Review of Income and Wealth
Series 42, Number 2, June 1996

MULTILATERAL COMPARISONS OF OUTPUT, PRODUCTIVITY, AND PURCHASING POWER PARITIES IN MANUFACTURING

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This paper presents multilateral comparisons of output, productivity and purchasing power parities in manufacturing, for 1975 and 1987. Two multilateral approaches are considered, namely the Geary–Khamis method and the generalized Theil–Törnqvist method based on the EKS procedure. The paper discusses the problem of lack of additive consistency of the generalized Theil–Törnqvist index and the absence of constant price comparisons within this framework. Some procedures that lead to near additive consistency are proposed. The empirical results show that multilateralization does not substantially affect the results and that both the Geary-Khamis and the generalized Theil–Törnqvist index give results comparable to the binary comparisons.

1. INTRODUCTION

The basic problem of international comparisons of output and productivity is the conversion of national currency values to a common unit. The official exchange rate is not suitable for this purpose since it does not adequately reflect real price differences between countries. One possible approach to international price comparisons is the industry of origin approach. It takes a sectoral perspective and compares producer price levels between countries. Since 1983, the International Comparisons of Output and Productivity (ICOP) project at the University of Groningen, has been engaged in research on comparisons of real output, purchasing power parities (PPPs) and productivity for different sectors of the economy.

So far the ICOP project has essentially had a binary character. Sectoral output and productivity of one country was compared with another (base) country, usually the United States, primarily based on the Fisher index. For a number of reasons multilateral comparisons may also be needed. First of all, since the Fisher binaries are not transitive, they may not produce a unique ranking of countries. Direct comparisons between countries may give different results when compared with indirect binary comparisons through other countries. This can lead to inconsistencies between alternative sets of comparisons for a given pair of countries. Second, it may be useful to summarize the output of all countries in a

Note: We are most grateful to Bart van Ark in helping us with the statistical base of this paper. Furthermore, we wish to acknowledge stimulating comments from Bart van Ark, Salem Khamis, Angus Maddison, Bill Shepherd, Eddy Szirmai and two anonymous referees. We are particularly grateful to Salem Khamis, who acted as a referee at a later stage, for his detailed and incisive comments.

common currency, for instance for regional comparisons. Such comparisons would also need to be independent of the currency selected. In addition, output comparisons at a set of reference prices, usually referred to as international prices, cannot be made with the Fisher index. Such international prices are especially useful for sectoral analysis of output and productivity.

The main purpose of this paper is to construct multilateral comparisons of output and productivity within the conceptual framework and the available data from the ICOP studies. The paper uses a limited number of countries for which data are available for the two principal benchmark years 1975 and 1987.[1] The countries covered are Argentina, Brazil, Mexico, India, the U.K., South Korea, Japan and the U.S. for 1975, and Australia, Germany, Indonesia, Japan, South Korea, the U.K. and the U.S. for 1987. The paper considers two methods for multilateralization, the Geary–Khamis index and the generalized Theil–Törnqvist index, also known as the Caves–Christensen–Diewert (CCD) index in the literature. A major deficiency of the latter index is that there are no constant price comparisons associated with it. This issue is pursued here and a major contribution of the paper is a procedure that provides a set of international prices associated with the index, leading to near-additive comparisons.

The outline of the paper is as follows. Section 2 describes the basic data from the ICOP studies used for multilateral comparisons. The third section discusses the two aggregation methods and the problem of additive consistency. Section 4 presents the results of multilateral measurement. Some results from experiments with additive consistency are discussed separately in Section 5. The final section provides some concluding remarks.

A general remark on the paper may be noted here. The paper merely describes the procedures employed in the study, and no attempt is made to evaluate the suitability of one aggregation procedure against another. Furthermore, the results are influenced by the present organization of the ICOP work and the type of data available from these studies. The study may thus be considered as a first step in the multilaterization of the ICOP work.

2. Sources and Methods for Manufacturing Comparisons

2.1. *Basic Sources*

The basic source for all ICOP comparisons of real output in manufacturing is the manufacturing production census. In most countries, it is the most detailed account of manufacturing production. Quantities and shipments values are shown by product, which allows the calculation of unit values. The advantage of the unit values is that they refer mainly to actual sales transactions, and that the census covers most of these transactions in the census year (Maddison and Van Ark, 1988). Furthermore, since the unit values have quantity weights attached to them, it is possible to indicate the share of total output for which price relatives have been calculated. ICOP's use of unit values is quite different from ICP's (International Comparisons Project, see Kravis, *et al.*, 1982) use of specification prices. The

[1]The choice of benchmark years for the ICOP studies is intricately related to the availability of detailed manufacturing census data for different countries.

ICOP procedure produces potentially larger problems with regard to differences in quality and product mix between countries. In a few cases, for instance for passenger cars, adjustments have been made for this mix-problem, using secondary sources.

Unit values are not available for all manufacturing products. In a number of industries the products are so heterogeneous that only the value of output by product is supplied. In other cases, unit values are available by product, but the heterogeneity of the product among countries prevents a meaningful comparison. It is important to note however, that industry of origin comparisons include basic and intermediate goods, products which are by definition excluded in expenditure comparisons. These products are in general more homogeneous between countries than consumer or investment goods (Maddison and Van Ark, 1988; Van Ark 1993), and are therefore relatively easy to compare. An additional problem is that some products and industries may be unique to one country, for instance aircraft in the United States. This implies that comparisons of unit values between countries will be restricted to a subset of products, and that the price relatives derived for these products are assumed to be representative for the larger aggregate.

