HUMAN CAPITAL: DETERIORATION AND NET INVESTMENT

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The current practice in national accounting is to exclude from national product investment in schooling and on-the-job training, except for direct costs of schooling which are included in consumption. Foregone earnings, which form the major part of investment in human capital, go unrecorded.

Much is to be gained in consistency and analytical clarity by treating human capital like physical capital in national accounting. Estimating the amount of foregone earnings net of deterioration, that is, net investment, is a step in this direction.

Using the framework of the life-cycle hypothesis of earnings, and assuming declining-balance deterioration of human capital, estimates of deterioration rates in respect of American males by race and education level are computed for 1960. Every such d is, however, the *minimum* consistent with the respective costs and benefits profile. Hence an upper limit d is assumed. The model generates for each costs and benefits profile and in respect of either d, a year-by-year series of net investment in human capital. These are used to obtain two estimates (which turn out to be close to each other) of aggregate net investment in American white males in 1960. On the basis of these estimates, aggregate net investment in human capital is found to be about equal to net investment in physical assets (including consumers' durables).

It is also found that the Denison method of estimating the contribution of the increase in human capital to economic growth understates this contribution by a ratio approximately equal to net investment in on-the-job training to returns to human capital. This was about 16 percent in 1960.

The object of this article is to investigate the problem of deterioration, or depreciation, of human capital, with a view to estimating net investment in schooling and on-the-job training (*OJT*) in U.S. males in 1960.

Deterioration will be used as a generic term covering all factors that reduce earning capacity with the passage of time such as obsolescence of acquired training, foregetfulness and decline of psycho-physiological powers as well as increased preference for leisure in old age. Unemployment, under-employment and non-participation in the labour force affect average earnings capacity for persons out of school, and they also affect the estimates of deterioration. In 1960 about 10 percent of the U.S. civilian white male population aged 25 to 64 not at school were either unemployed or not in the labour force, the proportion increasing with age.

The framework for the analysis is the by now well-known theory that *OJT* has an opportunity cost, and the life-cycle hypothesis of earnings which is superimposed upon it.[1]

Investment in *OJT* will be calculated indirectly, as the difference between earning capacity and observed earnings. It is important that all investment in schooling be included, otherwise the understating of the costs to which the benefits are attributed would result in a bias in the estimates of earning capacity and therefore of investment in *OJT*. Hence we use what are considered to be social costs of schooling which usually exceed private costs. Even so, we omit

[1]See Becker (1964, 1967), Ben-Porath (1967, 1970) and Mincer (1962, 1970, 1972).

one element in cost: foregone earnings during the period of compulsory education, that is, at ages 6 to 14. These represent the returns to the human capital carried by the child when entering school. While the omission may be justified when calculating private rates of return on the ground that society has decided that children shall not be gainfully employed under a certain age, it can only be justified in the case of social rates of return by the lack of the requisite data. The resulting inaccuracy in the estimate of investment is smaller the higher the education level since the omission decreases in importance in comparison with the total costs.

Since we are concerned with social rather than private returns, earnings should be gross of income tax. Strictly externalities should be taken into account but these are hard to quantify. It will be assumed that for a given education level, the rental value per annum per unit of human capital net of deterioration is the same for both schooling and *OJT* and remains constant throughout life. In view of the long life span this entails only a slight difference between the internal rates of return to schooling and *OJT*, and between these and the common rate of return calculated by equation (10) below.

Johnson (1970) used non-linear regression analysis in the simultaneous estimation of rates of deterioration and other parameters from a continuous earnings function. The fact that he used private costs while social costs are used in this article may account for some of the differences in the results. Our analysis uses a discrete time model that bears a strong resemblance to that put forward by Mincer (1962). Because of failure to allow for deterioration, and other reasons mentioned later on, Mincer found that investment in *OJT* becomes nil towards the middle of working life. In the model presented below, gross investment in *OJT* is positive throughout working life. It starts falling early in working life and becomes nil in the year of retirement, as envisaged in the life-cycle hypothesis of earnings. The model has its limitation: it only makes it possible to estimate the minimum rate of deterioration that fits a given costs and returns series, but this is not necessarily the true rate of deterioration. However, an upper limit to deterioration rate is set, and the model generates, in respect of each education level, for either deterioration rate, a year-by-year series of gross and net investment, earning capacity and capital stock.

Minimum deterioration rates are estimated in the second part of the paper, as well as lifetime and aggregate investment in human capital in respect of American males in 1960.

In the third part of the paper, the analysis is linked with social accounting and growth accounting. It is shown that consistency as well as analytical clarity require treating human capital on a more equal footing with physical capital. It is found that the amount of net investment in human capital which is at present not included in national income is well over 10 percent of national income, and the understatement of the returns to human capital is about 16 percent.

I. The Rate of Deterioration

We shall consider schooling to begin at age 6 as in the U.S.A. and retirement to occur at age 65. We thus have a series of 60 "payments", $I_1, I_2, \ldots, I_{60}$, the earlier part of which represents social costs of schooling (both direct, that is, tuition, and indirect, that is, foregone earnings). The latter part of the series is benefits, or earning differentials accruing to the particular level of education considered, *in comparison with persons who have had no schooling*. We use the term "payments" because both costs and benefits will be treated in the same way in many respects, though the former are positive quantities and the latter negative quantities.

We have no empirical evidence as to the formula for deterioration, that is whether it is straight-line or proportional or some other possible form, nor as to whether its rate remains constant throughout life. Mincer (1970, pp. 13–14) suggests there might be appreciation in early life, followed by deterioration which rises at first gently and then at an accelerating rate. In the absence of empirical evidence, we assume that a uniform rate of *proportional* deterioration applies to a given education level throughout schooling and working life. This assumption has the signal advantage that a rate of deterioration can be fitted to any series of I_t of the conventional form, that is, a series of costs followed by a series of benefits.

Let an investment A be made in year 1, then the value of the asset in year t is reduced to $A(1-d)^{t-1}$, d being the rate of deterioration. Let the return produced by the asset in any year be R times the undepreciated part remaining from that asset in that year. Thus the original investment yields $RA(1-d)^{t-1}$ in year t.

Consider now a single investment A in year 1, whose returns are not withdrawn but continue to be re-invested up to year t. Every such re-investment is treated as an asset in its own right. We want to calculate the accumulated amount of returns, the undepreciated capital and the amount of deterioration in year t.

Year 1: Investment, A.

Year 2: Deterioration on last year's capital, dA, unused capital at beginning of year, $A(1-d)$, returns on unused capital, $AR(1-d)$. These returns are re-invested. Unused capital at end of year 2, $A(1-d)+AR(1-d) = A(1+R)(1-d)$.

Year 3: Deterioration on last year's capital, $dA(1+R)(1-d)$. Unused capital at beginning of year, $A(1+R)(1-d)^2$. Returns on unused capital $AR(1+R)(1-d)$. These returns are re-invested, so unused capital at end of year, $A(1+R)(1-d)^2+AR(1+R)(1-d)^2 = A(1+R)^2(1-d)^2$.

We can generalize, and write for year t,

(1a) Deterioration, $dA[(1+R)(1-d)]^{t-2} = dAS^{t-2}$

(1b) Returns, $AR(1-d)[(1+R)(1-d)]^{t-2} = AR(1-d)S^{t-2}$

(1c) Unused capital, beginning of year, $A(1-d)S^{t-2}$

where $S = (1+R)(1-d)$.

