

FORECASTING BANK LIQUIDITY¹

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I. INTRODUCTION

SEVERAL forecasts of bank liquidity have been made during the post-war years, in connection with the work of the Norwegian Council for Monetary and Financial Policy. The Council has taken considerable interest in the prospective development of bank liquidity, as the liquidity constitutes one of the major factors motivating credit policy of the banks. Since the Norwegian bond and stock market plays a minor role in transferring savings directly from savers to investors, bank credit is a major source of financing real investments in the business sector. Consequently a curtailment or eventually an expansion of investments may be quite effectively accomplished through a governmental imposed reduction or augmentation of bank liquidity.² As liquidity may change also for other reasons, forecasts of bank liquidity became necessary as a guide to policy.

A forecast for 1950 and another for 1951 is discussed in this paper.³ The purpose of this discussion is to describe the forecasting methods that were applied and to analyse the results of the forecasts in the light of the actual development. We shall see that, although the analytical models used were very simple, the experience is quite encouraging.

The liquidity position of the banks cannot be exhaustively described by means of a single index. Only a detailed study of the composition of bank assets and debts and their relation to one another would reveal the true liquidity of the banks.

¹ Most of this paper was first written in Norwegian as a contribution to a volume in honour of Ragnar Frisch, *Sozialökonomien som hjelpemiddel i den økonomiske politik*. Reprint of *Statsøkonomisk Tidsskrift*, Oslo 1955. The translation and to some degree a further elaboration on this paper has been done at Stanford University under contract. I am indebted to my colleagues at Stanford University and other friends who have suggested improvements of the paper, and particularly to Mr. John Hooper.

² Until recently this could be accomplished only by open market operations and by Central Bank credit policy. Legal provisions for imposing reserve requirements were enacted by the Storting (Norwegian Parliament) on 28th June 1952.

³ Both forecasts were prepared in the Central Bureau of Statistics of Norway. The 1950 forecast was made by the author and Messrs. Odd Aukrust and Stein Rossen. The 1952 forecast was made by the author with the assistance of Messrs. Leiv Vidvei of the Central Bureau of Statistics and Leif Johansen of the Oslo University Institute of Economics.

However, with regard to the Norwegian private banks and savings banks, it is generally believed that their balance with the Central Bank plus their Treasury Bill holdings is a reasonably accurate indicator of their liquidity. The primary aim of the 1950 and 1952 forecasts was the estimation of the changes in these balances.¹

One of the major factors influencing bank liquidity in the post-war years was the large and persistent deficit on the balance of payments. Most of this deficit has been financed by currency reserves possessed by the Government or Bank of Norway at the end of the war, by government or the Central Bank borrowing from abroad, and by foreign aid, primarily from the United States, in the form of currency which has been granted to the Norwegian Government. The Government, in turn, sold this currency to importers through the medium of the Central Bank. By their sales of foreign currency, the Government and the Central Bank have reduced bank liquidity in the same manner as is done by open market operations. The quantitative significance of this factor may be illustrated by the fact that the aggregate deficit on the balance of payments for the years 1946-53 amounted to 5,400 millions of Kroner, i.e. almost one fourth of the gross national product in 1953.

Tables I and II show the various flows of financial assets and debts which were considered in the 1950 and 1952 forecasts. Each table implies a given set of definitional relationships between these variables. The model of the 1950 forecast included only the variables and relationships defined by Table 1. The 1952 model included the variables and relationships presented by Table II and, in addition, included four behaviour relationships. In both models the number of variables is considerably larger than the number of relationships, i.e. the models are characterized by a relatively large number of degrees of freedom. Therefore, the same number of variables as there are degrees of freedom had to be determined outside the model in order to arrive at a conclusion on the magnitudes of the remaining 'unknown' variables. The first group of variables we shall term data variables in order to distinguish them from the unknowns. Two criteria were used in the selection of data

¹ The government banks and the co-operative banks hold negligible amounts in the form of such assets, so that the total bank holdings of Central Bank deposits and of treasury bills could be considered identical with those of the private banks and the savings banks.

variables. The first was to select those variables which could be considered as government variables, i.e. variables whose values were fixed directly by the Government. The second was to select variables whose values were least difficult to estimate with the supplementary information available.

The magnitudes of the Government's data variables were either set in conformity with existing government plans or, in the absence of such plans, were set on the basis of certain reasonable assumptions. The estimates of the private data variables were based upon the information obtained on (a) the holdings of the various types of financial assets and debts and the amortization contracts of these assets and debts, (b) laws and restrictions pertaining to banks and to negotiable documents, (c) customary institutional practices, and (d) the magnitudes of the corresponding variables for previous years. Particularly, information of type (a) proved to be very useful.

Thus the forecasting method which was applied, attempts to utilize all information available on the behaviour relationships and on the magnitudes of the data variables for the purpose of obtaining the best estimate of the values of the unknowns. In this respect forecasting resembles a puzzle: by putting the various pieces together according to certain rules we may get a picture which cannot be obtained by looking at each of the pieces separately. Furthermore, by treating some of the government variables as independent, we can assume alternative values of these variables and obtain corresponding solutions for the unknowns. In this manner an estimate can be made of the effects to be expected as a consequence of different government policies.

In the 1952 forecast estimates of the latter type were made as follows: First, two government variables were treated as independent variables, viz. the surplus on the Government's budget and the deficit on the balance of payments. This could be done by introducing only as many data variables so as to leave the model with two degrees of freedom. By solving the equation system in respect to the independent variables each of the unknowns could be found as a function of these variables. Next, alternative values were assigned to the independent variables and corresponding solutions were computed for the unknowns.

In Section II the 1950 model is described, and the results of

the forecast are compared with corresponding *ex post* figures. In the same manner the 1952 forecast is discussed in Section III. The presentation concentrates on the formal aspects of the forecasts, i.e. on the internal structure and logic of the models. In order to demonstrate how these models were applied, we also discuss briefly how the numerical estimates were prepared.¹ In the concluding Section IV an attempt is made to appraise the forecasting method and to suggest possible improvements.

