ON MEASURING POVERTY

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This paper investigates the properties of various measures of poverty and of the "difficulty of alleviation of poverty". It is found that the ranking properties of both kinds of indices can be quite counter-intuitive and that they could be misleading if used for policy evaluation. An alternative index is proposed; it is compared to the other indices and seems to fare rather well. To illustrate, a special reference is made to S. Anand's recent article on poverty in Malaysia.

In a previous issue of this Review, S. Anand[1] describes and estimates for Malaysia several measures of poverty and of the economy's ability to eradicate it. The author concludes with a poverty profile of Malaysia and the recommendation that "in order to design efficient policies and projects to help the poor selectively, we need to identify smaller more homogeneous groups . . . with particularly high incidences of poverty".[2]

In this paper we show that the very natural redistribution rule implicit in the above prescription (i.e. to help the most poor first) would be put at an handicap with the rule of redistributing from the rich to the marginally poor if it was evaluated by some of the indices S. Anand proposes. This is true in particular of Sen's index which could even "recommend" disequalizing transfers between the poor. The indices of "difficulty of alleviation of poverty" he describes are also investigated from the point of view of their reactions to income transfers and the behaviour of some of them is best described as erratic.

We conclude the paper with the introduction of a "better" poverty index and use S. Anand's empirical results to estimate it for Malaysia. We start by deriving a few theoretical results to be used later on in the paper.

Let us call someone whose income is no larger than the poverty line a poor person and someone whose income is strictly larger than the poverty line a rich person. Quite naturally, the poverty measures will be interpreted as negative welfare indices.

A Family of Poverty Measures

The family of measures we are concerned with in this section is defined as follows:

Definition 1. "A Poverty Index based on the Rank Order of the Poor (P_r in short) is the normalized weighted sum of the income gaps $g_i = Z - y_i$ of the poor

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[1]S. Anand, "Aspects of Poverty in Malaysia," *Review of Income and Wealth*, Series 23, No. 1, March 1977, pp. 1–16.

[2]*ibid*, p. 16.

persons, with weights $q+1-i$;

$$P_r = A(Z, q, n) \sum_{i=1}^{q} g_i(q+1-i).\text{"}$$

Z is the poverty line, Y is the income vector with the convention that $y_1 \leq y_2 \ldots \leq y_n$, q is the number of the poor and n the total population.

The P_r family is a generalization of Sen's poverty index[3]

$$P = \frac{2}{(q+1)nZ} \sum_{i=1}^{q} g_i(q+1-i),$$

which embodies the further specification that $A(Z, q, n) = 2/[(q+1)nZ]$.

We now investigate the properties of the P_r family from the point of view of their reaction to transfers and pure income changes. Z and n are assumed to be given and we shall deal with single transfers from one person to another and single income changes. In addition, we consider only those transfers which preserve the relative ranking of the donor and the recipient. This is an innocuous restriction as the familiar anonymity property implicit in P_r's definition guarantees that there always exists a sequence of such transfers whose net effect on P_r is equivalent to a transfer which inverts the relative ordering of the two people involved. No assumption is made about the relative size of the mean income and the poverty line.

For a given n and Z, $A(Z, q, n)$ does not vary if the number of the poor does not change; the weighting system of P_r is such that the larger the gap of a poor person the larger its weight and the weight of every rich person is nil.

As a result, we have:

Proposition 1. "If the number of the poor does not vary, an equalizing transfer between poor persons, an increase in a poor person's income or a transfer from a rich person to a poor one decreases P_r; a disequalizing transfer between poor persons, a loss of income by a poor person or a transfer from a poor person to a rich one increases P_r."

P_r's response to a transfer or a pure change of income which affects the number of the poor is less straightforward as then both $A(Z, q, n)$ and the weights vary as functions of q. It is useful to define and investigate the effect on $P_r(Y)$ of a "*ceteris paribus* change in the number of the poor".