Assume that investments $I_1, I_2, \ldots I_{t-1}$ are made in years 1, 2, . . ., $t-1$, and returns are not withdrawn, but allowed to accumulate as above, then applying (1b), the returns in year t originating in investments made in years 1, 2, . . ., $t-1$, are as follows:

$$I_1R(1-d)S^{t-2},\ I_2R(1-d)S^{t-3},\ \ldots\ I_{t-1}R(1-d)S.$$

Hence, total returns accruing in year t, or *earning capacity* is

$$(2) \qquad Y_t = R(1-d)(I_1S^{t-2}+I_2S^{t-3}+\ \ldots\ +I_{t-2}S+I_{t-1})$$

This procedure is understandable if $I_1, I_2, \ldots, I_{t-1}$ are social costs of schooling. Y_t as given by (2) would give the earning capacity if the labour market was entered in year t. The question now arises: what if years $t-1, t-2, \ldots, t-i$, were working years, that is, $I_{t-1}, I_{t-2}, \ldots, I_{t-i}$ were benefits rather than costs, that is, they were negative quantities? The answer is: a benefit has an equal effect on future earning capacity as a cost of the same magnitude, except for the sign which would be negative, since it offsets an investment of the same magnitude. The same is true of the effect of a benefit on future deterioration and stock of unused capital. Hence equation (2) gives earning capacity in any year t provided the payments I_t are given the appropriate signs.

Gross investment in *OJT* in a working year t is defined as earning capacity *less* benefit or observed earnings differential, that is

$$(3) \qquad G_t = Y_t+I_t = R(1-d)(I_1S^{t-2}+I_2S^{t-3}+\ \ldots\ +I_{t-2}S+I_{t-1})+I_t$$

Deterioration in year t can be inferred from (1a)

$$(4) \qquad D_t = d(I_1S^{t-2}+I_2S^{t-3}+\ \ldots\ +I_{t-1})$$

Net investment is found by subtracting (4) from (3)

$$(5) \qquad N_t = G_t-D_t = (S-1)(I_1S^{t-2}+I_2S^{t-3}+\ \ldots\ +I_{t-1})+I_t$$

$$(6) \qquad K_t,\ \text{beginning of year}\ t = Y_t/R = (1-d)(I_1S^{t-2}+I_2S^{t-3}+\ \ldots\ +I_{t-1})$$

The following conditions are entailed by the life-cycle hypothesis of earnings:

$$(7) \qquad G_t \geqslant 0 \text{ for all } t < T$$

$$(8) \qquad G_t = 0 \text{ for } t = T$$

where T is the year of retirement, and G_t is as defined in (3).

The T conditions given by (7) and (8) to be satisfied by two of the variables d, R and S (the third being by definition a function of the other two), constitute a problem in non-linear programming, which fortunately can be solved by simple means (see Appendix II for a practical method of solution).[2] The principles underlying the method of solution are stated below and in Appendix I.

Let r be the internal rate of return for the stream of payments $I_1, I_2, \ldots, I_T$. Then r can be calculated from the following equation:

$$(9) \qquad \frac{I_1}{(1+r)}+\frac{I_2}{(1+r)^2}+\ \ldots\ +\frac{I_T}{(1+r)^T} = 0.$$

[2]It will be noted that inequality (7) is satisfied by all the years of schooling since in these years $I_t > 0$.

Divide equality (8) by $R(1-d)S^{T-1}$ and remembering that $S = (1+R)(1-d)$, we get:[3]

(10) $$\frac{I_1}{S} + \frac{I_2}{S^2} + \ldots + \frac{I_{T-1}}{S^{T-1}} + \frac{I_T}{S^T}\left(1+\frac{1}{R}\right) = 0$$

Comparing (10) with (9), it will be seen that S should be just a little over $(1+r)$. The difference arises from the factor $(1+(1/R))$ in the last term of the left-hand side of (10) (the introduction of which is equivalent to increasing the benefits), and varies inversely with R.

Now, neither d nor R is known. We start by assuming a given d, then find by trial and error an R such that S satisfies (10) and therefore (8). However, such a set of d and R may not satisfy (7), that is, though $G_T = 0$, some G_t $(t < T)$ may be negative, in which case, try a larger d and find the corresponding R and S. It is shown in Appendix I that to a larger d corresponds a smaller S (para. (c)), and to a smaller S corresponds a larger earning capacity $Y_t = R(1-d)P_{t-1}$ (para. (d)) and therefore a larger $G_t = R(1-d)P_{t-1}+I_t$. (P_{t-1} is the polynomial in S appearing in equations (2) and (6)).

Let us consider the significance of I_T/R which appears as a benefit in (10) but not in (9). Since in the year of retirement $G_T = Y_T+I_T = 0$ we have $-I_T = Y_T$ and $-I_T/R = Y_T/R = K_T$, that is, the stock of capital remaining at retirement. Hence the reason for $S > (1+r)$ is that in (10) the internal rate of return is $(S-1)$ which takes into account the undepreciated capital at the end of working life as a benefit, whereas in (9) the rate of return is r which does not take such residual capital into account. Such residual is akin to scrap value in physical assets, though unlike the latter it cannot be sold. Its existence is both a matter of casual observation and a logical requirement due to the use of proportional deterioration.

A number of propositions follow from the above:

(i) There are generally different sets of d and R that satisfy (7) and (8) for a given series of I_t. We can only speak of the *minimum* d. In other words, the model is underspecified. Determination of the true d requires empirical knowledge of or the making of assumptions[4] about e.g., the earning capacity Y_t (Appendix I, para. (d)),[5] or the proportion of earning capacity invested in *OJT* at a given age, (G_t/Y_t), (prop. (vi) below).

(ii) If benefits decrease in the course of working life, then d must be positive. It cannot be nil or negative. However, if benefits increase monotonically, d could be $\gtreqless 0$.

[3]Since there is a single change of sign of I_t, there can be no more than one positive solution for either $(1+r)$ or S.

[4]When Johnson (1970) assumes in his regression analysis that G_t/Y_t falls in a straight line from the year of participation in the labour force till retirement, he is making an assumption that is crucial for fixing the level of d. Calculating d for successive increments of schooling, the lowest level of comparison being 5–7 years, he finds d up to 0.09 for white males and up to 0.13 for non-white males.

[5]Since earning capacity increases with d, d cannot be very large for that would imply a very high earning capacity. This provides both a rough guide to setting an upper limit to d, and a hint as to how to obtain a more accurate estimate of d through empirical research.

Let benefits start dropping as from year n, that is $I_n < I_{n+1}$ (remember these are negative quantities), then[6]

$$G_{n+1}-G_n = (R-Rd-d)G_n+I_{n+1}-(1-d)I_n \tag{11}$$

For $d = 0$ we have,

$$G_{n+1}-G_n = RG_n+I_{n+1}-I_n \tag{12}$$

Since $R > 0$, $G_n \geqslant 0$, $I_{n+1} - I_n > 0$, we get $G_{n+1}-G_n > 0$, that is, gross investment in *OJT* cannot decrease to zero as required. It can be shown from (11) that the result holds *a fortiori* if $d < 0$.