An important advantage of the census is the great industry detail it shows. This provides an opportunity to reclassify industries amongst manufacturing branches to achieve a comparable classification between countries. Furthermore, the census is a relatively consistent source, since all information is gathered from a single survey of manufacturing establishments. This implies, for instance, that information on labour input is obtained from the same survey as gross output and census value added. A disadvantage of the census is that it does not always cover all manufacturing activities.[2] Most ICOP comparisons are based on the production census, mainly because it is a consistent source where output and labour input is concerned and also because it shows more detail than the national accounts.

2.2. *ICOP Methodology for Binary Comparisons*

The ICOP comparisons for manufacturing are implemented by valuing the output of each country at the prices of either country.[3] Purchasing power parities (PPPs) at the industry level are derived by weighting the unit values of individual products with the corresponding quantity in either of the two countries:

$$(1) \qquad \text{PPP}_j^{XU(X)} = \frac{\sum_{i=1}^{s} P_{ij}^X * Q_{ij}^X}{\sum_{i=1}^{s} P_{ij}^U * Q_{ij}^X}, \qquad \text{PPP}_j^{XU(U)} = \frac{\sum_{i=1}^{s} P_{ij}^X * Q_{ij}^U}{\sum_{i=1}^{s} P_{ij}^U * Q_{ij}^U}$$

where quantities Q_{ij} of country X and U are used as weights for the unit value prices P_{ij} of all matched products $i=1, \ldots, s$ in industry j. The two indices in (1) are the Paasche and Laspeyres price indices discussed in the general literature. In

[2]This does imply that real output in manufacturing may be understated if the census is used for comparative purposes. Labour productivity comparisons do not need to be affected, since employment from the census is directly related to the output involved.

[3]This paper only gives a short summary of some aspects of the ICOP methodology. See Van Ark (1993) for an extensive survey of ICOPs manufacturing comparisons.

most of the ICOP work, the geometric average of the two (the Fisher index) is taken as the preferred measure.[4]

A major problem with industry of origin comparisons concerns double deflation. Real value added, adjusted for price differences, can be derived by calculating real output and subtracting real intermediate input. The feasibility of double deflation depends on the availability of relative prices of output and inputs. In most countries, the manufacturing production census provides little detail on actual quantities and prices of intermediate inputs. In general, only some broadly defined categories, such as fuel and raw materials are available. In such circumstances, it is not possible to apply double deflation. In addition, small measurement errors in the price ratios of output and input can become magnified in the price ratio for value added, especially if the ratio of value added to output is small. Some experiments with input–output tables have been undertaken (Szirmai and Pilat, 1990; Van Ark, 1993; Pilat 1994), but these gave rather volatile results. Due to the practical and theoretical problems arising from the double deflation procedure, the ICOP project has so far adopted the single indicator approach by weighting industry PPPs, derived at the output level, by its corresponding value added.[5] Details of this procedure are provided in the next section.

3. Methods for Multilateral Comparisons

This section describes the actual aggregation methods employed in the study. Two basic requirements, transitivity and base invariance, are at the heart of the choice of aggregation method. The Geary–Khamis (GK) method as well as the generalized Theil–Törnqvist indices satisfy these properties. The GK method also produces constant price comparisons with the property of additive consistency. On the other hand, the generalized Theil–Törnqvist index preserves the binary Theil–Törnqvist indices by maintaining a degree of "characteristicity," achieved through the use of the EKS method on the binary Theil–Törnqvist indices.

3.1. *Aggregation Levels*

Comparisons of sectoral output can be made at different levels of aggregation. The finest detail available in the manufacturing censuses concerns individual products, for which quantities and values of production are available. These individual products can be classified by industries, which in turn are part of larger aggregates, i.e. the manufacturing branches. From the branch level, the final aggregation leads to the manufacturing sector as a whole. These aggregation levels are conceptually somewhat similar to the levels distinguished in the International Comparison Program (ICP, i.e. basic heading level, major expenditure categories and total GDP).

Due to the nature of the industry of origin approach and the methodology used in ICOP studies, multilateral methods are not conceptually suitable or operationally feasible at any level below the manufacturing branch level. Multilateral

[4]The reader may refer to Diewert (1992) for a comprehensive set of arguments in favour of using the Fisher index for binary comparisons.

[5]See Paige and Bombach (1959) for a justification of this approach.

comparisons at the product level are not feasible as the binary comparisons are not based on a pre-specified product list, but include as many products as possible which can be matched between each pair of countries. The product specifications therefore differ between countries. In addition, the product comparisons do not cover the manufacturing sector as a whole, but can only be made for a sample of products.

The comparisons at the next level of aggregation, the industry, are based on the available product matches within that industry. Only those industries are compared in which a substantial share of total output can be matched. This implies that the list of industries differs between pairs of countries, and that these "sampled" industries do not cover the manufacturing sector as a whole. Pilat and Prasada Rao (1991) considered this second level of aggregation for seven countries for 1975, and found that the results were only marginally different from those at the branch level, which is the final level of aggregation.

This paper mainly considers multilateral procedures at the manufacturing branch level. Manufacturing branches roughly correspond to the two- and three-digit level of the ISIC classification. The branches are comparable between countries, cover the whole manufacturing sector and have been standardized for most of the ICOP work. The branch level purchasing power parities from the binary studies are used as the price data for the multilateral procedures, and the branch value added is deflated with these parities to derive implicit quantity data, or "real value added."[6] The basic data used in this study are: p_{ij}, which equals the PPP for the i-th branch in country j, with the U.S. as its base; v_{ij}, which equals value added in national currency values in branch i and country j; and $q_{ij} = v_{ij}/p_{ij}$, which equals real value added in branch i and country j, expressed in U.S. dollars. These real values can be referred to as quantities, which is also the practice in the ICP work. The parities for each branch, p_{ij}, are referred to as prices from here on.