Define $\bar{S}$, a subset of the set of all income vectors as follows:

$$Y \in \bar{S} \text{ iff } q < n \text{ and } \sum_{i=1}^{q} g_i > 0.$$

For any given $Y \in \bar{S}$ and any given P_r, define $\bar{P}_r(Y)$ as:

$$\bar{P}_r(Y) = A(Z, q+1, n) \sum_{i=1}^{q} g_i(q+2-i)$$
$$= A(Z, q+1, n)\left[\frac{P_r(Y)}{A(Z, q, n)} + \sum_{i=1}^{q} g_i\right]$$

[3]A Sen. (76), "Poverty, an ordinal approach to measurement," *Econometrica*, Vol. 44, No. 2. March 1976, pp. 219–231. Like S. Anand, we are borrowing Sen's notation.

$\bar{P}_r(Y)$ can be viewed as the P_r of a distribution exactly like Y but for the fact that one rich person's income is assumed to be equal to Z. As far as P_r is concerned, such a distribution is different from Y only in the sense that the number of the poor is larger by one, as its $g_{q+1} = 0$ by construction.

$P_r(Y) - \bar{P}_r(Y)$ can therefore be interpreted as the change in $P_r(Y)$ due to a *ceteris paribus* decrease in the number of the poor and viewed as the change in $P_r(Y)$ due to a pure increase of income, a transfer from a rich person, or the limit value of an infinitesimal transfer from a poor person, all occurring to someone with income exactly equal to Z.

The usefulness of such a construction is best seen from the way it is put to use in the proofs of propositions 4 and 7 below.

Definition 2. "P_r is Positively Concerned with the Number of the Poor (PCNP) iff it decreases due to a *ceteris paribus* decrease in the number of the poor, i.e. $\bar{P}_r(Y) > P_r(Y)$, for all $Y \in \bar{S}$."

Definition 3. "P_r is Negatively Concerned with the Number of the Poor (NCNP) iff it increases due to a *ceteris paribus* decrease in the number of the poor, i.e. $\bar{P}_r(Y) < P_r(Y)$, for all $Y \in \bar{S}$."

We now show that the P_r family can be partitioned into three subsets; the subset of P_r's which are PCNP, the subset of those which are NCNP and a last subset of those which are neither.

By definition, a P_r belongs to the first subset iff, for all $Y \in \bar{S}$:

$$A(Z, q, n) \sum_{i=1}^{q} g_i(q+1-i) < A(Z, q+1, n) \sum_{i=1}^{q} g_i(q+2-i),$$

that is

$$\frac{A(Z, q, n)}{A(Z, q+1, n)} < 1 + \frac{\sum_{i=1}^{q} g_i}{\sum_{i=1}^{q} g_i(q+1-i)}$$

The lowest possible value of the right hand side over all Y is for a Y such that all the gaps are nil but one. This can easily be ascertained by checking that any departure from such a distribution would increase it. With such a Y, the only non-zero gap is g_1, its weight is q and the right hand side is

$$1 + \frac{g_1}{qg_1} = \frac{q+1}{q}, \qquad \forall g_1.$$

As a result, we have

Proposition 2. "P_r is PCNP iff $A(Z, q, n)$ is such that, for all Z, n, $q < n$:

$$\frac{A(Z, q, n)}{A(Z, q+1, n)} < \frac{q+1}{q}."$$

By definition, P_r belongs to the group of NCNP indices iff $\bar{P}_r(Y) < P_r(Y)$ for all $Y \in \bar{S}$.

