in the distribution of the level of income.

The counterfactual densities are obtained by applying the kernel method to appropriate samples. This technique has been derived from the one proposed by DiNardo, Fortin and Lemieux (1996).

Each observation is actually a vector $(y, z|t_y, t_z)$, composed of income y, a vector z of household characteristics and a date t at which, respectively, income

[5] In this paper the kernel function used is the normal.
[6] For the proof see Wand and Jones (1995).

and characteristics are observed, belonging to a joint distribution $F(y, z|t_y, t_z)$. The marginal density of income at one point in time, $f^t(y)$, can be obtained by integrating the density of income conditional on a set of household characteristics and on a date t, $f(y|z, t_y, t_z)$, over the distribution of household characteristics $F(z|t_y, t_z)$ at the date t:

$$(3) \qquad f^t(y) = \int_{z \in \Omega_z} dF(y, z | t_y = t, t_z = t)$$

$$= \int_{z \in \Omega_z} f(y | z, t_y = t, t_z = t)\, dF(z | t_y = t, t_z = t)$$

$$\equiv f(y | t_y = t, t_z = t)$$

where Ω_z is the domain of definition of household characteristics.

If all the variables are observed at two different times, e.g. t_1 and t_2, then two counterfactual densities can be obtained from (3): the counterfactual density of income at t_1 and characteristics at t_2, represented by $f(y|t_y = t_1, t_z = t_2)$:

$$(4) \qquad f(y | t_y = t_1, t_z = t_2) = \int_{z \in \Omega_z} dF(y, z | t_y = t_1, t_z = t_2)$$

$$= \int_{z \in \Omega_z} f(y | z, t_y = t_1, t_z = t_2)\, dF(z | t_y = t_1, t_z = t_2)$$

and analogously the counterfactual density of income at t_2 and characteristics at t_1.

Under the assumption that the structure of income conditional on the distribution of household characteristics does not depend on the time of the household characteristics:

$$(5) \qquad f(y | z, t_y = t_1, t_z = t_2) = f(y | z, t_y = t_1, t_z = t_1)$$

and under the assumption that the distribution of household characteristics conditional on the time of the characteristics does not depend on the date when incomes are observed:

$$(6) \qquad F(z | t_z = t_2, t_y = t_1) = F(z | t_z = t_2, t_y = t_2)$$

then the counterfactual density of income at t_1 and characteristics at t_2 is:

$$(7) \qquad f(y | t_y = t_1, t_z = t_2) = \int_{z \in \Omega_z} f(y | z, t_y = t_1)\, dF(z | t_z = t_2).$$

This counterfactual density indicates the density that would have prevailed if household characteristics had remained at their t_2 level and if the household income distribution had been the one observed in t_1 for households with those characteristics. General equilibrium effects are, indeed, excluded from the analysis, as the effects of changes in the distribution of z on the structure of income are not taken into account. What I estimate is the effect of movements between groups on the total density of income under the assumption that the distributions within each group do not change over time.

Assuming instead that:

$$f(y|z, t_y = t_2, t_z = t_1) = f(y|z, t_y = t_2, t_z = t_2) \tag{8}$$
$$F(z|t_z = t_1, t_y = t_1) = F(z|t_z = t_1, t_y = t_2)$$

the counterfactual density of income at t_2 and characteristics at t_1 is:

$$f(y|t_y = t_2, t_z = t_1) = \int_{z \in \Omega_z} f(y|z, t_y = t_2)\, dF(z|t_z = t_1). \tag{9}$$

This counterfactual density focusses on the within-group component of the observed movements by estimating the effect of changes in the distribution of income among households with the same characteristic on the distribution of income for the whole population, assuming that the household characteristic does not change over time.

The difference between the actual and the counterfactual density represents the effects, on the one hand, of changes in the distribution of the characteristics of the households (between-group component) and, on the other, of changes in the income structure of households with given characteristics (within-group component).

It is clear from equations (7) and (9) that the counterfactual densities can be obtained by estimating[7] the component densities non-parametrically:

- $f(y|z, t_y = t_i)$ is estimated by applying the kernel method to the appropriate sample in year t_i;
- $F(z|t_z = t_i)$ is estimated non-parametrically as the proportion of households with given characteristics in year t_i.

3. The Decomposition Method

For simplicity, in what follows I rewrite (3) for z being a discrete random variable:

$$\begin{aligned} f^t(y) &= \int_{z \in \Omega_z} dF(y, z|t_y = t, t_z = t) \\ &= \int_{z \in \Omega_z} f(y|z, t_y = t, t_z = t)\, dF(z|t_y = t, t_z = t) \\ &= \sum_z \alpha_z^t(y) f_z^t(y) \end{aligned} \tag{10}$$

where $\alpha_z^t(y) = F(z|t_y = t, t_z = t)$, the proportion of households in each group, and $f_z^t(y) = f(y|z, t_y = t, t_z = t)$, the density of income within each group. The total density of income, $f^t(y)$, can change over time, both because there is a movement of households between groups, i.e. values of $\alpha_z^t(y)$ change, and because the structure of income within each group changes, i.e. values of $f_z^t(y)$ vary. The variation in $f(y)$ from t_1 to t_2 is:

[7] An alternative estimation method for the counterfactual density of income at t_1 and characteristics at t_2 is proposed by DiNardo *et al.* (1996).

$$(11) \qquad f^{t_2} - f^{t_1} \cong df(t)\big|_{t=t_1} = f'(t)\,dt\big|_{t=t_1}.$$

From (10):

$$(12) \qquad f'(t) = \sum_z \alpha_z'(t) f_z(t) + \sum_z \alpha_z(t) f_z'(t).$$

Hence (11) is given by:

$$(13) \qquad f'(t)\,dt\big|_{t=t_1} = \sum_z \alpha_z'(t) f_z(t)\,dt\big|_{t=t_1} + \sum_z \alpha_z(t) f_z'(t)\,dt\big|_{t=t_1}.$$

