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CAN A PERPETUAL INVENTORY CAPITAL STOCK BE USED FOR PRODUCTION FUNCTION PARAMETER ESTIMATION?

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Due to lack of data on capital disappearance, we simply do not know the covariance of the capital stock with factor inputs and prices well enough to estimate production function parameters. Since replacement rates are rational economic decisions, the errors in a perpetual inventory capital stock vary systematically with the business cycle and such economic variables as rates of technical progress and interest rates. This introduces systematic errors into calculated parameters of production functions and rates of technical progress.

I. INTRODUCTION

Although capital inputs are required for growth accounting for estimating production functions, and for studies of substitution of one factor for another, there are few direct capital input measures available. The direct estimates available come from book value sources, which because of price adjustment or other problems, are seldom used. The capital stock estimates used in studies of productivity and factor substitution come from perpetual inventory sources derived from investment with the aid of various assumptions. The basic data series are *investment*, not capital stocks. Reasonable estimates of the capital stock would be possible if there were good data on the amounts of capital removed from the stock. However, in reality there is no data on the amounts of capital removed from the stock, forcing the use of the very arbitrary assumption that capital is removed at a uniform rate.

In this paper I will discuss some unrecognized difficulties in using perpetual inventory data to estimate production functions. At a minimum these introduce severe biases into the published estimates. At worse, it may be logically impossible to deduce the scope for substitution of capital for other inputs knowing only investment and factor prices. This is because there is no way of knowing how much of the observed investment went to replace old capital goods, and how much went to augment the capital stock.

The basic point might be illustrated by considering something as simple as the family automobile. Suppose you learn that a friend who had one automobile has responded to a fall in interest rates by buying a new automobile. How many automobiles does he have now? One plausible answer is given by the standard perpetual inventory model. He has now two, the original one and the new one.

Another plausible answer exists. He traded the old one in. If one is interested in estimating his consumption of automobile services (perhaps in automobile hours), for the services of the second to displace the first's services is even more plausible. The truth is that neither his stock of automobiles, nor his consumption of automobile services can be deduced merely from observing the purchase of a new car. If one cannot determine whether the interest rate induced investment in an automobile raised your friend's stock of automobiles, one cannot speculate about whether the interest rate drop increased the number of cars he owned, or accelerated his shift to more modern or fuel efficient cars, or merely led him to own a less deteriorated car. The same argument would apply to the deductions possible from observing a friend's computer purchase, or a firm's truck or computer purchases.

The traditional solution for the problem of no data about the amount of capital that is removed from the stock is to *assume* a replacement rate that is proportional to capital stock or is a function of age only. All capital investment beyond this is assumed to be for capital augmentation. The researchers using this procedure or data based on it have provided little evidence it is even approximately correct. Even more important than the resulting errors' absolute size is their covariance with the other variables, causing biased estimates of key production function parameters. From the viewpoint of scientific methodology to fill a hole in data by a convenient assumption is poor practice.

The basic organization of the paper will be to first show with a simple diagram how an estimate of the elasticity of capital input with respect to the price of capital services depends on an unknown variable, the fraction of new investment that represents a net addition to the capital stock. Depending on the unknown net addition to the capital stock, the apparent elasticity can range from a negative number through zero to a very high value, with virtually any value being consistent with the observable data. The point will then be made, again with a simple diagram, that for most econometric purposes the issue is not the absolute size of the errors in lengths of life or capital decline rates, but how these errors covary with the other variables of economic interest, especially factor price ratios. A quick review of the evidence on the accuracy of the basic data will show that the claims are at best that the capital stock estimates are of the correct order of magnitude, not that the errors (admitted to be large) are uncorrelated with the other relevant economic variables. The discussion will then move to what in economic theory determines replacement, and what this in turn implies for the covariance of the errors in the perpetual inventory method with other economic variables of interest.

II. Diagrammatic Discussion

In laboratory science courses it is customary to use error brackets as an aid to seeing whether particular features of the data might result from mere data errors. Thus, let us add error brackets to the observations used in measuring the elasticity of substitution. Let us assume that all other data is accurate, and concentrate on the problem of drawing conclusions from an initial year's data and the amount of investment.