Now assume that the series of benefits rises throughout, that is $I_n > I_{n+1}$. Equation (12) in which $d = 0$, can be rewritten

$$G_{n+1}-(1+R)G_n = I_{n+1}-I_n < 0$$

This implies that $G_{n+1} < (1+R)G_n$ which is consistent with the requirement that $G_{n+1} < G_n$. Hence a rising series of benefits is consistent with $d = 0$. It can similarly be shown from (11) that $d \lesseqgtr 0$ is consistent with a rising series of benefits.

(iii) From (8) and as explained on p. 283, the condition for $G_T = 0$ is that S must be larger than $1+r$ (r is the internal rate of return). Put $d = 0$, then $S = 1+R$ and should be larger than $1+r$, that is we should have $R > r$ for G_T to be nil. So if we put $d = 0$, $R = r$, we get $G_T < 0$. If moreover, the series of benefits bends down in the later stages of life, then by proposition (ii) $d = 0$ will result in $G_T > G_{T-1} > G_{T-2}$ and so on, as long as benefits decrease, that is $G_t < 0$ for some $t < T$. In sum, if we put $d = 0$ and $R = r$, neither condition (7) nor (8) will be satisfied. This explains why Mincer (1962) gets the result that investment in *OJT* falls to zero towards the middle of working life.

(iv) If we have a monotonically increasing benefits series, and the minimum d is either zero or negative, then by proposition (i), solutions involving positive d's are also possible. As mentioned above, additional information is required for finding the true d.

(v) The higher the d applied to a given series I_t, the lower the net investment and the larger the deterioration for any given year.

From (5) we have $N_t = (S-1)P_{t-1}+I_t$, (P_{t-1} being the polynomial in S). It is shown in Appendix I that as d rises S falls, and therefore P_{t-1} falls. Hence N_t falls.

Since $D_t = G_t-N_t$, and G_t rises while N_t falls, as d rises, as shown in Appendix I, para. (d), it follows that D_t rises.

(vi) The proportion of earning capacity invested in *OJT* increases with d.

From the definition of G_t, we have

$$\frac{G_t}{Y_t} = \frac{Y_t+I_t}{Y_t}$$

Since Y_t increases with d by Appendix I, paras. (c) and (d) and I_t, which is negative, remains constant, the conclusion follows.

[6]Calculate $G_{n+1}-SG_n$ from (3), substitute $(1+R)(1-d)$ for S and transfer terms.

(vii) Secular rise in costs and benefits. The series of payments I_t considered so far is derived from a cross-section. Assume that all costs and benefits rise at the rate g per year, then the true payments stream of an individual aged $5+t$ years in the year under study is not that given by the cross-section, but the following *time-series*:

$$I_1(1+g)^{-(t-1)},\ I_2(1+g)^{-(t-2)},\ \ldots,\ I_{t-1}(1+g)^{-1},\ I_t,$$

$$I_{t+1}(1+g),\ \ldots,\ I_T(1+g)^{T-t} \tag{13}$$

For every age $5+t$ ($t = 1, 2, \ldots, T$) we have to construct a time-series, having in common with the cross-section only one term, I_t. We shall now calculate r', d', G_t', Y_t', D_t' from the time-series and compare them with those derived from the cross-section.

(a) The internal rate of return r' for the time-series is given by $(1+r') = (1+r)(1+g)$.

Equate the present value of the series in (13) to zero as in (9), and multiply throughout by $(1+g)^t$; we get:

$$\frac{I_1}{(1+r')/(1+g)} + \frac{I_2}{[(1+r')/(1+g)]^2} + \ldots + \frac{I_t}{[(1+r')/(1+g)]^t} + \ldots + \frac{I_T}{[(1+r')/(1+g)]^T} = 0 \tag{14}$$

Comparing (14) with (9), we have, $(1+r) = (1+r')/(1+g)$, or

$$(1+r') = (1+r)(1+g) \tag{15}$$

(b) The rental value for the time-series, R', is equal to R. The depreciation rate d' is given by $(1-d') = (1-d)(1+g)$, or[7]

$$d' = d-g+gd \tag{16}$$

These results are obtained easily by setting out equation (10) for the series given in (13), substituting $S' = (1+R')(1-d')$ for S, multiplying throughout by $(1+g)^t$, and comparing with (10).

(c) Proceeding in like manner, it can be proved that, in the year under study for which the time-series and cross-section have one I_t in common,

$$Y_t' = Y_t \qquad G_t' = G_t \tag{17}$$

$$D_t' = \frac{d'}{d} D_t \tag{18}$$

Since gross investment is the same in the time-series as in the cross-section, but depreciation is smaller in the time-series, net investment calculated from the time-series must exceed that calculated from the cross-section. From (17) and (18), in the year under study,

$$N_t' = G_t - \frac{d'}{d} D_t \tag{19}$$

[7]It can be shown that the minimum d' for the time-series and the minimum d for the corresponding cross-section are related as in (16).

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II. Estimates for the U.S., 1960

Table 1 shows the minimum deterioration rates (estimated to the nearest percentage point) in respect of white and non-white American males. No deterioration rates have been calculated in respect of females in view of the fact that much non-participation in the labour force occurs at family-building ages and cannot be subsumed under deterioration. The social costs[8] and benefits data are borrowed from the article by Hines *et al.* (1960) and unpublished earnings tables derived from the *One in One Thousand Sample of the 1960 Census of Population* used in that article, kindly supplied by the authors.

The base-line for the white is no schooling,[9] but for the non-white 1–4 years of education. This is because of the extremely small number of non-white earners with no schooling in the sample. No deterioration rates are calculated for differential amounts of education, e.g., between 8 years and 12 years, for that would be assuming human capital to consist of different layers with different rates of deterioration (though it makes sense to calculate rates of return on differential amounts of education).

The minimum deterioration rate in respect of the white is .04 at all education levels except high school, for which it is .03. The findings about the non-white exhibit some irregularities which might be due to peculiarities of the non-white labour market. Thus there is just one education level (8 years) whose benefit series does not fall and in consequence minimum $d = 0$. Moreover, the 5–7 years of education group has two ages at which net investment in *OJT* becomes nil. This is because the benefit stream for this group peaks at the early age of 18–19.

The findings with regard to the white are more in conformity with the life-cycle theory of earnings, and a model year-by-year computation of Y_t, G_t, D_t, in respect of education level 16 years, for the minimum d of .04 is given in Table A1. As envisaged in the theory, the G_t/Y_t ratio drops gradually to nil in the year of retirement, and $N_t = G_t - D_t$ becomes negative long before retirement, at ages between 29 years and 37 years.[10] If we assume a rate of growth of .02 per annum, N_t becomes negative at ages 36, 51, 57, and 54 years for the four education levels. These ages do not change appreciably with the rate of deterioration assumed, and they remain the same for $d = .10$ as for the minimum d's shown in Table 1.

The ratio G_t/Y_t in the year of entry into the labour force is more sensitive to the d assumed.[11]

[8]Social costs include direct costs of tuition plus foregone earnings, equal to those of persons with no schooling of the same age. No foregone earnings are imputed to ages 6 to 14.

Entry into the labour force is assumed to occur at age 6+ schooling period+2 years, and retirement at age 67. However, in our calculations the gap between end of schooling and joining the labour force is omitted, so that we get a life-cycle of 60 years.