II. THE FORECAST FOR 1950

The numerical results of this forecast are reproduced in Table I. The *ex post* figures are also included (in parenthesis). The presentation is made in the form of a double accounting system, where the entries indicate major changes over the year 1950 in the financial status of five sectors, viz. the Central Bank, the Central Government, the Private Banks and Savings Banks, the Government Banks and Co-operative Banks, and finally a residual sector, called 'Other Sectors'. There is only one account for each of these sectors.

The claims and debts are classified into eight categories. Figures corresponding to these categories are found on the lines 2 through 9 in Table I. Here, the debit entries indicate increases in net claims, i.e. in claims minus the same kind of debts, and the credit entries indicate increases in net debts, i.e. debts minus the same kind of claims.

On line 1 of each account a balance is entered, which makes the sum of the debit entries equal to the sum of the credit entries. Thus, on this line a debit entry represents an increase in the aggregate net debts of a sector (decrease in the aggregate net claims) and a credit entry represents an increase in the aggregate net claims (decrease in the aggregate net debts).

The account for the Central Government sector is defined to include, *inter alia*, claims and debts of central government funds and of the social security system. Local governments are included in Other Sectors. The account for Other Sectors also include changes in the Norwegian net claims and debts toward other countries.

¹ These figures may be of limited interest outside of Norway. Nevertheless, they may serve to make the presentation more concrete and intelligible.

1. *The model*

The formal properties of the model applied in the 1950 forecast may be explained as follows. We conceive of the credit entries in Table I as negative debit entries. In principle we admit the possibility of having all transactions represented as entries in the account of each sector. Thus we obtain forty-five variables. Each entry is symbolized by the letter of the account where it is found and by a subscript which indicates the number of the asset. The result is the following system of variables.

a ₁	b ₁	c ₁	d ₁	e ₁
a ₂	b ₂	c ₂	d ₂	e ₂
a ₃	b ₃	c ₃	d ₃	e ₃
a ₄	b ₄	c ₄	d ₄	e ₄
a ₅	b ₅	c ₅	d ₅	e ₅
a ₆	b ₆	c ₆	d ₆	e ₆
a ₇	b ₇	c ₇	d ₇	e ₇
a ₈	b ₈	c ₈	d ₈	e ₈
a ₉	b ₉	c ₉	d ₉	e ₉

Table I defines thirteen independent relationships between these variables (since the row and column sums equal zero), i.e. there is a system of equations with thirty-two degrees of freedom. This means that the values of thirty-two data variables have to be determined by information from without the model in order to arrive at a conclusion with regard to the remaining thirteen unknowns. In the above matrix the variables treated as un-knowns are encircled. The others are data variables. Among the latter the government data variables lie within the rectangles.

TABLE I
Forecast of Changes in Holdings of Claims and Debts, 1950 (Ex Post Figures are indicated in brackets)
 (Millions of Kroner)

Sectors	Central Bank		Central Government		Private Banks and Saving Banks		Government Banks and Co-operative Banks		Other Sectors	
	a		b		c		d		e	
	Dr.	Cr.	Dr.	Cr.	Dr.	Cr.	Dr.	Cr.	Dr.	Cr.
1. Aggregate Net Claims (Claims minus Debts)	—	—	—	350	—	—	—	—	350	—
2. Foreign Currency	—	150	—	(520)	—	—	—	—	(520)	—
3. Treasury Bills	(150)	—	—	1,050	—	—	—	—	1,200	—
4. Bank Loans	—	—	400	(1,040)	—	—	—	—	(890)	—
5. Bank Deposits	—	—	(340)	—	—	400	—	—	—	—
6. Bonds and Stocks	—	—	—	—	500	—	500	—	—	1,000
7. Central Bank Deposits	—	—	—	—	(800)	—	(500)	—	—	(1,300)
8. Other Non-Cash Claims	—	—	—	—	150	—	—	—	—	150
9. Cash Holdings	—	—	100	(150)	300	—	500	500	100	—
	—	—	—	(130)	—	—	—	(270)	(290)	—
	250	—	400	—	—	550	—	—	—	100
	—	(10)	(330)	—	—	(580)	—	—	(260)	—
	—	(50)	500	—	—	—	—	—	—	500
	—	100	(1,040)	—	—	(10)	—	(230)	—	(750)
	—	(90)	—	—	—	—	—	—	100	—
	—	—	—	—	—	—	—	—	(90)	—
Sum	250	250	1,400	1,400	950	950	500	500	1,750	1,750
	(150)	(150)	(1,710)	(1,710)	(940)	(930)	(500)	(500)	(2,050)	(2,050)

In accordance with Table I we have the following equations (the data variables are those within brackets):

$$\begin{aligned}
 (1) \quad & e_1 + [a_1 + b_1 + c_1 + d_1] = 0 \\
 (2) \quad & e_2 + [a_2 + b_2 + c_2 + d_2] = 0 \\
 (3) \quad & c_3 + [a_3 + b_3 + d_3 + e_3] = 0 \\
 (4) \quad & e_4 + [a_4 + b_4 + c_4 + d_4] = 0 \\
 (5) \quad & c_5 + e_5 + [a_5 + b_5 + d_5] = 0 \\
 (6) \quad & c_6 + [a_6 + b_6 + d_6 + e_6] = 0 \\
 (7) \quad & a_7 + b_7 + c_7 + [d_7 + e_7] = 0 \\
 (8) \quad & d_8 + e_8 + [a_8 + b_8 + c_8] = 0 \\
 (9) \quad & a_9 + [b_9 + c_9 + d_9 + e_9] = 0 \\
 (10) \quad & a_7 + a_9 + [a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_8] = 0 \\
 (11) \quad & b_7 + [b_1 + b_2 + b_3 + b_4 + b_5 + b_6 + b_8 + b_9] = 0 \\
 (12) \quad & c_3 + c_5 + c_6 + c_7 + [c_1 + c_2 + c_4 + c_8 + c_9] = 0 \\
 (13) \quad & d_8 + [d_1 + d_2 + d_3 + d_4 + d_5 + d_6 + d_7 + d_9] = 0 \\
 (14) \quad & e_1 + e_2 + e_4 + e_5 + e_8 + [e_3 + e_6 + e_7 + e_9] = 0
 \end{aligned}$$

One of these equations may be derived from the rest; consequently, the system consists of only thirteen independent equations.