This condition is equivalent to:

$$\frac{A(Z, q, n)}{A(Z, q+1, n)} > 1 + \frac{\sum_{i=1}^{q} g_i}{\sum_{i=1}^{q} g_i(q+1-i)}.$$

The largest possible value for the right hand side, over all Y is for a Y such that all gaps have the same value, $g^* \neq 0$. This can again be ascertained by checking that any departure from such a distribution would cause it to decrease. For such a Y, the right hand side is

$$1+\frac{qg^*}{\frac{q(q+1)}{2}g^*}=\frac{q+3}{q+1}, \qquad \forall g^*.$$

We have established:

Proposition 3. "P_r is NCNP iff $A(Z, q, n)$ is such that, for all $Z, n, q>n$:

$$\frac{A(Z, q, n)}{A(Z, q+1, n)}>\frac{q+3}{q+1}."$$

The two last propositions together tell us that the upper bound on $[A(Z, q, n)]/[A(Z, q+1, n)]$ for P_r to be PCNP is $(q+1)/q$ and the lower bound for it to be NCNP is $(q+3)/(q+1)$. Both values converge to 1 but the latter is larger than the former for all $q>1$. Accordingly, the PCNP and NCNP properties are mutually exclusive.

If $A(Z, q, n)$ is such that neither of the two above conditions is satisfied, $P_r(Y)$ will increase with a *ceteris paribus* decrease in the number of the poor for some Y and decrease for some other Y. The set of P_r's which are ambiguous in this respect constitutes the third subset of the P_r family.

Note that there exists no P_r which is insensitive to a *ceteris paribus* change in the number of the poor, for all Y. This would require $P_r(Y)=\bar{P}_r(Y)$, for all Y, that is

$$\frac{A(Z, q, n)}{A(Z, q+1, n)}=\frac{\sum_{i=1}^{q} g_i(q+2-i)}{\sum_{i=1}^{q} g_i(q+1-i)},$$

which is clearly impossible considering the domain of $A(\cdot)$.

Let us now prove:

Proposition 4. "If $\bar{P}_r(Y)>P_r(Y)$ for some Y with $n-q>1$, then there exists a sequence of transfers containing a transfer from a poor person to a richer poor person which decreases P_r."

Proof. To construct such a sequence, first bring a rich person's income down to Z by a transfer to another rich person. This is feasible as $n-q>1$. Call the result Y'.

$$P_r(Y')=\bar{P}_r(Y)=A(Z, q+1, n)\sum_{i=1}^{q} g_i(q+2-i)$$

Now transfer ε from any poor person to that person with income Z. Call the result Y''. If ε is chosen small enough, the rank of the donor, i^*, is not affected.

$$P_r(Y'')=A(Z, q, n)\left[\sum_{i=1}^{q} g_i(q+1-i)+\varepsilon(q+1-i^*)\right]$$

For $\varepsilon \to 0$, $P_r(Y'')\to P_r(Y)<\bar{P}_r(Y)=P_r(Y')$.

The limiting process involved is legitimate as for $\varepsilon > 0$, $A(Z, q, n)$ does not vary and $P_r(Y'')$ is a linear function of ε.

Y'' has been obtained by performing on Y' a disequalising transfer between poor persons and P_r has decreased. Q.E.D.

Two things happened during the last transfer: a discrete drop in P_r of size $\bar{P}_r(Y) - P_r(Y)$ as the number of the poor decreases by one and a continuous increase in P_r due to the widening of the donor's gap. As the increase in P_r is a function of ε and converges to zero with ε, there is always an ε small enough for the net effect to be a decrease.

Following Sen's approach,[4] let us propose a few desirable properties of a poverty measure. The following pair of three statements expresses, under the form of weak and strong requirements respectively, the basic properties that one can intuitively expect a poverty measure to satisfy.[5] The first two requirements of each pair are of egalitarian inspiration and the third one is obviously Paretian.

Requirement A. "*Ceteris paribus* no transfer of income from a poor person to anyone richer should decrease a poverty measure."

Requirement B. "*Ceteris paribus* no transfer of income to a poor person from a rich person who stays rich should increase a poverty measure."

Requirement C. "*Ceteris paribus* no increase in a poor person's income should increase a poverty measure."

Requirement A′. "*Ceteris paribus* a transfer of income from a poor person to anyone richer should increase a poverty measure."

Requirement B′. "*Ceteris paribus* a transfer of income to a strictly poor person from a rich person who stays rich should decrease a poverty measure."