[9]Ideally, earnings of persons of a given education level should be compared with the hypothetical earnings of persons with neither schooling nor *OJT*. However, to estimate such an earnings profile, we require data which are not available: rate of return to *OJT* of persons without education, and the depreciation rate of human capital carried by the same persons. The procedure followed here results in reducing investment in *OJT* per educated person by an amount roughly equal to that invested by a person with no education.

[10]Johnson gets $N_t < 0$ at higher ages than these.

[11]If we assume $d = 0.10$ in respect of white males of all education levels, the ratio becomes 0.68, 0.68, 0.58 and 0.57 for the four education levels.

TABLE 1

MINIMUM DETERIORATION RATES, RENTAL VALUE, PROPORTION OF EARNING CAPACITY REINVESTED WHEN ENTERING LABOUR FORCE, AND AGE AT WHICH NET INVESTMENT = 0 BY EDUCATION LEVEL FOR AMERICAN MALES, 1960 (BASED ON CROSS-SECTION DATA).

	White				Non-White			
Years of Education[a]	5–7	8	12	16	5–7	8	12	16
Minimum deterioration rate, d	0.04	0.04	0.03	0.04	0.12	0	0.07	0.02
Rental value on unused capital R^{b}	0.222587	0.229968	0.208062	0.192897	0.372481	0.133605	0.239880	0.114234
Proportion of earning capacity invested in *OJT* in year of entry into labour force, G_t/Y_t[b]	0.59	0.60	0.43	0.43	0 .64	0.43	0.49	0.60
Age at which net investment, $G_t - D_t = 0$[b]	30	30	31	37	(18, 48)	65	41	37

[a]Base-line for white: no schooling; for non-white: 1–4 years.

[b]Calculated at the minimum depreciation rates.

TABLE 2

GROSS INVESTMENT AND DETERIORATION PER HEAD,[a] BY AGE,[b] OVER LIFETIME AND OVER WORKING LIFE, BY EDUCATION LEVEL, AMERICAN WHITE MALES, 1960 (BASED ON CROSS-SECTION DATA).

Dollars

Years of education	5–7			8			12			16		
Age	G	D	D^s	G	D	D^s	G	D	D^s	G	D	D^s
14–15	812	250	193	1,029	303	215						
16–17	1,000	698	177	1,285	364	199						
18–19	862	350	163	1,215	435	184	1,573	526	398			
20–21	689	389	151	844	489	170	1,364	589	375			
22–24	687	414	136	888	526	153	1,268	640	349	3,094	1,588	1,139
25–29	640	461	116	776	579	130	1,131	711	308	3,052	1,818	967
30–34	327	462	94	450	582	106	721	738	265	2,445	1,997	788
35–44	178	410	71	308	531	80	606	724	213	1,850	2,045	591
45–54	139	326	47	251	448	53	552	690	158	1,352	1,912	393
55–64	21	244	31	52	350	35	155	605	115	332	1,576	262
Over lifetime	23,657	19,026		31,448	24,742		50,600	33,738		116,123	85,526	
$G - D$ over lifetime $= K_T$		4,631			6,706			16,862			30,597	
Over working life	16,925	18,290	4,326	23,456	23,965	4,827	31,745	31,943	19,273	72,252	79,792	24,499
$G - (D - D^s)$ over working life		2,961			4,318			19,075			16,959	

[a]Calculated at minimum deterioration rates shown in table 1. Base-line: no schooling.
[b]Averages for age-groups.
G, gross investment.
D, amount of deterioration of human capital.
D^s, amount of deterioration of human capital formed by schooling only.
$G - (D - D^s)$ = net investment in *OJT*.
K_T, capital stock at retirement.

TABLE 3

Gross Investment and Deterioration Per Head,[a] by Age,[b] over Lifetime and over Working Life by Education Level, American White Males, 1960 (d = 0.10 for all Education Levels) (based on Cross-Section Data).

Dollars

Year of education	5–7			8			12			16		
Age	G	D	D^s	G	D	D^s	G	D	D^s	G	D	D^s
14–15	1,188	626	438									
16–17	1,446	744	354	1,830	910	396						
18–19	1,388	876	287	1,867	1,088	321	2,801	1,754	1,188			
20–21	1,272	973	232	1,577	1,224	263	2,730	1,960	962			
22–24	1,310	1,040	178	1,677	1,316	202	2,761	2,134	739	5,471	3,969	2,500
25–29	1,330	1,152	117	1,643	1,447	132	2,788	2,371	485	5,768	4,541	1,640
30–34	1,019	1,155	69	1,322	1,455	78	2,437	2,458	286	5,420	4,985	968
35–44	788	1,022	33	1,099	1,327	37	2,274	2,404	136	4,862	5,090	464
45–54	605	807	12	895	1,111	13	2,063	2,263	47	4,002	4,693	162
55–64	273	568	4	425	821	5	1,089	1,842	16	1,848	3,595	56
Over lifetime	50,179	46,829		65,839	60,886		120,881	109,359		229,390	207,723	
$G - D$ over lifetime $= K_T$		3,350			4,953			11,522			21,668	
Over working life	42,343	44,989	4,342	56,698	58,970	5,152	97,842	103,382	12,400	176,926	193,393	27,474
$G - (D - D^s)$ over working life		1,696			2,880			6,860			11,007	

[a]Base-line: no schooling.
All other footnotes as in table 2.

Lifetime Investment

Tables 2 and 3 show that total net investment over a lifetime is just a fraction of total gross investment and deterioration.[12] It will be seen that as d is raised from the minimum levels to 0.10, G and D almost double, but total net investment falls moderately. It is also seen from the two tables that over working life total deterioration exceeds total gross investment in *OJT*. This is to be expected, because after schooling years deterioration is incurred on investment due to schooling as well as to *OJT*. To calculate total net investment in *OJT* over working life, deterioration on capital formed by schooling, as estimated at the end of the schooling period, has to be subtracted from total deterioration. (See columns headed D^s for estimates of the average deterioration per annum on capital formed by schooling at different ages.) With this adjustment, we get positive total net investment in *OJT* over working life.

Estimates similar to those appearing in tables 2 and 3 have been computed for the non-white. They show sharp fluctuations of net investment with level of education instead of a gradual rise. This reflects the peculiarities of the benefits series already mentioned.

Aggregate Investment in OJT

Estimates of gross and net investment in *OJT* derived from cross-section costs and benefits in white male population in 1960 and based on the minimum deterioration rates are set out in table 4, panel (a). White males account for about three-quarters of the returns to labour. No similar estimates for non-whites have been made, not so much on account of the peculiarities of the findings referred to, as because the base line is 1–4 years, which would result in an underestimate of the investment. The aggregates G, D and D^s are calculated by multiplying the averages for different age-groups given in table 2 by the corresponding (out of school) populations. The lower education levels show negative aggregate net investment in *OJT* (in spite of the positive total over an individual's working life, as shown in table 2) because of the weighting by population: at the lower ages, when gross investment is high and deterioration low, there are relatively few people out of school; large numbers are continuing their education.