The solutions for e_1 , e_2 , c_3 , e_4 , c_6 , a_9 , b_7 , and d_8 can be found directly from the equations (1), (2), (3), (4), (6), (9), (11), and (13). The solutions for the remaining unknowns can also be found without difficulty. It may suffice to reproduce the solution for c_7 , i.e. the increase in the balance of the Central Bank deposits belonging to Private Banks and Savings Banks. This variable was, as already pointed out, a major unknown to be determined by the 1950 forecast. Its solution is given as follows:

$$(15) \quad c_7 = (a_1 + b_1) + (a_2 + b_2) + (a_3 + a_4 + a_5 + a_6 + a_8 + b_3 + b_4 + b_5 + b_6 + b_8) - (d_7 + e_7) - (c_9 + d_9 + e_9).$$

In this equation, the sum $(a_1 + b_1)$ represents the deficit on the Central Bank and government accounts (more exactly, the deficit plus the real investments of these sectors); the sum $(a_2 + b_2)$ represents the net purchase of foreign currency by the Central Bank and Government; the sum $(d_7 + e_7)$ represents the net increase in Central Bank deposits belonging to Government Banks and Co-operative Banks and Other Sectors; the sum

$(c_9 + d_9 + e_9)$ represents the net increase in the holdings of cash by Private Banks and Savings Banks, Government Banks and Co-operative Banks, and Other Sectors; and finally all of the remaining variables on the right side of equation (15) represent transactions causing a net outflow of cash from the Government and the Central Bank.

2. *The Estimation of Data*

We shall now describe in detail how the value for each of the data variables in Table I was estimated.

The following government variables were known to be equal to or approximately equal to zero (and for our purposes were regarded as zero): the change in the aggregate net claims belonging to the Central Bank (a_1); the net purchase of treasury bills by the Central Bank (a_3), by the Government Banks and Co-operative Bank (d_3) and by Other Sectors (e_3); the net change in bank loans to the Central Bank (a_4) and the Government (b_4); the net increase in the bank deposits owned by the Central Bank (a_5); and the net increase in the government cash holdings (b_9).

Some other variables were set equal to zero because it was believed that the Government would make them equal to zero or because for analytical purposes they could be assumed to equal zero. These were:

(1) Net purchase of foreign currency by Private Banks and Savings Banks (c_2) and by the Government and Co-operative Banks (d_2). At the beginning of 1950 the holdings of foreign currency by these banks were at a minimum so that they could not be decreased. Neither would the banks be permitted to substantially increase their currency reserves. Thus, it was expected that these items (c_2 and d_2) would remain at or near zero.

(2) Net increase of government deposits in Private Banks and Savings Banks (b_5). From experience it was known that these deposits were fairly constant in the short run, and there were no reasons to expect a substantial change in 1950.

(3) The increase in the Central Bank holdings of bonds and stocks (a_6) and of other non-cash claims (a_8). Although the assumption that these variables would equal zero appeared valid on the basis of past experience, some doubt was expressed with regard to a_8 .

The magnitudes of the remaining government variables in Table I were set in accordance with the National Budget (which represents a projection of national accounts entries for a prospective year) and the government fiscal budget, or on the basis of other available information which could be obtained about the prospective monetary and fiscal policy. A surplus on the Government's budget, amounting to 350 millions of Kroner, was indicated in the National Budget for 1950, and it was assumed that the Government would increase its net claims by the same amount ($b_1 = -350$). Furthermore, the National Budget for 1950 indicated a balance of payment's deficit of 1,200 millions of Kroner. Since the foreign currency reserves of the Central Bank at the beginning of 1950 were relatively small, it was believed that these reserves could not be reduced by more than a maximum of 150 millions of Kroner ($a_2 = -150$). Due to the assumptions regarding the currency reserves of other sectors, this implied that the Government would have to supply foreign currency to the amount of 1,050 millions of Kroner to cover the remaining part of the balance of payments deficit ($b_2 = -1,050$). The estimate of the change in government loans previously obtained by treasury bills was simple since the Government had stated that, in effect, no more treasury bills would be sold and that no renewals of previous issues would be granted. The repayment contracts were such that with no renewals a repayment of the entire 400 millions of Kroner of outstanding treasury bills could be expected in 1950 ($b_3 = 400$). The estimate of the change in the government holdings of (or indebtedness in) bonds and stocks could be made in a corresponding manner since no government loans were planned for 1950 and the amounts and repayment contracts of existing government loans were known. These repayments were estimated at 100 millions of Kroner ($b_4 = 100$). The estimate of the change in Other Non-Cash Claims of the Government was more difficult to make. These consist of government lending to government enterprises and various other forms of government lending. On the basis of information contained in the Government's fiscal budget, this entry was estimated at 500 millions of Kroner, although this is a very crude approximation ($b_5 = 500$).

Some of the private variables were also of such a nature that they could be assumed to be equal to or approximately equal to zero. These variables were: (1) The change in the aggregate net

claims of Private Banks and Savings Banks (c_1) and of Government Banks and Co-operative Banks (d_1); (2) the increase in the holdings of Other Non-Cash Claims by Private Banks and Savings Banks (c_8); (3) the increase of the bank deposits and Central Bank deposits belonging to Government Banks and Co-operative Banks (d_5 and d_7); and (4) the increase in the holdings of cash by banks (c_9 and d_9). These entries are according to experience of such magnitudes that they could be disregarded with negligible consequences for the outcome of the forecast.

The other private data variables were estimated as follows:

(1) Very little information was available about the net increase in the holdings of bonds and stocks by Other Sectors and the net increase in the Central Bank deposits owned by this residual sector. It seemed probable that Other Sectors would increase their net bond holdings and that their Central Bank deposits would be correspondingly reduced. However, figures for previous years suggested that these amounts would not be large. On this basis it was assumed that Other Sectors would have a net increase in their holdings of bonds amounting to 100 millions of Kroner and an equally large decrease in their Central Bank deposits ($e_6=100$ and $e_7=-100$).

(2) Statistics for preceding years indicated a clear tendency of an increase in the loans granted by Government Banks and Co-operative Banks relative to the loans granted by Private Banks and Savings Banks. There were reasons to believe that this tendency would continue in 1950. Consequently, it was estimated that loans granted by Private Banks and Savings Banks would increase by 500 millions of Kroner ($c_4=500$) which was somewhat less than in the preceding year, and the increase of the loans granted by Government Banks and Co-operative Banks was estimated to be the same amount ($d_4=500$), about as in 1949.