Requirement C′. "*Ceteris paribus* an increase in a strictly poor person's income should decrease a poverty measure."

In proposition 4, $\bar{P}_r(Y) > P_r(Y)$ for some Y implies that P_r is either PCNP or ambiguous. Then it follows from proposition 4 that:

Proposition 5. "P_r satisfies requirement A only if it is NCNP."

The next result is provided without its rather tedious formal proof[6] as it is intuitively obvious:

Proposition 6. "P_r satisfies requirement A' if it is NCNP."

A transfer from a poor person to someone richer is a transfer to either a strictly poor person who stays poor or to a rich person or to someone with income Z or to a strictly poor person who becomes rich. In the first two cases, the weight of the donor is larger than the one of the recipient (which is zero if he is rich) and $A(Z, q, n)$ does not vary. It is clear that then P_r must increase. In the last two cases, the same effect of the weighting system as above is reinforced by the increase in P_r due to the reduction in the number of the poor. In all cases P_r increases.

We can also prove:

Proposition 7. "If $\bar{P}_r(Y) < P_r(Y)$ for some Y with $n - q > 1$, then there exists a sequence of transfers such that requirement B is violated."

[4]Our requirements A' and C' are Sen's requirements T and M, respectively. See Sen, *op. cit.* p. 219.

[5]It is clear that if an index fails to satisfy the weak version of a requirement it also fails to satisfy the strong one and that if an index satisfies the strong version it also satisfies the weak one.

[6]See D. Thon, "On a class of poverty measures," Hull Economic Research Paper, No. 39, University of Hull, October 1978, for a proof.

Proof. First bring a rich person's income down to Z by a transfer to a richer rich person. Call the result Y'. Then perform the inverse transfer. This last transfer violates requirement B as:

$$P_r(Y') = \bar{P}_r(Y) < P_r(Y). \qquad \text{Q.E.D.}$$

As in proposition 7, $\bar{P}_r(Y) < P_r(Y)$ implies that P_r is either NCNP or ambiguous, we have:

Proposition 8. "P_r satisfies Requirement B only if it is PCNP."

It is tedious to prove but intuitively clear[7] that:

Proposition 9. "P_r satisfies Requirement B' if it is PCNP."

If the recipient of a transfer from a rich person who stays rich is originally strictly poor and is still poor after transfer, P_r decreases as his gap decreases. If he is rich after transfer, we have the added drop in P_r by $\bar{P}_r(Y) - P_r(Y)$. In all cases requirement B' is satisfied.

The next result can easily be established along the same lines as the previous ones (proof omitted):

Proposition 10. "P_r satisfies Requirement C only if it is PCNP and then it satisfies Requirement C'."

Propositions 5, 8 and 10 together imply the following strong case against the P_r family:

Proposition 11. "No P_r satisfies Requirements A and (B or C)."

We have now collected all the technical results we need to begin the investigation of a few familiar poverty measures.

Measuring Poverty

The most usual measures of poverty are the Incidence of poverty (or Head-Count), $I = q/n$ and the aggregate income gap, $\text{GAP} = \sum_{i=1}^{q} g_i$. We shall also consider two members of the P_r family, Sen's index which will shortly be shown to be PCNP and one of the many conceivable NCNP P_r's, which is given here only in order to exemplify the family:

$$p = \left[\frac{(n+0.5)(n+1.5)2}{n(n+1)Z}\right] \frac{1}{(q+0.5)(q+1.5)} \sum_{i=1}^{q} g_i(q+1-i)$$

p is such that $[A(Z, q, n)]/[A(Z, q+1, n)] = (q+2.5)/(q+0.5)$ which is larger than $(q+3)/(q+1)$ for $q \geq 0$. By proposition 3, it is indeed NCNP. The term in square brackets normalizes p in such a way that it is contained into [0, 1], like P.

If a weight of $(q+1)/2$ is given to each gap in P instead of $(q+1-i)$, we get what Anand[8] calls the Sen measure with unit weights. It is thus defined as

$$P_e = \frac{\sum_{i=1}^{q} g_i}{nZ}$$

As it ranks exactly like GAP for a given n and Z, it will not be considered here; it will reappear in the last part of the paper.