Since most of the ICOP studies use the U.S. as the base country, the branch level PPPs used here are not transitive in the traditional sense. However, the U.S. can be considered as a "star" country, implying that comparisons between any two countries can be achieved through the U.S. The main problem is one of base invariance, i.e. if all ICOP comparisons were based on another "star" country, one would expect to find different results and possibly a different ranking of countries. However, until recently, few comparisons with other base countries had been carried out, implying that multilateralization is necessarily achieved with the current data set.

3.2. *Aggregation Procedures*

For aggregation, two procedures were a natural choice. The first of these is the Geary–Khamis (GK) method. It is the principal aggregation method for much of the work on international comparisons of real expenditure. The second method is the generalized Theil–Törnqvist index, which was proposed by Caves, Christensen, and Diewert (1982a, b) for temporal and spatial comparisons of prices,

[6]The procedures used to derive PPPs for the branch level have been documented extensively in other studies, e.g. Maddison and Van Ark (1988), Szirmai and Pilat (1990), Van Ark (1993) and Van Ark and Pilat (1993).

output and productivity. Much of the justification of this method derives from theoretical work on output and productivity indices. An important point of interest is to verify whether the multilateral comparisons derived from these methods deviate significantly from the present binary ICOP comparisons.

Geary-Khamis Method

This method was first proposed by Geary (1958) and later pursued by Khamis (1970, 1984). It is based on the twin concepts of "purchasing power parities" of currencies and "international (average) prices" of commodities, denoted by PPP_j and P_i, respectively. The Geary-Khamis method defines these unknown parities and international prices using the following system of interdependent equations, for each currency j $(1, 2, \ldots, M)$ and commodity i $(1, 2, \ldots, N)$:

$$\text{PPP}_j = \frac{\sum_{i=1}^{N} P_{ij} Q_{ij}}{\sum_{i=1}^{N} P_i Q_{ij}} \tag{2}$$

and

$$P_i = \sum_{j=1}^{M} \frac{P_{ij}}{\text{PPP}_j} \left[\frac{Q_{ij}}{\sum_{j=1}^{M} Q_{ij}} \right]. \tag{3}$$

The system is solved using one of the currencies as the numeraire. If the U.S. is used as the numeraire, the system provides PPPs that indicate the equivalence of its currency to the U.S. dollar. The results are invariant to the choice of the base currency in the sense that the PPP for any two countries would still be the same when a currency other than the U.S. dollar would be used as the basis to solve the system. The GK method provides "additively consistent" results, in the sense that for each country j:

$$\frac{\sum_{i=1}^{N} P_{ij} Q_{ij}}{\text{PPP}_j} = \sum_{i=1}^{N} P_i Q_{ij}. \tag{4}$$

The left-hand side of the expression is the national value aggregate deflated by PPP_j, leading to a value aggregate in the base currency unit. The right-hand side shows the value aggregate derived using international prices, expressed in base country currency units and country j's quantities. The formula implies that the converted total equals the sum of the deflated values for sub-sectors. This is an important feature of the GK procedure.

The Generalized Theil-Törnqvist Index

This index is defined in two stages. The first stage involves the computation of the standard Theil-Törnqvist index for binary comparisons. This index was first proposed by Törnqvist (1936) and later discussed by Theil (1965, 1974). The Theil-Törnqvist price index, TT^p_{kj}, for j with country k as base is given by:

$$\text{TT}^p_{kj} = \prod_{i=1}^{N} \left[\frac{P_{ij}}{P_{ik}} \right]^{(v_{ik} + v_{ij})/2}, \tag{5}$$

where v_{ij} represents the value share of i-th commodity in country j. The index is essentially a weighted geometric average of the price relatives with average value shares as weights. The TT index satisfies the country reversal test and a host of other properties. In addition, it has desirable economic theoretic properties as discussed by Diewert (1976) and Caves, Christensen, and Diewert (1982a, b), and it is "superlative" and "exact." The TT index is usually numerically very close to the Fisher index, due to the similarity in their definitions. Both are geometric averages of indices based on base and current period share weighted averages of price relatives.

However, the TT index does not give transitive results. Caves, Christensen, and Diewert proposed to obtain a generalized TT index through the application of the EKS technique. This procedure, proposed by Elteto and Koves (1964) and Szulc (1964), produces transitive multilateral comparisons from a matrix of non-transitive binary comparisons, maintaining a degree of "characteristicity." The EKS method is based on a log-linear least-squares criterion in deriving transitive indices that deviate the least from the corresponding binary indices.

The generalized TT index for two countries j and k is given by:

$$\mathrm{TT}^{G}_{kj} = \prod_{l=1}^{M} [\mathrm{TT}_{kl} * \mathrm{TT}_{lj}]^{1/M}. \tag{6}$$

The generalized TT index in (6) provides a PPP between country j's currency and that of country k that is directly comparable with that from the GK method. The TT^{G} index for a pair of countries (k, j) is a simple geometric average of all indirect comparisons between country j and k, through a bridge country l, of the form $\mathrm{TT}_{kl} * \mathrm{TT}_{kj}$.