(3) In conformity with the latter estimate the net sale of bonds by Government Banks and Co-operative Banks was estimated to be 500 millions of Kroner ($d_6=-500$). This estimate rested on the assumption that these banks could not finance an increase in their loans by other means than issuance of new bonds.

(4) It was expected that the total amount of Central Bank notes held by other Sectors would increase approximately in

proportion to the gross national product and that the increase of the latter would be equal to the amount indicated by the National Budget for 1950. Thus, the increase was estimated to be 100 millions of Kroner ($e_9=100$).

3. *An Appraisal of the Forecast*

The reader is cautioned that the 1950 forecast was prepared in August 1950 so that figures for the preceding months of the year were available for improving the forecast. However, it is believed that the major results would have been about the same had the forecast been prepared at the beginning of 1950.

The reliability of the *ex post* figures is not uniform for all variables. The figures on the accounts of the Central Bank and the other two groups of banks are excellent. Most of the figures found on the government account are quite reliable, but some, e.g. the figure for Other Non-Cash Claims, are not entirely satisfactory. Some of the figures on the account of Other Sectors are less satisfactory. The figures for this sector are computed as residuals so that errors in sectors (a) through (d) are reflected in these figures. Due to the deficiencies of these figures, and since the estimate of the change in liquidity for the Private Banks and Savings Banks was the primary object of the 1950 forecast, we shall compare the forecast with the *ex post* results only for the Central Bank deposits and for the sum of Central Bank deposits and treasury bill holdings of these banks.

The forecast indicated, as Table I shows, a decline of 550 millions of Kroner in the Central Bank deposits of the Private Banks and Savings Banks. The *ex post* figure shows a decline of 580 millions of Kroner, indicating that the forecast was remarkably close. For the other indicator of the liquidity for these banks, viz. the sum of their Central Bank deposits and their holdings of treasury bills, the forecast indicates a decrease by 950 millions of Kroner as compared with the actual decline of 920 millions of Kroner. In the report where this projection was first presented, it was concluded: 'The figure which is indicated in the example, 950 millions of Kroner, may be considered as a probable indicator of the liquidity reduction which can be expected, although the number is certainly subject to a considerable margin of error in both directions.'¹ In the same presentation these forecast results were compared with the

¹ *Innstilling fra Penge-og finansrådet*, 30th August 1950. Vedlegg 2, p. 17.

corresponding *ex post* figures for the months of January through July 1950. During this period the sum of cash,¹ Central Bank deposits, and treasury bills owned by the Private Banks and Savings Banks had decreased by only 168 millions of Kroner. It was nevertheless concluded that it would not be unreasonable, after allowing for seasonal movements in this figure, to believe that the decrease would amount to 950 millions of Kroner for the whole year.²

This conclusion was qualified by the assumptions that the Central Bank and the Government would not counteract or augment the decline in bank liquidity. Possible deviations from the estimates of the data variables were discussed, *inter alia*, deviations from the data figures for the deficit of the balance of payments and for the budget surplus. It was concluded, '... that these deviations could with equal probability go in either direction'.³ In fact, the government sold bonds for a net amount of 150 millions of Kroner, instead of repaying a net amount of 100 millions of Kroner as was assumed. Had this been foreseen when the forecast was prepared and had the other data variables been as in Table I, the forecast for the reduction in the holdings of liquid assets would have come out 250 millions of Kroner larger.

III. THE FORECAST FOR 1952

The numerical results of the 1952 forecast are reproduced in Table II, together with the corresponding *ex post* figures (in brackets).

The upper half of this table is prepared in the form of a matrix which records the change over the year 1952 in the internal non-cash claims and debts between four sectors, viz. the Government, the Central Bank, the Banks and Other Sectors. In this matrix the column for a sector shows the gross increase in the claims of this sector on the other three sectors (the diagonal elements equal zero), and the row for a sector shows the gross increase in the debts towards the other three sectors. (A negative

¹ Cash was included because no separate statistics of Central Bank deposits held by these banks were available. The change in cash was negligible and so did not influence the results.

² *Innstilling fra Penge-og finansrådet*, 30th August 1950. Vedlegg 2, p. 18.

³ *Ibid.* p. 17

figure of course indicates a decrease of the claims and the debts, respectively.)

The lower half of the table shows for each of the four sectors and for the nation as a whole the net increase in claims (including gold) of various kinds (lines 6 through 9) and in the aggregate claims (line 10).

The net increase in all Government claims (sum column *a*) is equal to the surplus on the Government budget (after deduction for all expenditures for goods and services). The sum of the increases in all claims (sum line 10) is equal to net foreign investment in the familiar national accounts.

The sectors of Table II are defined somewhat differently from those of Table I. The most important differences are that in Table II the sector Government includes local governments, the sector Banks includes all banks except the Central Bank, and Other Sectors embrace only those internal sectors (not foreign countries) which are not included elsewhere.

This method of presentation was first proposed by Professor Ragnar Frisch. A more detailed description of the method is published in an appendix to the report of the Council for Monetary and Financial Policy.¹

1. *The Model*

The variables and definitional relationships of the model applied when preparing the 1952 forecast are defined by Table II. For the sake of simplicity we disregard column *e* in lines 1 to 4 and also lines 5 and 6. We symbolize each entry in Table II by the letter of the column and by a subscript indicating the line. The result is the following system of thirty-two variables:

—	b_1	c_1	d_1	
a_2	—	c_2	d_2	
a_3	b_3	—	d_3	
a_4	b_4	c_4	—	
a_7	b_7	c_7	d_7	e_7
a_8	b_8	c_8	d_8	e_8
a_9	b_9	c_9	d_9	e_9
a_{10}	b_{10}	c_{10}	d_{10}	e_{10}

Table II defines the following relations between these variables (the data variables in brackets):

¹ *Innstilling fra Penge-og finansrådet*, 13th January 1952, pp. 68–87.