Point A in Figure 1 shows the previous period's inputs of labor and capital (assumed known through some unspecified method). Let us make the problem of estimating the elasticity of substitution relatively simple by assuming it is

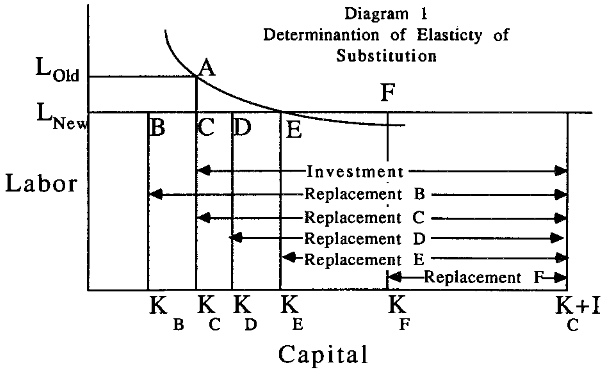

Figure 1. Determinantion of Elasticity of Substitution

known that technology has not changed (perhaps as a direct result of observation of production techniques) and that the capital input definition being used is appropriate for measuring the capital intensity of production (see Miller, April 1989, for further discussion). Data has been expressed per unit of output so that the economy can be visualized as moving along an isoquant. The current labor input (L_{new}) and the investment (1) since period 1 is known. The relative price of capital services has dropped and the estimated response to this price change will be used to estimate the elasticity of substitution. The labor input after the change in relative prices is measured and it is known the firm is operating somewhere along the horizontal line through L_{new}. The question is where on that line. Until the point is known, the elasticity can not be calculated.

Again it is useful to follow the procedure of the physics laboratory and indicate the data points that are consistent with the available observation. A direct capital input measurement is not available, and the only relevant facts known are the initial stock, and the investment. The new input is the old input plus the investment and minus the amount of the stock that has disappeared, *R* (for replacement). The possible new points include (among others) *B*, *C*, *D*, *E*, and *F*. Since the amount of capital that has disappeared is unknown, there is no logical basis for choosing between these points. Even with accurate information on the fraction of the capital that had disappeared over a long period of time, there is no reason to presume that the amount that disappeared over a short period was the average long term amount (and good reasons in economic theory for believing the short run could differ from the long run).

At point *C* the estimate of replacement requirements equals gross investment, leaving the capital stock unchanged. If the economy is at *C* there has been no substitution of capital for labor, and the observation that relative factor prices have changed implies a zero elasticity, *D* represents a point where the replacement was somewhat over estimated, but not enough to leave the capital stock unchanged. If the estimates make the economy appear to be at *D*, events are interpreted as indicating that there has been little substitution of capital for labor, and that the elasticity is low.

Point *B* represents the case where the estimated replacement requirement exceeds the gross investment, causing the apparent capital stock to decrease. This could easily happen if there was no technical progress, but the assumed replacement requirements were based on an earlier period of appreciable technical progress and high obsolescence.

Point *F* represents the opposite case. At *F*, events are interpreted as involving much substitution of capital for labor, and a high elasticity. E happens to be where the economy actually is but there is no way with the actual data to discover this and to calculate a correct capital figure.

The simple situation depicted in Figure 1 applies to any single period of time. If we cannot separate points *B*, *C*, *D*, *E* and *F* from one another because of lack of capital data, there really is not much scope for estimating marginal rates of substitution, substitution elasticities, rates of technical change etc. This very simple point does not appear to be recognized since there is a steady flow of econometrically sophisticated studies estimating parameters of production functions.

There is a well established custom of modeling production as involving perfectly malleable capital jelly which disappears at a constant rate, without thinking about what really happens when a real world firm decides to purchase a new machine because technology or factor prices has changed. Does it add the new machine to the factory leaving the others with their old role, or is an old machine replaced?

If most capital disappearance was due to physical causes that were independent of economic factors (relative prices, the stage of the business cycle, the rate of technical progress etc.) it might be plausible to assume a constant rate of disappearance. Alas, this is not a defensible position. Most capital goods appear to have very long (a century at least) physical lives if they are maintained and any components that wear out are replaced.[1] Structures are commonly observed to last over a century. These physical lives are much greater than the lengths of life used in perpetual inventory studies. The importance of these observations is that older capital goods stay around for a long time and our data series cover less than one physical lifetime for structures, and at best perhaps a couple of physical lifetimes for most machines (the lengths of time series used will be discussed further below).

Most capital appears to be replaced not because it became inoperable (which would imply its removal from the stock), but because it was replaced with something better (which raises the question of what happens to the old good and what is the effect on the gross stock of the replacement), possibly because of changed circumstances.

