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ALTERNATIVE LAND-PRICE INDEXES FOR COMMERCIAL PROPERTIES IN TOKYO

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The System of National Accounts (SNA) requires separate estimates for the land and structure components of a commercial property. Using transactions data for the sales of office buildings in Tokyo, a hedonic regression model (the "builder's model") was estimated and this model generated an overall property price index as well as subindexes for the land and structure components of the office buildings. The builder's model was also estimated using appraisal data on office building real estate investment trusts (REITs) for Tokyo. These hedonic regression models also generated estimates for net depreciation rates, which can be compared. Finally, the Japanese government constructs annual official land prices for commercial properties based on appraised values. The paper compares these official land prices with the land prices generated by the hedonic regression models based on transactions data and on REIT data. The results reveal that commercial property indexes based on appraisal and assessment prices lag behind the indexes based on transaction prices.

JEL Codes: C2, C23, C43, D12, E31, R21

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1. INTRODUCTION

When estimating commercial property price indexes, we are confronted with the following two problems: how to incorporate quality adjustments in the estimation method and which data source to use in the estimation procedure.

Research studies on commercial property price indexes have emphasized the problem of data selection when formulating indexes. Traditionally, transaction prices (also called "market prices" in the literature) have usually been used to estimate price indexes. However, the number of commercial property market transactions is extremely small. Furthermore, even if a sizable number of transaction

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prices can be obtained, the heterogeneity of the properties is so pronounced that it is difficult to compare like with like, and thus the construction of reliable constant-quality price indexes becomes very difficult.

Under such circumstances, many commercial property price indexes have been constructed using either appraisal prices from the real estate investment market or by using assessment prices for property tax purposes. The rationale for these price indexes is that, since appraisal prices and assessment prices for property tax purposes are regularly surveyed for the same commercial property, indexes based on these surveys hold most characteristics of the property constant,[1] thus greatly reducing the heterogeneity problem as well as generating a wealth of data.

However, while appraisal prices look attractive for the construction of price indexes, they are somewhat subjective; that is, exactly how are these appraisal prices constructed? Thus these prices lack the objectivity of market selling prices. Such considerations have led to the development of various arguments concerning the precision and accuracy of appraisal and assessment prices when used in measuring price indexes (on these issues, see Shimizu and Nishimura, 2006). In particular, the literature on this issue has pointed out that an appraisal-based index will typically lag actual turning points in the real estate market.[2] Geltner *et al*. (1994) clarified the structure of bias in the National Council of Real Estate Investment Fiduciaries (NCREIF) Property Index, a representative U.S. index based on appraisal prices. In a later study, Geltner and Goetzmann (2000) estimated an index using commercial property transaction prices and demonstrated the magnitude of errors and the degree of smoothing in the NCREIF Property Index. These problems plague not only the NCREIF Property Index, but all indexes based on appraisal prices, including the MSCI–Investment Property Databank (IPD) Index.

With specific reference to Japan's real estate bubble period, Nishimura and Shimizu (2003), Shimizu and Nishimura (2006), and Shimizu *et al*. (2012) estimated hedonic price indexes based on commercial property and indexes based on residential housing transaction prices, contrasted them with indexes based on appraisal prices, and statistically laid out their differences. An examination of the estimated results revealed that during the bubble period, when prices climbed dramatically, indexes based on appraisal prices did not catch up with transaction price increases. Similarly, during the period of falling prices, appraisal-based indexes did not keep pace with the decline in prices.

Furthermore, in the case of appraisal prices for investment properties, a systemic factor of appraiser incentives emerges as an additional problem. This problem differs intrinsically from the lagging and smoothing problems that arise in appraisal-based methods. Specifically, the incentive problem involves inducing

[1]Two important characteristics that are not held constant are the age of the structure and the amount of capital expenditure on the property between the survey dates. Changes in these characteristics are an important determinant of the property price.

[2]Another problem with appraisal based indexes is that they tend to be smoother than indexes that are based on market transactions. This can be a problem for real estate investors, since the smoothing effect will mask the short-term riskiness of real estate investments. However, for statistical agencies, smoothing short-term fluctuations will probably not be problematic.

higher valuations from appraisers in order to bolster investment performance (on this point, see Crosby *et al*. 2010).

In this connection, Bokhari and Geltner (2012) and Geltner and Bokhari (2018) estimated quality-adjusted price indexes by running a time dummy hedonic regression using transaction price data. Geltner (1997) also used real estate prices determined by the stock market in order to examine the smoothing effects of the use of appraisal prices. Finally, Geltner *et al*. (2010), Shimizu *et al*. (2015), Shimizu (2016), and Diewert and Shimizu (2017) proposed various estimation methods for commercial property price indexes using real estate investment trust (REIT) data.

In this paper, we will examine the three alternative data sources suggested in the literature that enable us to construct land-price indexes for commercial properties: (i) sales transactions data; (ii) appraisal data for REITs; and (iii) assessed values of land for property taxation purposes. We will utilize these three sources of data for commercial properties in Tokyo over 44 quarters covering the period Q1:2005 to Q4:2015 and compare the resulting land prices.

Section 2 explains our data sources. Sections 3 and 4 use sales transactions data and a hedonic regression model that allows us to decompose sale prices into land and structure components. The model of structure depreciation used in Section 3 is a single geometric rate and Section 4 generalizes this model to allow for multiple geometric rates. Section 5 implements the same hedonic regression model using the same transactions dataset, but we switch to a piecewise linear depreciation model. Section 6 compares the alternative depreciation schedules.

It will turn out that the land-price series that are generated using quarterly transactions data are very volatile and thus they may not be suitable for statistical agency use. Thus, in Section 7, we look at some alternative methods for smoothing the raw land-price indexes.

Section 8 estimates a hedonic regression model using quarterly appraisal values for 41 Tokyo office buildings over the sample period. Since we have panel data for this application, our hedonic regression model is somewhat different from our earlier models.

Section 9 estimates quality-adjusted land prices for commercial properties using tax assessment data. Section 10 compares our land price indexes from the three sources of data. Section 11 constructs overall property price indexes for Tokyo commercial properties using the models estimated in the previous sections; that is, we combine the land-price indexes with a structure-price index to obtain overall property price indexes. We also estimate a traditional log price time dummy hedonic regression model and compare the resulting index with our overall indexes. Section 12 concludes.

In summary: there are two main purposes for our paper: (i) using hedonic regression techniques, we show how overall property price indexes as well as land-price indexes for commercial office buildings in Tokyo can be constructed using information on property sales and property appraisal information for REITs, and we compare the resulting land-price indexes with a land-price index based on property tax assessment data; and (ii) we show how the hedonic regressions can be used to estimate commercial property depreciation rates. Our focus is on decomposing property values into land and structure components, because this decomposition is required in the Balance Sheets for the System of National Accounts (SNA) and it is also required in order to calculate the Total Factor Productivity of the Commercial Office Sector.

2. Data Description

This study compiled the following three types of micro-data relating to commercial properties in the Tokyo office market: (i) the transaction price data compiled by the Japanese Ministry of Land, Infrastructure, Transport, and Tourism (MLIT); (ii) the appraisal prices periodically determined in the Tokyo office REIT market; and (iii) the "official land prices" (OLPs) surveyed by the MLIT since 1970. OLPs are based on appraisals that are released on January 1st of each year. In Japan, asset taxes relating to land, such as inheritance taxes and fixed assets taxes, are assessed on the basis of these OLPs. Thus official land prices are considered as assessment data for tax purposes. As official land prices are exclusively based on surveys of land prices, they do not include structure prices.[3]

Using the first two data sources, land-price indexes were estimated using the "builder's model." These land-price indexes will be compared with those estimated using OLPs in Section 5.

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Our analysis covers the period from 2005 to 2015. The data variables compiled are listed in Table 1.

Table 2 shows a summary of the statistical parameters for the three data sources; that is, transaction prices, REIT appraisal prices, and OLPs. The compiled data consisted of 1,907 MLIT transaction prices, 1,804 REIT prices, and 6,242 MLIT official land prices (i.e. OLPs).[4]

3. The Builder's Model: Preliminary Results Using Transactions Data

We will use the MLIT commercial office building transactions data in this section and in Sections 4–7.

The *builder's model* for valuing a commercial property postulates that the value of a commercial property is the sum of two components: the value of the land on which the structure sits plus the value of the commercial structure.

In order to justify the model, consider a property developer who builds a structure on a particular property. The total cost of the property after the structure is completed will be equal to the floor-space area of the structure, say S square

[3]For the details on how appraisal and assessment prices are made in Japan, see Shimizu (2016).

[4]It should be noted that the average amount of each property characteristic differs substantially in many cases across our three samples. These differences may account for some of the differences in the land indexes that are generated by the three sources of data. We note that the MLIT data are sparse for some quarters. The lowest number of observations were quarters 18, 26, and 32, with 16, 20, and 26 observations, respectively.

TABLE 1
Variables from the Three Data Sources

Symbols	Variables	Contents	Unit(s)
V	Price	Transaction price and appraisal price in total	million yen
L	Total land area	Land area of building	m^2
S	Total floor space	Floor space of building	m^2
A	Age of building at the time of transaction	Age of building at the time of transaction/appraisal	years
H	Number of stories	Number of stories in the building	stories
DS	Distance to the nearest station	Distance to the nearest station	meters
TT	Travel time to central business district	Minimum railway riding time in daytime to Tokyo Central Station	minutes
WD_k	Location (ward) dummy	kth ward = 1, other district =0 ($k = 0, \ldots, K$)	(0, 1)
D_t	Time dummy (quarterly)	tth quarter = 1, other quarter = 0 ($t = 0, \ldots, T$)	(0, 1)

TABLE 2
Summary Statistics (with Standard Deviations in Parentheses)

	MLIT	REIT	OLP
V: selling price of office building (million yen)	394.18 (337.76)	6,686.60 (4,055.60)	1,264.3 (1,304.1)
S: structure floor area (m^2)	834.00 (535.19)	8,509.70 (5,463.90)	–
L: land area (m^2)	239.27 (135.08)	1,802.10 (1,580.20)	229.94 (217.18)
H: total number of stories	5.75 (2.14)	10.12 (3.30)	–
A: age (years)	24.23 (10.61)	19.14 (6.80)	–
DS: distance to nearest station (meters)	387.65 (238.45)	308.29 (170.04)	347.24 (254.79)
TT: Time to Tokyo Central Station (minutes)	19.63 (8.23)	15.88 (5.10)	21.74 (8.54)
PS: structure construction price per square meter (million yen)	0.2347 (0.0103)	0.2359 (0.0102)	–
Number of observations	1,907	1,804	6,242

meters (m^2) times the building cost per square meter, β_t during quarter or year t, plus the cost of the land, which will be equal to the cost per square meter, α_t during quarter or year t, times the area of the land site, L. Now think of a sample of properties of the same general type, which have prices or values V_{tn} in period t[5] and structure areas S_{tn} and land areas L_{tn} for $n = 1, \ldots, N(t)$, where $N(t)$ is the number of observations in period t. Assume that these prices are equal to the sum of the land and structure costs plus error terms ε_{tn}, which we assume are independently normally distributed with zero means and constant variances. This leads to the

[5]The period index t runs from 1 to 44, where period 1 corresponds to Q1:2005 and period 44 corresponds to Q4:2015.

following *hedonic regression model* for period t, where the α_t and β_t are the parameters to be estimated in the regression[6]:

$$(1) \qquad V_{tn} = \alpha_t L_{tn} + \beta_t S_{tn} + \varepsilon_{tn}, \qquad t = 1, \ldots, 44, \quad n = 1, \ldots, N(t).$$

Note that the two characteristics in our simple model are the quantities of land L_{tn} and the quantities of structure floor space S_{tn} associated with property n in period t and the two *constant-quality prices* in period t are the price of a square meter of land α_t and the price of a square meter of structure floor space β_t.