I can approximate the following components of (13) by:

$$(14) \qquad \alpha_z'(t)\,dt \cong \alpha_z(t_2) - \alpha_z(t_1)$$

$$f_z'(t)\,dt \cong f_z(t_2) - f_z(t_1).$$

Hence the variation in f is approximately given by:

$$(15) \qquad f^{t_2} - f^{t_1} \cong \sum_z [\alpha_z(t_2) - \alpha_z(t_1)] f_z(t)\big|_{t=t_1} + \sum_z \alpha_z [f_z(t_2) - f_z(t_1)]\big|_{t=t_1}$$

$$= \underbrace{\left\{\sum_z [\alpha_z(t_2) f_z(t_1)] - \sum_z [\alpha_z(t_1) f_z(t_1)]\right\}}_{\text{between group}}$$

$$+ \underbrace{\left\{\sum_z [\alpha_z(t_1) f_z(t_2)] - \sum_z [\alpha_z(t_1) f_z(t_1)]\right\}}_{\text{within group}}.$$

Each component of (15) can be estimated with the non-parametric method as explained in the previous section.

4. Summary Indices

The coefficients needed to summarize the observed movements are of two kinds. First, an index is needed to take into account the changes in the density of a given group over time, i.e. an index that summarizes how much any two given densities differ: a coefficient of distance or divergence. Second, an index is needed to take into account the existing "distance" between given groups in which a society can be partitioned at one point in time, i.e. an index that tracks the moving apart of some densities classified according to some characteristic of the households: the polarization index.

Several coefficients have been suggested in the statistical literature for measuring the distance and divergence between probability distributions. The approach taken in this work follows Ali and Silvey (1966).

Two probability distributions F_1 and F_2 are given on the real line, with corresponding densities f_1 and f_2, absolutely continuous with respect to Lebesgue measure and with respect to each other. The measures computed belong to a general class based on the ratio of the densities:

$$(16) \qquad \phi(y) = \frac{f_2(y)}{f_1(y)}.$$

If F_1 and F_2 are the same then $\phi(y) \equiv 1$. As F_1 and F_2 move apart, $\phi(y)$ takes larger values on a set of decreasing F_1-probability and increasing F_2-probability, and smaller values on a set of increasing F_1-probability and decreasing F_2-probability. Looking at the expectation of $\phi(y)$ with respect to F_1—$E_1(\phi)$—we can see that $E_1(\phi) = 1$ for all F_1 and F_2; hence the coefficient of the F_1-dispersion of ϕ could be a measure of divergence of F_2 from F_1 as it would increase as F_1 and F_2 move apart. The proposed form of the coefficient of divergence is based on these intuitions. Ali and Silvey state four properties that a coefficient of divergence should satisfy and prove that these are met by any coefficient of the following form[8]:

$$E\{C(\phi)\} \tag{17}$$

where C is a continuous convex function on $(0, \infty)$. Notice that the expectation of a convex function of a real random variable measures its dispersion to a greater or lesser extent depending on the nature of this function. Hence depending on the specification of the convex function different measures are obtained.

1. When $E\{C(\phi)\} = E\{(\phi - 1)\log\phi\}$ the measure is the Jeffreys measure of divergence:

$$J(1,2) = \int (f_2(y) - f_1(y)) \log \frac{f_2(y)}{f_1(y)} \, dy. \tag{18}$$

2. For $E\{C(\phi)\} = E\{-\log\phi\}$ and $E\{C(\phi)\} = E\{\phi\log\phi\}$ the measures are the Kullback and Leibler measures of discriminatory information, $I(1,2)$ and $I(2,1)$, respectively:

$$I(1,2) = \int f_1(y) \log \frac{f_1(y)}{f_2(y)} \, dy \tag{19}$$

$$I(2,1) = \int f_2(y) \log \frac{f_2(y)}{f_1(y)} \, dy.$$

   Jeffreys, and Kullback and Leibler measures are based on the Shannon–Wiener definition of information: the extent to which two populations differ depends on how difficult it is to discriminate between them with the best test. The Kolmogorov measures, analyzed next, are measures of distance. Indeed:

3. When $E\{C(\phi)\} = \frac{1}{2}E(\sqrt{\phi} - 1)^2$ the measure is the Kolmogorov measure of distance, namely:

$$Ko = \frac{1}{2}\int (\sqrt{f_2(y)} - \sqrt{f_1(y)})^2 \, dy. \tag{20}$$

4. When $E\{C(\phi)\} = \frac{1}{2}E|\phi - 1|$ the measure is the Kolmogorov measure of variation distance, namely:

$$Kov = \frac{1}{2}\int |f_2(y) - f_1(y)| \, dy. \tag{21}$$

[8]The expectation considered by Ali and Silvey is really a generalized expectation, E^*, that is defined even if $\phi = \infty$. For simplicity I avoid this notation but everything holds even in the case where $\phi = \infty$.

For the second type of index, the index of social distance,[9] I use that suggested by Esteban and Ray (1994) as well as a modification that I propose.

Esteban and Ray introduce a model of individual attitudes in a society and use four axioms to narrow down the set of allowable measures. The notation is the following: $(\eta, y) \equiv (\eta_1, \ldots, \eta_N; y_1, \ldots, y_N)$ is a distribution for any positive integer N if $y \in \Re^N$, $y_i \neq y_j$, $\forall i, j$ and $\eta > 0$. The total population associated with (η, y) is given by $\sum_{i=1}^{N} \eta_i$. Φ is the space of all distributions. A polarization measure is a mapping $ER: \Phi \to \Re_+$. In particular, Esteban and Ray suppose that each individual is subject to two forces: on the one hand, he identifies with those he considers to be members of his own group, $I: \Re_+ \to \Re_+$ represents the identification function; and on the other hand, he feels alienated from those he considers to be members of other groups, $a: \Re_+ \to \Re_+$ is the alienation function. An individual with income y feels alienation $a(\delta(y, y'))$ from an individual with income y'. $\delta(y, y')$ is a measure of distance between the two incomes. For Esteban and Ray this is simply the absolute distance $|y - y'|$. The joint effect of the two forces is given by the effective antagonism function, $T(I, a)$, and total polarization in the society is postulated to be the sum of all the effective antagonisms:

$$ER(\eta, y) = \sum_{i=1}^{N} \sum_{j=1}^{N} \eta_i^{1+\alpha} \eta_j T(I(\eta_i), a(\delta(y_i, y_j))). \tag{22}$$