We are not sure that the minimum rates are the true rates. Let us recalculate the net investment in *OJT* for $d = 0.10$ taken tentatively as the upper limit for all education levels.[13] Table 5, panel (a), shows that while the change in d causes gross investment and deterioration to more than double, net *OJT* hardly falls. This is due not only to the low sensitivity of net investment to the rate of deterioration but also to a kind of offsetting variation. The fall in net investment in the three lower education levels is largely offset by an *increase* in the net investment in the highest education level. This increase can be explained as follows: net

[12]Total lifetime net investment is equal to K_T, residual capital at retirement. It can be shown that $K_t = \sum_1^t N_i - G_t$. Since $G_T = 0$, $K_T = \sum_1^T N_i$.

[13]The high investment/earning capacity ratios at the ages of entry into the labour force for $d = 0.10$ given in footnote 11 p.286, and the correspondingly higher earning capacities at those ages, indicate that it may reasonably be taken as the upper limit. Estimates of aggregate net investment would not be much different if a higher rate, say 0.15, were applied to all educational levels or to the lowest level only.

TABLE 4

AGGREGATE POST-SCHOOL GROSS INVESTMENT, DETERIORATION, AND NET INVESTMENT, AMERICAN WHITE MALES, 1960
(Assuming minimum rates of deterioration)[a]

$ million

Years of education[b]	5–7	8	12	16	Total
(a) *Based on cross-section social costs and earnings*					
G	2,522	4,085	12,520	13,084	32,211
D	3,044	5,608	11,437	12,225	32,314
D^s	415	886	3,932	4,348	9,581
$G-(D-D^s)$	−107	−637	5,015	5,207	9,478
(b) *Assuming social costs and earnings increase at 2 per cent per annum*[c]					
G	2,522	4,085	12,520	13,084	32,211
D	2,101	2,916	4,117	6,357	15,491
D^s	287	461	1,416	2,261	4,425
$G-(D-D^s)$	708	1,630	9,819	8,988	21,145

Sources: U.S. Census of Population, Final Report PC(2)5B, Subject Reports, Educational Attainments, 1964, and *Statistical Abstract of the United States*, 1960, p.105; 1963, p.116. Table 2 above.

[a]See table 1. Base-line: no schooling.

[b]In computing the aggregates indicated by G, D and D^s, population of education levels not covered by the above education groups has been allocated among them as follows:
Population of 1–4 years schooling: half to no schooling and half to 5–7 years group.
Population of 9–11 years schooling: half to 8 years group and half to 12 years group.
Population of 13–15 years schooling: half to 12 years group and half to 16 years group.

[c]Deterioration rates d' are related to d underlying estimates in panel (a) above as follows $(1-d') = 1.02(1-d)$. Hence d'/d for the four education levels are 0.52, 0.52, 0.36, 0.52 respectively. D and D^s in panel (b) are obtained by multiplying the corresponding amounts in panel (a) by these coefficients.

G, D, D^s, see footnotes to table 2.

investment in *OJT* $= G_t-(D_t-D_t^s)$ (where D_t^s is deterioration at age $5+t$ on capital formed by schooling). As d rises $N_t = G_t - D_t$ falls moderately, while at the lower ages, D_t^s rises considerably. (See tables 2 and 3.) Hence net investment in *OJT*, $G_t-(D_t-D_t^s)$, increases at the lower ages. Since the age structure of the higher education levels is weighted in favour of lower ages in view of the secular increase in education, the result follows.

Estimates of net investment in *OJT* are also made on the assumption of a secular rise in costs and benefits of 0.02 per annum. For this purpose we derive for each education-age group a time-series related to the relevant cross-section as shown in (13).[14]

[14]The assumption of a uniform rate of increase of 0.02 per annum is an approximation Hines *et al.* (1970, p.332) have found that schooling resource costs in public elementary and secondary schools increased by 3 per cent between 1955 and 1967. They give, however, no estimate of the change in the direct costs of higher education. Becker (1964, p.141) suggests that the rate of change is not the same for all education levels. Estimates by H. P. Miller quoted in Mincer (1972) show that between the years 1956 and 1966 the rate of increase of income differed as between age-education groups and in many cases was higher than 0.02 per annum. These years were chosen because of their similar cyclical positions.

TABLE 5

AGGREGATE POST-SCHOOL GROSS INVESTMENT, DETERIORATION AND NET INVESTMENT

AMERICAN WHITE MALES, 1960

(ASSUMING UNIFORM RATE OF DETERIORATION OF 0.10)[a]

$ million

Years of education[b]	5–7	8	12	16	Total
(a) *Based on cross-section social costs and earnings*					
G	4,386	11,447	37,968	30,186	83,987
D	5,013	12,897	37,669	29,881	85,460
D^s	372	696	4,358	5,491	10,917
$G-(D-D^s)$	−255	−754	4,657	5,796	9,444
(b) *Assuming social costs and earnings increase at* 2 *per cent per annum*					
G	4,386	11,447	37,968	30,186	83,987
D	4,111	10,576	30,889	24,502	70,078
D^s	306	591	3,574	4,502	8,973
$G-(D-D^s)$	581	1,462	10,653	10,186	22,882

Sources: same as for table 4 and table 3.

[a]Applies to panel (a) only, the deterioration rate for panel (b), derived as indicated in footnote c of table 4, is 8.2 per cent. Hence for panel (b) $d'/d = 0.82$.

[b]As in table 4.

G, D, D^s, see footnotes to table 2.

Proposition (vii) gives us the relation between the estimates based on a cross-section and those based on a time-series derived from it: Gross investment is the same for both, but deterioration is smaller for the time-series.

Panel (b) of tables 4 and 5 shows that the estimates of net investment in *OJT* under the assumption of a secular increase of 0.02 per annum are more than twice those calculated from cross-sections. Moreover, total net investment increases as the d assumed is raised. This is because the combined effect of age-structure and increased deterioration on human capital formed by schooling is much stronger now than in the case of cross-sections. (See table 3.)

Investment in Schooling

Estimates are obtained by multiplying enrolments at different ages by the appropriate social costs. Social costs include tuition costs and foregone earnings (earnings of persons of the same age but without schooling), as well as an interest charge on the human capital accumulated in the student, computed at the internal rate of return. This gives us gross investment $G_t = Y_t + I_t$ during the schooling year (see equation (3) above and table A1). To get net investment, we have to subtract deterioration incurred during the year on the human capital carried by the student. We get higher estimates[15] than those obtained by conventional

[15]Our estimates would be even higher if we included foregone earnings at ages of compulsory education.

methods (see e.g., Schultz (1961) and Lewis Solomon (1972)) which take social costs as equal to direct costs *plus* foregone earnings. The latter are equal to earnings of a person aged $5+t$ who left school at age $5+(t-1)$, *less* earnings of a student aged $5+t$. But the earnings of a person who left school at age $5+(t-1)$ do not represent his full earning capacity, since he invests part of it in *OJT*. Hence our gross (net) estimates exceed the conventional ones roughly by gross (net) investment in *OJT* of a person aged $5+t$ who left school at age $5+(t-1)$. Our approach is in keeping with our position that investment in *OJT* should be included both in national income and in the returns to human capital (see pp. 295–6 below).