In a net capital series most capital disappearance is not due to the physical disappearance of the capital goods, but is due to a reduction in the amount of capital they are considered to represent due to obsolescence or deterioration.

[1]This durability arises because the materials used (concrete, bricks, steel, aluminum, copper, plastics, and wood) will retain their shape and strength almost forever if protected from exposure to oxygen, moisture, termites, fire, and catastrophic destruction. Most machines consist of a frame on which a number of moving parts are attached, and if parts are replaced as they wear out the machine can last virtually forever.

Year to year changes in the stock are very sensitive as to exactly when these obsolescence and deterioration adjustments are considered to occur. Is it really logical to assume that obsolescence occurs at a uniform rate when the stock estimate's purpose is to estimate a rate of technical progress believed to be changing? Likewise, with some obsolescence caused by changing relative prices making the current stock of the wrong design for current price ratios, is it really proper to assume this occurs at the historical average rate (built into the usual estimates) while simultaneously using more rapid price factor price changes to estimate elasticities of substitution (see "Factor Price Change Caused Capital Input Errors" below)?

Role of Covariances

Even if evidence was presented that the errors in the perpetual inventory method are small enough that the basic series is usable for some purposes, this would not imply that the estimates are suitable for estimating such parameters as elasticities of substitution, rates of productivity growth, and biases in technical change. The standard statistical methods estimate these parameters from covariances either of the factor shares, the factor inputs, or the factor prices with other variables. Where the price of capital services is used in a cost function, it will often be found that it was computed from dividing the capital input into the total income from capital (or doing corresponding exercises with indices). The values of the estimated parameters depend not on the size of the capital stock at any given time, but on whether the capital input or the price of its services is closely correlated with other variables such as factor prices or outputs. Claims that estimates of variables (such as a capital stock) are on average correct do not imply that covariances calculated using the variables will be even approximately correct. Where there is a net stock with the disappearance of the capital series allocated over the full life of the goods, the covariance is very sensitive to how this is done. An allocation that covaries with a particular explanatory variable (or a failure to recognize such a covariance that really occurs) will affect the calculated covariances.

These covariances are sensitive not merely to whether a machine by the end of its life has been fully removed from the capital stock, but to exactly when it was removed from the stock or its services from the estimates of capital services. With output held constant, any time that capital input diminishes, the current econometric methods assume (in a stochastic manner of course) that either technology has changed to reduce the capital input or that some other factor has been substituted for the capital no longer used. Which factor is believed to have substituted for capital depends on how well the decrease in capital services correlates with either increases in the services of other factors, or with changes in the prices of the other factors relative to the prices of capital services. With perpetual inventory data the parameter estimates are very sensitive to exactly when (if ever) goods are removed from the stock, or a reduction in their services recognized.

Many hypotheses (including rational replacement and mechanical rule replacement) appear consistent with available time series data. At best, the average length of life used is correct (U.S. Bureau of Labor Statistics, 1983; Hulten and Wykoff, 1981; Griffin, 1979, Appendix 1). Small shifts in the lifetimes of capital goods (say plus or minus five years) are consistent with the available evidence, and with the estimated *average* lengths of life being correct. Yet such small shifts correspond to wide swings in the fraction of investment used for replacement and hence in the increase in the capital stock. In the estimation of net stocks, small shifts in the allocation of obsolescence or deterioration between adjacent years are consistent with the available evidence, but may have large effects on covariances with other variables.

The problem is illustrated by Figure 2. This shows two possible capital stock series. Series *A* is flat. The other (series *B*) has the same average length of life and value for capital. However, in accordance with economic theory, firms use more capital services when the cost of capital services is low. Both capital stock series are consistent with a particular investment series and the available data on *average* length of life. Economists simply cannot distinguish between the two cases, and hence cannot estimate covariances.

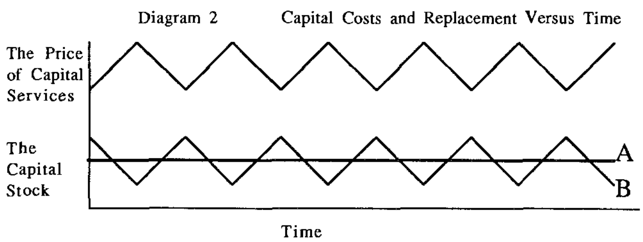

Figure 2. Capital Costs and Replacement Versus Time

Notice how the capital input's covariance with the price of capital service differs between the two cases. Curve *A* shows no covariance between the capital input and the price of capital services. Curve *B* shows a strong covariance. If the two time series for capital input over time cannot be distinguished, it would appear impossible to use the existing data to distinguish between the hypothesis that the price elasticity of demand for capital services was zero and the hypothesis that it was quite high. The data is inadequate to even test the theory that elasticities have the right sign since an inverted version of *B* would still be consistent with the investment and length of life data.