The measure that satisfies the axioms introduced by Esteban and Ray has the following expression:

$$ER(\eta, y) = k \sum_{i=1}^{N} \sum_{j=1}^{N} \eta_i^{1+\alpha} \eta_j \delta(y_i, y_j) = k \sum_{i=1}^{N} \sum_{j=1}^{N} \eta_i^{1+\alpha} \eta_j |y_i - y_j| \tag{23}$$

for some constants $k > 0$ and $\alpha \in [1, 1.6]$ that indicates the degree of sensitivity to polarization.[10]

This index of polarization is computed empirically as follows:

$$ER(\alpha) = \sum_{i=1}^{N} \sum_{j=1}^{N} \pi_i^{1+\alpha} \pi_j |\mu_i - \mu_j| \tag{24}$$

π_i and μ_i represent respectively the relative frequency[11] and the conditional mean in group i for a density of the logarithm of income $f(y)$, namely:

$$\pi_i = \int_{y_{i-1}}^{y_i} f(y)\, dy \tag{25}$$

$$\mu_i = \frac{1}{\pi_i} \int_{y_{i-1}}^{y_i} y f(y)\, dy.$$

In other words, what is computed empirically is the degree of polarization in a society, where it is assumed that everybody in each given group possesses an income equal to the mean of the group.

[9]Wolfson's measure of polarization does not apply as it is a measure of bipolarization.

[10]With $\alpha = 0$ the Esteban and Ray index of polarization is proportional to the Gini coefficient normalized using the logarithm of income and not the mean.

[11]The population weights η_i, $i = 1, \ldots, N$ are replaced by the population frequencies. The constant k is hence set to $k = [\sum_{i=1}^{N} \eta_i]^{-(2+\alpha)}$.

I propose a modification[12] of *ER* that can compute the level of social distance within a given society without assuming that everybody in each group has an income equal to the mean, and that instead considers a characteristic, other than income, to generate the group partition, e.g. region of residence, age, education.

The idea behind the modification is a direct application of the method described in the previous paragraphs. The total density of income, $f^t(y)$, at any point in time, is given by the sum of the densities of each group, weighted by the relative frequency of each group:

$$f^t(y) = \int_{z \in \Omega_z} dF(y, z \mid t_{y,z} = t) \qquad (26)$$

$$= \int_{z \in \Omega_z} f(y \mid z, t_y = t)\, dF(z \mid t_z = t).$$

The polarization index has to register the moving apart of the densities classified according to some characteristics of the household that forms the groups and differences in the frequencies between the groups. Each individual identifies with those of his own group and feels alienated from those he considers to be members of other groups, as Esteban and Ray noted, but now the groups are identified by these other characteristics and not by levels of income. Hence the index of polarization that Esteban and Ray proposed is modified in order to take into account the distance between the distributions of income of each group. I propose to use as measure of distance between two distributions the Kolmogorov measure of variation distance[13]:

$$Kov_{if} = \frac{1}{2}\int \left| f_i(y) - f_j(y) \right| dy \qquad (27)$$

and compute the following polarization index obtained from (23):

$$PK(\alpha) = \sum_{i=1}^{N} \sum_{j=1}^{N} \pi_i^{1+\alpha} \pi_j\, Kov_{ij} \qquad (28)$$

[12]Esteban, Gradin and Ray (1997) and Gradin (2001) have already proposed a modification of $ER(p)$, to correct for not having included in the analysis the inequality within each group, and the overlapping of the groups that has the effect of overestimating the level of observed polarization. In particular:

$$P(\alpha, \beta) = ER(\alpha) - \beta\varepsilon$$

where

$$\varepsilon = G(f) - G(\mu)$$

the difference between the Gini coefficient computed on the ungrouped, $G(f)$, and grouped data, $G(\mu)$. β is the parameter that indicates the importance given to the approximation error.

[13]Kolmogorov measure of variation distance is a measure of the lack of overlapping between groups *i* and *j*. $Kov_{ij} = 0$ if $f_i(y) = f_j(y)\ \forall y$, it reaches the maximum, $Kov_{ij} = 1$, if $f_i(y)$ and $f_j(y)$ do not overlap. The distance is sensitive to changes of the distributions only when both take positive values, being insensitive to changes whenever one of them is zero. It will not change if the distributions move apart, provided that there is no overlapping between them or that the overlapping part remains unchanged. Kolmogorov measure of variation distance was also used by Weitzman (1970).

$PK(\alpha)$ ranges between 0 and $(\frac{1}{2})^{1+\alpha}$. The maximum is achieved when there are only two groups of the same size with no overlapping. The index can be normalized to take values between [0, 1] by multiplying it by $2^{1+\alpha}$.

5. An Application to the Distribution of Income in Italy

Several studies have already analyzed the Italian distribution of income. The importance of monitoring its evolution through time and tracking where different groups of the population are located on the income scale is well recognized. The *Consiglio Nazionale dell'Economia e del Lavoro* (National Council of Economy and Labor) promoted several reports on the distribution among factors, persons and households (Rossi, 1993, 1994, 1996, 1998).

The reason for additional research on this topic is the following. Inequality has increased from 1987 to 1995 as the distribution of income for the whole country has become more disperse (the Gini coefficient of the distribution of logarithm of income increased from 0.0350 in 1987 to 0.0380 in 1995). Looking at the estimate of the distribution of the logarithm of income among persons and its regional decomposition, we can see that the increase in the dispersion of the density was not the same in all areas. Inequality decreased in the northern area (the Gini index decreased from 0.0312 to 0.0306), slightly increased in the central (from 0.0302 to 0.0317), and dramatically increased in the southern area (from 0.0352 in 1987 to 0.0415 in 1995).