The hedonic regression model defined by equation (1) applies to new structures. But it is likely that a model that is similar to equation (1) applies to older structures as well. Older structures will be worth less than newer structures due to the depreciation of the structure. Assuming that we have information on the age of the structure n at time t, say $A(t,n)$, and assuming a geometric (or declining balance) depreciation model, a more realistic hedonic regression model than that defined by equation (1) is the following *basic builder's model*[7]:

$$(2) \qquad V_{tn} = \alpha_t L_{tn} + \beta_t (1-\delta)^{A(t,n)} S_{tn} + \varepsilon_{tn}; \qquad t = 1, \ldots, 44, \quad n = 1, \ldots, N(t),$$

where the parameter δ reflects the *net geometric depreciation rate* as the structure ages by one additional period. Thus if the age of the structure is measured in years, we would expect an annual *net* depreciation rate to be between 2 and 3 percent.[8] Note that equation (2) is now a non-linear regression model whereas equation (1) was a simple linear regression model.[9] The period-t constant-quality price of land will be the estimated coefficient for the parameter α_t and the price of a unit of a newly built structure for period t will be the estimate for β_t. The period-t quantity of land for commercial property n is L_{tn} and the period-t quantity of structures for commercial property n, expressed in equivalent units of a new structure, is $(1-\delta_t)^{A(t,n)} S_{tn}$, where S_{tn} is the floor-space area of commercial property n in period t.

Note that the above model is a *supply-side model* as opposed to the *demand-side model* of Muth (1971) and McMillen (2003). Basically, we are assuming competitive suppliers of commercial properties so that we are in Case (a) of Rosen 1974, p. 44), where the hedonic surface identifies the structure of supply. This

[6]Other papers that have suggested hedonic regression models that lead to additive decompositions of property values into land and structure components include Clapp 1980, pp. 257–8), Bostic *et al.* 2007, p. 184), de Haan and Diewert (2011), Diewert (2010, 2008), Francke 2008, p. 167), Koev and Santos Silva (2008), Rambaldi *et al.* (2010), Diewert *et al.* (2015, 2011), Diewert and Shimizu (2015b, 2016, 2017), and Rambaldi *et al.* (2016).

[7]This formulation follows that of Diewert (2010, 2008), de Haan and Diewert (2011), Diewert *et al.* (2015), and Diewert and Shimizu (2015b, 2016, 2017) in assuming that the property value is the sum of land and structure components but that movements in the prices of structures are proportional to an exogenous structure-price index. This formulation is designed to be useful for national income accountants who require a decomposition of property value into structure and land components. They also need the structure index, which in the hedonic regression model needs to be consistent with the structure-price index that they use to construct structure capital stocks. Thus the builder's model is particularly suited to national accounts purposes (see Diewert and Shimizu, 2015a; Diewert *et al.*, 2016).

[8]This estimate of depreciation is regarded as a net depreciation rate because it is equal to a "true" gross structure depreciation rate less an average renovations appreciation rate. Since we do not have information on renovations and major repairs to a structure, our age variable will only pick up average gross depreciation less average real renovation expenditures.

[9]We used Shazam to perform the non-linear estimations (see White, 2004). Note that equation (2) is estimated as a single non-linear regression using the data for all 44 quarters.

assumption is justified for the case of newly built offices but it is less well justified for sales of existing commercial properties.

There is a major problem with the hedonic regression model defined by equation (2): the multicollinearity problem. Experience has shown that it is usually not possible to estimate sensible land and structure prices in a hedonic regression such as that defined by equation (2) due to the multicollinearity between lot size and structure size.[10] Thus, in order to deal with the multicollinearity problem, we draw on *exogenous information* on the cost of building new commercial properties from the MLIT and we assume that the price of new structures is equal to an official measure of commercial building costs (per square meter of building structure), p_{St}. Thus we replace β_t in equation (2) by p_{St} for $t = 1, \ldots, 44$. This reduces the number of free parameters in the model by 44.

Experience has also shown that it is difficult to estimate the depreciation rate before obtaining quality-adjusted land prices. Thus, in order to get preliminary land-price estimates, we temporarily assumed that the annual geometric depreciation rate δ in equation 2 was equal to 0.025. The resulting regression model becomes one defined by the following equation:

$$(3) \quad V_{tn} = \alpha_t L_{tn} + p_{St}(1-0.025)^{A(t,n)} S_{tn} + \varepsilon_{tn}; \qquad t=1,\ldots,44, \quad n=1,\ldots,N(t).$$

The final log likelihood (LL) for this **Model 1** was −13,328.15 and the R^2 value was 0.4003.[11]

In order to take into account possible neighborhood effects on the price of land, we introduce *ward dummy variables*, $D_{W,tnj}$, into the hedonic regression given in equation (3). There are 23 wards in Tokyo special district; therefore, we made 23 ward or locational dummy variables.[12] These 23 dummy variables are defined as follows:

for $\quad t=1,\ldots,44, \quad n=1,\ldots,N(t), \quad j=1,\ldots,23$

$$(4) \qquad D_{W,tnj} \equiv \begin{cases} 1 & \text{if observation } n \text{ in period } t \text{ is in ward } j \text{ of Tokyo;} \\ 0 & \text{if observation } n \text{ in period } t \text{ is not in ward } j \text{ of Tokyo.} \end{cases}$$

We now modify the model defined by equation (3) to allow the *level* of land prices to differ across the wards. The new non-linear regression model is as follows[13]:

$$(5) \qquad \begin{aligned} V_{tn} &= \alpha_t \left(\sum_{j=1}^{23} \omega_j D_{W,tnj} \right) L_{tn} + p_{St}(1-0.025)^{A(t,n)} S_{tn} + \varepsilon_{tn}; \\ & t=1,\ldots,44, \quad n=1,\ldots,N(t). \end{aligned}$$

[10]On the multicollinearity problem, see Schwann (1998) and Diewert *et al*. (2015, 2011).

[11]Our R^2 concept is the square of the correlation coefficient between the dependent variable and the predicted dependent variable.

[12]The 23 wards (with the number of observations in brackets) are as follows: 1, Chiyoda (191); 2, Chuo (231); 3, Minato (205); 4, Shinjuku (203); 5, Bunkyo (97); 6, Taito (122); 7, Sumida (74); 8, Koto (49); 9, Shinagawa (69); 10, Meguro (28); 11, Ota (64); 12, Setagaya (67); 13, Shibuya (140); 14, Nakano (39); 15, Suginami (39); 16, Toshima (80); 17, Kita (30); 18, Arakawa (42); 19, Itabashi (35); 20, Nerima (40); 21, Adachi (19); 22, Katsushika (18); 23, Edogawa (25).

[13]From this point on, our non-linear regression models are nested; that is, we use the coefficient estimates from the previous model as starting values for the subsequent model. Use of this nesting procedure is essential to obtaining sensible results from our non-linear regressions. The non-linear regressions were estimated using Shazam (see White, 2004).

Not all of the land time dummy-variable coefficients (the α_t) and the ward dummy-variable coefficients (the ω_j) can be identified. Thus we impose the following normalization on our coefficients:

$$\alpha_1 = 1. \tag{6}$$

The final LL for the model defined by equations (5) and (6) was −12,956.60 and the R^2 value was 0.5925. Thus there was a large increase in the R^2 value and a huge increase in the LL of 371.55 over the previous model. However, many of the wards had only a small number of observations and thus it is unlikely that our estimated ω_j for these wards would be very accurate.

In order to deal with the problem of too few observations in many wards, we used the results of the above model to group the 23 wards into four combined wards based on their estimated ω_j coefficients. The *Group 1 high-priced wards* were 1–3 and 13 (their estimated ω_j coefficients were greater than 1), the *Group 2 medium-high-priced wards* were 4–6, 9, and 14 ($0.6 < \omega_j \leq 1$), the *Group 3 medium-low-priced wards* were 7, 8, 10, 12, 15, and 16 ($0.4 < \omega_j \leq 0.6$), and the *Group 4 low-priced wards* were 11 and 17–23 ($\omega_j \leq 0.4$).[14] We reran the non-linear regression model defined by equations (5) and (6) using just the four combined wards (call this **Model 2**); the resulting LL was −12,974.31 and the R^2 value was 0.5850. Thus combining the original wards into grouped wards resulted in a small loss of fit and a decrease in LL of 17.71 when we decreased the number of ward parameters by 19. We regarded this loss of fit as an acceptable tradeoff.

In our next model, we introduce some non-linearities into the pricing of the land area for each property. The land-plot areas in our sample of properties run from 100 to 790 m^2. Up to this point, we have assumed that land plots in the same grouped ward sell at a constant price per square meter of lot area. However, it is likely that there is some non-linearity in this pricing schedule; for example, it is likely that very large lots sell at an average price that is below the average price of medium sized lots. In order to capture this non-linearity, we initially divided up our 1,907 observations into seven groups of observations based on their lot size. The Group 1 properties had lots less than 150 m^2, the Group 2 properties had lots greater than or equal to 150 m^2 and less than 200 m^2, the Group 3 properties had lots greater than or equal to 200 m^2 and less than 300 m^2, and so on, up to the Group 7 properties, which had lots greater than or equal to 600 m^2. However, there were very few observations in Groups 4–7, so we added these groups to Group 4.[15] For each observation n in period t, we define the four *land dummy variables*, $D_{L,tnk}$, for $k = 1, \ldots, 4$, as follows:

$$D_{L,tnk} \equiv \begin{array}{ll} 1 & \text{if observation } tn \text{ has land area that belongs to Group } k; \\ 0 & \text{if observation } tn \text{ has land area that does not belong to Group } k. \end{array} \tag{7}$$

These dummy variables are used in the definition of the piecewise linear function of L_{tn}, $f_L(L_{tn})$, defined as follows:

[14] The estimated combined ward-relative land-price parameters turned out to be as follows: $\omega_1 = 1.3003$, $\omega_2 = 0.75089$, $\omega_3 = 0.49573$, and $\omega_4 = 0.25551$. The sample probabilities of an observation falling in the combined wards were 0.402, 0.278, 0.177, and 0.143, respectively.

[15] The sample probabilities of an observation falling in the seven initial land size groups were as follows: 0.291, 0.234, 0.229, 0.130, 0.050, 0.034, and 0.033.

$$
\begin{aligned}
f_L(L_{tn}) \equiv {} & D_{L,tn1}\lambda_1 L_{tn} + D_{L,tn2}[\lambda_1 L_1 + \lambda_2(L_{tn}-L_1)] \\
& + D_{L,tn3}[\lambda_1 L_1 + \lambda_2(L_2-L_1) + \lambda_3(L_{tn}-L_2)] \\
& + D_{L,tn4}[\lambda_1 L_1 + \lambda_2(L_2-L_1) + \lambda_3(L_3-L_2) + \lambda_4(L_{tn}-L_3)],
\end{aligned} \tag{8}
$$

where the λ_k are unknown parameters and $L_1 \equiv 150$, $L_2 \equiv 200$, and $L_3 \equiv 300$. The function $f_L(L_{tn})$ defines a *relative valuation function for the land area of a commercial property* as a function of the plot area.

The new non-linear regression model is as follows:

$$
V_{tn} = \alpha_t \left(\sum_{j=1}^{4} \omega_j D_{W,tnj} \right) f_L(L_{tn}) + p_{St}(1-\delta)^{A(t,n)} S_{tn} + \varepsilon_{tn}; \quad t = 1, \dots, 44, \quad n = 1, \dots, N(t). \tag{9}
$$

Comparing the models defined by equations (5)[16] and (9), it can be seen that we have added an additional four *land-plot size parameters*, $\lambda_1, \dots, \lambda_4$, to the model defined by equation (5) (with only four ward dummy variables). However, looking at equation (9), it can be seen that the 44 land time parameters (the α_t), the four ward parameters (the ω_j), and the four land-plot size parameters (the λ_k) cannot all be identified. Thus we impose the following identification normalizations on the parameters for **Model 3** defined by equations (9) and the following:

$$
\alpha_1 \equiv 1; \quad \lambda_1 \equiv 1. \tag{10}
$$

Note that if we set all of the λ_k equal to unity, Model 3 collapses down to Model 2. The final LL for Model 3 was an improvement of 59.65 over the final LL for Model 2 (for adding three new marginal price of land parameters), which is a highly significant increase. The R^2 value increased to 0.6116 from the previous model's R^2 value of 0.5850. The parameter estimates turned out to be $\lambda_2 = 1.4297$, $\lambda_3 = 1.2772$, and $\lambda_4 = 0.2973$. For small land-plot areas of less than 150 m^2, we set the (relative) marginal price of land equal to 1 per square meter. As the lot sizes increased from 150 m^2 to 200 m^2, the (relative) marginal price of land increased to $\lambda_2 = 1.4297$ per square meter. For the next 100 m^2 of lot size, the (relative) marginal price of land decreased to $\lambda_3 = 1.2772$ per square meter. For lot sizes greater than 200 m^2, the (relative) marginal price of land decreased to 0.2973 per square meter. Thus the average cost of land per square meter initially increases and then tends to decrease as the lot size becomes larger.