III. The Perpetual Inventory Formula

How has the extensive research on elasticities of substitution and total factor productivity been conducted if no one actually knows how much capital disappears? The answer, of course, is that in the grand tradition of economics, the required data has been assumed. Capital disappearance has been taken to depend on only the age of the capital goods, with a given fraction of each vintage disappearing in a particular year. In practice, the percentage disappearing has usually been taken to be the same for all vintages.

In a general form the perpetual inventory method holds that the capital stock $K_t = S_1 I_1 + S_2 I_2 + S_3 I_3, \ldots, S_n I_n = \sum_0^t S_i I$ where S_i is the weight given to investment of vintage (age) i, with each S_i being a constant.

In most applications, this is specialized to: $K_t = (1-\delta)K_{t-1} + I$, with δ being the percentage of the stock disappearing each year (a constant).

Differentiating K_t with respect to I reveals the simple result that $dK/dI = 1$, since the retirement rate and the size of the initial capital stock are predetermined. In words, short run variations in investment are interpreted as producing identical changes in the size of the capital stock. Reversing the reasoning, the short run variations in measured capital input are due to short run changes in investment. Apparent correlations between capital input and other variables such as factor prices (interest rates) may reflect the variations in gross investment with these variables, or a common factor (such as the business cycle) impacting both investment and factor prices or factor shares of income.

Of course, once the analysis moves beyond a single year, the past investment has some impact on the current stock, but the weights given to the investment in the immediately preceding years are so small that little of the year to year variation in the capital stock caused by changing investment is offset by the much more slowly changing estimated capital disappearance.

While the perpetual inventory model asserts that $dK/dI = 1$, no evidence exists that this is correct. Many observers upon noticing that new machines are installed would expect a large offset through removal of older machines. If the capital removed equals the capital brought in (as when one machine is removed when another is added, and both are considered to embody the same quantity of capital), $dK/dI = 0$. If the new goods represent less capital than the old goods, dK/dI is actually negative. These values appear at least as plausible as the usual value of unity in a perpetual inventory.

To some a negative dK/dI is implausible. How could a burst of investment reduce the capital stock? Suppose a gross capital concept is being used where the quantity of capital a old good represents is considered to be the amount the good represented when produced (as Miller, April 1989 urges for factor input studies). Also suppose technical progress has reduced the amount of capital embodied in a new state-of-the-art machine.[2] When new machines embody a technology that is less capital intensive than the existing machines, higher investment actually serves to reduce the capital stock.

The importance of the extreme case of dK/dI being negative is that a reduction in interest rates, or in the prices of capital goods can cause a surge of investment as obsolete technologies are replaced (see Miller, Fall 1988 for more on decreases in the price of capital services increasing the capital input). If the perpetual inventory shows an increase in capital input when the input has actually decreased, the estimates of the elasticity of demand for capital services with respect to interest rates will be very misleading.

[2]This regularly occurs when the speed of the machinery or vehicle is increased, or the technical obstacles to large machines are overcome. For instance, the jet airplane was capital saving because its greater speed made a few airplanes able to generate the same passenger miles previously provided by a larger number of planes.

Major questions in applying the perpetual inventory formula include whether to adjust the quantity of investment for improved quality and what capital disappearance rate to use? (Does it for instance include obsolescence?) The latter question is really one of what characteristic of capital is to be considered to have disappeared as capital ages, and what units of capital are to be measured. These are complex questions which the author has discussed elsewhere (Miller, 1983; 1985; April-June 1988; April 1989; July 1989; 1990). In essence the answers depend on the purpose for which the capital stock estimates are desired. The author believes a reproduction cost based gross stock is most appropriate for studies of factor substitution. A marginal product (or service price) weighted measure of services closely related to the net stock is appropriate for growth and productivity studies. However, these questions will not be discussed further here.

IV. Empirical Evidence on the Perpetual Inventory Method

There are reasons to question the accuracy of the absolute length of life values used in constructing perpetual inventory estimates. Till recently most such U.S. data could be traced back to lifetimes published for tax purposes in 1942 (U.S. Treasury, 1942), which meant that, at best, the information must have come from retirements during the thirties. In turn, the machines then being retired would have often been built before World War I and the structures in the previous century. It is not hard to imagine significant errors in such data. More recent research is often based on work by Hulten and Wykoff (1981) which is more recent, but as they would admit, imperfect.