Rather than a simple increase in inequality it is more appropriate to describe the evolution of the Italian distribution of income during those years as subject to an increase in polarization between the northern, central and southern areas (Figure 1) since the income distributions of the geographic areas moved apart. The distributions of the three areas are very similar in shape in 1987, even if located at different levels of the income scale. The south composes the left-hand tail of the distribution of the whole country, while the north contributes the most to the density of the right-hand tail. In 1995, these considerations are still valid, but there has been a moving apart of the densities accompanied by a dramatic change in the shape of the distributions, particularly in the density of the south. What caused these movements? Can we measure the rise in the distance between the distributions of the geographic areas and discover what determined it?

This section contains a tentative answer to these questions. The data used are those of the Bank of Italy, SHIW, from the years 1987 and 1995. The incomes are expressed in real terms by correcting for inflation using CPI (base 1990). To take into account the composition and the number of members of the households I use the Carbonaro equivalence scale.[14] The distribution of income among persons has been derived assuming that household income is shared equally among all the members of each household and assuming that each member is endowed with the level of equivalent income previously computed. The estimation of the

[14]This scale can be expressed in a parametric form:

$$y_i^e = \frac{y_i}{n_i^e}$$

where n_i is the number of members of household i and $e = 0.711$.

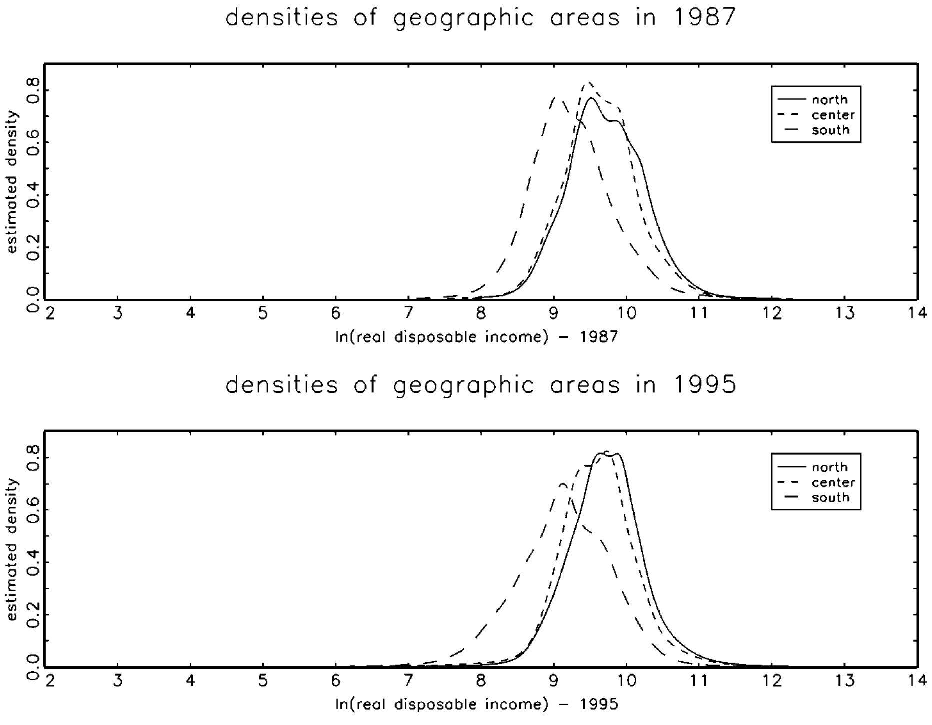

Figure 1. Distribution of the Logarithm of Equivalent Income Among Persons

equivalent distributions of the logarithm[15] of income among persons is obtained by applying the non-parametric method described earlier to the SHIW samples. In the 1987 survey 8,027 households with a total of 25,098 individuals and 13,549 earners were interviewed; the corresponding values for 1995 are 8,135 households, 23,924 individuals and 14,699 earners. The definition of income analyzed is net yearly income in thousands of lira composed of employee income (including fringe benefits), self-employed income, capital income (including non-cash property income), pensions and arrears, transfers and other revenues. Brandolini and Cannari (1994) analyzed the quality of these data and reported that this is the same as the corresponding surveys in other countries.

To understand the causes of the observed movement in the distributions I decompose the total population into subgroups according to the following classification:

- number of earners (one, two, three or more earners in the household);
- education (head of the household has no schooling degree, elementary, junior high, high school degree, laurea or more);
- employment status (the head of the household is not employed, employee, self-employed, retired).

[15]To estimate the distribution of the logarithm of income I had to eliminate negative and zero income from the data.

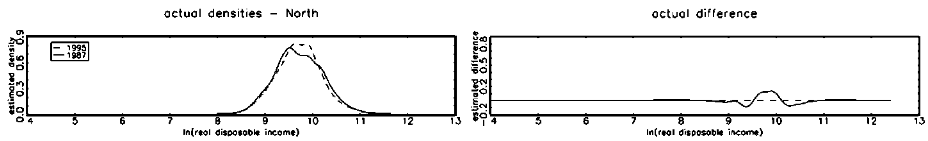

Figure 2. Distribution of the Logarithm of Equivalent Income, North of Italy

5.1. *Northern Italy*

The evolution of the distribution for northern Italy from 1987 to 1995 is characterized by a shift of the density towards central levels of income from the two tails and the appearance of two peaks with the same mass (Figure 2) in the center of the density. These movements are clearly highlighted in the graph on the right side of the same figure where the difference between the two densities is plotted. The value of the mean in the two years did not change, 9.710 in 1987 and 9.704 in 1995, while inequality as measured by the Gini coefficient decreased from 0.0312 in 1987 to 0.0306 in 1995. The decompositions by population subgroups are described in Figure 3. The results can be classified in two separate groups: on the one hand are the decompositions obtained by taking into account the within-group component of the whole movement and, on the other, are those resulting from the counterfactuals of the between-group changes. The main finding is that changes in the characteristics of the households did not have a large influence on the evolution of the density of income during these years, since most of the observed variations can be attributed to the within-income schedule which underwent a dramatic modification process. The between-group component of number of earners decomposition had some effect due to the combined effect of increasing female participation rate and the tendency of adult working children to leave the family later; the percentage of households with two earners, indeed, increases from 44.56 percent in 1987 to 47.86 percent in 1995, and those with three or more earners from 15.87 percent in 1987 to 16.87 percent in 1995.