The *footprint* of a building is the area of the land that directly supports the structure. An approximation to the footprint land area for property n in period t is the total structure area S_{tn} divided by the total number of stories in the structure H_{tn}. If we subtract the footprint land area from the total land area, TL_{tn}, we obtain the *excess land*,[17] EL_{tn} defined as follows:

[16]We compare equation (9) to the modified equation (5) where we have only four combined-ward dummy variables in the modified equation (5) rather than to the original 23 ward dummy variables.

[17]This is land that is usable for purposes other than the direct support of the structure on the land plot. Excess land was first introduced as an explanatory variable in a property hedonic regression model for Tokyo condominium sales by Diewert and Shimizu 2016, p. 305).

$$(11) \qquad EL_{tn} \equiv L_{tn} - (S_{tn}/H_{tn}); \qquad t=1,\ldots,44, \quad n=1,\ldots,N(t).$$

In our sample, excess land ranged from 1.083 m^2 to 562.58 m^2. We grouped our observations into five categories, depending on the amount of excess land that pertained to each observation. Group 1 consists of observations tn where $EL_{tn} < 50$; Group 2 observations such that $50 \leq EL_{tn} < 100$; Group 3 observations such that $100 \leq EL_{tn} < 150$; Group 4 observations such that $150 \leq EL_{tn} < 300$; and Group 5 observations such that $EL_{tn} \geq 300$.[18] Now define the excess-land dummy variables, $D_{EL,tnm}$, as follows:

for $\quad t=1,\ldots,44, \quad n=1,\ldots,N(t), \quad m=1,\ldots,5,$

$$(12) \quad D_{EL,tnm} \equiv \begin{cases} 1 & \text{if observation } n \text{ in period } t \text{ is in excess-land Group } m; \\ 0 & \text{if observation } n \text{ in period } t \text{ is not in excess-land Group } m. \end{cases}$$

We will use the above dummy variables as adjustment factors to the price of land. As will be seen, in general, the more excess land a property possessed, the lower was the average per meter squared value of land for that property.[19]

The new **Model 4** excess-land non-linear regression model is as follows:

$$(13) \quad V_{tn} = \alpha_t \left(\sum_{j=1}^{4} \omega_j D_{W,tnj} \right) \left(\sum_{m=1}^{5} \chi_m D_{EL,tnm} \right) f_L(L_{tn}) + p_{St}(1-\delta)^{A(t,n)} S_{tn} + \varepsilon_{tn}; \quad t=1,\ldots,44, \quad n=1,\ldots,N(t).$$

However, looking at equations (13) and (8), it can be seen that the 44 land-price parameters (the α_t), the four combined-ward parameters (the ω_j), the four land-plot size parameters (the λ_k), and the five excess-land parameters (the χ_m) cannot all be identified. Thus we imposed the following identifying normalizations on these parameters:

$$(14) \qquad \alpha_1 \equiv 1; \quad \lambda_1 \equiv 1; \quad \chi_1 \equiv 1.$$

Note that if we set all of the χ_m equal to unity, Model 4 collapses down to Model 3. The final LL for Model 4 was an improvement of 23.99 over the final LL for Model 3 (for adding four new excess-land parameters), which is a significant increase. The R^2 value increased to 0.6207 from the previous model's R^2 value of 0.6116. The χ_m parameter estimates turned out to be $\chi_2 = 0.9173$, $\chi_3 = 0.7540$, $\chi_4 = 0.7234$, and $\chi_5 = 0.8611$. Thus excess land does reduce the average per meter price of land.

The non-linear estimating equations for Model 5 are exactly the same as those defined by equations (13) except that we estimated the geometric depreciation rate δ instead of assuming that it was equal to 0.025. The final LL increase for Model 5 (for adding one new parameter) was 50.58, which was highly significant. However,

[18]The sample probabilities of an observation falling in the five excess-land size groups were as follows: 0.352, 0.343, 0.149, 0.114, and 0.041.

[19]The excess-land characteristic was also used by Diewert and Shimizu (2016) and Burnett-Issacs *et al*. (2016) in their studies of condominium prices. The same phenomenon was observed in these studies: the more excess land that a high rise property had, the lower was the per meter land price.

the estimated δ turned out to be 0.00165, with a standard error of 0.00152, which is too low. The R^2 value for this model was 0.6399.

It is likely that the height of the building affects the quality of the structure. In our sample of commercial property prices, the height of the building (the H variable) ranged from three stories to 14 stories. Thus initially, we had 12 building-height categories. Define the building-height dummy variables, $D_{H,tnh}$, as follows:

for $\quad t=1, \ldots, 44, \quad n=1, \ldots, N(t), \quad h=3, \ldots, 14,$

$$D_{H,tnh} \equiv \begin{array}{ll} 1 & \text{if observation } n \text{ in period } t \text{ is in a building that has height } h; \\ 0 & \text{if observation } n \text{ in period } t \text{ is not in a building that has height } h. \end{array} \tag{15}$$

Due to the small number of observations in the last five height categories, we combined these dummy variables into a single height category that included all buildings of height 10–14 stories; that is, the new $D_{H,tn10}$ was defined as $\sum_{h=10}^{14} D_{H,tnh}$. The new **Model 6** non-linear regression model is as follows:

$$\begin{aligned} V_{tn} &= \alpha_t \left(\sum_{j=1}^{4} \omega_j D_{W,tnj} \right) \left(\sum_{m=1}^{5} \chi_m D_{EL,tnm} \right) f_L(L_{tn}) \\ &\quad + p_{St}(1-\delta)^{A(t,n)} \left(\sum_{h=3}^{10} \phi_h D_{H,tnh} \right) S_{tn} + \varepsilon_{tn}; \quad t=1, \ldots, 44, \quad n=1, \ldots, N(t). \end{aligned} \tag{16}$$

In addition to the normalizations in equation (14), we also imposed the normalization $\phi_3 \equiv 1$. Note that if we set all of the μ_h equal to unity, the new model collapses down to Model 5.

The final LL for the new model was −12,649.26, a big improvement of 190.83 over the final LL for Model 5 (for adding seven new height parameters). The R^2 value increased to 0.7036 from the previous model's R^2 value of 0.6207. The ϕ_4 to ϕ_{10} parameter estimates turned out to be 1.2071, 1.4599, 1.5720, 1.5114, 2.0950, 2.3062, and 2.5437, respectively. Recall that ϕ_3 is set equal to 1. It can be seen that the structure value of a property increased (with one exception) as the height of the building increased. The estimated geometric depreciation rate for this model was $\delta = 0.0212$ (with a standard error of 0.0020). This is a very reasonable estimate for a wear-and-tear depreciation rate.

Recall that we used building height as a quality-adjustment factor for the structure portion of the property value. In our next model, we use the building height as a possible quality-adjustment factor for the land component of the property. Consider two adjacent commercial office properties with the same lot size and building footprint, except that property A has a ten-story tower while property B has a modest four-story office building. In theory, the land plot for each property should be valued at its best potential use, but the local market may not be able to support two high-rise buildings in the same area. Hence the land component of property B may not be valued at the same level as that of property A, due to an accident of history. Moreover, placing a high-rise building on property B may lead to a decline in the land value of property A due to an impairment of views (or sunlight) from the higher stories of property A. In any case, we will introduce one new building-height parameter μ, which reflects the possible changes in land value due to the height H of the building on the property. Thus Model 7 is defined as the following non-linear regression model:

$$\begin{aligned} V_{tn} = \alpha_t \left(\sum_{j=1}^{4} \omega_j D_{W,tnj} \right) \left(\sum_{m=1}^{5} \chi_m D_{EL,tnm} \right) \left(1 + \mu(H_{tn} - 3)\right) f_L(L_{tn}) \\ + p_{St}(1-\delta)^{A(t,n)} \left(\sum_{h=3}^{10} \phi_h D_{H,tnh} \right) S_{tn} + \varepsilon_{tn}; \quad t = 1, \ldots, 44, \quad n = 1, \ldots, N(t). \end{aligned} \tag{17}$$

For identification purposes, we imposed the following restrictions on the parameters in equation (17):

$$\alpha_1 \equiv 1; \quad \lambda_1 \equiv 1; \quad \chi_1 \equiv 1; \quad \phi_3 \equiv 1. \tag{18}$$

The final LL for Model 7 was −12,640.40, an improvement of 8.86 LL points over the final LL for Model 6 (for adding one new parameter μ). The R^2 value increased to 0.7063 from the previous model's R^2 value of 0.7036. The estimated depreciation rate δ was 3.41 percent, with a standard error of 0.0077. The estimated $\phi_4, \ldots, \phi_{10}$ were equal to 1.11, 1.31, 1.32, 1.11, 1.83, 2.01, and 2.12 (recall that ϕ_3 was set equal to 1). The estimate for μ was 0.1135, with a standard error of 0.0339. Thus as the building height increases by one story, the land value appears to increase by approximately 11 percent. Thus some of the extra cash flow generated by an extra story for the structure appears to leak over into the land value of the property.[20]

This completes our description of our preliminary hedonic regression models for Tokyo office buildings. In the following section, we will extend these preliminary models by estimating more complex depreciation schedules.

4. The Builder's Model with Multiple Geometric Depreciation Rates

In the following model, we allowed the geometric depreciation rates to differ after each 10-year interval (except for the last interval).[21] We divided up our 1,907 observations into five groups of observations based on the age of the structure at the time of the sale. The Group 1 properties had structures with a structure age less than 10 years, the Group 2 properties had structure ages greater than or equal to 10 years but less than 20 years, the Group 3 properties had structure ages greater

[20]As a referee pointed out, this result may be due to omitted characteristics that are correlated with the building height. It should be pointed out that our estimate for μ in our final model is 0.0602 instead of 0.1135.

[21]The analysis in this section and the subsequent section follows the approach taken by Diewert *et al.* (2017). Geltner and Bokhari (2018) estimate a much more flexible model of commercial property depreciation using U.S. transaction data by allowing an age dummy variable for each age of building. This methodological approach generates a combined land and structure depreciation rate, whereas our approach will generate depreciation rates that apply only to the structure portion of the property value.

than or equal to 20 years but less than 30 years, the Group 4 properties had structure ages greater than or equal to 30 years but less than 40 years, and the Group 5 properties had structure ages greater than or equal to 40 years.[22] For each observation n in period t, we define the five *age dummy variables*, $D_{A,tni}$, for $i = 1, \ldots, 5$, as follows:

$$(19)\quad D_{A,tni} \equiv \begin{cases} 1 & \text{if observation } tn \text{ has a structure age that belongs to age group } i; \\ 0 & \text{if observation } tn \text{ has a structure age that does not belong to age group } i. \end{cases}$$

These age dummy variables are used in the definition of the *aging function*, $g_A(A_{tn})$, defined as follows[23]:

$$(20)\quad \begin{aligned} g_A(A_{tn}) \equiv\ & D_{A,tn1}(1-\delta_1)^{A(t,n)} + D_{A,tn2}(1-\delta_1)^{10}(1-\delta_2)^{(A(t,n)-10)} \\ & + D_{A,tn3}(1-\delta_1)^{10}(1-\delta_2)^{10}(1-\delta_3)^{(A(t,n)-20)} \\ & + D_{A,tn4}(1-\delta_1)^{10}(1-\delta_2)^{10}(1-\delta_3)^{10}(1-\delta_4)^{(A(t,n)-30)} \\ & + D_{A,tn5}(1-\delta_1)^{10}(1-\delta_2)^{10}(1-\delta_3)^{10}(1-\delta_4)^{10}(1-\delta_5)^{(A(t,n)-40)}. \end{aligned}$$

Thus the annual geometric depreciation rates are allowed to change at the end of each decade that the structure survives.