The generalized TT index provides a set of transitive comparisons that are also base invariant, in the sense that all countries are treated symmetrically in the formula. The index seems to retain the flavour of the TT index and yet gives transitively consistent results.[7]

The Generalized TT-index and Additive Consistency

The generalized TT indices can be used in defining the PPPs for different currencies, for instance, with the U.S. dollar as the numeraire currency. The parities can subsequently be used for the conversion of the national currency values into the numeraire currency. However, the generalized TT method is not useful for comparative analysis of output and productivity across manufacturing branches. This is primarily due to the absence of a set of "reference" prices in the TT method, similar to the "international prices" in the GK method. As a result the TT method does not give additive results, which has limited the applicability of this procedure in international comparisons. A set of "international average prices" based on the Theil–Törnqvist parities can however be derived using various methods. It is possible to define international average prices using either the GK definition in (2) or explore other alternatives such as the Gerardi or Rao type

[7]Indices similar to that in (6) can also be derived from other binary index formula, such as the Fisher or GK binary indices.

international prices. A detailed description of these methods and their justification is available in Pilat and Prasada Rao (1991).[8]

Alternative 1: Geary–Khamis type international prices. The first possible set of prices can be defined for commodity i, using the Geary framework with the TT parities, as follows:

$$P_i = \sum_{j=1}^{M} \frac{P_{ij}}{\text{PPP}_j^{\text{TT}}} \frac{Q_{ij}}{\sum_{j=1}^{M} Q_{ij}}, \tag{7}$$

where P_i is the international price of i-th commodity based on the Geary framework, derived as a quantity weighted average of the converted prices, and where PPP_j^{TT} is the generalized Theil–Törnqvist PPP.

Alternative 2: Rao-type international prices. An alternative set of prices can be derived by using an alternative definition used in Prasada Rao (1990). This set of international prices is given by:

$$P_i = \prod_{j=1}^{M} \left[\frac{P_{ij}}{\text{PPP}_j^{\text{TT}}}\right]^{v_{ij}/\Sigma_{j=1}^{M} v_{ij}}. \tag{8}$$

Each international price for i-th commodity is defined as a weighted geometric average of the price of i-th commodity in each country, using value shares as weights.

Alternative 3: Gerardi-type international prices. Another set of international prices may be defined using an unweighted geometric mean of country prices, after converting them into a common currency unit, following a method proposed by Gerardi (Eurostat, 1983). For each commodity i, these international prices are defined as:

$$P_i = \prod_{j=1}^{M} \left[\frac{P_{ij}}{\text{PPP}_j^{\text{TT}}}\right]^{1/M}. \tag{9}$$

In defining these sets of prices the same PPPs, with the U.S. dollar as the reference currency, were used. The three alternatives can be used to revalue the real quantities, i.e. to derive constant price aggregates for different countries. Since the prices are independently derived using the generalized TT index, there is no underlying guarantee of additive consistency. This means that the sum of the revalued quantities may differ from the PPP-converted value aggregate. We return to this issue below.

4. Empirical Results of Multilateral Measurement

This section discusses results from the application of multilateral methods described in Section 3, for the two benchmark years 1975 and 1987. As mentioned above, the choice of these years and countries covered is driven by the ICOP work to date. The main purpose of the empirical exercise is two fold. The first aspect

[8]The main justification for the use of equations (7), (8) and (9) in Pilat and Prasada Rao (1991) is based on a regression framework with least-squares properties, similar to that used in earlier studies by Khamis (1984) and Prasada Rao and Selvanathan (1991).

TABLE 1

BINARY AND MULTILATERAL PPPs FOR MANUFACTURING, 1975

	Binary PPPs				Multilateral PPPs		
	Paasche (national weights, nat.cur./ U.S.$)	Laspeyres (U.S. weights, nat.cur./ U.S.$)	Fisher (geometric average, nat.cur./ U.S.$)	Theil–Törnqvist (nat.cur./ U.S.$)	Geary–Khamis (nat.cur./ Int.$)	Theil–Törnqvist (nat.cur./ Int.$)	Exchange Rate (nat.cur./ U.S.$)
Argentina	34.43	42.75	38.37	39.68	32.67	38.98	36.57
Brazil	6.91	8.77	7.79	7.91	7.77	7.96	8.13
Mexico	11.97	15.60	13.67	13.79	13.14	13.79	12.50
India	6.70	12.77	9.25	8.93	7.90	8.77	8.65
U.K.	0.436	0.499	0.466	0.464	0.462	0.469	0.452
South Korea	437.0	584.8	505.5	499.4	464.0	501.9	484.0
Japan	196.4	269.5	230.1	230.3	224.0	233.2	296.8
U.S.	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Source: Paasche, Laspeyres and Fisher PPPs from Pilat and Hofman (1990) for Argentina; Van Ark (1993) for Brazil, Mexico and India; Van Ark (1989) for the U.K.; Pilat (1995) for South Korea, and Pilat (1994) for Japan; Binary Theil–Törnqvist and multilateral results based on procedure explained in text.

is to examine the effect of multilateralization on the basic binary comparisons undertaken in the project. The second is to examine the feasibility of finding a constant price comparisons counterpart to the generalized Theil–Törnqvist index so that real output comparisons can be made at different levels of aggregation within the manufacturing sector.

4.1. *Purchasing Power Parities for 1975 and 1987*

Tables 1 and 2 show the binary and multilateral PPPs for 1975 and 1987, respectively, as well as the exchange rate. For 1975 eight countries could be included, namely, Argentina, Brazil, Mexico, India, Japan, South Korea, the U.K. and the U.S. These countries show a substantial range in income and productivity levels, which makes the comparison more interesting, since multilateral procedures often have relatively large effects for low income countries. Particularly for India, the country with the lowest income level in this sample, the spread between the index at own weights (the Paasche index) and that at U.S. quantity weights (the Laspeyres index) is large, reflecting substantial variability in quantity and price relatives, which in turn reflect differences in the structure of the manufacturing sector as well as in the relative price levels.[9]

A general feature of Table 1 is that the PPPs based on various formulae are relatively close to the exchange rate, indicating that relative price levels in manufacturing were relatively close in 1975 for these countries. This is at variance with the normal ICP results for total GDP, where PPPs and exchange rates are

[9]Von Bortkiewicz (1923) has shown that the Laspeyres–Paasche spread can be decomposed in three elements, namely: (1) the variability in relative prices in the two countries; (2) the variability in relative quantities in the two countries; (3) the correlation between the two countries. This indicates that for India relative to the U.S., the Laspeyres–Paasche spread reflects substantial differences in price and quantity structures with the United States.