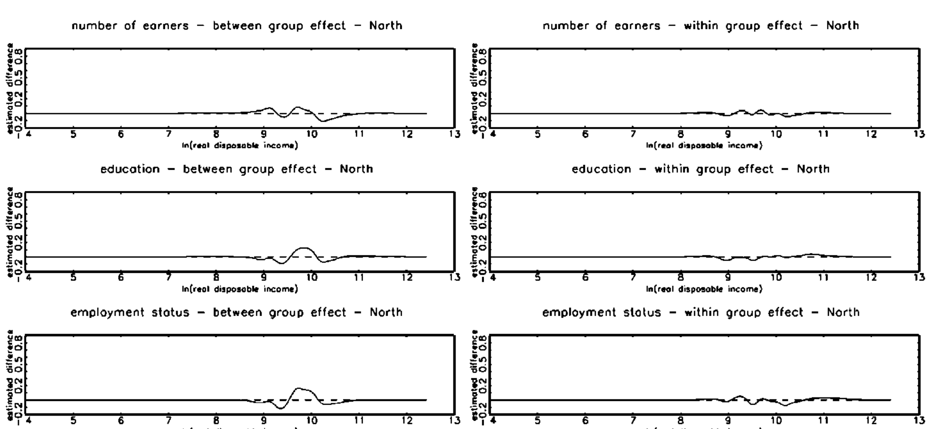

Figure 3. Distance Among 1987 Estimated Density and 1995 Counterfactual Densities Obtained by Applying the Between- and Within-group Decomposition, North of Italy

The changes in the densities of the different groups can be summarized as follows.[16] From the analysis of the densities within each group composed according to the number of earners, we can see that the densities are ordered on the income scale according to the number of earners in both years, as expected, and that over time there has been an increase in the dispersion of income in the distribution of one-earner households, with the opposite holding for households with two or more earners. The distributions for the grouping based on the educational degree of the head of the household are ordered according to the level of education and over time the most evident changes occurred among the two extreme groups with a clear shift of density towards higher level of income in both densities. The groups that witnessed the greatest transformations in the shape of their densities are those obtained by grouping according to the employment status of the head of the household: on the one hand there are the employee and the retired heads, on the other the self-employed and the not employed heads. Members of the first group experienced an improvement in their relative position, particularly retired heads who, owing to the pension system of those years, were able to retire early and with a relatively high pension; the densities of the second group, instead, moved towards lower levels of income, with this shift being dramatic for the self-employed and more moderate for the not employed heads.

The values of the measures of divergence and distance between the distribution of 1987 on the one hand and the distribution of 1995—actual and counterfactual—on the other are reported in Table 1, where in brackets is the

TABLE 1

Measures of Divergence and Distance Between the Actual Distribution of 1987 and the Actual/Counterfactual Distribution of 1995, North of Italy

North	Jeffreys Divergence	Kullback–Leibler Divergence	Kolmogorov Distance	Kolmogorov Variation Distance
Actual	0.0316	0.0159	4.2094	0.0615
Earners between	0.0404	0.0201	3.5243	0.0617
	(+27.754)	(+26.385)	(−16.274)	(+0.361)
Earners within	0.0086	0.0041	0.6690	0.0268
	(−72.719)	(−74.396)	(−84.108)	(−56.391)
Education between	0.0342	0.0170	3.9874	0.0630
	(+8.071)	(+6.883)	(−5.273)	(+2.397)
Education within	0.0148	0.0074	0.8346	0.0281
	(−53.089)	(−53.465)	(−80.173)	(−54.281)
Employment between	0.0384	0.0192	6.2700	0.0727
	(+21.515)	(+20.970)	(+48.953)	(+18.165)
Employment within	0.0396	0.0210	1.4066	0.0444
	(+25.017)	(+32.382)	(−66.584)	(−27.891)

Percentage of the change of estimated value with respect to the one computed on the actual densities in parentheses.

percentage of the change of estimated value with respect to the one computed on the actual densities. All measures report a reduction in the distance and divergence between the distribution of 1987 and the counterfactual of 1995, according to the decomposition obtained by taking into account the within-group changes,

[16]The graphs of the densities of the groups are omitted here but are available upon request.

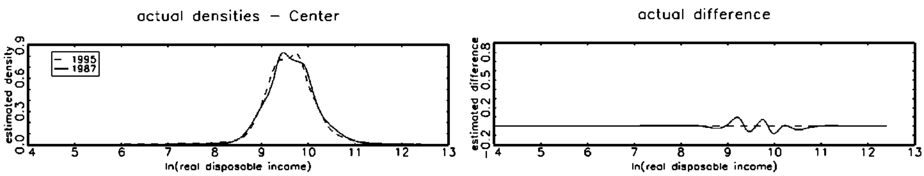

Figure 4. Distribution of the Logarithm of Equivalent Income, Center of Italy

and an increase if the decomposition is the result of the between-group movements. The only exceptions are the measures of divergence for employment, within-group decomposition, where an increase of the value is observed. The Kolmogorov measure of distance does not agree for number of earners and education (between-group decomposition) since it shows a decrease in the distance value. Regarding the intensity of the explanatory effects, all the measures of distance and divergence report the greatest reductions when the counterfactual density of 1995 is obtained with the within-group effect of numbers of earners, implying that this factor explains from 56 percent (according to Kolmogorov variation distance) to 84 percent (according to Kolmogorov distance) of the observed movements in the distribution of personal equivalent income from 1987 to 1995 in northern Italy.