However, this approach has to contend with one problem: How to classify students by the education level they ultimately achieve. Take a pupil aged 10. Unless we know what his final education level will be, we cannot tell what the social cost of his education is for the year since internal rates of return and rates of deterioration are different for different education levels. One way out would be to forecast the proportions of pupils of different ages who will achieve different education levels, so as to get a cross-classification by age and expected final education level. The appropriate social costs can then be applied to the different categories. To simplify matters, we shall apply to all persons at school table A1 which relates to 16 years of education, for which the rate of return is 0.145, the lowest for all the education levels. Computations for the same education level but with $d = 0.10$ instead of 0.04 have also been made. Estimates of investment in schooling for American white males corresponding to these deterioration rates are given in table 6, cols. A.

Two alternative estimates assuming increase in costs and benefits of 0.02 per annum are shown in cols. B. It will be seen that the rate of deterioration makes little difference to the estimate of net investment, but the assumption of secular growth of costs and benefits raises the estimate appreciably (line 3).

Aggregate Net Investment in Schooling and OJT

To get total net investment we add to net investment in persons at school, net investment in *OJT* as computed in tables 4 and 5 (see table 6, line 4), and subtract deterioration of investment in schooling incurred by persons out of school (line 5). This deterioration should be distinguished from that in line (2) which relates to persons at school.

Net investment in human capital thus calculated is about 50 percent larger when we assume secular growth of costs and benefits at 0.02 per annum than when we do not make this assumption. As to the sensitivity of the estimates to the rates of deterioration assumed, raising these rates of deterioration from the minima (at which net investment is a maximum) to 0.10 reduces net investment by over 5 percent when costs and benefits are assumed to grow at 0.02 per annum, and by about 4 percent when this assumption is not made.

III. Implications for Growth Accounting

The above analysis and estimates may be helpful in setting up a rational system of growth accounting. I have shown elsewhere (1971) that the method

TABLE 6
AGGREGATE INVESTMENT IN SCHOOLING AND *OJT*
AMERICAN WHITE MALES, 1960

$ million

	Estimated at			
	Minimum deterioration rates[a]		Deterioration rate 0.10[b]	
	A	B	A	B
1. Gross investment in schooling	42,228	42,228	50,473	50,473
2. Deterioration during schooling	5,529	2,875	13,834	11,344
3. Net investment during schooling	36,699	39,353	36,639	39,129
4. Net investment in *OJT*	9,478	21,145	9,444	22,882
5. *Less* Post-school deterioration of investment in schooling	9,581	4,425	10,917	8,973
6. Net investment in schooling and *OJT*	36,596	56,073	35,166	53,038

[a]See table 1 for minimum deterioration rates at different schooling levels, underlying estimates of investment and deterioration during working life. Estimates of investment and deterioration during schooling are based on table A1.

[b]Applies to col. A only. For col. B, d = 0.082.

A. Estimates based on cross-section data.

B. Estimates based on the assumption that social costs and benefits increase at the rate of 0.02 per annum.

initiated by Denison (1962, 1967) for measuring the contribution of education to growth actually covers the effects of both schooling and *OJT* except that it understates the contribution of the latter, by reducing the weights in the proportion of net investment in *OJT* in the base year to returns to human capital in the base year. I also suggested a system of national accounting and growth accounting which is the logical outcome of admitting education and *OJT* as factors in growth. A step towards implementing such a system can now be taken in the light of the findings presented above.

In one variation of the Denison method (see Bowman (1964)) the contribution of human capital to the increase in national income between years I and II is measured by $\Sigma w_i(b_i - a_i)$, where i is education level, a_i and b_i number of employed persons of education level i in years I and II respectively, w_i is the difference between average earnings of persons of education level i and persons with no education. The peculiarities of the weight w_i are revealed by means of a simple model.

Let there be one level of education and three stages in life. Persons who get no schooling work in all three stages and get no *OJT*. Other persons are educated in stage 0, work and get *OJT* in stage 1 and work without getting *OJT* in stage 2, at the end of which they retire. There is no unemployment. There are n

educated persons at each stage. Let C be the investment in schooling and I_1 and I_2 the observed earnings differentials between educated persons and persons without schooling. $I_2 > I_1$ because of *OJT*. The internal rate of return r, assumed to be the same for both schooling and *OJT*, is given by[16]

$$C = \frac{I_1}{(1+r)} + \frac{I_2}{(1+r)^2} \tag{20}$$

Assume there is no deterioration of human capital, and let f be the constant stream of returns to schooling that would have accrued in the absence of *OJT*. Let investment in *OJT* (which takes place in stage 1) be j. Then f and j are related as follows:

$$\text{Stage 1:} \quad I_1 = f - j \tag{21}$$

$$\text{Stage 2:} \quad I_2 = f + (1+r)j \tag{22}$$

This is because in stage 1, earnings j are foregone for the sake of acquiring *OJT*, and in stage 2 they are recovered together with the appropriate returns. We can calculate f and j from these two equations.

The weight w_i used in the Denison method is the average per educated person of observed returns, that is

$$\frac{1}{2n}(nI_1 + nI_2) = \frac{1}{2n}\{n(f-j) + n[f + j(1+r)]\} = f + \frac{jr}{2} \tag{23}$$

(by substitution from (21) and (22)).[17] This understates the true average returns to human capital which should be calculated as follows:

Stage 1: returns to formal education per person, f.

Stage 2: returns to formal education per person, f, *plus* returns due to investment in *OJT* in stage 1, $j(1+r)$.

Hence average returns to human capital per educated person:

$$\frac{1}{2n}\{nf + n[f + j(1+r)]\} = f + \frac{j(1+r)}{2} \tag{24}$$

This exceeds the Denison weight by $j/2$, that is, the average investment in *OJT* per educated person.[18] It may be considered legitimate to subtract the investment in *OJT* in the base year and to calculate returns to human capital net of this investment, but this is inconsistent with the treatment of investment in schooling. The latter is not deducted from the returns to human capital.

The conclusions of this simple model can be extended to our more complex multi-period model which involves deterioration of human capital. The amount of returns to human capital omitted is net investment in *OJT* (table 6, line 4),

[16]In contrast to the assumption in the previous sections, we are here taking r rather than R to be the same for both types of investment. In view of the short time horizon, this results in a sizeable difference in the rental values.

[17]It will be clear that if we use f as weight, we get only the contribution of schooling to growth. This is the essence of the method used by Schultz (1961).

[18]A similar conclusion is reached by Bowman (1968, p. 228, footnote).

and the reduction in the Denison weight is this amount divided by the number of persons out of school.

If we accept human capital as a source of growth, then clarifying the relation between the various quantities involved requires treating investment in education and *OJT* on the same footing as physical investment. This entails the adoption of several measures:

(1) The weight used in calculating returns to human capital and the contribution of human capital to growth should include net investment in *OJT*. An estimate of the relative size of the omission is obtained as follows: The net investment in *OJT* of white American males in 1960 is, on the assumption of a secular growth of costs and benefits of 0.02 per annum, about \$22 billion (table 6, line 4). The returns to the human capital carried by the same group are on the formula $\Sigma w_i a_i$, \$132.5 billion (calculated from earnings and population data). Hence the understatement is of the order of 16 percent.[19]

(2) In any year, investment in both schooling and *OJT* should be included in National Product under "Investment". The present accounting practice is to include direct costs of schooling under private and public consumption and ignore foregone earnings.[20]

From table 6, line 6 we see that net investment in human capital is between \$53 and \$56 billion. However, direct cost of schooling for white males already included in Net National Product is about \$14 billion. Hence national income should be increased by \$39–\$42 billion (this is on account of white males only), which amounts to about 10 percent of the National Income of \$414.5 billion in 1960.