The new **Model 8** non-linear regression model is as follows:

$$(21)\quad \begin{aligned} V_{tn} = \ & \alpha_t \left(\sum_{j=1}^{4} \omega_j D_{W,tnj}\right)\left(\sum_{m=1}^{5} \chi_m D_{EL,tnm}\right)\left(1+\mu(H_{tn}-3)\right) f_L(L_{tn}) \\ & + p_{St} g_A(A_{tn}) \left(\sum_{h=3}^{10} \phi_h D_{H,tnh}\right) S_{tn} + \varepsilon_{tn}; \quad t=1, \ldots, 44, \quad n=1, \ldots, N(t). \end{aligned}$$

We imposed the normalizations $\alpha_1 \equiv 1$, $\lambda_1 \equiv 1$, $\chi_1 \equiv 1$, and $\phi_3 \equiv 1$. Note that Model 8 collapses down to Model 7 if $\delta_1 = \delta_2 = \delta_3 = \delta_4 = \delta_5 = \delta$. Thus the number of unknown parameters in Model 8 increased by four over the number of parameters in Model 7. The final LL for Model 8 was −12,631.21, an improvement of 9.19 over the final LL for Model 7 (for adding four additional parameters). The R^2 value increased to 0.7091 from the previous model's R^2 value of 0.7063. The estimated depreciation rates (with standard errors in brackets) were as follows: $\delta_1 = 0.0487\,(0.0111)$, $\delta_2 = 0.0270\,(0.0097)$, $\delta_3 = 0.0096\,(0.0106)$,[24] $\delta_4 = 0.0403\,(0.0154)$, and $\delta_5 = -0.0319\,(0.0185)$. Thus properties with structures

[22]There were only 28 properties that had an age greater than 50 years, so these properties were combined with the age 40–50 properties.

[23]A_{tn} is the same as $A(t,n)$. The aging function $g_A(A_{tn})$ quality-adjusts a building of age A_{tn} into a comparable number of units of a new building.

[24]Recall that these depreciation rates are net depreciation rates. As surviving structures approach their middle age, renovations become important and thus a decline in the net depreciation rate is plausible. The pattern of depreciation rates is similar to the comparable geometric depreciation rates that were observed for detached houses in Richmond (a suburb of Vancouver, Canada) by Diewert *et al.* (2017).

that are over 40 years old tend to have a negative depreciation rate; that is, the value of the structure tends to *increase* by 3.19 percent per year.[25]

There are two additional explanatory variables in our dataset that may affect the price of land. Recall that DS was defined as the distance to the nearest subway station and TT as the subway running time in minutes to the Tokyo station from the nearest station (see Tables 1 and 2). DS ranges from 0 to 1,500 m, while TT ranges from 1 to 48 minutes. Typically, as DS and TT increase, the land value decreases.[26] Model 9 introduces these new variables into the previous non-linear regression model given in equation (21) in the following manner:

$$
\begin{aligned}
V_{tn} = {} & \alpha_t \left(\sum_{j=1}^{4} \omega_j D_{W,tnj} \right) \left(\sum_{m=1}^{5} \chi_m D_{EL,tnm} \right) \left(1+\mu(H_{tn}-3)\right) \left(1+\eta(DS_{tn}-0)\right) \\
& \times \left(1+\theta(TT_{tn}-1)\right) f_L(L_{tn}) + p_{St} g_A(A_{tn}) \left(\sum_{h=3}^{10} \phi_h D_{H,tnh} \right) S_{tn} + \varepsilon_{tn}; \\
& t=1,\ldots,44, \quad n=1,\ldots,N(t).
\end{aligned} \tag{22}
$$

Thus two new parameters, η and θ, are introduced. If these new parameters are both equal to 0, then Model 9 collapses down to Model 8.

The final LL for Model 9 was −12,614.70, an improvement of 16.51 over the final LL for Model 8 (for adding two additional parameters). The R^2 value increased to 0.7142 from the previous model's R^2 value of 0.7091. The estimated walking-distance parameter was $\eta = -0.00023$ (0.000066), which indicates that the land value of commercial property does tend to decrease as the walking distance to the nearest subway station increases. However, the parameter for the estimated travel time to Tokyo Central Station was $\theta = 0.0209$ (0.0053), which indicates that the land value increases on average as the travel time to the Central Station increases, a relationship that was not anticipated. All of the estimated parameter coefficients and their t-statistics are listed in Table 3.

Recall that α_1 was set equal to 1. The sequence of coefficients $\alpha_1, \alpha_2, \ldots, \alpha_{44}$ comprises our estimated quarterly commercial office building price index for the land component of the property value. It can be seen that this land-price index is quite volatile due to the sparseness of commercial property sales and the heterogeneity of the properties. In a subsequent section, we will show how this volatile land-price index can be smoothed in a fairly simple fashion.

Turning to the other estimated coefficients, it can be seen that the ward-relative land-price parameters, ω_1-ω_4, decline (substantially) in magnitude as we move from the first more expensive composite ward to the less expensive composite wards. The marginal value of land parameters, λ_1 (set equal to 1), λ_2, λ_3, and λ_4, exhibit the same inverted-U pattern that emerged in Model 3 (and persisted through all of the subsequent models). The excess-land parameters, χ_1 (set equal to 1), χ_2, χ_3, χ_4, and χ_5, show that excess land is generally valued less than footprint land, but the

[25]This phenomenon has been observed in the housing literature before; that is, older heritage houses that have been extensively renovated may increase in value over time rather than depreciate as they age. Diewert *et al*. (2017) found that Richmond house structures appreciated by 2.4 percent per year after age 50.

[26]See Diewert and Shimizu (2015b), where these relationships also held for Tokyo detached houses.

TABLE 3
Estimated Coefficients for Model 9

Coefficient	Estimate	*t*-Statistic	Coefficient	Estimate	*t*-Statistic	Coefficient	Estimate	*t*-Statistic
α_2	1.5461	7.183	α_{25}	0.7937	3.739	ω_4	0.1229	3.510
α_3	1.6315	8.815	α_{26}	0.8785	3.185	λ_2	1.4212	4.160
α_4	1.5339	8.763	α_{27}	1.2128	4.792	λ_3	1.6805	7.867
α_5	1.4198	6.250	α_{28}	1.2489	7.593	λ_4	0.5771	4.199
α_6	1.7653	7.833	α_{29}	1.3320	6.819	χ_2	0.9070	24.025
α_7	2.1552	7.893	α_{30}	1.0175	4.038	χ_3	0.7634	17.948
α_8	1.7566	7.338	α_{31}	1.1255	5.402	χ_4	0.8278	16.228
α_9	2.2697	7.743	α_{32}	1.5853	6.808	χ_5	0.9551	10.654
α_{10}	2.6226	7.593	α_{33}	1.2028	5.776	μ	0.0602	2.675
α_{11}	2.4724	7.271	α_{34}	1.4807	5.755	η	−0.0002	−3.533
α_{12}	2.4234	7.194	α_{35}	1.4105	7.359	θ	0.0209	3.924
α_{13}	2.4672	7.425	α_{36}	1.5614	6.432	ϕ_4	1.3063	6.503
α_{14}	1.7139	5.754	α_{37}	1.6905	7.738	ϕ_5	1.6760	10.260
α_{15}	1.7080	7.011	α_{38}	1.3886	6.174	ϕ_6	1.7117	10.687
α_{16}	2.0576	7.009	α_{39}	1.7169	7.528	ϕ_7	1.6865	10.216
α_{17}	1.4671	6.664	α_{40}	1.9941	7.675	ϕ_8	2.3615	13.392
α_{18}	0.8818	3.656	α_{41}	1.3744	6.291	ϕ_9	2.5553	14.103
α_{19}	0.6900	3.341	α_{42}	1.6397	6.161	ϕ_{10}	2.8092	13.714
α_{20}	1.1983	5.128	α_{43}	1.5263	6.219	δ_1	0.0484	5.221
α_{21}	0.9835	5.383	α_{44}	2.0459	7.031	δ_2	0.0252	2.955
α_{22}	1.1742	4.981	ω_1	0.6518	6.535	δ_3	0.0060	0.656
α_{23}	1.2670	5.864	ω_2	0.3483	5.455	δ_4	0.0389	2.925
α_{24}	0.9392	4.903	ω_3	0.2493	4.841	δ_5	−0.0312	−1.876

decline in land value as excess land increases is not monotonic. The building-height land parameter $\mu = 0.0602$ is no longer as large as it was in Model 5, but an extra story of building height still adds 6 percent to the land value of the structure, which is a significant premium for extra building stories. The parameter for the walking distance to the nearest subway station, $\eta = -0.00023$, seems small, but it tells us that if the property is 1,000 m away from the nearest station, then the land value of the property is expected to fall by 23 percent compared to a nearby property. The parameter for the travel time to Tokyo Central Station, $\theta = 0.0209$, has a counter-intuitive sign; it is possible that this variable is correlated with other characteristics determining land prices and hence is not reliably determined. The height parameters, $\phi_3 = 1$ and $\phi_4 - \phi_{10}$, are very significant determinants of structure value; the value of the structure increases almost monotonically as the number of stories increases. Finally, the decade-by-decade estimated geometric depreciation rates, $\delta_1 - \delta_5$, show much the same pattern as was shown by the results for the previous model. Overall, the results of Model 9 seem to be reasonable.

In the following section, we will see if changing the model of depreciation from one based on multiple geometric depreciation rates to a piecewise linear model of depreciation leads to a significant change in our estimated land price index.

5. The Builder's Model with Piecewise Linear Depreciation Rates

Thus far, we have assumed that geometric depreciation models can best describe our data. In this section, we check the robustness of our results by assuming alternative depreciation models.

Recall that the structure aging (or survival) function for Model 9, $g_A(A_{tn})$, was defined by equation (20). In this section, we switch from a geometric model of depreciation to a straight-line or linear depreciation model. Thus for **Model 10**, we defined the aging function as follows:

$$g_A(A_{tn}) \equiv (1 - \delta A_{tn}), \tag{23}$$

where δ is the straight-line depreciation rate. Our new non-linear regression model is the same as the previous model defined by equation (22), except that the function g_A is defined by equation (23). The starting parameter values were taken to be the final parameter values from Model 7, except that the initial δ was set equal to 0.01 and the initial values for the parameters η and θ were set equal to 0.

The final LL for Model 10 was −12,635.83 and the R^2 value was 0.7078. The estimated straight-line depreciation rate was $\delta = 0.01357$ (0.00127). This model generated reasonable parameter estimates and the imputed value of the structure component of the property value was positive for all observation.[27]

The straight-line model of depreciation is not very flexible. Thus, following the approach used by Diewert and Shimizu (2015b), we implement a piecewise linear

[27]This does not always happen for straight-line depreciation models; that is, for properties with very old structures, the imputed value of the structure can become negative if the estimated depreciation rate is large enough. This phenomenon cannot occur with geometric depreciation models, which is an advantage of assuming this form of depreciation.

depreciation model. Recall the definitions given in equation (19), which defined the five age dummy variables, $D_{A,\,tni}$, for $i = 1, \ldots, 5$. We use these age dummy variables to define the piecewise linear aging function, $g_A(A_{tn})$, as follows:

$$
\begin{aligned}
(24)\quad g_A(A_{tn}) \equiv\ & D_{A,tn1}(1-\delta_1 A_{tn}) + D_{A,tn2}(1-10\delta_1-\delta_2(A_{tn}-10)) \\
& + D_{A,tn3}(1-10\delta_1-10\delta_2-\delta_3(A_{tn}-20)) \\
& + D_{A,tn4}(1-10\delta_1-10\delta_2-10\delta_3-\delta_4(A_{tn}-30)) \\
& + D_{A,tn5}(1-10\delta_1-10\delta_2-10\delta_3-10\delta_4-\delta_5(A_{tn}-40)).
\end{aligned}
$$

The Model 11 non-linear regression model is the same as the model defined by equation (22), except that the function g_A is defined by equation (24). The starting parameter values were taken to be the final parameter values from Model 10, except that the new depreciation parameters $\delta_1, \ldots, \delta_5$ were all set equal to the final straight-line depreciation rate δ estimated in Model 10. If all 5 δ_i are set equal to a common δ, then Model 11 collapses down to Model 10.