$$\begin{aligned}
 (1) \quad & a_2 + a_7 - a_{10} + [a_3 + a_4 + a_8 + a_9 - b_1 - c_1 - d_1] = 0 \\
 (2) \quad & -a_2 + b_7 + b_8 - c_2 + [b_1 + b_3 + b_4 + b_9 - b_{10} - d_2] = 0 \\
 (3) \quad & c_2 + c_4 - c_{10} - d_3 + [c_1 + c_7 + c_8 + c_9 - a_3 - b_3] = 0 \\
 (4) \quad & -c_4 + d_3 + d_7 + d_8 - d_{10} + [d_1 + d_2 + d_9 - a_4 - b_4] = 0 \\
 (5) \quad & e_7 + e_9 - e_{10} + [e_8] = 0 \\
 (6) \quad & a_7 + b_7 + d_7 - e_7 + [c_7] = 0 \\
 (7) \quad & b_8 + d_8 + [a_8 + c_8 - e_8] = 0 \\
 (8) \quad & e_9 + [a_9 + b_9 + c_9 + d_9] = 0
 \end{aligned}$$

TABLE II

Forecast of Changes in Holdings of Claims and Debts, 1952
 (Ex Post Figures are indicated in brackets)

(Millions of Kroner)

Sectors	Government	Central Bank	Banks	Other Sectors	Sum
	a	b	c	d	e
Claims and Debts					
Gross Increase in Internal Non-Cash Claims on:					
Government (1)	— (—)	— (-1)	-155 (227)	-35 (-166)	-190 (60)
Central Bank (2)	495 (338)	— (—)	-580 (-315)	— (-162)	-85 (-139)
Banks (3)	235 (502)	— (-11)	— (—)	320 (658)	555 (1,149)
Other Sectors (4)	315 (172)	— (-87)	1,300 (1,174)	— (—)	1,615 (1,259)
Sum (5)	1,045 (1,012)	— (-99)	565 (1,086)	285 (330)	1,895 (2,329)
Net Increase in Internal Non-Cash Claims (Sum of Column minus Sum of Line) (6)	1,235 (952)	85 (40)	10 (-63)	-1,330 (-929)	— (—)
Increase in Net Claims on Foreign Countries (7)	-20 (113)	115 (217)	10 (51)	45 (-373)	150 (8)
Increase in Cash Holdings (8)	30 (-5)	200 (-250)	— (-7)	170 (262)	— (—)
Increase in Gold Holdings (9)	— (—)	— (—)	— (—)	— (—)	— (—)
Net Increase in all Claims (including Gold) (10)	1,245 (1,060)	— (7)	20 (-19)	-1,115 (-1,040)	150 (8)

In addition the following behaviour relationships were included:

$$\begin{aligned} (9) & \quad a_7 = f_1(e_{10}) \\ (10) & \quad b_7 = f_2(e_{10}) \\ (11) & \quad d_7 = f_3(e_{10}) \\ (12) & \quad d_8 = f_4(e_{10}) \end{aligned}$$

The functions (9) through (11) describe the policy which the Currency Control was expected to pursue when determining the change in the net holdings of foreign currency owned by the Government (a_7), the Central Bank (b_7), and Other Sectors (d_7), at different magnitudes of the surplus on the balance of payments (e_{10}). Function (12) describes the relationship which the Council for Monetary and Financial Policy expected to exist between the increase in cash holdings by Other Sectors (d_8) and the surplus on the balance of payments (e_{10}).¹

One of the equations (1) through (12) may be derived from the other eleven so the system consists of only eleven independent equations, i.e. it has twenty-one degrees of freedom. In order to close the equation system, twenty-one of the variables have to be considered as data whose magnitudes must be determined by information from outside the model. Then a solution can be found for each of the remaining unknown variables. Instead of closing the system, it was left with two degrees of freedom. The two variables, a_{10} and e_{10} , were considered as independent and the unknowns as dependent. Then the system could be solved for each of the unknowns in respect to the independent variables. Alternative values were inserted for a_{10} and e_{10} , and consequently the corresponding solutions for each of the unknowns could be found. In this manner the quantitative effects on liquidity of different alternatives with regard to the budget policy (a_{10}) and the balance of payments policy (e_{10}) were investigated. This was done because the possibility of alternative budget and balance of payments policies was still open and the monetary policy had to be adapted to the decision made on the choice among these alternatives.

The two indicators of bank liquidity which the Council

¹ These functions will be further discussed in connection with the description of the manner in which their quantitative form was estimated.

considered as being most important, were the increase in the claims on the Central Bank held by Banks (c_2) and the increase in the claims on the Banks held by Other Sectors (d_3). The solutions for these variables are:

$$(13) \quad c_2 = (f_1(e_{10}) + f_2(e_{10}) - f_4(e_{10})) - (a_{10} + b_{10}) + (a_3 + a_4 + a_9) + (b_3 + b_4 + b_9) - (c_1 + c_8) - (d_1 + d_2) + e_8$$

and

$$(14) \quad d_3 = (f_1(e_{10}) + f_2(e_{10}) - f_4(e_{10})) - (a_{10} + b_{10}) + (a_4 + a_9) + (b_4 + b_9) + (c_4 + c_7 + c_9 - c_{10}) - (d_1 + d_2) + e_8$$

Equation (13) is derived from equations (1), (2), (7), (9), (10), and (12), and equation (14) is derived from (1), (2), (3), (7), (9), (10), and (12). The other unknowns may be found in a corresponding manner.

In equations (13) and (14) the first parenthesis symbolizes the influence of the balance of payments policy, the second symbolizes the budget policy, and finally, the third and fourth parenthesis symbolize the monetary policy (performed partly by the Government and partly by the Central Bank).

The expressions (13) and (14) may be simplified by aggregating the behaviour relations so that $F(e_{10}) = f_1(e_{10}) + f_2(e_{10}) - f_4(e_{10})$. Furthermore by introducing the numerical values assigned to the various data variables (which will be discussed later), we get:

$$(15) \quad c_2 = -a_{10} + F(e_{10}) + 740$$

and

$$(16) \quad d_3 = -a_{10} + F(e_{10}) + 1,640$$

By means of these equations and the figures in Table III, we can find the values of c_2 and d_3 at alternative values of e_{10} and a_{10} .