[7]See D. Thon, *op. cit.* for a proof.

[8]S. Anand, *op. cit.* p. 10..

I and GAP alike ignore the distribution of income among the poor. In particular they both are insensitive to a transfer from a poor person to a richer poor person who stays poor, a violation of requirement A'. If the recipient becomes rich, GAP satisfies requirement A' but I decreases and violates requirement A. It is easy to check that I violates both requirements B' and C'.

P is a P_r with

$$A(Z, q, n) = \frac{2}{(q+1)nZ};^{9}$$

it follows that

$$\frac{A(Z, q, n)}{A(Z, q+1, n)} = \frac{q+2}{q+1}$$

which is smaller than $(q+1)/q$ for $q>0$. By proposition 2, P is PCNP, by proposition 5 it violates requirement A[10] and by propositions 9 and 10 it satisfies requirements B' and C'.

In the proof of proposition 4 which sets the stage for proposition 5, the recipient of the transfer which violates requirement T has initial income Z and the transfer is infinitesimal. Neither of those conditions is necessary for a transfer to violate requirement A. All we need is that the recipient's income be increased from "not too far below" to "not too far above" the poverty line.[11] To make this clear, let us give an example. Suppose Y is such that the incomes of the poor are $(1, 2, 3, 4, 5)$, $q = 5$, $n = 10$, $Z = 5.1$. Then $P = 0.271$. If 0.5 is then transferred from the poorest to the richest poor person, the incomes of the poor become $(0.5, 2, 3, 4)$, $q = 4$ and $P = 0.258$. This also shows that according to P, a transfer from the poorest person to someone else can very well increase Welfare, a dramatic violation of Rawl's Principle. It also illustrates the fact that P can attribute a higher Welfare level to a distribution which is Lorenz dominated by another one.

The implications of P's violation of requirement A seem serious. It makes P anti-egalitarian at the expense of the poorest. If evaluated by P, a policy of redistributing from the very poor to the better off poor could very well score an increase in Welfare. P could also decrease by more if a given sum was redistributed from the rich to the relatively well off poor rather than to the very poor (a defect that P shares with I but from which GAP is exempt). In such circumstances, P's rank weighting system has become a liability.

Prima facie, a NCNP P_r like p is not very attractive as it is designed to score a loss of Welfare if there is one less poor person, *ceteris paribus*. Nevertheless, p would be quite "well behaved" in evaluating the systematic redistribution of a given sum from the rich to the poor. Unlike a PCNP P_r, its behaviour does not constitute an invitation to redistribution to the marginally poor rather than to the very poor, quite to the contrary. True, it is liable to be misleading in advising about

[9]See Sen, *op. cit.* p. 224 or S. Anand, *op. cit.* p. 8.

[10]Contrary to what Sen expected. See Sen, *op. cit.* p. 220. Sen points out that P can violate requirement A and proposes a weaker requirement in "Social Choice: a Re-examination," *Econometrica*, Vol. 45, No. 1, January 1977, p. 77, n 52.

[11]This statement applies to all PCNP P_r's and *mutatis mutandis* to the violation of requirements B and C by a NCNP P_r.

the desirability of a rich-to-poor transfer—whose recipient becomes rich and where there is someone left strictly poor—in a context where the only alternative to such a transfer is the status quo, but in the case considered here, the policy maker may have some egalitarian alternatives. Like all P_r's, p drops to zero when the last poor person crosses the poverty line.

This behaviour is not entirely unattractive and could be paraphrased by the following "ideology": "As long as there is someone left strictly poor, transfers which make some other poor person rich are not to be encouraged. The really poor are to be taken care of first. Disequalizing transfers between the poor are out of question. Only when all the poor people have reached the poverty line is one allowed to think of making any of them rich. This is not a poverty problem any more."