The final LL for Model 11 was −12,614.35, which was an increase in LL of 21.48 over the Model 10 LL. The R^2 value for Model 11 was 0.7143.[28] The estimated piecewise linear depreciation rates (with standard errors in brackets) were as follows: $\delta_1 = 0.0393\,(0.0057)$, $\delta_2 = 0.0125\,(0.0049)$, $\delta_3 = 0.0302\,(0.0041)$,[29] $\delta_4 = 0.0159\,(0.0054)$, and $\delta_5 = -0.0135\,(0.0074)$. Thus, as was the case with the multiple geometric depreciation rate model, properties with structures that are over 40 years old tend to *increase* in value by 1.35 percent per year. All of the estimated parameter coefficients for Model 11 and their *t*-statistics are listed in Table 4.

Comparing the estimated coefficients in Tables 3 and 4, it can be seen that the parameter estimates for Models 9 and 11 were very similar, except that there were some differences in the estimated depreciation rates δ_1 to δ_5. However, in the following section, we will show that these two models based on multiple depreciation rates generate aging functions g_A that approximate each other reasonably well. Thus both models describe the underlying data to the same degree of approximation.

6. Comparing Alternative Models of Depreciation

The determination of depreciation schedules for commercial office buildings is important for tax purposes, for investors, and for the estimation of commercial office structure stocks, which in turn feed into the computation of the Multifactor Productivity of the Commercial Office Sector. Thus, in this section, we compare the single geometric rate (Model 7), straight-line (Model 10), multiple geometric rate (Model 8), and piecewise linear (Model 11) depreciation schedules.

[28] Recall that the LL for the comparable geometric model of depreciation, Model 9, was −12,614.70 and the R^2 value for Model 9 was 0.7142. Thus the descriptive power of both models is virtually identical.

[29] Recall that these depreciation rates are net depreciation rates. As surviving structures approach their middle age, renovations become important and thus a decline in the net depreciation rate is plausible. The pattern of depreciation rates is again similar to the comparable geometric depreciation rates that were observed for detached houses in Richmond (a suburb of Vancouver, Canada) by Diewert *et al*. (2017).

TABLE 4
Estimated Coefficients for Model 11

Coefficient	Estimate	*t*-Statistic	Coefficient	Estimate	*t*-Statistic	Coefficient	Estimate	*t*-Statistic
α_2	1.5529	7.227	α_{25}	0.7911	3.552	ω_4	0.1211	3.669
α_3	1.6342	8.878	α_{26}	0.8702	3.090	λ_2	1.4159	4.134
α_4	1.5352	8.846	α_{27}	1.2100	4.945	λ_3	1.6789	8.081
α_5	1.4210	6.373	α_{28}	1.2474	7.587	λ_4	0.5667	4.002
α_6	1.7646	7.895	α_{29}	1.3276	6.854	χ_2	0.9090	23.966
α_7	2.1559	7.705	α_{30}	1.0091	3.855	χ_3	0.7654	17.928
α_8	1.7560	7.255	α_{31}	1.1229	5.239	χ_4	0.8313	15.173
α_9	2.2680	7.632	α_{32}	1.5835	6.670	χ_5	0.9644	9.593
α_{10}	2.6239	7.483	α_{33}	1.1892	5.837	μ	0.0595	2.671
α_{11}	2.4706	7.299	α_{34}	1.4767	5.744	η	−0.0002	−3.973
α_{12}	2.4236	7.196	α_{35}	1.4063	7.278	θ	0.0212	3.972
α_{13}	2.4715	7.362	α_{36}	1.5521	6.370	ϕ_4	1.2947	6.828
α_{14}	1.7192	5.818	α_{37}	1.6954	7.796	ϕ_5	1.6571	10.556
α_{15}	1.7004	7.001	α_{38}	1.3819	6.117	ϕ_6	1.6872	10.906
α_{16}	2.0585	7.234	α_{39}	1.7117	7.498	ϕ_7	1.6708	10.098
α_{17}	1.4660	6.624	α_{40}	1.9944	7.622	ϕ_8	2.3273	13.861
α_{18}	0.8829	3.554	α_{41}	1.3680	6.354	ϕ_9	2.5212	14.271
α_{19}	0.6842	3.258	α_{42}	1.6344	6.043	ϕ_{10}	2.7717	13.774
α_{20}	1.1944	5.085	α_{43}	1.5117	6.324	δ_1	0.0393	6.920
α_{21}	0.9789	5.400	α_{44}	2.0454	6.897	δ_2	0.0125	2.524
α_{22}	1.1714	5.079	ω_1	0.6493	6.794	δ_3	0.0030	0.737
α_{23}	1.2619	5.899	ω_2	0.3460	5.720	δ_4	0.0159	2.962
α_{24}	0.9390	5.026	ω_3	0.2473	5.061	δ_5	−0.0135	−1.826

TABLE 5
THE GEOMETRIC, STRAIGHT-LINE, MULTIPLE GEOMETRIC, AND PIECEWISE LINEAR AGING FUNCTIONS

A	$g_G(A)$	$g_{SL}(A)$	$g_{MG}(A)$	$g_{PL}(A)$	A	$g_G(A)$	$g_{SL}(A)$	$g_{MG}(A)$	$g_{PL}(A)$
0	1.0000	1.0000	1.0000	1.0000	28	0.3782	0.6200	0.4695	0.4942
1	0.9659	0.9864	0.9516	0.9607	29	0.3653	0.6064	0.4667	0.4912
2	0.9329	0.9729	0.9055	0.9214	30	0.3528	0.5928	0.4485	0.4753
3	0.9011	0.9593	0.8617	0.8821	31	0.3408	0.5793	0.4311	0.4594
4	0.8703	0.9457	0.8200	0.8427	32	0.3291	0.5657	0.4143	0.4435
5	0.8406	0.9321	0.7803	0.8034	33	0.3179	0.5521	0.3982	0.4276
6	0.8119	0.9186	0.7425	0.7641	34	0.3071	0.5386	0.3827	0.4117
7	0.7842	0.9050	0.7066	0.7248	35	0.2966	0.5250	0.3678	0.3958
8	0.7574	0.8914	0.6724	0.6855	36	0.2865	0.5114	0.3535	0.3799
9	0.7316	0.8779	0.6398	0.6461	37	0.2767	0.4978	0.3397	0.3640
10	0.7066	0.8643	0.6237	0.6337	38	0.2672	0.4843	0.3265	0.3481
11	0.6825	0.8507	0.6080	0.6212	39	0.2581	0.4707	0.3138	0.3323
12	0.6592	0.8371	0.5927	0.6087	40	0.2493	0.4571	0.3236	0.3457
13	0.6367	0.8236	0.5777	0.5962	41	0.2408	0.4436	0.3336	0.3592
14	0.6150	0.8100	0.5632	0.5838	42	0.2326	0.4300	0.3441	0.3727
15	0.5940	0.7964	0.5490	0.5713	43	0.2246	0.4164	0.3548	0.3862
16	0.5737	0.7829	0.5352	0.5588	44	0.2170	0.4028	0.3659	0.3997
17	0.5541	0.7693	0.5217	0.5463	45	0.2096	0.3893	0.3773	0.4132
18	0.5352	0.7557	0.5085	0.5339	46	0.2024	0.3757	0.3890	0.4266
19	0.5170	0.7421	0.4957	0.5214	47	0.1955	0.3621	0.4012	0.4401
20	0.4993	0.7286	0.4927	0.5184	48	0.1888	0.3485	0.4137	0.4536
21	0.4823	0.7150	0.4898	0.5153	49	0.1824	0.3350	0.4266	0.4671
22	0.4658	0.7014	0.4868	0.5123	50	0.1762	0.3214	0.4399	0.4806
23	0.4499	0.6878	0.4839	0.5093	51	0.1702	0.3078	0.4536	0.4941
24	0.4346	0.6743	0.4810	0.5063	52	0.1643	0.2943	0.4678	0.5076
25	0.4197	0.6607	0.4781	0.5032	53	0.1587	0.2807	0.4824	0.5210
26	0.4054	0.6471	0.4752	0.5002	54	0.1533	0.2671	0.4974	0.5345
27	0.3916	0.6336	0.4723	0.4972					

These depreciation schedules are equal to the aging functions $g_A(A)$ defined by $g_G(A) \equiv (1-\delta)^A$, $g_{SL}(A) \equiv (1-\delta A)$, $g_{MG}(A)$, where g_{MG} is equal to g_A defined by equation (20), and $g_{PL}(A)$, where g_{PL} is the g_A defined by equation (24) and the age variable $A = 0,1,2,\ldots,54$. The resulting depreciation schedules are listed in Table 5 and plotted in Figure 1.

The straight-line depreciation schedule is represented by the aging function $g_{SL}(A)$; it is the straight line in Figure 1. The depreciation schedule for the geometric model of depreciation is represented by the convex curved line in Figure 1. It can be seen that these single-rate depreciation schedules are rather different. The multiple geometric rate depreciation schedule is the lower of the two broken lines in Figure 1, while the piecewise linear depreciation schedule is the slightly higher broken line. It can be seen that these two multiple depreciation rate schedules approximate each other fairly well.[30] It can also be seen that the single geometric rate depreciation schedule provides a rough approximation to the two multiple rate schedules up to age 40, but then the schedules diverge.

The sequence of parameters α_t for $t = 2,3,\ldots,44$ (along with $\alpha_1 \equiv 1$) listed in Tables 3 and 4 provides alternative land-price indexes generated by the MLIT transaction data. It can be seen that these alternative indexes are virtually identical

[30]This is to be expected. As the number of separate depreciation rates in each of these models tends to 43, the two schedules will converge to a common schedule.

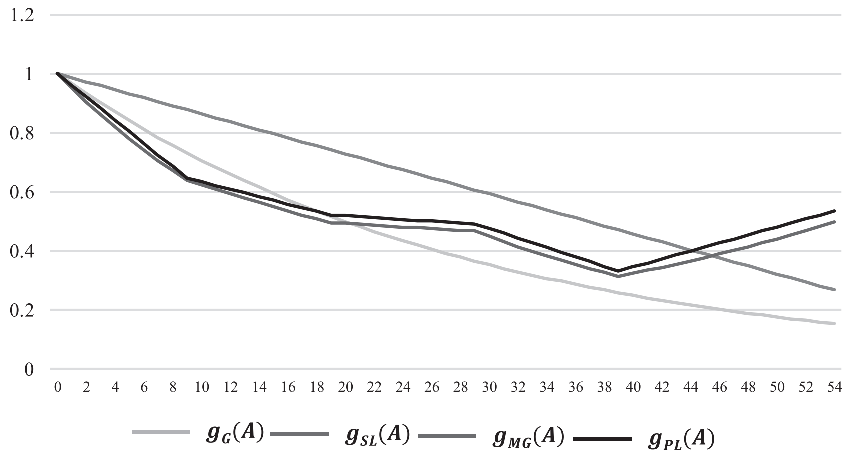

Figure 1. Alternative Aging Functions

(they cannot be distinguished on a chart) and hence only one of these alternative models of depreciation needs to be considered in what follows. Since the log likelihood of the piecewise linear depreciation model (Model 11) was slightly higher than the multiple geometric depreciation rate model (Model 10), we will use the α_t sequence generated by Model 11 as our MLIT land-price series in subsequent sections. We will label this series for quarter t as PL^t_{MLIT}.