2. The Estimation of Behaviour Relationships

The relations (9) through (11), which describe the behaviour of the Currency Control, are based on the fact that when the holding of foreign currency is controlled by the Government, the Control Agency will have a fairly definite conception of how it will allocate among a few major sectors of the economy an increase or decrease in the net claims of foreign countries. It was recognized that these relationships may change over time, but for the purpose of a one-year forecast, it was considered safe to assume that the relationships would be stable. For the same

reason their form could not be estimated by regression analysis utilizing *ex post* statistics. Instead, the method used was that of interviewing the officials of the Currency Control.

By means of these interviews an attempt was made to estimate how much the various sectors would be permitted to increase their net claims on foreign countries in case the balance of payments amounted to alternatively a deficit of 450 millions of Kroner, a deficit of 50 millions of Kroner, and a surplus of 350 millions of Kroner. The values a_7 , b_7 , and d_7 at these alternatives were discussed with officials of the Currency Control and in meetings of the Council for Monetary and Financial Policy. The numerical estimates resulting from these discussions are presented in Table III. It should be noticed that only three points of each of the curves representing the functions (9) through (11) were estimated.

TABLE III
Estimates of the Behaviour Relationships
(Millions of Kroner)

	The surplus of the balance of payments (e_{10})		
	-450	-50	350
1. The increase in net claims on foreign countries belonging to the Government (a_7)	-20	-20	80
2. The increase in net claims on foreign countries belonging to the Central Bank (b_7)	-360	-10	190
3. The increase in net claims on foreign countries belonging to Other Sectors (d_7).	-80	-30	70
4. The increase in cash held by Other Sectors (d_9)	240	195	150
5. $F(e_{10})=(1+2-4)$	-620	-225	120

The reasoning by which these estimates were arrived at is as follows:

(1) The Council felt that the actual deficit or surplus on the balance of payments would amount to a figure in between the three alternative values which were chosen and that the figures

for a_7 , b_7 and d_7 for intermediate values of e_{10} could be estimated by linear interpolation.

(2) The repayment of government debt to foreign countries was estimated on the basis of the statistics of foreign claims and debts as of 31st December 1951. It was found that this repayment would amount to approximately 80 millions of Kroner at any of the alternative values for the balance of payments. For the surplus alternative of the balance of payments ($e_{10}=350$), no other changes in the Government's foreign claims or debts were expected, so that the Government's net claims on foreign countries would increase by 80 millions of Kroner ($a_7=80$). In case of a balance of payments deficit it was assumed that the Government would have to spend 100 millions of Kroner of a recently-granted British loan, i.e. the Government's net claims toward foreign countries would decline by 20 millions of Kroner (see Table III).

(3) The net claims on foreign countries owned by the Central Bank (b_7) were believed to depend on the need for the repayment of its debt, the depletion of its reserves of liquid foreign currency, and the increase in its debit balance with the European Payments Union (EPU). Considering these factors it was expected that the total net foreign claims owned by the Central Bank would decline by 360 millions of Kroner at the alternative of a 450 millions of Kroner deficit on the balance of payments, and by 50 millions of Kroner at a deficit of 50 millions of Kroner. Furthermore, it was expected that at a balance of payments surplus of 350 millions of Kroner the Central Bank would increase its net claims on other countries by 190 millions of Kroner.

(4) It was expected that the net foreign claims belonging to Other Sectors would decline by 80 millions of Kroner ($d_7=80$) if the balance of payments deficit was 450 millions of Kroner. At deficit of 50 millions of Kroner it was believed that the net foreign claims of Other Sectors would decline by 30 millions of Kroner. For the surplus alternative it was expected that there would be a total increase in net foreign claims owned by Other Sectors amounting to 70 millions of Kroner.

Function (12) describes the change in the cash preference of Other Sectors. For the purpose of the liquidity forecast it was found plausible to assume that the cash holding of Other Sectors would increase in proportion to the total internal

availabilities of goods and services, i.e. the sum of the gross national product and the import surplus, excluding certain transactions not requiring cash. It was further assumed that the increase of the gross national product would be approximately the same for the three alternatives of the balance of payments which were considered, and a separate forecast of this increase was made. Under these assumptions the increase in the cash holdings was computed at each alternative of the balance of payments surplus (e_{10}).¹

3. *The Estimation of Data*

The numerical values of the data variables were, in general, estimated in the same manner as for the 1950 forecast, so that a detailed description of the estimation procedure should not be required. However, it must be pointed out that the major assumptions about the monetary policy in 1952 were (1) no change, except repayments according to contracts, in the Government's claims on or debts to the Banks and, (2) no change in the Central Bank's holdings of internal claims, i.e. the government variables b_1 , b_3 , and b_4 are equal to zero. These unrealistic assumptions were made for the purpose of estimating the probable development of bank liquidity in case no conscious effort was made by the Government or the Central Bank to counteract or strengthen this development.² For analytical purposes it was further assumed that the Central Government would deposit in the Central Bank its entire budget surplus except what was required for amortization of debt and other payments previously decided upon. Under this assumption the entire increase in the Government's claims on the Banks would have to come from the local governments. There was no information on the extent to which government enterprise (included in Other Sectors) would increase or decrease their Central Bank deposits. In light of this difficulty it was assumed that no change would occur in these claims.³ The increase in

¹ Since the total availabilities of goods and services in 1952 were calculated at 1951 prices, the report of the Council pointed out that the cash holdings would increase more than indicated in Table III, if prices rose.

² The policy which the majority of the Council advocated in its report was, *inter alia*, government borrowing by treasury bills from the Private Banks and Savings Banks and government lending to the Government Banks. The possible liquidity effects of such a policy could easily be calculated by modifying the numerical values of the data variables b_1 ; b_3 ; and b_4 .

³ This estimate later proved to be in error by 162 millions of Kroner.

bank claims on the Government was estimated under the assumption that only the amortization of debt would change the value of this variable.¹ Finally it was assumed that the net increase in the Banks' claims on Other Sectors would be about the same as in 1951. This assumption, which might appear rather dubious, does not affect the forecast of c_2 , but is essential for the forecast of d_3 .