5.2. *Central Italy*

The results of the estimations for central Italy are described in Figure 5. The density of the logarithm of income in 1995 looks symmetric to the one of 1987 (Figure 4): the mass of the distribution moved towards higher levels of income with the mode rising from 9.4 to 9.75. The mean decreased slightly (from 9.630 in 1987 to 9.588 in 1995), and inequality, measured by the Gini coefficient, increased slightly (from 0.0302 in 1987 to 0.0317 in 1995). Central Italy was subject to the smallest changes in the shape of the distribution of equivalent income

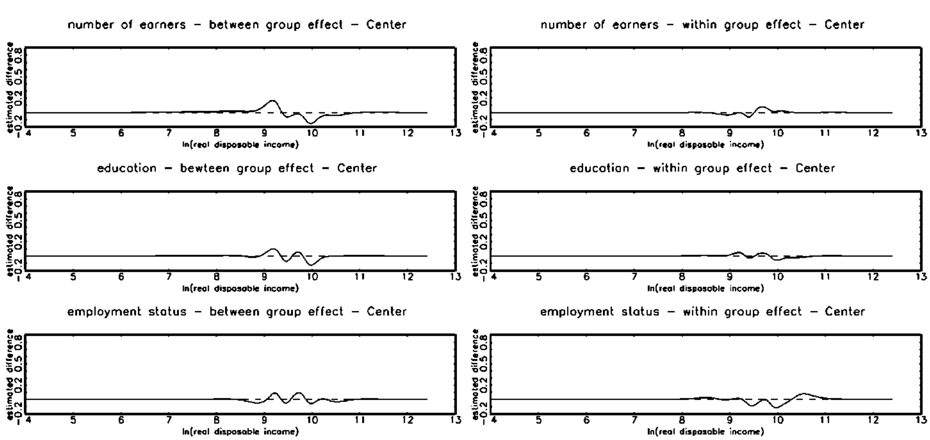

Figure 5. Distance Among 1987 Estimated Density and 1995 Counterfactual Densities Obtained by Applying the Between- and Within-group Decomposition, Center of Italy

among persons. Given the results of the decomposition method, the observed movements can be attributed to the following factors. According to the decomposition based on the number of earners in the households, the movements that occurred within the densities of each earning group had a greater impact in reducing the distance between the distributions in the years of analysis. The densities are ordered on the income scale according to the number of earners. From 1987 to 1995 the density of two and three or more earners leveled off, with a decrease in the mode and a greater increase in mass on the left-hand tail seen, especially for the first group. The shift in the mode of the density can be attributed to the same movement observed in the density of the households with two earners. Regarding the composition of the population, the households with one earner decreased from 46 percent in 1987 to 35 percent in 1995, with the percentage increasing in the other two groups, 41–46 percent for two earners and 13–19 percent for three or more earners. For the grouping based on educational characteristics the greatest changes come from the within-group decomposition. The groups slightly modified their composition from 1987 to 1995 with an increasing proportion of households having heads with a high school degree and diploma. The densities of the educational groups are ordered on the income scale depending on the degree awarded; during the years analyzed there was an increase in the level of polarization among them since the densities moved apart. The extreme groups displayed a dramatic shift in the opposite direction, toward the left for those with no education and toward the right for those with laurea or more. The results obtained by the analysis of the movements between and within employment status groups represent an exception, since the counterfactual density of 1995 obtained by substituting the relative frequencies of the groups of 1987 is closer to the estimated density of 1987 than the counterfactual of 1995, which was obtained by taking into account the distribution of income that was estimated for each group in 1987. Both the relative frequencies and the densities of each group obtained according to the employment status of the head of the households were subject to significant changes from 1987 to 1995. The percentages of heads that were employees decreased from 49.43 to 37.05 percent, self-employed increased from 11.74 to 16.08 percent, and retired increased from 38.10 to 41.27 percent while those not employed increased from 0.74 to 5.60 percent of the total population. The changes in the shape of the total density are due, on the one hand, to the shift toward lower levels of income of the mode of the density of employees, and, on the other, to the rightward movement of the densities of retired heads which is the only group better off in 1995 than it was in 1987. The net losers are instead the self-employed whose density moved dramatically towards lower levels of income.

The measures of distance and divergence (see Table 2) register a decrease in the observed value for all decompositions based on within-group changes, except for employment where an increase is observed. The evidence for the between-groups decomposition is more mixed: increasing values for all measures for number of earners; decreasing values for the measures of divergence; and increasing values for the measures of distance for education. All measures of distance attribute the biggest reduction to the changes that occurred within the distributions of educational groups, 62 percent according to Kolmogorov distance and 42 percent

TABLE 2

MEASURES OF DIVERGENCE AND DISTANCE BETWEEN THE ACTUAL DISTRIBUTION OF 1987 AND THE ACTUAL/COUNTERFACTUAL DISTRIBUTION OF 1995, CENTER OF ITALY

Centre	Jeffreys Divergence	Kullback–Leibler Divergence	Kolmogorov Distance	Kolmogorov Variation Distance
Actual	0.0351	0.0193	2.1951	0.0474
Earners between	0.0916	0.0507	7.2610	0.0854
	(+160.935)	(+162.823)	(+230.774)	(+79.995)
Earners within	0.0073	0.0036	1.1151	0.0286
	(−79.234)	(−81.426)	(−49.201)	(−39.797)
Education between	0.0323	0.0173	3.1671	0.0499
	(−7.926)	(−10.103)	(+44.277)	(+5.272)
Education within	0.0070	0.0037	0.8327	0.0273
	(−79.929)	(−80.945)	(−62.068)	(−42.418)
Employment between	0.0240	0.0121	2.2933	0.0462
	(−31.597)	(−37.370)	(+4.469)	(−2.680)
Employment within	0.0495	0.0261	3.3592	0.0594
	(+40.923)	(+35.545)	(+53.026)	(+25.195)

Percentage of the change of estimated value with respect to the one computed on the actual densities in parentheses.

according to Kolmogorov variation distance. Jeffreys measure, as well, registers the greatest reduction, 80 percent, when the counterfactual density of 1995 is obtained by taking into account the densities of each educational group of 1987. According to Kullback–Leibler, instead, the greatest effect is due to changes within the densities of number of earners, 81 percent.