TABLE 2

BINARY AND MULTILATERAL PPPs FOR MANUFACTURING, 1987

	Binary PPPs				Multilateral PPPs		
	Paasche (national weights, nat.cur./ U.S.$)	Laspeyres (U.S. weights, nat.cur./ U.S.$)	Fisher (geometric average, nat.cur./ U.S.$)	Theil–Törnqvist (nat.cur./ U.S.$)	Geary–Khamis (nat.cur./ Int.$)	Theil–Törnqvist (nat.cur./ Int.$)	Exchange Rate (nat.cur./ U.S.$)
Japan	148.5	202.9	173.6	175.7	171.7	177.4	144.6
Germany	2.16	2.25	2.21	2.20	2.24	2.24	1.80
Korea	576.8	848.7	699.6	698.2	678.4	705.1	822.6
U.K.	0.670	0.748	0.708	0.702	0.696	0.698	0.612
Australia	1.412	1.576	1.492	1.495	1.461	1.477	1.430
Indonesia	826.9	1355.6	1058.8	1025.1	957.9	1008.0	1644.0
U.S.	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Source: Paasche, Laspeyres and Fisher for Japan and Germany from Van Ark and Pilat (1993), for Korea from Pilat (1995), for the U.K. from Van Ark (1993), for Australia from Pilat, Prasada Rao and Shepherd (1993), and for Indonesia from Szirmai (1993); Binary Theil–Törnqvist and multilateral results based on procedure explained in text.

generally close for developed countries, but not for developing countries where the PPPs are generally low relative to the exchange rate.

The 1987 benchmark includes a different set of countries. Several developing countries, such as Brazil, India and Mexico produce their manufacturing censuses with considerable delays. These countries could therefore not be included in the latest benchmark. Table 3 shows the PPPs for 1987 for six countries. Compared to the exchange rate, the PPPs in 1987 are relatively high for the four industrialized countries in the sample, reflecting the rather low value of the U.S. dollar after the 1985 devaluation. The two developing countries both have relatively low PPPs. The Indonesian rupiah devalued against the dollar in 1985, whereas the Korean won appreciated somewhat since 1986, but maintained a relatively low price level in spite of that. The general broad alignment of PPPs and the exchange rate may be attributable to the nature of the manufacturing sector, as most goods in this sector are heavily traded. One would therefore expect the manufacturing PPPs to be relatively close to the exchange rate.

In 1975, there is a close correspondence between the two multilateral methods for all the countries with the exception of Argentina and India. This difference may be attributable to the Gerschenkron effect (Gerschenkron, 1962), though the differences are not as pronounced as one might expect in such cases (as is the case with the ICP results). For the 1987 comparison, there is no appreciable difference between the generalized Theil–Törnqvist and Geary–Khamis PPPs. It is quite surprising how close the GK and generalized TT parities are for Indonesia.[10]

From the point of view of the ICOP work, a more salient feature of the results is the close agreement between binary Fisher PPPs, which are used throughout the

[10]This implies that the traditional arguments against the Geary–Khamis methods may not be applicable in the case of manufacturing sector comparisons. However, it would be interesting to see what the inclusion of India in the 1987 comparisons might lead to. Some work to this effect is currently in progress.

ICOP work, and the PPPs from the binary Theil–Törnqvist, multilateral GK and generalized Theil–Törnqvist indices. It is to be expected that the Fisher and Theil–Törnqvist indices are close and from the nature of the construction of the generalized TT indices these should be expected to be close to their multilateral counterparts.[11] What is important to note, however, is that the GK PPPs are also close to all of these.

Partly the explanation for this may come from the fact that the exercise multilateralizes from the branch level upwards, i.e. a certain amount of aggregation has already taken place up to that point and the aggregation up to the branch level is only based on Fisher binaries. More importantly, the general agreement in the PPPs from different methods may be due to the tradeable nature of manufactured goods and therefore a similarity of relative price structures may be embedded in the price data. From the point of view of the ICOP work, these results are quite reassuring, as they suggest that there may not be a need for a comprehensive multilateral exercise.

4.2. *Value Added and Productivity Comparisons*

Tables 3 and 4 show relative census value added and productivity based on the two multilateral procedures. For 1975, reflecting the differences in PPPs in

TABLE 3

RELATIVE OUTPUT AND PRODUCTIVITY IN MANUFACTURING MULTILATERAL COMPARISONS, 1975, U.S.A. = 100

	Real Census Value Added		Real CVA per Person Employed		Real CVA per Hour Worked	
	Geary–Khamis	Theil–Törnqvist	Geary–Khamis	Theil–Törnqvist	Geary–Khamis	Theil–Törnqvist
Argentina/U.S.A.	3.21	2.69	28.59	23.96	24.63	20.64
Brazil/U.S.A.	8.92	8.71	41.73	40.74	38.23	37.33
Mexico/U.S.A.	3.78	3.60	38.77	36.96	35.37	33.72
India/U.S.A.	2.20	1.98	6.67	6.01	5.47	4.92
U.K./U.S.A.	18.08	17.79	41.59	40.92	43.94	43.24
Korea/U.S.A.	1.18	1.09	13.75	12.71	9.81	9.07
Japan/U.S.A.	46.60	44.77	65.03	62.47	59.55	57.21

Source: Value added, employment and hours worked based on sources quoted in Table 1, and converted with multilateral PPPs from Table 1.