4. *The Numerical Results*

The Council for Monetary and Financial Policy first considered two alternative values for the independent variable a_{10} and three alternative values for the other independent variable e_{10} , which have already been discussed. The values for a_{10} were 1,345 millions of Kroner and 1,145 millions of Kroner. (The expenditures for defence were assumed to be 200 million of Kroner lower at the first than at the second alternative.) If we combine these two alternatives with the three alternatives for e_{10} , we get six alternative sets of values for a_{10} and e_{10} . Insert these sets into equation (15) and (16), and we arrive at six alternative forecasts for respectively the increase in claims on the Central Bank held by Banks (c_2) and the increase in claims on the Banks held by Other Sectors (d_3), see Table IV.

TABLE IV

Forecast of the Increase in Claims on the Central Bank held by Banks and in Claims on the Banks by Other Sectors at Different Alternatives for a_{10} and e_{10} .
(Millions of Kroner)

Object of the Forecast	e_{10}/a_{10}	1,345	1,145
The increase in claims on the Central Bank held by the Banks (c_2)	-450	-1,225	-1,025
	-50	-830	-630
	350	-485	-285
The increase in claims on the Banks held by Other Sectors (d_3)	-450	-325	-125
	-50	70	270
	350	415	615

In addition, the Council made forecasts under the assumption that a_{10} equalled 1,245 millions of Kroner and e_{10} equalled 150

¹ The Council however, recommended in contrast to this assumption that the banks be given the opportunity to purchase treasury bills.

millions of Kroner. For this set of values for the independent variables, which the Council termed 'a probable and possible alternative', we get a forecast for $c_2 = -580$ millions of Kroner and a forecast for $d_3 = 320$ millions of Kroner. The recommendations for the Council were largely based on and conditioned by this alternative.

Commenting on these results, the Council concluded that the decline in the claims of the Banks on the Central Bank would not be small if the Government or the Central Bank would act as assumed.¹ In fact the liquidity development was counteracted by a government loan of 400 millions of Kroner to the Government Bank (the Council's recommendation was 300 to 350 millions of Kroner). Furthermore, the Government borrowed 280 millions of Kroner by selling treasury bills to the Private Banks and Savings Banks (this kind of borrowing was also recommended by the Council). Had these transactions been accounted for in the forecasts, i.e., had a_3 been set 400 millions of Kroner higher ($a_3 = 635$) and c_1 been set 280 millions of Kroner higher ($c_1 = 125$) than they were, the forecast of c_2 would have been 120 millions of Kroner higher for all alternatives (the numerical values of the figures would have been 120 millions of Kroner lower).

Equation (14) shows that the increase in claims on the Banks belonging to Other Sectors (d_3) is independent of a_3 and c_1 , so that the forecast of d_3 would not have changed even if the Government's loans to and borrowings from the Banks had been included in the forecast.

In the report of the Council for Monetary and Financial Policy the numerical solutions for the other unknown variables are also presented at the various alternatives for a_{10} and e_{10} . However, for our purpose it should not be necessary to discuss these latter solutions.

5. *An Appraisal of the Forecast*

The *ex post* figures which are presented in Table II, are (generally) of a better quality than those of Table I. Also in Table II the *ex post* figures for the Government and Other Sectors are somewhat unreliable, particularly those for the latter sector. However, figures for the increase in claims between, on the one hand, these sectors and, on the other hand,

¹ *Innstilling fra Penge-og finansrådet*, 13th January 1952, p. 38.

the Central Bank, may be considered quite reliable. Also, figures which indicate claims between the Banks and the Government and Other Sectors, respectively, are of a fairly high quality. Figures for claims between Norway and foreign countries are good except for those related to Other Sectors. In general, figures related to the Central Bank and the Banks are best. In fact, most of these figures are entirely reliable.

In attempting to appraise the numerical results of the 1952 forecast, we shall, as in the preceding section, limit the analysis to the variables c_2 and d_3 (the increase in the Banks' claims on the Central Bank and in the indebtedness of Banks towards Other Sectors). We shall first make some short comments on the alternatives presented in Table IV, and then we shall analyse other alternatives which are relevant to the problem. For some of the other unknown variables the difference between forecast and fact is small, but for most of them the difference is larger than in the case of c_2 and d_3 . The estimates of data seem to deviate more from the *ex post* results than the forecast of the unknown variables. In this respect the results are similar to those of the 1950 forecast. The explanation seems to be that errors in the estimates of data variables tend to neutralize each other.

The Council pointed out that among the forecast of c_2 (the increase in the Banks' claims on the Central Bank) shown in Table IV, the alternatives which represented the largest reduction of liquidity were scarcely plausible. In these cases the Banks would have rediscounted short-term loans in the Central Bank or sold bonds and hence, the reduction of liquidity would have been correspondingly smaller than indicated by the table.¹ This inconsistency could have been avoided if an additional behaviour equation had been included in the model, viz. an equation which describes the relationship between the change in indebtedness of the Banks to the Central Bank (a_3) and the change in the Banks' claims on the Government (c_1) at alternative magnitudes of the change in the deposits of the Banks with the Central Bank (c_2). In this case the model would have had one less degree of freedom and it would not have been necessary to make the unrealistic assumptions which were made in respect to a_3 and c_1 .²

¹ Ibid, p. 31.

² The assumptions regarding the monetary policy were unrealistic because, *inter alia*, one of the major objects of monetary policy was to maintain a low level of interest rates.

In fact the Banks reduced their claims on the Central Bank by 315 millions of Kroner ($c_2 = -315$) in 1952. Thus, the actual reduction was somewhat larger than indicated by the lowest alternative in Table IV, but lower than the next lowest alternative. This seems to indicate that, since the Council in advance considered the largest reduction alternatives in Table IV as unlikely, the results of the 1952 forecast provided a good basis for the deliberations of the Council. The attempt to make a somewhat less conditional forecast was not equally successful. The decrease in the claims of Banks on the Central Bank which was forecast at 'a possible and probable' alternative for a_{10} and e_{10} ($c_2 = -580$) is 265 millions of Kroner larger than the actual decrease. However, if the actual government lending to the Government banks and the actual government borrowing by treasury bills from the private banks and savings banks (both recommended by the Council) had been considered in the forecast ($a_3 = 635$ and $c_1 = 125$) the numerical results of the forecast would, instead of the three lowest reduction alternatives in Table IV, have been the figures $c_2 = -510$, $c_2 = -365$, and $c_2 = -165$ and instead of $c_2 = -580$ at a 'possible and probable' alternative for a_{10} and e_{10} , the forecast would have been $c_2 = -460$. These results are all of the same order of magnitude as the actual outcome. Finally, if both the actual government lending to and borrowing from the Banks and the actual values of $a_{10} = 1,060$ and $e_{10} = 8$ had been considered in the forecast the result of the forecast would have been a reduction of 380 millions of Kroner in the Banks' claims on the Central Banks ($c_2 = -380$). This is closer to reality than any of the other alternatives which we have discussed. The Ministry of Finance might have computed all these alternative forecasts on the basis of the 1952 model before its decisions on borrowing policy were made. It is not known whether such computations actually were made.