"Productivity" and "Economic" Concepts

The terms "capital", "deterioration", "earning capacity" and "investment" have been used here, as in all the writings on the life-cycle hypothesis of earnings and *OJT* referred to, in a "productivity" sense rather than in an "economic" sense. Thus "capital stock" at any point of time is what is left over from the original stock of capital after deduction of deterioration according to a formula which expresses the reduction in the productive capacity of capital over time. We come across this approach to physical capital in such works as Denison (1962, 1967) which attempt to account for increases in national product by assigning returns to increments in the quantities of factors of production. The "economic" definition of "wealth" is the discounted value of future returns. This, of course, need not be quantitatively equal to "capital stock" in the "productivity" sense. It can, however, be shown that "capital stock" and the other "productivity" concepts are equivalent to their "economic" counterparts if, in the former approach deterioration is of the proportional type and in the

[19]Consistency as well as realism require that when applying the Densison method, the secular growth of earnings should be taken into account. Either the weights of a year half-way between I and II or an average of the weights for years I and II should be used. If the data available are for only one terminal year, the weights for the other year could be estimated by applying a secular rate of growth to the benefits of the first year. This would raise the estimate of the contribution of human capital to growth.

[20]See Seers and Jolly (1966) for a discussion of the current practice of omitting foregone earnings from national income.

latter approach the stream of costs and benefits is discounted at a rate $(S-1)$, and capital residual at retirement, K_T, is treated as a benefit.[21]

The depreciation allowance in national accounting is only a rough approximation to the "productivity" concept of depreciation. In the U.S.A. it is based on businessmen's depreciation allowance in which tax regulations and guidelines are used in arriving at length of life and method of spreading cost,[22] and therefore involves both an under-estimate (it is based on original cost), and an over-estimate due to the use of accelerated depreciation.[23]

IV. Conclusion

It has been argued in the above pages that human capital should be treated like physical capital, in particular gross investment should be distinguished from net investment; investment in schooling and *OJT* should be included in national income. This is, however, not to deny the essential differences between the two types of capital. Lindsay (1971) has argued that human capital differs from other forms of wealth in that it is not embodied in a separate asset, but manifests itself in the raising of the rate of earnings, and therefore affects the allocation of time between leisure on the one hand, and work and training on the other. In particular the supply of labour increases with the rate of earnings in so far as the latter is the result of investment in human capital. Hence an upward bias in the estimation of the rate of return to human capital. Though the truth of this assertion is conceded, it has not been possible to take this factor into account.

Another simplification has been used which also results in an upward bias in the rates of return. This is to attribute earning differentials between a given education level and no education to investment in schooling and *OJT* alone, thus assuming homogeneity among white males of abilities, social background and other factors that influence earnings.

The net investment in white males in 1960 was $53–56 billion which is about 70 percent of the net national investment (including consumers' durables). If we assume that net investment in non-white males and in females (both white and non-white) is roughly in proportion to their earnings, then we may conclude that total net human investment in 1960 was roughly equal to net investment in other assets.

In view of the fact that it was possible to identify only minimum deterioration rates, and because net investment is much less sensitive to the rates of deterioration than gross investment, it has been easier to obtain an estimate of net investment than of gross investment. Estimates of the latter must wait upon a more accurate determination of rates of deterioration.

Appendix I

(a) *Within limits,* $P_{t-1} = I_1S^{t-2}+I_2S^{t-3}+\ldots+I_{t-2}S+I_{t-1}$, *the polynomial in S appearing in equations* (2)–(6), *varies directly with S.*

[21]See my article (1973) for a comparison of the two sets of concepts. It is also shown there that the difference between "wealth" in year t and year $(t\text{-}1)$, calculated in the manner prescribed above, is equal to net investment N_t as defined in this paper.

[22]Except for agriculture in which the perpetual inventory method is used.

[23]See Abraham (1969) p. 35.

Call Z the expression on the left-hand side of (10), and put $R = R^* > 0$. Let there be h years of schooling, so that the first h terms are positive. The remaining $T-h$ terms are negative. By Descartes' Rule of Signs there is at most one value of $S > 0$ that satisfies $Z = 0$. Let $Z = 0$ at $S = S^*$. Then it can be shown that Z varies directly with S in the neighbourhood of S^*:

Consider Z as S is increased above S^* and R is kept equal to R^*. The negative terms (having $t > h$) which have the higher powers of S in the denominator, become small relative to the positive terms which have the lower powers of S in the denominator. Hence for some $S > S^*$, we have $Z > 0$. But Z changes sign at $S = S^*$. Therefore for $S > S^*$ we get $Z > 0$, and for $S < S^*$ we get $Z < 0$. It follows that in the neighbourhood of $S = S^*$, Z varies directly with S.

The reason for emphasizing that all values of S considered are close to each other is that at S significantly different from S^*, Z may decrease as S rises, since Z may be moving towards a local minimum or away from a local maximum.

For any given costs and benefits series, we are concerned with values of S that make $Z = 0$. Now these values of S vary (inversely) with R, but as can be seen from (10), this variation in S is very small, so that all S that correspond to $R \neq R^*$ (as determined above) are very close to S^*. Hence if R is kept equal to R^* and S varies within limits, Z varies directly with S.

Now Z_{T-1} equals Z less the last term which is negative and whose absolute value decreases as S increases. Hence Z_{T-1} must *a fortiori* increase as S increases. Since $P_{T-1} = S^{T-1}Z_{T-1}$, P_{T-1} must also vary directly with S. We can thus continue to omit negative terms and get shorter polynomials P_t whose value varies directly with S. When we get P_t such that $t \leq h$, that is, all terms are positive, the proposition is easily seen to hold, and the above proof is not required.

(b) *For different values of S that satisfy* (8) *or* (10), $R(1-d)$ *varies inversely with S.*

From (8) we have

(A1) $$R(1-d)P_{T-1} = -I_T \text{ (a positive constant)}$$

By para. (a) above, the value of P_{T-1} varies directly with S. Hence $R(1-d)$ must vary inversely with S.

(c) *For different values of S that satisfy* (8) *or* (10), *d varies inversely with S.*

By the definition of S, we have $R(1-d) = S-1+d$. If S rises, $R(1-d)$ falls, by para. (b). Hence d must fall.

(d) *For any t,* $Y_t = R(1-d)P_{t-1}$ *varies inversely with S.*

It is easily seen that

(A2) $$P_t = SP_{t-1}+I_t$$

Put $t = T-1$ and rearrange terms

(A3) $$P_{T-2} = (P_{T-1}-I_{T-1})/S$$

Consider $Y_{T-1} = R(1-d)P_{T-2}$. Substituting for P_{T-2} from (A3) and then for $R(1-d)P_{T-1}$ from (A1), we get,

(A4) $$Y_{T-1} = -\frac{1}{S}[I_T+R(1-d)I_{T-1}]$$

(where I_T, $I_{T-1} < 0$). As S increases, $R(1-d)$ falls (by para. (b)). So the denominator increases and numerator decreases.

It can be shown in like manner that $Y_{T-2} = (1/S)[Y_{T-1} - R(1-d)I_{T-2}]$ varies inversely with S, and we can thus work backwards to smaller t.