Table 1, real value added based on the Geary–Khamis procedure is higher than that of the Theil–Törnqvist index. The multilateral comparison for 1975 is dominated by the manufacturing sectors of Japan and the U.S. The U.K. manufacturing sector was only 18 percent of that in the U.S., and those of the developing countries in the sample were even smaller. South Korea's manufacturing sector was particularly small compared to the U.S. in 1975.

[11]Under some general conditions, the Fisher and Theil-Törnqvist indices provide numerical values that are very close to each other. Diewert (1976, 1992) has established that these two indices provide exact index numbers for quadratic and translog aggregator functions respectively, and are therefore expected to be of similar magnitude.

TABLE 4

RELATIVE OUTPUT AND PRODUCTIVITY IN MANUFACTURING MULTILATERAL COMPARISONS, 1987, U.S.A. = 100

	Real Census Value Added		Real CVA per Person Employed		Real CVA per Hour Worked	
	Geary-Khamis	Theil-Törnqvist	Geary-Khamis	Theil-Törnqvist	Geary-Khamis	Theil-Törnqvist
Japan/U.S.A.	51.82	50.14	77.29	74.79	68.28	66.07
Germany/U.S.A. (a)	26.54	26.64	66.11	66.35	77.43	77.71
Korea/U.S.A.	4.70	4.52	27.30	26.27	18.90	18.18
U.K./U.S.A.	13.86	13.82	54.50	54.33	59.02	58.83
Australia/U.S.A.	2.72	2.69	49.40	48.85	51.09	50.52
Indonesia/ U.S.A. (a)	1.10	1.05	10.76	10.22	n.a.	n.a.

Source: Value added, employment and hours worked from sources quoted in Table 2, and converted with multilateral PPPs from Table 2.

Note: (a) The Germany/U.S.A. and Indonesia/U.S.A. comparison excludes establishments with less than 20 employees. The value added figures for Germany and Indonesia shown in Table 8 are also exclusive of these establishments, but the U.S. data in that table include establishments with less than 20 employees. The Germany/U.S.A. and Indonesia/U.S.A. ratios shown in the current table can therefore not be derived directly from Table 8.

The labour productivity comparison is shown in the final columns of Table 3. A distinction is made between comparisons of value added per person employed and comparisons of value added per hour worked. For some countries, notably South Korea, the comparison at the hours worked level gives much lower productivity levels than that per person employed. Surprising are the relatively high productivity levels of Brazil and Mexico, especially compared to South Korea, which is regarded as a much more successful manufacturer than Brazil and Mexico. However, the spread in relative productivity levels between manufacturing branches and between manufacturing establishments is very large in South Korea (Szirmai and Pilat, 1990; Pilat, 1995). Although the average productivity level in 1975 was still very low, some manufacturing branches had achieved much higher productivity levels. In addition, in 1975 South Korea had experienced less than 15 years of successful industrialization. Since it started at very low productivity levels, it could not yet have achieved high productivity levels, unlike Brazil and Mexico, which already had much longer histories of industrialization. The Japanese productivity level was at approximately two-thirds of the U.S. in 1975, but some manufacturing branches were already at par with U.S. productivity levels (Szirmai and Pilat, 1990). The Indian data only reflect productivity of the formal sector of manufacturing. Inclusion of its informal sector results in productivity levels of only 2 percent of those in the U.S. (van Ark, 1991).

Table 4 summarizes relative value added and productivity in the six countries for 1987. The relative size of the Japanese and South Korean manufacturing sector has risen substantially since 1975. The size of South Korean manufacturing increased from slightly more than 1 percent of the U.S. in 1975 to more than 4.5 percent in 1987. The U.K. manufacturing sector decreased in relative size, reflecting its massive restructuring in the early 1980s.

The dominance of the U.S. in the sample increased somewhat, mainly because of the inclusion of Germany.[12]

Productivity in Japan, South Korea and the U.S. has also risen substantially since 1975. However, it is important to note here that the two benchmarks are independent estimates and are likely to be inconsistent.[13] Germany had a substantially higher productivity level than Japan in 1987 (Van Ark and Pilat, 1993). South Korea had made significant progress since 1975, especially in certain manufacturing branches. Australia and the U.K. are relative laggards in productivity terms, especially compared to other advanced countries (see also, Van Ark, 1993). Indonesia had not yet reached the same productivity level as South Korea had in 1975, although its manufacturing sector was similar in size to South Korea's in 1975.

Within their manufacturing sectors, Japan and South Korea continue to show large differences in relative productivity performance. Their success as manufacturing exporters is primarily based on their performance in a limited number of important sub-sectors of manufacturing, especially in high-tech industries like machinery, transport equipment and electronics. The productivity levels of Germany, the U.K. and Australia are in a much narrower range than those of Japan and South Korea. German relative productivity levels in manufacturing branches are roughly between 50 and 90 percent of the U.S. level, with most sectors at 70 and 80 percent of U.S. productivity. In the U.K., the range is between 45 and 90 percent, and in Australia the levels are between 40 and 75 percent of the U.S. level. Japan and South Korea on the other hand, show much larger variations in relative productivity levels. Both have very weak productive performance in the resource intensive industries, such as food products, and wood, furniture and fixtures. Both are also relatively weak in wearing apparel, but relatively strong in textiles and leather and footwear. South Korea has relative low productivity in chemicals, in spite of its chemical industry drive in the 1970s. In both countries, the top performance is in basic and fabricated metals and in the investment industries.