While one might argue that constant replacement rates are unreasonable assumptions for all purposes, this is not the argument of this paper. They, and the associated perpetual inventory method may be quite reasonable for some purposes, such as estimating approximate capital stocks, or capital output ratios. Such capital output ratios may be useful in estimating how much investment would be required to permit an economy to expand using its customary production techniques. These were the purposes for which the perpetual inventory technique was developed (Goldsmith, 1951, 1956). For such purposes, the *absolute magnitude* of the errors may be large, but acceptable.

Creamer (1961) and Kuznets (1961, Appendix D) have compared perpetual inventory estimates with available census benchmarks. The evidence is that over periods of about a decade, both methods give reasonably similar results. However, this evidence is insufficient to show that year to year errors are small, or that the errors are uncorrelated with other relevant variables.

Much more encouraging is a recent set of comparisons by Gorman *et al.* (1985, p. 44) of the U.S. Bureau of Economic Analysis estimates of gross stocks of fixed private capital in historical cost valuation and Internal Revenue Service estimates of gross book values of depreciable assets from selected years for corporations and for proprietorships and partnerships combined from 1959-81. These show a reasonable degree of agreement with the largest year to year disagreements being 3 percent (1979-80). There are no year to year comparisons for industries or for structures or equipment separately. Also there are no comparisons for net stocks after depreciation (caused by obsolescence and deterioration). The lack is important since there are strong theoretical reasons to believe that when technical progress or price changes are unusually rapid the rate of obsolescence is also unusually rapid, causing the errors in the stock (and associated flow of services) adjusted for obsolescence to be correlated with the rate of technical progress.

The problems discussed in this paper relate primarily to time series estimates of production function parameters since the correlation of perpetual inventory capital stock errors with other variables appears much less serious for cross-sectional observations, although cross-sectional observations are exposed to their own sources of error.[3]

Rational Replacement versus Constant Replacement

The assumption of a constant replacement rate might not be too unreasonable if there was no evidence against it. Alas, assuming that replacement is not influenced by factors other than the age and size of the capital stock conflicts with a well developed economic theory that makes replacement a rational economic decision involving such variables as possible savings in variable costs, capital good prices, and interest rates (Smith, 1961; Salter, 1966; Feldstein and Rothschild, 1974; Nickell, 1978).

The constant replacement rate theory has been tested (Feldstein and Foot, 1971; Eisner, 1972) and the evidence decisively rejects it. The results appear to be consistent with a rational replacement model. Even more important "replacement and modernization expenditures . . . moved up and down with expansion investments" (Eisner, 1972, p. 304), a result also reported by Feldstein and Foot. Bitros and Kelejian (p. 276) report that high gross investment promotes scrappage. The showing that times of high expansion investment are also times of high replacement, suggests that dK/dI is less than 1.

The above research (done to predict investment) defines replacement investment to be the value of the investment undertaken to replace an item (the capital in the replacement), while for capital stock estimation one wishes to know the amount of the capital represented by the good replaced. However, since old capital is replaced when replacement investment is made, showing that the two categories of investment move together does provide evidence that dK/dI is less than one.

[3]Probably the most serious problem is that location theory shows that industrial location depends on relative factor prices, i.e. industries locate where the prices of factors they use most are cheapest. Evidence on varying factor inputs across locations with respect to factor prices can easily be interpreted as evidence regarding factor substitution holding output mix constant. For instance, the low utilization of energy in certain states where energy costs are high is primarily due to specialization in non-energy intensive products. A cross-sectional regression will be dominated by these locational effects, but will yield a coefficient that might be interpreted as evidence of substitution while holding output constant. For instance, if the lower energy use in high energy price states is interpreted as evidence that rising energy prices over time will be met by substitution without a fall in living standards, the conclusion is incorrect because the observed low energy use depends on a product mix that would not be possible for a whole economy operating in an environment of energy scarcity. The low energy use in Connecticut manufacturing is due to a concentration on products that require little energy, with the necessary energy intensive raw materials being imported from elsewhere. If the whole economy faced the prices Connecticut now faces, it could not produce the final products without the raw materials (copper wire requires copper), making policy based on a high elasticity of substitution (estimated from cross-sectional data) incorrect.