5.3. *Southern Italy*

The estimation of the densities in southern Italy is reported in Figure 7. The behavior of this geographic area differs dramatically from the previous two: there has been a shift in the mass from the center of the distribution towards lower levels of income and partially from the extreme right tail to the left, as highlighted in the graph on the right side of Figure 6 where the distance between the two densities is plotted. Furthermore, this area is the one that witnessed the greatest changes in the shape of the density of the logarithm of equivalent income among persons from 1987 to 1995. For all the decompositions of total population into groups with different characteristics, the counterfactual density of 1995 obtained by taking into account the changes within the densities of each component group almost coincide with the estimated density of 1987. According to employment status the counterfactual density of 1995 built with the relative frequencies of 1987 is able to explain the same proportion of the observed shifts of mass of the

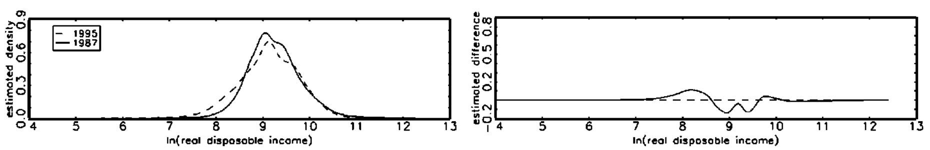

Figure 6. Distribution of the Logarithm of Equivalent Income, South of Italy

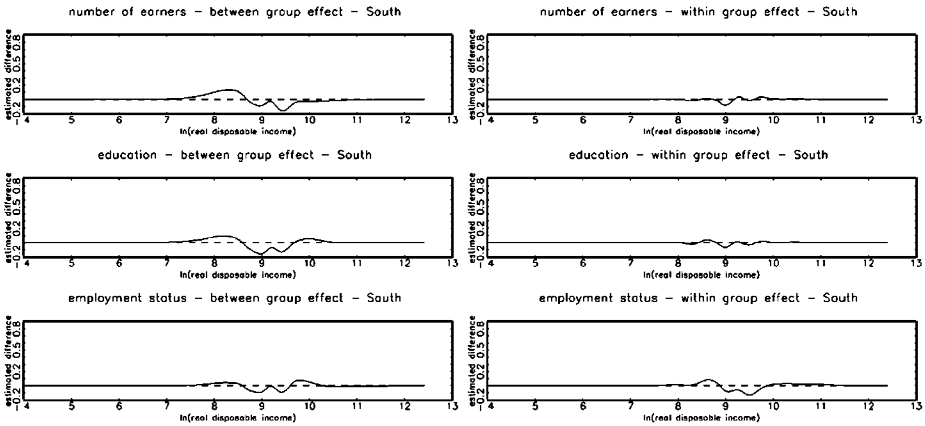

Figure 7. Distance Among 1987 Estimated Density and 1995 Counterfactual Densities Obtained by Applying the Between- and Within-group Decomposition, South of Italy

within-group decomposition. The composition of the population based on the number of earners of the households is strikingly different from that of the other geographic areas of Italy. The great majority of households have only one earner in both years, 58 percent in 1987 and 48 percent in 1995. There was an increase in the number of households with two earners, owing to the increase in the female participation rate, but the weight of this group is still 8 percent and 6 percent lower than northern and central Italy, respectively. The densities of the groups are ordered on the income scale according to the number of earners and from 1987 to 1995 there was a large increase in the dispersion within each density, with a shift of mass from the center to the left tail. The biggest movement towards lower level of income is registered for the density of the households with two earners. This episode can be attributed to households that were one-earner in 1987, becoming two-earners in 1995, with the contribution of the second earner to the total income of the household not being high enough compared with households that were already two-earners in 1987. The densities of the different educational groups are ordered according to the degree awarded, but less so in 1995 since there was a convergence towards the same levels of income of all the densities and an increase in the dispersion within them. The increase in the density on the left tail of the total distribution is due to the shift of density on the left tail in the distributions of junior high and high school heads of household. The weight of the different employment status groups changed over time: the relative frequencies of employee heads was the only one that decreased (from 53.46 to 36.46 percent); all the others increased (self-employed from 12.06 to 13.61 percent, retired from 32.56% to 39.12 percent and not employed from 1.92 to 10.81 percent). Note that the relative frequency of not employed heads is twice as high as the corresponding one for the groups in the other geographic areas, owing to the differences in geographic unemployment rates. By analyzing the movements within the densities of the groups, we can see that the net losers are the self-employed and the not employed heads, since the densities of these groups moved left. The behavior of the distribution of the retired heads of the households in this geographic area differs from that observed in the other areas: the distribution did not shift right but was simply a movement of density from left and right of the mode towards the center. The pension system, indeed, did not favor the households of the south but rather redistributed income towards the richer areas of the country.

TABLE 3

Measures of Divergence and Distance Between the Actual Distribution of 1987 and the Actual/Counterfactual Distribution of 1995, South of Italy

South	Jeffreys Divergence	Kullback–Leibler Divergence	Kolmogorov Distance	Kolmogorov Variation Distance
Actual	0.1551	0.0887	5.5212	0.1012
Earners between	0.2157	0.1252	7.9705	0.1191
	(+39.043)	(+41.204)	(+44.361)	(+17.695)
Earners within	0.0073	0.0035	0.7287	0.0274
	(−95.288)	(−96.013)	(−86.801)	(−72.858)
Education between	0.1529	0.0873	6.8531	0.1085
	(−1.411)	(−1.614)	(+24.122)	(+7.249)
Education within	0.0078	0.0042	0.4635	0.0252
	(−94.846)	(−95.283)	(−91.604)	(−75.046)
Employment between	0.0472	0.0251	3.6713	0.0671
	(−69.581)	(−71.745)	(−33.506)	(−33.696)
Employment within	0.0477	0.0244	4.8222	0.0727
	(−69.246)	(−72.507)	(−12.659)	(−28.179)

Percentage of the change of estimated value with respect to the one computed on the actual densities in parentheses.

According to the measures of divergence and distance, Table 3, all the counterfactual densities obtained by taking into account the changes that occurred within the densities of the groups produce a decrease in the estimated value. The maximum effect on divergence is reached when the distribution of 1995 incorporates the changes that occurred within number of earners distributions. According to this decomposition, indeed, the divergence between the density of 1987 and the counterfactual of 1995 is reduced by 95 percent for the Jeffreys measure, and 96 percent for the Kullback–Leibler measure. The maximum effect on distance is instead reached when the distribution of 1995 incorporates the changes that occurred within educational distributions: 92 percent for Kolmogorov measure, and 75 percent for Kolmogorov variation. In this area, even the movements between the groups had a relevant effect on increasing the distance. The estimated values of the measures of distance and divergence, indeed, decrease for the between-group decompositions based on employment status of the head of the household. The measures do not agree when the grouping is obtained according to education (between-group effect). With these counterfactual densities, the measures of divergence register a decrease in the value for education, whereas the measures of distance register an increase.