5. Additive Consistency and the Generalized Theil–Törnqvist Index

This section looks at the possibility of associating a set of constant price comparisons with the generalized Theil–Törnqvist index. Section 3 presented three sets of international average prices, that can be combined with the TT index. This is at variance with the GK approach, where the PPPs and international prices are defined simultaneously through an interdependent system of equations that guarantees additive consistency. The prices derived for the TT index do not satisfy the property of additive consistency, however.

[12] Germany refers here to former Federal Republic of Germany.

[13] For the U.K., van Ark (1993) provides an extensive discussion of the differences between the two benchmark estimates. For Japan and South Korea, Pilat (1994) provides some comparisons over time.

TABLE 5

INTERNATIONAL PRICES FOR MANUFACTURING BRANCHES, 1975, Int.$

	Full Geary–Khamis Procedure	Generalized TT PPPs with Geary–Khamis Prices	with Rao Prices	with Gerardi Prices
1. Food Products	0.909	0.892	0.721	0.749
2. Beverages	0.992	0.973	0.961	0.942
3. Tobacco Products	0.885	0.864	0.608	0.705
4. Textile Mill Products	1.053	1.022	0.882	0.995
5. Wearing Apparel	1.066	1.051	1.021	1.036
6. Leather, Rubber & Plastics	0.919	0.901	1.010	1.012
7. Wood, Furniture & Fixtures	1.376	1.353	1.561	1.460
8. Paper, Printing & Publishing	1.152	1.134	1.311	1.309
9. Chemicals, Petroleum & Coal	1.137	1.115	1.189	1.193
10. Non-Met. Mineral Products	0.989	0.967	0.884	0.889
11. Basic & Fabricated Metals	0.994	0.975	0.988	0.965
12. Machinery, Transp. Equipment	0.873	0.859	0.944	0.983
13. Elec. Machinery & Equipment	0.890	0.874	0.923	0.941
14. Other Manufacturing	1.011	0.996	1.003	1.012
Total Manufacturing	1.000	0.982	1.020	1.030

Source: Basic data from sources quoted in Table 1, calculations based on formulas in text.

The international average prices associated with the TT index for the years 1975 and 1987 are shown in Tables 5 and 6. The price inputs in the calculations are in the form of branch level PPPs with the U.S. as the base country. In view of this, the international prices can be interpreted as international average prices that are normalized to make the PPP of the U.S. dollar equal to unity, when all the branch level prices in the U.S. are taken to be unity. In this sense, they may be interpreted as "relative prices," associated with each branch, while taking total manufacturing as unity. An international price below unity therefore implies a relatively low price for the corresponding branch. The final three columns of both tables show the international prices derived using the generalized TT PPPs in conjunction with the three definitions in equations (7), (8) and (9).[14]

While no generalizations can be drawn on the basis of this empirical exercise, some general remarks can be made and the results can be confronted with *a priori* expectations. The GK international prices from the full Geary–Khamis method and the Geary averaging procedure based on TT parities suggest that it is feasible to obtain meaningful international prices along with the TT parities. The Rao and Gerardi prices seem to be close to each other, showing that the value shares of different branches are stable across countries, which implies near equal weight to each country's prices in the averaging process.

[14]The final row corresponding to total manufacturing is not an international price by definition. These numbers are ratios of U.S. manufacturing value added in national currency to the constant price aggregate derived from the international prices and quantities. This ratio is equal to one for the full GK system, indicating the additively consistent nature of the method. For the TT methods, the figures suggest a deviation from unity of about 3 percent in 1975 and up to 1.5 percent in 1987.

TABLE 6

INTERNATIONAL PRICES FOR MANUFACTURING BRANCHES, 1987, Int.$

	Full Geary-Khamis Procedure	Generalized TT PPPs with Geary-Khamis Prices	with Rao Prices	with Gerardi Prices
1. Food Products	1.103	1.092	1.091	1.111
2. Beverages	1.029	1.021	0.990	1.030
3. Tobacco Products	0.864	0.858	0.792	0.713
4. Textile Mill Products	1.040	1.025	0.991	1.026
5. Wearing Apparel	1.038	1.028	1.041	0.992
6. Leather & Footwear	1.007	0.995	0.842	0.875
7. Wood, Furniture & Fixtures	1.215	1.205	1.173	1.375
8. Paper, Printing & Publishing	1.050	1.042	1.138	1.108
9. Chemicals, Petroleum & Coal	1.087	1.077	1.140	1.143
10. Rubber & Plastic Products	0.880	0.871	0.892	0.886
11. Non-Met. Mineral Products	0.995	0.982	0.920	0.922
12. Basic & Fabricated Metals	1.011	1.000	1.051	1.040
13. Machinery, Transp. Equipment	0.876	0.868	0.845	0.858
14. Elec. Machinery & Equipment	0.945	0.933	0.937	0.933
15. Other Manufacturing	1.004	0.998	1.019	1.021
Total Manufacturing	1.000	0.991	1.007	1.015

Source: Basic data from sources quoted in Table 2, calculations based on formulas in text.

The general relatives implied by the prices are roughly consistent with expectations. A high international price (above 1) implies that the U.S. is a relatively cheap producer in that sector. For these branches, international dollar prices are higher than U.S. dollar prices. In both years this is the case in wearing apparel, wood, furniture and fixtures, paper, printing and publishing and chemicals, petroleum and coal products. For branches with low international prices (below 1), the U.S. is a relatively expensive producer. This was primarily the case in tobacco products, rubber and plastic products, non-metallic mineral products, machinery and transport equipment and in electrical machinery and equipment.