When a gross stock is being estimated or when capital input is based on original cost reduced for loss of capacity (as Miller, April 1989 argues to be appropriate for factor input studies and estimation of substitution elasticities) the quantity of capital represented by the good being replaced is of the same order of magnitude as that of the goods displacing them, and failure to allow for induced replacement can cause errors.

It might appear that when a net capital stock was being estimated (say for growth accounting), the older goods whose retirement is caused by purchase of new goods represent only a fraction of the stock embodied in the new goods, and the errors resulting from not detecting the induced retirement might be tolerable. However, the event causing retirement of the older goods on the margin of being retired (a technical change or a price change) may also increase the obsolescence of the infra-marginal goods, causing a large error in the estimated net stock.

Since estimation of production functions depends on the errors in the capital stock estimates being independent of other variables in the equations, evidence that either investment affects replacement, or investment and capital disappearance are both affected by the same variables, creates severe (and apparently unrecognized) identification problems. Thus evidence that items included in income to capital such as profits (Eisner, p. 304), the internal availability of funds (p. 56), or other components of the user cost of capital (Feldstein and Foot, p. 56) affect replacement investment is quite important. In many studies income to capital (as reported in the national income statistics) directly enters into the calculated cost of capital services. Thus there will be a systematic correlation between the errors in the capital stock (due to assumed replacement equaling actual replacement) and the cost of capital services, biasing any estimates of the effect of capital service prices.

As Feldstein and Foot (1971) point out, the curve giving the present value of costs versus the time of replacement is very flat in the vicinity of the optimum length of life. This flatness is important because it indicates that the exact timing of replacement can be determined by short run considerations, many of which may be correlated with factor prices or the business cycle. For instance, firms may adjust the exact timing of their replacement to the availability of funds from retained earnings or other outside sources. If, as their evidence suggests, retained earnings are highest in boom years when sales are highest, there will be a spurious correlation of the apparent capital stock with output.

V. Econometric Implications

The complexity of production functions being estimated has gradually increased over time without anyone asking whether the quality of the data is able to support the increasing requirements placed on it. As the number of variables to be estimated has grown, the length of the data series used has tended to decline (probably because data series for all the variables of interest can not be obtained except for recent years).

For instance, Berndt and Khaled (1979), Diewert and Wales (1987), and Anderson (1981) all use 1947–71 data; Brown and Christensen (1981) uses 1947–74 data; Denny, Fuss, and Waverman (1981) use 1949-71 data for the U.S. and 1961-75 data for Canada; Jorgenson and Fraumeni (1981) use data from an even shorter period 1958-74. Studies by these authors typically estimate elasticities of substitution between several factors and often simultaneously estimate rates of technical progress or factor saving biases. During these years there were only a few business cycles, possibly one or two equipment replacement cycles, and not even a full replacement cycle for structures.

One might legitimately wonder how it is possible to estimate all these parameters from simple time serious of output, factor inputs, and factor prices. In general, even with perfect data, it is impossible. A given set of data can usually be described by many different combinations of shifts in the production function and movements along it. Experiments with plausible parameter values (reported in Barnhart and Miller (1990)) show that plausible levels of covariance between variables and the errors in a perpetual inventory capital stock can lead to large errors in estimating production function parameters.

Differentiating Between Hypothesizes

In general, the different hypotheses can not be separated with the part of the available data that reflects trends. Econometric procedures that claim to be able to separate input variations due to output growth, technical change, and changing relative factor prices require knowing the covariances of the capital input with other variables over periods of time that represent only a fraction of the physical lengths of life of the goods involved. Yet, as noted, the available investment data is consistent with virtually any values for the covariances of capital input with other variables.

As an example, consider a simple model where there are believed to be four factors of production, i.e. capital, labor, materials, and energy. Suppose it is observed that with output constant that the input of labor has decreased and that the price of labor services relative to the prices of other factors has risen. This observation could be explained by price induced substitution of capital for labor, materials for labor, energy for labor, or by an infinite variety of combinations of these effects. With each explanation would be an estimate of the elasticity of substitution of capital for labor, materials for labor, and energy for labor. If there were only two data points (reflecting the start and the end of the period), it would be recognized that these hypotheses could not be distinguished.

In practice how are these hypotheses distinguished? Fortunately, there are more than two data points. Abstracting from the econometric details, the choice between the hypotheses can not be based on the broad trends in labor and capital input relative to the relevant factor prices (for as noted broad trends do not give enough data).