The following table summarizes the behaviour of the poverty measures with respect to all the requirements; a cross indicates that the requirement is violated.

	A	B	C	A'	B'	C'
I	×			×	×	×
GAP $-P_e$				×		
PCNP $P_r(P)$	×			×		
NCNP P_r		×	×		×	×

This concludes our critical review of a few poverty measures. The picture is rather dismal as none of them satisfies our three quite basic requirements in their strong version (and all but GAP even violate one or more weak requirements). Even if one restricted oneself to the pure distribution problem, no index would satisfy the two relevant strong requirements A' and B'. In the last section we propose a poverty measure which satisfies all the above requirements. We first turn to the difficulty of alleviating poverty.

Measuring the "Difficulty of Alleviation of Poverty"

S. Anand describes three indices meant to evaluate (negatively) the society's ability to eliminate poverty and estimates their value for Malaysia, both globally and by racial group.[12]

The standard measure is the percentage of the GNP needed to close the poverty gap. We call it

$$E = \frac{\sum_{i=1}^{q} g_i}{\sum_{i=1}^{n} y_i}$$

S. Anand modifies the above to take into account income inequality among the poor by introducing P's weighting system in E. This gives

$$M = \frac{2}{(q+1)n\mu} \sum_{i=1}^{q} g_i(q+1-i) = \frac{Z}{\mu}P,$$

where μ is the average income of the population. The third measure, F, focuses on

[12]See S. Anand, *op. cit.* p. 10. It is not clear what interpretation is to be given to the index if computed for a single racial group.

the relation between the gaps of the poor and the income of the rich:

$$F = \frac{\sum_{i=1}^{q} g_i}{\sum_{i=q+1}^{n} y_i}$$

The difficulty of alleviation of poverty in a given country depends on a large number of economic and political elements and any attempt at measuring it by considering only the existing income distribution is necessarily defective for obvious reasons that are not worth belabouring here. If one restricts oneself to such a limited informational basis, one can only hope for a simple descriptive relationship between existing needs (e.g. income gaps) and resources (i.e. some income concept) available to eradicate poverty completely. It is reasonable then to specify that E, M and F are meant to take the above finite values if $Z < \bar{y}$ and infinite value if $Z \geq \bar{y}$.

Because the concept of difficulty of alleviation of poverty is rather vague, it is difficult to formulate more than a few minimal requirement that a measure should satisfy. It seems reasonable to postulate at least that an increase in anyone's income should decrease the measure and that no disequalizing transfer should decrease it if the donor is a poor person (and particularly if the recipient is a rich person after the transfer is performed).

This last very mild requirement disqualifies both M and F. Consider a transfer from a poor person to a richer poor person who becomes rich. M can increase or decrease (and then violate our requirement) like P of which it is a linear function, for a given n and z. It is difficult to justify M's ambiguous behaviour in such circumstances and even to rationalize the introduction of the rank order weighting system in the first place. Given the conceptual distinction between the measurement of poverty and the measurement of how difficult it would be to eradicate it by transfers, carefully spelled out by S. Anand himself,[13] it is suggested that the distribution of income among the poor is relevant for the former but not the latter.

If the income gap of the recipient of the above transfer is originally g_{i^*} and the size of the transfer is t, the post-transfer GAP is $\sum_{i=1}^{q} g_i + t - g_{i^*}$; the total income does not vary and the total income of the rich becomes $\sum_{i=q+1}^{n} y_i + Z + t - g_{i^*}$, where g_i and y_i refer to the original distribution. We have $t > g_{i^*}$. Then, whether F increases or decreases depends on the relative magnitude of

$$\frac{\sum_{i=1}^{q} g_i}{\sum_{i=q+1}^{n} y_i} \quad \text{and} \quad \frac{\sum_{i=1}^{q} g_i + t - g_{i^*}}{\sum_{i=q+1}^{n} y_i + Z + t - g_{i^*}}$$

As a little manipulation will show, the former can be larger or smaller than the latter and thus F has the same defect as M in the case considered.