Tables 7 and 8 provide an indication of the extent of additive consistency, or lack of it, associated with the three sets of international prices and the generalized TT PPPs. The first row shows census value added based on the complete Geary-Khamis calculations. The second row shows manufacturing value added directly based on the conversion of national currency values with the Generalized Theil-Törnqvist PPP. Normally this would be all that could be derived using the TT parities. However, by application of the international prices three other rows can be derived, based on constant prices valuations. The absolute and relative price discrepancies show the extent of additive consistency of these sets of international prices.

Examining the differences between the three sets of international prices, it appears that the Rao-system of prices provides lower discrepancies than the other two methods. The discrepancies for 1975 appear to be larger than for 1987. This may be due to the particular group of countries, and potentially also to the higher data quality of the 1987 comparison. An interesting observation is also that in

TABLE 7

MANUFACTURING CENSUS VALUE ADDED IN MULTILATERAL COMPARISONS
(1975 Comparison, values in million International $)

	U.S.	Argentina	Brazil	Mexico	India	U.K.	Korea	Japan
1. Geary Khamis system	442,487	14,191	39,476	16,727	9,732	80,014	5,238	206,214
2. Converted with TT PPP	442,487	11,892	38,545	15,946	8,761	78,730	4,842	198,114
3. TT PPPs/GK prices	434,583	13,920	38,738	16,409	9,528	78,563	5,127	202,449
Abs. discrepancy (2-3)	7,904	−2,028	−193	−463	−767	167	−285	−4335
Rel. discrepancy (%)	1.79	−17.05	−0.50	−2.90	−8.75	0.21	−5.89	−2.19
4. TT PPPs/Rao prices	451,119	13,291	38,751	16,287	9,126	81,008	5,003	208,918
Abs. discrepancy (2-4)	−8,632	−1,399	−206	−341	−365	−2,278	−161	−10,804
Rel. discrepancy (%)	−1.95	−11.76	−0.53	−2.14	−4.17	−2.89	−3.33	−5.45
5. TT PPPs/Gerardi prices	455,694	13,607	39,382	16,508	9,444	82,504	5,180	212,620
Abs. discrepancy (2-5)	−13,207	−1,715	−837	−562	−683	−3,774	−338	−14,506
Rel. discrepancy (%)	−2.98	−14.42	−2.17	−3.52	−7.80	−4.79	−6.98	−7.32

Source: Row 1 based on complete Geary-Khamis system, row 2 based on conversion of total manufacturing value added in national currencies with the generalized Theil-Törnqvist PPP; row 3 to 5 based on summation of revalued manufacturing branch value added. Revaluation according to formulas (7) to (9). Basic data derived from sources quoted in Table 1.

TABLE 8

MANUFACTURING CENSUS VALUE ADDED IN MULTILATERAL COMPARISONS
(1987 Comparison, values in million International $)

	U.S.	Japan	Germany	Korea	U.K.	Australia	Indonesia
1. Geary-Khamis system	1,165,750	604,073	279,863	54,807	161,579	31,655	11,931
2. Converted with TT PPP	1,165,748	584,516	280,899	52,735	161,031	31,305	11,338
3. TT PPPs/GK prices	1,154,942	598,083	277,144	54,227	160,022	31,349	11,813
Abs. discrepancy (2-3)	10,806	−13,559	−3,729	−1,492	1,039	−48	−475
Rel. discrepancy (%)	0.93	−2.32	1.33	−2.83	0.65	−0.15	−4.19
4. TT PPPs/Rao prices	1,173,884	603,619	279,063	54,255	161,201	31,721	11,693
Abs. discrepancy (2-4)	−8,136	−19,095	1,809	−1,520	−140	−419	−356
Rel. discrepancy (%)	−0.70	−3.27	0.64	−2.88	−0.09	−1.34	−3.14
5. TT PPPs/Gerardi prices	1,182,963	605,873	281,188	54,497	162,416	32,019	11,947
Abs. discrepancy (2-5)	−17,215	−21,350	−316	−1762	−1354	−717	−609
Rel. discrepancy (%)	−1.48	−3.65	−0.11	−3.34	−0.84	−2.29	−5.37

Source: Row 1 based on complete Geary-Khamis system, row 2 based on conversion of total manufacturing value added in national currencies with the generalized Theil-Törnqvist PPP; row 3 to 5 on summation of revalued manufacturing branch value added. Revaluation according to formulas (7) to (9). Basic data derived from sources quoted in Table 2.

most cases the revaluation of "real quantities" with the three sets of international prices lead to higher value added (i.e. negative discrepancies) than the direct conversion with the Theil-Törnqvist PPPs.

Prasada Rao and Pilat (1991) have shown that the system can be made completely additive, by defining modified sets of international prices. However, these modified prices may diverge significantly from the non-additive estimates, which implies that the structure of relative prices is affected in the process. If the discrepancies are relatively small it may therefore be preferable to use the non-additive international prices.[15]

[15]The presence of statistical discrepancies in national accounts at constant prices is not unusual, particularly for countries experiencing high levels of inflation.

6. Concluding Remarks

This paper has presented results from the multilateralization of the ICOP work on manufacturing output and productivity. The results from the multilateral procedures do not differ significantly from the binary work, though the results are somewhat sensitive to the aggregation procedure employed. This is particularly the case for countries with small manufacturing sectors and rather different price structures, such as Argentina, India, Indonesia and South Korea. The results are conditioned by the nature of the ICOP data and the level of aggregation at which the multilateralization has been attempted. The paper has explored some methodological issues relating to constant price comparisons associated with the generalized TT index. The results suggest that near-additive consistent "international average prices" can be defined that leave the TT parities intact. This issue deserves further exploration.

The results show large variations of manufacturing productivity levels between countries. Among manufacturing branches, the variation is also quite substantial. Further work may be required to effectuate the multilateralization of ICOP work from the product or industry level.

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