Strictly speaking, the proof applies only where $I_t < 0$, that is, during working life. But this is where the proof is particularly needed. We have to show that G_t (equation (3)), varies directly with d, so that d can be increased sufficiently to make all $G_t > 0$. Now G_t are always positive for schooling years (because all I_t for schooling years are positive), so the proof is needed only in respect of working years as far as this matter is concerned.

Still, in all our computations of G_t at different levels of d, G_t varies directly with d (inversely with S) during all schooling years. This is in view of the relatively large rise in $R(1-d)$ which accompanies a given fall in S. We shall therefore assume the proposition to hold throughout life.

Appendix II

In this Appendix a simple algorithm will be expounded for determining the minimum deterioration rate d and the corresponding rental value R that produce non-negative gross investment ($G_t \geq 0$), for every year between the beginning of schooling and retirement, except for the year of retirement T when $G_T = 0$.

But first we have to explain how, for a given d and R which do not necessarily satisfy the above conditions, we can calculate, year by year, earning capacity, gross investment and deterioration. Such computations are illustrated in table A1.

The figures I_t in col. 2 are positive at the lower ages when they represent social costs and negative during working life, when they represent excess earnings over white males of the same age but with no schooling. The series of earning differentials has been mostly calculated by interpolation between the averages for age groups.

Starting from row 2, earning capacity in a given year equals $(1-d)$ times the earning capacity in the preceding year plus $R(1-d)$ times the gross investment made in the preceding year, i.e., $Y_t = (1-d)Y_{t-1} + R(1-d)G_{t-1}$. It can be verified that this is equivalent to equation (2). G_t as well as D_t (deterioration), are easily calculated from the definitions given at the end of table A1.

It is advisable that the operations set out in table A1 be programmed for handling by computer, as the identification of the minimum d requires some trial and error, which implies that table A1 has to be computed several times, using different R's (except for the last column D_t, which is not necessary for the determination of the minimum d). If earnings fall in old age, then we start with a positive d, say 0.10. The R which makes $G_{60} = 0$ is such that $(1+R)(1-d)$ is just a little greater than $1+r$, r being the internal rate of return. The difference is of the order of 0.001. Since it is unlikely to hit upon the right R at the first attempt, try two R's, R_1 and R_2, close to each other. Suppose, as is very likely, that for both R_1 and R_2, $G_{60} \neq 0$. To find the right R, interpolate between R_1 and R_2, or if necessary extrapolate, so as to make $G_{60} \simeq 0$. It is found in practice that for small variations R bears an almost linear relation to G_{60}. The closer the

TABLE A1

EARNING CAPACITY, GROSS INVESTMENT AND DETERIORATION BY AGE
AMERICAN WHITE MALES, 16 YEARS OF EDUCATION, 1960
(d = 0.04, R = 0.19289675)

Dollars

(1) Age	(2) I_t	(3) Y_t	(4) G_t	(5) D_t
6	470	—	470	—
7	470	87	557	19
8	470	187	657	40
9	470	301	771	65
10	470	432	902	93
11	470	581	1,051	126
12	470	753	1,223	163
13	470	949	1,419	205
14	734	1,174	1,908	254
15	734	1,480	2,214	320
16	779	1,831	2,610	395
17	779	2,241	3,020	484
18	2,475	2,711	5,186	586
19	2,475	3,562	6,037	770
20	2,802	4,538	7,340	980
21	2,802	5,716	8,518	1,235
22	−3,995	7,064	3,069	1,526
23	−4,256	7,350	3,094	1,588
24	−4,517	7,629	3,112	1,648
25	−4,778	7,900	3,122	1,707
26	−5,038	8,162	3,124	1,763
27	−5,363	8,415	3,052	1,818
28	−5,688	8,643	2,955	1,867
29	−6,013	8,845	2,832	1,910
30	−6,338	9,015	2,677	1,947
31	−6,662	9,150	2,488	1,977
32	−6,800	9,245	2,445	1,997
33	−6,938	9,328	2,390	2,015
34	−7,076	9,398	2,322	2,030
35	−7,213	9,452	2,239	2,042
36	−7,350	9,488	2,138	2,049
37	−7,487	9,504	2,017	2,053
38	−7,623	9,498	1,875	2,052
39	−7,615	9,465	1,850	2,045
40	−7,607	9,429	1,822	2,037
41	−7,599	9,389	1,790	2,028
42	−7,591	9,345	1,754	2,019
43	−7,583	9,296	1,713	2,008
44	−7,575	9,242	1,667	1,996
45	−7,567	9,181	1,614	1,983
46	−7,558	9,112	1,554	1,968
47	−7,549	9,036	1,487	1,952
48	−7,540	8,950	1,410	1,933
49	−7,501	8,853	1,352	1,912
50	−7,461	8,749	1,288	1,890
51	−7,421	8,638	1,217	1,866
52	−7,381	8,517	1,136	1,840
53	−7,341	8,387	1,046	1,812
54	−7,301	8,245	944	1,781

TABLE A1—*continued*

	(d = 0.04 R = 0.19289675)		Dollars	
(1) Age	(2) I_t	(3) Y_t	(4) G_t	(5) D_t
55	−7,261	8,090	829	1,747
56	−7,221	7,920	699	1,711
57	−7,181	7,733	552	1,670
58	−7,141	7,526	385	1,626
59	−6,964	7,296	332	1,576
60	−6,787	7,066	279	1,526
61	−6,610	6,835	225	1,476
62	−6,433	6,603	170	1,426
63	−6,256	6,370	114	1,376
64	−6,079	6,137	57	1,326
65	−5,902	5,902	0	1,275
Total			116,123	85,526

Sources: Hines, Tweeten and Redfern (1970), together with earnings data derived from *One in One thousand Sample of the 1960 Census of Population*, supplied by the authors.

Figures have been rounded to the nearest $1.

I_t, social costs of education when positive, observed returns (excess earnings over white males with no schooling) when negative,

Y_t, earning capacity $= (1-d)Y_{t-1} + R(1-d)G_{t-1}$

G_t, gross investment $= Y_t + I_t$

D_t, deterioration $= (d/R(1-d))G_t$.

approximation to zero, the more accurate will R have to be: it may have to be specified to six decimal places or more.

It is not enough that G_{60} should be zero. All the other G_t should be non-negative, and if they are, we should ensure that d is a minimum. Hence we have to try either higher or lower d's than the one we started with depending upon the signs of the G_t. It should be noted that for sets of d and R which make $G_{60} = 0$, $S = (1+R)(1-d)$ increases as d falls, and *vice versa*, the change in S being very small in comparison with the change in d. Thus in the case of table A1, d is 0.04 and $S = 0.14518$. If d is raised to 0.10, S becomes 0.14514. It should be noted that 0.04 is the minimum d corresponding to the series I_t of table A1.

If the earnings profile does not bend down, then it is likely that d is non-positive, so it is advisable to start with $d = 0$, then try $d < 0$ or $d > 0$ as indicated by the sign of the G_t obtained.

It will be clear by now that table A1 can be used to calculate the internal rate of return r. The higher d the closer is S to $1+r$. In fact we do not need to calculate r separately. All we need is to find a set of R and d that satisfy the required conditions. A slightly overestimated r is obtained as a by-product.

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