While the econometric studies use sophisticated methods and duality theory, understanding is facilitated by focusing on the logic of the process. If there is a high covariance of the labor/capital input ratio with the relative prices of capital and labor and no relationship of the labor to materials or labor to energy input ratios with the relative prices of labor and energy or labor and materials, the conclusion would be that the fluctuations in labor input were caused by the changing relative prices of capital and labor. The econometric procedure supplying the production function whose elasticities best fitted the data would assign a high cross elasticity to demand between capital and labor and a low cross elasticity with the other variables.

The covariances are critical, not the absolute value of inputs, or even the absolute values of the trends. Knowing that the covariance of the capital/labor input ratio with the relative prices of labor and capital is heavily affected by systematic errors in the capital input due to the correlation of capital replacement or capital idleness with relative factor prices, we should have very little confidence in the estimates of the elasticity of substitution coming out of the procedure.

Business Cycle Related Errors

Very worrisome is the exclusion of business cycle effects from most empirical studies, even though for the few years being studied the errors in capital input are likely to be correlated with the stage of the cycle, as are estimates of relative factor prices.

Business cycle theory provides several reasons for investment and either factor prices or factor inputs to be correlated, without the correlation reflecting either movement along the production function or shifts in it. Investment may either cause the cycle (via Keynesian effects) or respond to the cycle (via the accelerator) producing spurious correlations with either labor input, or factor price ratios.

Most production functions are estimated assuming that output is always the maximum possible given available factor inputs. Yet it is well known that economies operate closer to capacity at some times than at others. At the low points in the cycle output is down as are the inputs of capital, labor, energy, and materials, although there is no reason to believe the reductions in all inputs are proportional.

Very interesting in this context is the showing by the Bureau of Labor Statistics (1983, p. 28) that capacity utilization explains about 80 percent of the total variation in output per unit of capital in manufacturing for 1948-81. Output per unit of capital is the reciprocal of the capital input per unit of output; a key element in many econometric calculations. (Even studies that appear to use the prices of capital services use capital quantities in calculating these prices.) With capacity utilization essentially a surrogate for the business cycle, any correlation of factor price ratios with the business cycle could give rise to spurious correlations, which would affect estimated substitution elasticities.

What sort of biases could arise from the business cycle correlated errors in capital input? It is known that some factor prices are more sensitive to the cycle than others. In particular wages are sticky, and seldom decline in depressions. Raw material prices do decline in depressed times. Energy prices might be expected to decline in recessions like other material prices, but in recent years sudden surges in energy prices appear to have caused recessions. Most important of all, profits and interest rates, major components in the price of capital services, vary with the cycle.

Consider what conclusions might be drawn from observing that the cost of capital services dropped sharply relative to the prices of labor services while the capital to labor ratio rose. Given this data, an economist trained in neoclassical theory would conclude that capital had been substituted for labor due to changes in relative prices and could calculate an elasticity of substitution. If this sequence of events occurred repeatedly, he would have more confidence in his conclusion and estimated elasticities. However, as noted this pattern could also be caused by the normal operations of the business cycle and the resulting pattern of capital input errors. Similar problems could arise from other cyclical patterns in the pricing of factor inputs or in their use.

Although there is an extensive literature showing the distribution of income among the factors of production varies over the business cycle (for instance Nolan's 1987 paper in this journal), econometricians estimating production functions write as if none of the cyclical effects that are the subject of the income distribution literature existed, even though their estimates of production function parameters in the translog method are often calculated from the regression of factor shares on factor prices using a time period short enough so that the data must be dominated by cyclical effects and by data errors.

Those whose estimates are oriented towards measuring productivity typically limit the damage from cyclical effects by making peak to peak comparisons. This is not done by many trying to estimate elasticities of substitution among several factors. Unfortunately, increasing the number of data points by using poorer data merely leads to believing that something is known when it is not, which is worse than not knowing.

Factor Price Change Caused Capital Input Errors

Recently there has been some discussion of the possibility that the OPEC price increases may have made part of the capital stock obsolete without this being detected in the capital stock statistics (Baily, 1981). The effect has generally been treated as a possible one-time event. What has not been noticed is that obsolescence is a normal result of changing factor price ratios, and that it will bias time series estimates of the elasticity of substitution of capital for other factors. For instance, Berndt and Wood's (1975, 1979) papers reporting energy capital complementarity do not mention this source of bias, although it is a very serious problem for their work and a possible contributor to their paradoxical energy-capital complementarity finding (Miller, 1986).