E, on the other hand unambiguously increases, as

$$\frac{\sum_{i=1}^{q} g_i}{\sum_{i=1}^{n} y_i} < \frac{\sum_{i=1}^{q} g_i + t - g_{i^*}}{\sum_{i=1}^{n} y_i}$$

In conclusion, it is suggested that the "difficulty of alleviation of poverty" is best measured by a simple ratio between the amount of resources needed and the

[13]See S. Anand, *op. cit.* p. 10.

amount available, like E. Another reasonable candidate is

$$\frac{\sum_{i=1}^{q} g_i}{\max\{0, \sum_{i=1}^{n} y_i - nZ\}},$$

which is self explanatory and ranks like E in the pure distribution case but not otherwise.

Suggestions and Conclusions

Consider the following poverty index obviously inspired by Sen's:

$$P^* = \frac{2}{n(n+1)Z} \sum_{i=1}^{q} g_i(n+1-i)$$

Like P, P^* reaches its lower bound (0) when no one is poor and its upper bound (1) when everyone has zero income. P^* satisfies all requirements A', B', C' and therefore A, B, C, as can easily be checked.

P^* therefore seems to have some merit; gauged by our requirements, it dominates all the other indices. Whether the requirements are reasonable or not is, of course, left to the reader to judge. Note that P^* does not "take the number of the poor into account", the way P does. P^* can easily be checked to be insensitive to a *ceteris paribus* change in the number of the poor, as defined above.

This raises the issue of how advisable it is after all to insist on taking the number of the poor into explicit consideration when measuring poverty. It seems that the conflict between such an approach and satisfying requirement A is unavoidable. As long as the index is designed to record a gain in welfare only when someone crosses the poverty line from below there will always be, for some initial income distribution, a transfer from a poor person which accomplishes that and which is small enough to cause a violation of requirement A.

In a sense, it is a pity as intuitively, poverty can very reasonably be viewed as a state which adversely affects the welfare of an individual subjected to it regardless of the size of the gap. On the other hand it probably appeals to our intuition that whether someone has income Z or $Z+\varepsilon$ does not matter to him and that a poverty index should be consistent with that fact. As we have shown, this second intuition is the correct one if requirement A is to be satisfied.

To illustrate, we give the values of P^* for Malaysia, globally and by racial group. It is possible to compute them from S. Anand's numerical results[14] without having to go back to the original data.

The following table reproduces some of S. Anand's results and the last column gives the values of P^* which can be computed from q/n, P and P_e as follows:

$$P^* = \frac{2}{n(n+1)Z} \sum_{i=1}^{q} g_i(n+1-i)$$

$$= \frac{2}{n(n+1)Z} \sum_{i=1}^{q} g_i(q+1-i) + \frac{2}{n(n+1)Z}(n-q) \sum_{i=1}^{q} g_i.$$

The first term $\approx P(q/n)$.

[14] S. Anand, *op. cit.* p. 11.

As $P_e = 1/Zn \sum_{i=1}^{q} g_i$, the second term is $P_e[2(n-q)]/(n+1)$. As n and q are large numbers, it is harmless to assume: $(n-q)/(n+1) \simeq 1-(q/n)$. This gives: $P^* = P(q/n) + 2 \cdot P_e[1-(q/n)]$; all the necessary numerical values are available from S. Anand's results.

	q/n	P_e	P	P^*
Peninsular Malaysia	0.402	0.145	0.200	0.254
Malays	0.562 (1)	0.219 (1)	0.294 (1)	0.357 (2)
Chinese	0.183 (4)	0.050 (4)	0.072 (4)	0.095 (4)
Indians	0.334 (3)	0.097 (3)	0.137 (3)	0.175 (3)
Others	0.433 (2)	0.215 (2)	0.288 (2)	0.368 (1)

The number in brackets represents the ranking of the four racial groups according to each index.

It is noteworthy that P^*'s ranking is different from the unanimous ranking of the other indices. This perhaps suggests that the issues raised in this article are of some practical importance.