7. Smoothing the MLIT Land-Price Series

In Figure 2, it can be seen that our Model 11 estimated land price series, $PL^t_{\mathrm{MLIT}} \equiv \alpha_t$, is somewhat volatile. This is due to the fact that commercial properties are very heterogeneous and we have relatively few transactions per quarter. Thus the raw series PL_{MLIT} does not accurately represent the *trend* in commercial land prices in Tokyo; the raw series requires some smoothing in order to model the trends in land prices.[31] Patrick (2017) found the same problem for Irish house price sales and we will follow his example and smooth the raw series.[32]

We used the LOWESS non-parametric smoother in Shazam in order to construct a preliminary smoothed land-price series, PL_{S}, using PL_{MLIT} as the input series.[33] We used the cross-validation criterion to choose the smoothing parameter,

[31]The volatility in our raw series could be a real phenomenon in that land prices are inherently volatile. If this is the case, it would be useful for statistical offices to publish the unsmoothed series as well as the smoothed series. As noted by Geltner *et al.* (2014), property investors would find unsmoothed property price indexes useful in order to evaluate the riskiness of property investments.

[32]Patrick initially smoothed his series by taking a 3-month rolling average of the raw index prices for Ireland. He found that the resulting index was still too volatile to publish and he ended up using a double-exponential smoothing procedure.

[33]The initial smoothed series was divided by the quarter 1 value so that the resulting normalized series equalled 1 in quarter 1. Recall that quarter 1 is the first quarter in 2005 and quarter 44 is the last quarter in 2015.

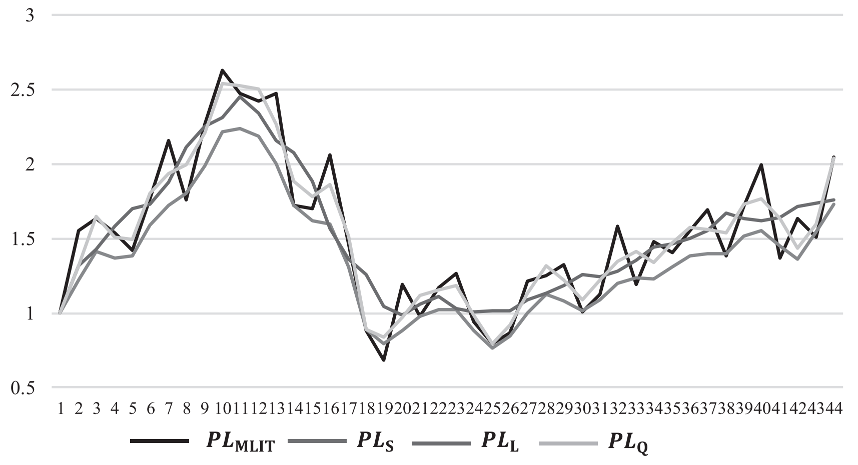

Figure 2. The MLIT Land Prices, the LOWESS Smoothed Prices, and the Linear and Quadratic Smoothed Prices

which turned out to be $f = 0.12$. The series PL_{MLIT} and PL_{S} are listed in Table 6 and plotted in Figure 2.

The jagged black line in Figure 2 represents the unsmoothed land price index PL_{MLIT} that we estimated from Model 11, while the lowest line represents the LOWESS non-parametric smoothed series PL_{S} that was generated using Shazam. It can be seen that while PL_{S} is reasonably smooth, it is not quite centered; that is, it is consistently below the raw series. Thus we considered some alternative methods for smoothing the raw series.

Henderson (1916) was the first to realize that various moving-average smoothers could be related to rolling-window least squares regressions that would exactly reproduce a polynomial curve. Thus we apply his idea to derive the moving-average weights that would be equivalent to fitting a linear function to five consecutive quarters of a time series, which we represent by the vector $\boldsymbol{Y}^T \equiv [y_1, \dots, y_5]$, where $\boldsymbol{Y}^T$ denotes the transpose of a vector $\boldsymbol{Y}$. Define the five-dimensional column vectors $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$ as $\boldsymbol{X}_1 \equiv [1,1,1,1,1]^T$ and $\boldsymbol{X}_2 \equiv [-2,-1.0,1,2]^T$. Define the (5×2)-dimensional $\mathbf{X}$ matrix as $\mathbf{X} \equiv [\boldsymbol{X}_1, \boldsymbol{X}_2]$. Denote the linear smooth of the vector $\boldsymbol{Y}$ by $\boldsymbol{Y}^*$. Then least squares theory tells us that $\boldsymbol{Y}^* = \mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\boldsymbol{Y}$. Thus the five rows of the 5×5 projection matrix $\mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T$ give us the weights that can be used to convert the raw $\boldsymbol{Y}$ series into the smoothed $\boldsymbol{Y}^*$ series. For our particular example, the five rows of the projection matrix are as follows: row one $= (1/10)[6, 4, 2, 0, -2]$; row two $= (1/10)[4, 3, 2, 1, 0]$; row three $= (1/5)[1, 1, 1, 1, 1]$; row four $= (1/10)[0, 1, 2, 3, 4]$; row five $= (1/10)[-2, 0, 2, 4, 6]$. Note that row three tells us that the third component of the smoothed vector $\boldsymbol{Y}^*$ is equal to $y_3^* = (1/5)(y_1 + y_2 + y_3 + y_4 + y_5)$. a simple equally weighted moving average of the raw data for five periods. Thus the way in which this smoothing method could be applied in practice to 44 consecutive quarters of PL_{MLIT} data is as follows. The first three components of the smoothed series would use the inner products of the first three rows of the projection matrix $\mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T$ times the first

TABLE 6

The MLIT Land Prices, PL_{MLIT}, the LOWESS Smoothed Land Prices, PL_{S}, and the Linear and Quadratic Smoothed Land Prices, PL_{L} and PL_{Q}

Quarter	PL_{MLIT}	PL_{S}	PL_{L}	PL_{Q}
1	1.00000	1.00000	1.00000	1.00000
2	1.55293	1.22256	1.31711	1.31711
3	1.63422	1.41343	1.42867	1.64505
4	1.53523	1.36721	1.58159	1.50881
5	1.42096	1.38616	1.70218	1.49669
6	1.76462	1.58816	1.72654	1.80134
7	2.15588	1.72313	1.87309	1.93802
8	1.75601	1.80146	2.11368	1.99351
9	2.26798	1.98565	2.25488	2.20673
10	2.62393	2.21322	2.30843	2.54089
11	2.47061	2.23370	2.45153	2.52436
12	2.42362	2.18881	2.34178	2.49936
13	2.47153	2.00272	2.15709	2.26334
14	1.71923	1.72301	2.07466	1.88127
15	1.70045	1.61503	1.88313	1.78365
16	2.05848	1.59538	1.56540	1.86242
17	1.46597	1.31167	1.35840	1.51611
18	0.88287	0.88680	1.25719	0.88721
19	0.68422	0.79234	1.04127	0.83499
20	1.19442	0.88117	0.98237	0.97427
21	0.97889	0.97860	1.05818	1.11980
22	1.17144	1.02013	1.10914	1.15441
23	1.26194	1.02133	1.02848	1.18481
24	0.93901	0.88339	1.00673	0.98500
25	0.79111	0.76458	1.01445	0.79266
26	0.87016	0.84384	1.01155	0.92135
27	1.21003	1.00334	1.08928	1.13215
28	1.24743	1.12503	1.13287	1.31487
29	1.32764	1.08376	1.18341	1.21856
30	1.00910	1.01178	1.25811	1.08766
31	1.12286	1.09153	1.24647	1.21854
32	1.58349	1.19563	1.27629	1.34877
33	1.18925	1.23645	1.35573	1.41007
34	1.47675	1.22737	1.44159	1.33842
35	1.40632	1.31129	1.46396	1.47429
36	1.55214	1.38556	1.50250	1.57230
37	1.69536	1.39737	1.54949	1.56217
38	1.38194	1.39867	1.66709	1.53537
39	1.71167	1.51667	1.63026	1.72640
40	1.99436	1.54872	1.61806	1.76603
41	1.36798	1.45003	1.64401	1.63230
42	1.63437	1.36017	1.71076	1.43488
43	1.51167	1.54133	1.73534	1.59740
44	2.04541	1.73013	1.75991	2.03579

five components of the PL_{MLIT} series. This would generate the first three components of the smoothed series PL_{L}^t, for $t = 1, 2, 3$. For $t = 3, 4, \ldots, 42$, define $PL_{\text{L}}^t \equiv (1/5)[PL_{\text{MLIT}}^{t-2} + PL_{\text{MLIT}}^{t-1} + PL_{\text{MLIT}}^{t} + PL_{\text{MLIT}}^{t+1} + PL_{\text{MLIT}}^{t+2}]$. Thus for all observations t except for the first two and last two, the smoothed series PL_t would be defined as the simple centered moving average of five consecutive PL_{MLIT} observations with equal weights. The final two observations would be defined as the inner products of rows four and five of $\mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T$ with the last five observations in the PL_{MLIT} series. In practice, as the data of a subsequent period became available, the last two observations in the existing series would be revised—but after receiving the data of two subsequent periods, there would be no further revisions; that is, the final smoothed value of an observation would be set equal to the centered five-period moving average of the raw data.

We implemented the above procedure but the above algorithm does not ensure that the value of the smoothed series in the first quarter of the sample is equal to 1 and so the generated series had to be divided by a constant to ensure that the first observation in the smoothed series was equal to unity. We found that this division caused the smoothed series to lie below the raw series for the most part.[34] Patrick 2017, pp. 25–6) found that a similar problem occurred with his initial simple moving-average smoothing method. He solved the problem by setting the smoothed values equal to the actual values for the first two observations when he applied his second smoothing method. We solved the centering problem in a similar manner: we set the initial value of the smooth equal to the corresponding raw number (so that $PL_{\mathrm{L}}^{1} \equiv PL_{\mathrm{MLIT}}^{1}$) and we set the second value of the smooth equal to the average of the first and third observations in the raw series (so that $PL_{\mathrm{L}}^{2} \equiv (1/2)[PL_{\mathrm{MLIT}}^{1} + PL_{\mathrm{MLIT}}^{3}]$. For the quarter 3 value of the smooth, we used the simple five-term centered moving average so that $PL_{\mathrm{L}}^{3} \equiv (1/5)[PL_{\mathrm{MLIT}}^{1} + PL_{\mathrm{MLIT}}^{2} + PL_{\mathrm{MLIT}}^{3} + PL_{\mathrm{MLIT}}^{4} + PL_{\mathrm{MLIT}}^{5}]$ and we carried on using this moving average until quarters 43 and 44, where we used rows four and five of the matrix $\mathbf{X}(\mathbf{X}^{T}\mathbf{X})^{-1}\mathbf{X}^{T}$ defined above for our moving-average weights. The resulting smoothed series PL_{L}^{t} is listed in Table 6 and plotted in Figure 2. It can be seen that it does a good job of smoothing the initial PL_{MLIT}^{t} series.

We also applied the same least squares methodology to a rolling-window five-term quadratic regression model. Define the five-dimensional column vectors $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$ as before and define $\boldsymbol{X}_3 \equiv [4, 1, 0, 1, 4]^{T}$. Define the (5×3)-dimensional $\mathbf{X}$ matrix as $\mathbf{X} \equiv [\boldsymbol{X}_1, \boldsymbol{X}_2, \boldsymbol{X}_3]$. Denote the quadratic smooth of the vector $\boldsymbol{Y}$ by $\boldsymbol{Y}^{**}$. Again, least squares theory tells us that $\boldsymbol{Y}^{**} = \mathbf{X}(\mathbf{X}^{T}\mathbf{X})^{-1}\mathbf{X}^{T}\boldsymbol{Y}$. The five rows of the new 5×5 projection matrix $\mathbf{X}(\mathbf{X}^{T}\mathbf{X})^{-1}\mathbf{X}^{T}$ give us the weights that can be used to convert the raw $\boldsymbol{Y}$ series into the smoothed $\boldsymbol{Y}^{**}$ series. The five rows of the new projection matrix are as follows: row one $= (1/35)[31, 9, -3, -5, 3]$; row two $= (1/35)[9, 13, 12, 6, -5]$; row three $= (1/35)[-3, 12, 17, 12, -3]$; row four $= (1/35)[-5, 6, 12, 13, 9]$; and row five $= (1/35)[3, -5, -3, 9, 31]$. Now repeat the steps that were used to construct the linear smooth PL_{L}^{t} to construct a preliminary quadratic smooth PL_{Q}^{t}, except that the new 5×5 projection matrix $\mathbf{X}(\mathbf{X}^{T}\mathbf{X})^{-1}\mathbf{X}^{T}$ replaces the previous one. A final PL_{Q}^{t} series was constructed by replacing the first two values in the smoothed series by the same initial two values that we used to construct the final versions of PL_{L}^{1} and PL_{L}^{2}. The resulting smoothed series PL_{Q}^{t} is listed in Table 6 and plotted in Figure 2. It can be seen that PL_{Q}^{t} is not nearly as smooth as the linear smoothed series PL_{L}^{t} but, of course, it is a lot closer to the unadjusted

[34]A similar problem of a lack of centering occurred when we implemented the LOWESS smoothing procedure; that is, we had to divide by a constant to make the first component of the smoothed series equal to one. As a result, the LOWESS smoothing tended to lie below the raw series, as can be seen in Figure 2.

series PL^t_{MLIT}. For our particular dataset, we would recommend the linear smoother over the quadratic smoother.[35]

We now turn to the construction of land prices using commercial property appraisal data.