5.4. *Indices of Social Distance Among Geographic Areas*

Indices that track the moving apart of the densities classified according to the geographic area of the head, hence of social distance, are included in Tables 4 and 5.

TABLE 4

Esteban and Ray Polarization Index Among Actual Distributions of 1987 and the Actual/Counterfactual Distributions of 1995

ER	$\alpha = 1$	$\alpha = 1.3$	$\alpha = 1.6$
1987	0.0789	0.0586	0.0437
1995	0.0979	0.0730	0.0546
Earners between	0.1005	0.0748	0.0560
Earners within	0.0746	0.0555	0.0415
Education between	0.0953	0.0710	0.0531
Education within	0.0848	0.0632	0.0473
Employment between	0.0849	0.0632	0.0473
Employment within	0.0826	0.0615	0.0460

Table 4 reports the indices of social distance measured as polarization with the index suggested by Esteban and Ray. Regional polarization in Italy increases regardless of the value of the coefficient of degree of sensitivity to polarization, α. The values of the *ER* indices, indeed, pass from 0.0789 to 0.0979 ($\alpha = 1$), from 0.0586 to 0.0730 ($\alpha = 1.3$), and from 0.0437 to 0.0546 ($\alpha = 1.6$) in the estimated distributions.

To understand what determined polarization, the *ER* index is computed using the counterfactual densities. If the value obtained with one counterfactual density is equal to the 1987 value it is possible that the social distance between the geographic areas was determined by the factor used to build this particular density.

When the polarization measure is computed on the counterfactual densities, the values of *ER* decrease for all groupings except earners (between-group decomposition). The lowest values of polarization that can be reached are obtained with the within-group modifications of the densities from 1987 to 1995. In particular, we see that the increase in the level of polarization is due to the effect of the changes that occurred, in those years, to the income distribution within number of earners, since the value is the closest to the one of 1987. The growing distance among Italian geographic areas is, hence, attributed to the different behavior of the distributions of earners within each area: increasing dispersion within the distribution of household with only one earner in the north and

TABLE 5

Esteban and Ray Normalized Polarization Index Modified by Using Kolmogorov Measure of Variation Distance Among Actual Distributions of 1987 and the Actual/Counterfactual Distributions of 1995

PK Normalized	$\alpha = 1$	$\alpha = 1.3$	$\alpha = 1.6$
1987	0.246	0.225	0.206
1995	0.274	0.251	0.230
Earners between	0.276	0.253	0.233
Earners within	0.234	0.214	0.197
Education between	0.264	0.242	0.223
Education within	0.256	0.235	0.216
Employment between	0.260	0.238	0.219
Employment within	0.245	0.224	0.206

in the center; and movements toward lower levels of income for the density of household with two earners in the south.

The values assumed by the modified polarization index, PK (Table 5) report an increase in the distance between the regional densities for all values of the coefficient of degree of sensitivity to polarization. Normalized PK indeed increases from 0.246 to 0.274 if $\alpha = 1$, from 0.225 to 0.251 if $\alpha = 1.3$ and from 0.206 to 0.230 if $\alpha = 1.6$. The indices computed using the counterfactual densities agree with the signs of the corresponding one of the Esteban and Ray index. The value closest to that of the 1987 one is obtained for the decompositions within the densities of employment status groups. Hence, according to this index the increase in the level of social distance observed among Italian geographic areas from 1987 to 1995 was caused by the changes of the income schedules within employment status groups, due, in particular, to the pension system that redistributed income towards the richer areas of the country.

6. Conclusions

Economists have been studying welfare distributions within societies for several decades. However, the main questions that have been addressed are mainly related to the dispersion exhibited by the distributions under analysis, both as an important issue *per se* and for the implied consequences for economic growth, for social welfare, and so on. This approach is not entirely satisfactory since it ignores additional useful information that can be obtained by looking at the whole welfare distribution instead of simply focussing on measures related to its second moment.

The methods available to estimate the density function can be classified in two groups: there are parametric methods, where one assumes that the observed sample is drawn from a given density and what is estimated are the parameters that characterize the density; and there are non-parametric techniques, which allow one to obtain an estimate without assuming the functional form of the density *a priori*. I prefer the second approach since it does not get rid, in advance, of all the irregularities of the densities, and clumping and multimodalities are relevant information for economic reasoning. However, the estimated densities may not be enough for further analysis, for they do not consist of a single number that summarizes what transpired over time between distributions in one or in different countries.

Therefore, in this paper, I have introduced a new method better able to offer a clear picture of what has happened to any distribution and why. In particular, I have proposed a new decomposition technique of within- and between-group components and introduced various indices for summarizing the results. The approach to the decomposition analysis taken here differs from the classical one of additively decomposable inequality indices, as it is more appropriate to the purpose of monitoring which factors modified the entire distribution, tracking where precisely on the distribution these factors had an effect, and pinpointing what determined the variations in social distance between groups observed in a given society.

I have applied the proposed technique to Italian data. The main finding of the decomposition method is that changes in household characteristics did not have a large influence on the evolution of the Italian density of income within each geographic area in the period under examination. Instead, most of the observed variation can be attributed to the within-group income schedule which underwent a dramatic change. The exceptions are changes in the employment status of household heads and number of earners, which are due respectively to the rise in the proportion of retired and unemployed heads on the one hand, and to the increase in the female participation rate and to the tendency of adult working children to leave the family later, on the other.

Several extensions of this work are possible. One can think of analyzing separately the different components of household income. More interestingly, one can apply the method developed here to income mobility, in order to decompose the total observed mobility into movements between and movements within groups. Finally, the modified Esteban and Ray index of polarization could be used to measure social distance in a mobility framework.

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