The debt of the Banks towards Other Sectors actually increased by 658 millions of Kroner ($d_3 = 658$) in 1952. Thus, the increase was somewhat larger than at even the largest alternative in Table IV ($d_3 = 615$). For $a_{10} = 1,245$ millions of Kroner and $e_{10} = 150$ millions of Kroner ('a possible and probable' alternative) the forecast becomes an increase of 320 millions of Kroner, i.e. about one half of the actual increase. For $a_{10} = 1,060$ millions of Kroner and $e_{10} = 8$ millions of Kroner (the actual figures for

1952) the forecast is an increase of 400 millions of Kroner.¹ Thus for d_3 the forecasting procedure does not work as well as for e_2 .

A change in the claims and the indebtedness of the Banks has different effects on their behaviour at different total magnitudes of these claims and debts. Therefore, in appraising the various forecasts for c_2 and d_3 , it is necessary to bear in mind that the Banks at the beginning of 1952 owned about 1,300 millions of Kroner in claims on the Central Banks and were indebted to Other Sectors by approximately 9 billions of Kroner. Most of the deviations between forecast and actual results, which we have discussed, are small in relation to these totals. Furthermore, the purpose of the forecasts should be remembered. In the report of the Council for Monetary and Financial Policy it is stated that 'Such estimates concerning the future are always subject to uncertainties and most of the figures are subject to a margin of error. The main purpose has not been to arrive at exact quantitative results, but to indicate the direction of change and the order of magnitudes which may be expected.'² These purposes appear to have been fulfilled quite well in regard to variables c_2 and d_3 . Whether this occurred by chance or because the forecasting method is good, we cannot know for certain; nevertheless, the experience with this method appears quite encouraging.

IV. CONCLUSION

Monetary policy must be based on some kind of liquidity forecast and it may be enlightening to compare the results of our methods with the results of alternative forecasting procedures. We can conceive of four simpler procedures which might have been used in forecasting the increase in the claims on the Central Bank held by the Banks.

(1) Assume no change in the claims on the Central Bank held by Banks at the beginning of the year.

(2) Project for the forecasting year the same change in the claims on the Central Bank held by Banks as that which occurred in the preceding year.

¹ The variable d_3 is independent of government lending to and borrowing from banks, i.e., independent of a_3 and c_1 (see equation (14)).

² Ibid, p. 21.

(3) Assume that the increase (decrease) in the Banks' claims on the Central Bank equals the deficit (surplus) on the Government's budget plus the surplus (deficit) on the balance of payments. By using the same symbols as in the 1952 forecast this amounts to $c_2 = -a_{10} + e_{10}$, where a_{10} and e_{10} are considered as independent variables.

(4) Disregard a_9 , b_3 , b_4 , b_9 , b_{10} , c_7 , c_8 , and e_8 . Assume that the increase in the cash holdings of Other Sectors is independent of the balance of payments surplus, so that we have d_8 instead of $f_4(e_{10})$ and substitute $(e_{10} - d_7)$ for $[f_1(e_{10}) + f_2(e_{10})]$. We then get $c_2 = -a_{10} + e_{10} + [a_3 + a_4 - c_1 - d_1 - d_2 - d_7 - d_8]$.

A comparison of the procedures 1 through 3 with the results of the methods used in Sections I and II follows. In addition, procedure 4 is compared with the method described in III. The comparison is limited to the forecasts of the change in the claims of the Banks on the Central Bank.

TABLE V
*Alternative Forecasts of the Change in the Claims on the
Central Bank held by Banks*
(Millions of Kroner)

Forecasting procedure	1950	1952
Procedure 1	0	0
Procedure 2	515	300
Procedure 3	-1,550	-680
Procedure 4	¹ -600
The procedures of Tables I and II	-550	-580
Actual Results	-580	-315

¹The figure has been arrived at by assuming $d_8 = 200$. All other data were set as in Section III.

Table V shows that the simpler models implied in procedures 1-3 for both 1950 and 1952 do not perform nearly as well as the models which have been the subject of this paper. Procedure 4 yields practically as good a result as the procedure implied in Table II. It should be superfluous to make a corresponding comparison for the variable d_3 .

The efficiency of the forecasting procedures cannot be evaluated on the basis of such comparisons alone. The major criteria upon which an efficiency consideration must be based, is the

usefulness of the various procedures for those who plan and decide on economic policy. A comparison of the respective procedures on this basis leads to the conclusion that the methods discussed in Sections II and III are by far superior to the more primitive procedures mentioned above. Furthermore, the method of Section III is superior to that of Section II, partly because of its behaviour relationships, and partly because of the fact that its book-keeping relationships are defined in clearer and more meaningful manner than those of Section II.

The forecasts presented in Tables I and II were, of necessity, based on rather unsatisfactory statistics. They might have been improved if a more detailed breakdown of *ex post* statistics by transactions and by sectors had been available. This defect is now being remedied by the Central Bureau of Statistics of Norway which is preparing for 1953 and succeeding years, an *ex post* table similar to Table II, but with eight types of transactions and eleven sectors, and it is planned that henceforth corresponding tables will be prepared regularly. Another source of improvement would be better information on repayment contracts, particularly with regard to long-term financial assets.

One of the weaknesses inherent in the 1952 model is that its behaviour relations are, in principle, only valid for 1952. If the underlying conditions are different from those in 1952, the structure of these relationships will also be different. Therefore, the forecasting relations (15) and (16) cannot be applied to other years in order to test the quality of the model. It is believed that further studies of behaviour relationships may lead to improvements although the possibilities of obtaining relationships which are stable over longer periods of time seem to be rather remote.