8. The Builder's Model Using Property Appraisal Data

As was indicated in Section 2, we have quarterly appraisal data for 41 commercial office REIT office buildings located in Tokyo for the 44 quarters starting at Q1:2005 and ending at Q4:2015, which, of course, is the same period that was covered by the MLIT selling-price data. We will implement the builder's model for this dataset in this section.

The builder's model using appraisal data is somewhat different from the builder's model using selling-price data. The panel nature of the REIT data means that we can use a single property-specific dummy variable as a variable that concentrates all of the location attributes of the property into a single variable; that is, we do not have to worry about how close to a subway line the property is, or how many stories the building has, or how much excess land is associated with the property. The single property-specific dummy variable will take all of these characteristics into account.

There are 41 separate properties in our REIT dataset. For each of our 44 quarters, we assume that the 41 properties appear in the appraised property value for property n in period t, V_{tn}, in the same order. Our initial regression model is the following one, where the variables have the same definitions as in equation (2) except that ω_n is now the *property n sample average land price* (per square meter) rather than a ward-n relative price of land:

$$(25) \qquad V_{tn} = \sum_{n=1}^{41} \omega_n L_{tn} + p_{St}(1-0.025)^{A(t,n)} S_{tn} + \varepsilon_{tn}; \qquad t=1,\ldots,44, \quad n=1,\ldots,41.$$

Thus, in Model 1 above, there are no quarter-t land-price parameters in this very simple model with 41 unknown property average land-price ω_n parameters to estimate. Note that the geometric (net) depreciation rate in the model defined by equation (25) was assumed to be 2.5 percent per year.

The final LL for this model was −14,968.77 and the R^2 value was 0.9426. Thus the 41 property average price parameters ω_n explain a large part of the variation in the data.

In Model 2, we introduce quarterly land prices α_t into the above model. The new non-linear regression model is as follows:

$$(26) \qquad V_{tn} = \sum_{n=1}^{41} \alpha_t \omega_n L_{tn} + p_{St}(1-0.025)^{A(t,n)} S_{tn} + \varepsilon_{tn}; \qquad t=1,\ldots,44, \quad n=1,\ldots,41.$$

[35]A quadratic Henderson-type smoother would be much smoother if we lengthened the window. But a longer window would imply a longer revision period before the series would be finalized. Since the linear smoother with window length 5 seems to do a nice job of smoothing, we would not recommend moving to a longer window length for this particular application.

Not all of the quarterly land-price parameters (the α_t) and the average property price parameters (the ω_n) can be identified. Thus we impose the following normalization on our coefficients:

$$\alpha_1 = 1. \tag{27}$$

We used the final parameter values for the ω_n from Model 1 as starting coefficient values for Model 2 (with all α_t initially set equal to 1).[36] The final LL for Model 2 was −13,999.00, a huge improvement of 969.77 for adding 43 new parameters. The R^2 value was 0.9804. Thus the 41 property average price parameters ω_n and the 43 quarterly average land-price parameters α_t explain most of the variation in the data.

Model 3 is the following non-linear regression model:

$$V_{tn} = \alpha_t \omega_n L_{tn} + p_{St}(1-\delta)^{A(t,n)} S_{tn} + \varepsilon_{tn}; \qquad t = 1, \dots, 44, \quad n = 1, \dots, 41 \tag{28}$$

where δ is the annual geometric (net) depreciation rate. The normalization given in equation (27) is also imposed. Thus Model 3 is the same as Model 2 except that we now estimate the single geometric depreciation rate δ.

We used the final parameter values for the α_t and ω_n from Model 2 as starting coefficient values for Model 3 (with δ initially set equal to 0.025). The final LL for this model was −13,993.47, an increase of 5.53 for one additional parameter, and the R^2 value was 0.9806. The estimated geometric (net) depreciation rate was $\delta = 0.01353$.[37] The estimated coefficients and their t-statistics are listed in Table 7. Recall that α_1 was set equal to 1. The sequence of land price (per square meter) α_t, for $t = 1,2,\dots,44$, is our estimated sequence of quarterly Tokyo land prices, PL^t_{REIT}, which appears in Figure 3.

Note that the implied standard errors on the quarterly land-price coefficients, the α_t, are fairly large, whereas they are fairly small for the property coefficients, the ω_n. This means that our estimated land-price indexes, $PL^t_{\text{REIT}} = \alpha_t$, are not reliably determined. Note also that our estimated geometric depreciation rate δ is only 1.35 percent per year, which is much lower than our estimated depreciation rate from Model 7 in Section 3, which was 3.41 percent per year. One factor that may help to explain this divergence in estimates of wear-and-tear depreciation is that appraisers take into account capital expenditure on the properties. However, our current database did not contain information on capital expenditure and it is likely that not having capital expenditure as an explanatory factor affected our estimates for the depreciation rate. In our previous study of land prices using REIT data for Diewert and Shimizu (2017), we adjusted our non-linear regressions for capital expenditure and found that the resulting estimated quarterly wear-and-tear

[36]The reader may well wonder why we estimated the ω_n in Model 1 rather than first estimating the α_t in Model 1. When this alternative strategy was implemented, we found that the resulting Model 2 did not converge to the "right" parameter values; that is, the resulting R^2 value was very low. This is the reason for following our nested estimation methodology, in which each successive model uses the final coefficient values from the previous model. It is not possible to simply estimate our final models in one step and obtain sensible results.

[37]We also estimated the straight-line depreciation model counterpart to Model 3. The resulting estimated straight-line depreciation rate δ was equal to 0.01317 (t-statistic = 45.73). The R^2 value for this model was 0.9806 and the final LL was −13,989.83. The resulting land-price series was very similar to the one generated by Model 3.

TABLE 7
The Estimated Coefficients for Model 3 Using REIT Data

Coefficient	Estimate	*t*-Statistic	Coefficient	Estimate	*t*-Statistic	Coefficient	Estimate	*t*-Statistic
α_2	1.0268	2.121	α_{31}	1.0645	1.403	ω_{15}	2.1471	10.600
α_3	1.0637	1.479	α_{32}	1.0594	1.429	ω_{16}	5.8157	44.703
α_4	1.1045	1.541	α_{33}	1.0498	1.466	ω_{17}	5.8961	40.816
α_5	1.1499	1.532	α_{34}	1.0393	1.334	ω_{18}	4.0615	34.128
α_6	1.1987	1.607	α_{35}	1.0341	1.380	ω_{19}	5.4266	39.534
α_7	1.2473	1.569	α_{36}	1.0294	1.242	ω_{20}	5.7298	36.905
α_8	1.2994	1.636	α_{37}	1.0277	1.226	ω_{21}	1.0098	40.629
α_9	1.3450	1.735	α_{38}	1.0299	1.200	ω_{22}	4.0731	47.336
α_{10}	1.3882	1.904	α_{39}	1.0350	1.253	ω_{23}	2.0521	25.603
α_{11}	1.4422	2.027	α_{40}	1.0425	1.315	ω_{24}	2.5844	38.974
α_{12}	1.4904	2.052	α_{41}	1.0564	1.270	ω_{25}	1.0869	41.419
α_{13}	1.5082	1.995	α_{42}	1.0700	1.244	ω_{26}	1.2409	22.634
α_{14}	1.4990	2.075	α_{43}	1.0874	1.286	ω_{27}	2.0714	23.201
α_{15}	1.4751	2.103	α_{44}	1.1078	1.827	ω_{28}	0.7289	32.147
α_{16}	1.4419	2.238	ω_1	3.8704	31.383	ω_{29}	0.6271	7.422
α_{17}	1.3976	1.721	ω_2	4.8678	48.918	ω_{30}	3.1068	39.453
α_{18}	1.3423	1.838	ω_3	1.7514	27.642	ω_{31}	1.7773	32.149
α_{19}	1.2892	1.705	ω_4	2.3099	26.957	ω_{32}	5.8748	41.597
α_{20}	1.2428	1.522	ω_5	1.8451	27.751	ω_{33}	1.5201	20.558
α_{21}	1.2108	1.634	ω_6	3.7399	30.589	ω_{34}	3.4731	30.059
α_{22}	1.1766	1.583	ω_7	2.6487	32.409	ω_{35}	2.1225	27.539
α_{23}	1.1543	1.419	ω_8	3.2710	28.703	ω_{36}	6.2429	48.012
α_{24}	1.1375	1.485	ω_9	4.8665	47.654	ω_{37}	4.2053	22.829
α_{25}	1.1166	1.414	ω_{10}	4.9867	41.462	ω_{38}	2.6778	21.825
α_{26}	1.1007	1.479	ω_{11}	1.1427	16.089	ω_{39}	3.0139	23.805
α_{27}	1.0967	1.314	ω_{12}	2.3817	20.345	ω_{40}	2.9460	12.591
α_{28}	1.0908	1.401	ω_{13}	1.1255	15.765	ω_{41}	1.8028	15.349
α_{29}	1.0799	1.437	ω_{14}	0.8444	14.470	δ	0.0135	4.437
α_{30}	1.0683	1.349						

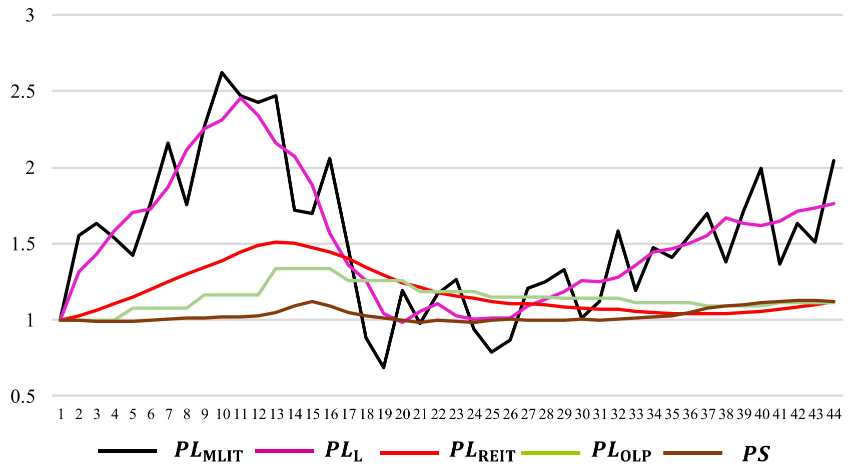

Figure 3. The Alternative Land-Price Series and the Price of Structures [Colour figure can be viewed at wileyonlinelibrary.com]

geometric depreciation rate was 0.005, which implied an annual (single) geometric depreciation rate of about 2 percent.[38]

In the following section, we will estimate our final land-price series for Tokyo commercial office buildings using official estimates for the land values of commercial properties for taxation purposes.

9. Estimating Land Prices for Commercial Properties Using Tax Assessment Data

In this section, we will use the OLP data described in Section 2. We have 6,242 annual assessed values for the land components of commercial properties in Tokyo covering the 11 years from 2005 to 2015. We will label these years as $t = 1, 2, \ldots, 11$. The assessed land value for property n in year t is denoted as V_{tn}.[39] We have information on which ward each property is located in and the ward dummy variables $D_{W,tnj}$ are defined by the definitions given in equation (4). The land-plot area of property n in year t is denoted by L_{tn} and the subway variables DS_{tn} and TT_{tn} are defined as in Section 2. The number of observations in year t is $N(t)$.