The Adjustment Lags Problem

A widely recognized problem is that the shift to the technique appropriate to new factor prices takes time, due to the need to replace the old machines using the old technology. Thus, the change in long run factor proportions is understated, as is the substitution elasticity. Distributed lag methods can be used *if the adjustment rate is independent of factor prices.*

Unfortunately, replacement of capital goods in response to factor price changes is not an easily modeled constant rate process, but one whose rate depends on the costs of services from new capital goods (which depend on relative factor prices and such variables as the prices of capital goods, interest rates, and tax rules). Following increased investment due to lower capital costs, it is very hard, possibly impossible, to separate an acceleration of the shift to a new technique from a shift in the optimum technique itself.

It should be noted that when replacement occurs, it often affects only "marginal" capital goods, but the deterioration or obsolescence that made a good marginal often occurred many years earlier. A significant innovation may have occurred twenty years earlier (causing replacement of some goods that were marginal then), and reducing the rent of existing capital goods. Deterioration and other innovations gradually reduce rent of the goods built before the innovation. Finally, a reduction in interest rates makes replacement economical. The new goods will be considered to represent much more capital input (net) than the old goods because they are newer (and presumably less deteriorated and obsolete). They represent an increase in rent defined capital input. Does the final incorporation of an innovation that has been best practice for twenty years provide information about how the equilibrium capital intensity is affected by the factor price change that finally made replacement economical? It clearly does not, yet econometric models, even with lagged adjustment, would interpret replacement as evidence of a shift to a more capital intensive technique of production caused by lower prices for capital services and calculate elasticities.

VI. Implications and Conclusions

There are alternatives to the perpetual inventory method for obtaining capital estimates. Direct estimates may be available from census questions, accounting data (accounting procedures normally provide for removing items from the stock when they are retired or sold), insurance valuations, etc. These methods, although used from time to time, have usually been rejected because they were believed (probably correctly) to be less accurate than perpetual inventory methods. When the goal is merely to answer the simple questions about how large the capital stock is, or what is the capital output ratio, the correct criterion for selecting a measurement method is its accuracy.

However, when the capital measurements are intended to help measure something else, such as rates of technical progress or elasticities of substitution, the criterion is what measurement system will give the best estimates of the parameters of interest. Here the key question is less the absolute values of the errors, but whether the errors are correlated with the independent variables to be estimated? Some of the alternative sources may contain errors that are less correlated with the other variables of interest. (Remember that an absolute error that remains constant over time has no effect on the calculation of covariances.)

The goal of econometric production function estimation is usually given as learning about technical constraints and how these change over time. The reader should remember that there is a whole discipline, called engineering, concerned with technology. If it is concluded that current data is inadequate for discovering the scope for factor substitution, a feasible alternative is to rely on engineering knowledge.

Conclusions

Calculating a capital stock (or a flow of capital services) with the aid of a perpetual inventory requires an item of information that is simply lacking, the amount of capital disappearing each year. Without this one has little hope of calculating the various covariances needed for econometric production function parameter estimates.

The argument here is not merely that there is a wide range of uncertainty about lengths of lives, a problem that conceptually could be dealt with by trying different lengths of life in the models and seeing how the answers were altered. The real problem is that if errors exist (and they are virtually certain to) their size and direction will be correlated with the independent variables of interest. Even if the average lengths of life used in our perpetual inventory calculations are correct (i.e. over the length of time used in a study, the algebraic sum of the errors is zero) the correlation of the errors with the independent variables may be such that it is impossible to identify and estimate the parameters of interest through time series studies.

Any correlation of investment rates (and the perpetual inventory capital stock calculated from them) with a ratio of capital costs to prices of other factors (i.e. low interest rates, low prices for capital goods, favorable tax treatment, high wages) may be due to a shift in the optimum production technique towards using more capital (a high marginal rate of substitution), or to the optimal technique remaining unchanged but the rate of shifting to this technique increasing, or to increased replacement of deteriorated goods because their replacement is now economic, or to any combination of the above. Any given rate of gross investment is explainable by a wide range of combinations of the above effects. It is implausible that these different causes can be distinguished if data series which are less than several replacement cycles in length are used.

The custom of writing papers with an elaborate theoretical section at the front, with the data used being mentioned only in footnotes or in a data appendix is unfortunate. There are some problems with data derived from perpetual inventory, and relegating the data derivation to appendices or footnotes has prevented readers from appreciating just how frail the evidence for the empirical conclusions really are.

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