[38]In the multiple geometric depreciation rate model estimated by Diewert and Shimizu (2017), the various rates averaged out to an annual rate of 2.6 percent per year. Our earlier study covered the 22 quarters starting at Q1:2007 and ending at Q2:2012. The correlation coefficient between the price of land series in this model in Diewert and Shimizu (2017) and the above Model 3 price of land series for the overlapping 22 quarters is 0.9901, so these two studies using REIT appraisal data show much the same trends in Tokyo commercial property land prices even though the estimated wear-and-tear depreciation rates are different. Note that in addition to wear-and-tear depreciation, depreciation due to the early demolition of a structure before it reaches "normal" retirement age should be taken into account. Our current study does not estimate this extra component of depreciation. However, Diewert and Shimizu (2017) estimated demolition depreciation for Tokyo commercial office buildings at 1.2 percent per year.

[39]The units of measurement used in this section are 100,000 yen.

Our initial regression model is the following one, in which we regress the property land value on the ward dummy variables times the land-plot area:

$$(29)\qquad V_{tn}=\left(\sum_{j=1}^{23}\omega_j D_{W,tnj}\right)L_{tn}+\varepsilon_{tn};\qquad t=1,\ldots,11,\quad n=1,\ldots,N(t).$$

Thus, in Model 1 above, there are no year-t land-price parameters in this very simple model and ω_j is an estimate of the average land price (per square meter) in ward j for j = 1,…,23. The final LL for this model was −67073.91 and the R^2 value was 0.3647.

In Model 2, we introduce annual land prices α_t into the above model. The new non-linear regression model is as follows:

$$(30)\qquad V_{tn}=\alpha_t\left(\sum_{j=1}^{23}\omega_j D_{W,tnj}\right)L_{tn}+\varepsilon_{tn};\qquad t=1,\ldots,11,\quad n=1,\ldots,N(t).$$

Not all of the 11 annual land-price parameters (the α_t) and the 23 ward average property relative price parameters (the ω_n) can be identified. Thus we impose the normalization $\alpha_1 = 1$.

We used the final parameter values for the ω_n from Model 1 as starting coefficient values for Model 2 (with all α_t initially set equal to 1). The final LL for Model 2 was −67,022.90, an increase of 51.01 for adding 43 new parameters. The R^2 value was 0.3748.

In our next model, we allowed the price of land to vary as the lot size increased. We divided up our 6,242 observations into five groups based on their lot size. The Group 1 properties had lots of less than 100 m2, the Group 2 properties had lots greater than or equal to 100 m^2 and less than 150 m^2, the Group 3 properties had lots greater than or equal to 150 m^2 and less than 200 m^2, the Group 4 properties had lots greater than or equal to 200 m^2 and less than 300 m^2, and the Group 5 properties had lots greater than or equal to 300 m^2.[40] For each observation n in period t, we define the five *land dummy variables*, $D_{L,tnk}$, for k = 1,…,5 as follows:

$$(31)\qquad D_{L,tnk}\equiv\begin{array}{ll}1 & \text{if observation } tn \text{ has a land area that belongs to Group } k;\\ 0 & \text{if observation } tn \text{ has a land area that does not belong to Group } k.\end{array}$$

Define the constants L_1–L_4 as 100, 150, 200, and 300, respectively. These constants and the dummy variables defined by equation (31) are used in the definition of the following piecewise linear function of L_{tn}, $f(L_{tn})$:

$$(32)\qquad \begin{aligned} f(L_{tn})\equiv\ & D_{L,tn1}\lambda_1 L_{tn}+D_{L,tn2}[\lambda_1 L_1+\lambda_2(L_{tn}-L_1)]\\ &+D_{L,tn3}[\lambda_1 L_1+\lambda_2(L_2-L_1)+\lambda_3(L_{tn}-L_2)]\\ &+D_{L,tn4}[\lambda_1 L_1+\lambda_2(L_2-L_1)+\lambda_3(L_3-L_2)+\lambda_4(L_{tn}-L_3)]\\ &+D_{L,tn5}[\lambda_1 L_1+\lambda_2(L_2-L_1)+\lambda_3(L_3-L_2)+\lambda_4(L_4-L_3)+\lambda_5(L_{tn}-L_4)].\end{aligned}$$

[40]The sample probabilities of an observation falling in the five land size groups were as follows: 0.171, 0.285, 0.175, 0.178, and 0.191.

Model 3 was defined as the following non-linear regression model:

$$V_{tn} = \alpha_t \left(\sum_{j=1}^{23} \omega_j D_{W,tnj} \right) f(L_{tn}) + \varepsilon_{tn}; \qquad t = 1, \dots, 11, \quad n = 1, \dots, N(t). \tag{33}$$

We imposed the normalizations $\alpha_1 = 1$ and $\lambda_1 = 1$ so that all of the remaining parameters in equation (33) could be identified. These normalizations were also imposed in Model 4 below.

We used the final parameter values for the α_t and ω_j from Model 2 as starting coefficient values for Model 3 (with all λ_k initially set equal to 1). Thus Model 3 adds the four new marginal prices of land, λ_2, λ_3, λ_4, and λ_5, to Model 2. The final LL for Model 3 was −66,044.02, an increase of 978.88 for adding four new parameters. The R^2 value was 0.4668.

Our final land-price model added the subway variables to Model 3. Thus Model 4 was defined as the following non-linear regression model[41]:

$$\begin{aligned} V_{tn} &= \alpha_t \left(\sum_{j=1}^{23} \omega_j D_{W,tnj} \right) \left(1 + \eta(DS_{tn} - 50)\right) \left(1 + \theta(TT_{tn} - 4)\right) f(L_{tn}) + \varepsilon_{tn}; \\ & t = 1, \dots, 11, \quad n = 1, \dots, N(t). \end{aligned} \tag{34}$$

Thus Model 4 has added two new subway parameters, η and θ, to Model 3. We used the final parameter values for the α_t,ω_j and λ_k from Model 3 as starting coefficient values for Model 4 (with η and θ initially set equal to 0). The final log likelihood for Model 4 was −65,584.56, an increase of 459.46 for adding two new parameters. The R^2 value was 0.5401. The estimated coefficients for this model are listed in Table 8. The α_t sequence of estimated parameters (along with $\alpha_1 \equiv 1$) forms an annual (quality-adjusted) OLP series. For comparison purposes, we repeat each α_t four times and convert the annual OLP series into the quarterly OLP series, PL^t_{OLP}. It will be listed and compared with our final transactions-based MLIT land-price series PL^t_{MLIT} and its linear smooth PL^t_{L} along with our final REIT-based land-price series PL^t_{REIT} in the following section.

It can be seen that the standard errors on the estimated annual land prices α_t are fairly small; recall that they were fairly large for the REIT-based quarterly land-price series, PL^t_{REIT}. Except for λ_3, it can be seen that the λ_k decrease monotonically as k increases; this indicates that the marginal price of land decreases as the land-plot size increases. The two estimated subway parameters, η and θ, both have the expected negative sign and are reasonable in magnitude. Since we do not have additional information on the height or size of the buildings, we cannot add more explanatory variables to the Model 4 regression.

10. Comparing Land-Price Indexes from Different Sources

Table 6 lists the land-price series based on MLIT transactions PL^t_{MLIT} and its linear smooth, PL^t_{L}. Table 7 lists the REIT-based land-price series PL^t_{REIT} and the

[41]The minimum value for the distance to the nearest subway station DS_{tn} was 50 m and the minimum value for the subway running time from the nearest station to the central Tokyo subway station was 4 minutes.

TABLE 8
THE ESTIMATED COEFFICIENTS FOR MODEL 4 USING ANNUAL TAX ASSESSMENT DATA

Coefficient	Estimate	*t*-Statistic	Coefficient	Estimate	*t*-Statistic	Coefficient	Estimate	*t*-Statistic
ω_1	181.560	26.429	ω_{14}	172.950	14.037	α_5	1.2595	27.892
ω_2	164.660	25.788	ω_{15}	146.350	9.085	α_6	1.1865	27.452
ω_3	236.800	26.434	ω_{16}	213.590	21.443	α_7	1.1486	28.356
ω_4	263.190	24.031	ω_{17}	91.988	10.182	α_8	1.1382	27.710
ω_5	124.920	17.418	ω_{18}	83.365	17.515	α_9	1.1120	26.484
ω_6	126.740	22.236	ω_{19}	145.590	7.685	α_{10}	1.0919	25.122
ω_7	77.712	7.866	ω_{20}	193.350	6.975	α_{11}	1.1154	25.699
ω_8	84.417	8.496	ω_{21}	86.169	9.314	λ_2	0.7011	6.032
ω_9	137.330	18.302	ω_{22}	87.688	11.699	λ_3	−0.3331	−2.907
ω_{10}	230.320	14.959	ω_{23}	64.602	6.739	λ_4	0.3568	7.247
ω_{11}	101.550	12.494	α_2	1.0751	27.758	λ_5	0.1440	18.745
ω_{12}	195.970	13.212	α_3	1.1643	28.318	η	−0.000740	−27.937
ω_{13}	385.910	23.858	α_4	1.3399	28.916	θ	−0.022807	−45.349

OLP series PL^t_{OLP} can be constructed using the estimated α_t listed in Table 8. These four series, along with the official construction price series P_{St}, are listed in Table 9 and plotted in Figure 3.

It can be seen that the land-price series based on transactions data, PL^t_{MLIT}, and its linear smooth, PL^t_{L}, paint a very different picture of land-price movements as compared to the series based on appraisal values for commercial land in Tokyo, PL^t_{REIT}, and the series based on property tax assessed values, PL^t_{OLP}. As was noted in Section 1, appraisal prices tend to lag behind the movements in transaction prices and they also smooth the sales data. The same phenomenon evidently applies to assessed value prices. Figure 3 shows that the price indexes for commercial land based on appraisal and assessed values fluctuate far less than the index-based actual transactions prices. However, it can be seen that the appraisal and assessed value series do tend to move in the same direction as the transactions prices, but with a lag. The figure also shows the problem with the transactions-based series: its quarter-to-quarter fluctuations are massive. But it also can be seen that the linear smoothed series PL^t_{L} (which is essentially a centered five-quarter moving average of the unsmoothed series PL^t_{MLIT}) captures the trend in transactions prices quite well. This series can be finalized after a two-quarter delay. Our preferred land price series is the linear smoothed transaction series PL^t_{L}.

In the following section, we will use the MLIT and REIT data to construct alternative commercial property price indexes; that is, we will aggregate the land- and structure-price data into overall property price indexes and compare these indexes with other indexes that are simpler to construct.

11. A Comparison of Alternative Commercial Property Price Indexes

Recall that in Section 3, the MLIT value of property n in quarter t was defined as V_{tn} in period t and the corresponding property land and structure areas were defined as S_{tn} and L_{tn} for $n = 1, \ldots, N(t)$ and $t = 1, \ldots, 44$. In the property price literature, a frequently used index of overall property prices is the period average of the individual property values V_{tn} divided by the corresponding structure area S_{tn}. Thus define the (preliminary) quarter t mean property price P^t_{MEANP} as follows:

$$(35) \qquad P^t_{\mathrm{MEANP}} \equiv (1/N(t))\sum_{n=1}^{N(t)} V_{tn}/S_{tn}; \qquad t = 1, \ldots, 44.$$

The final mean property price index for quarter t, P^t_{MEAN}, is defined as the corresponding preliminary index P^t_{MEANP} divided by P^1_{MEANP}; that is, we normalize the series defined by equation (35) to equal 1 in quarter 1.

As could be expected, the mean property price series P^t_{MEAN} is rather volatile and so in order to capture the trends in Tokyo commercial property prices, it is necessary to smooth this series. We used the same linear smoothing procedure that was explained in Section 7 to construct the smoothed land-price series PL^t_{L}. Thus we set the initial value of the smoothed mean series, P^1_{MEANS}, equal to the corresponding unsmoothed value P^1_{MEAN}. We set the quarter-2 value of the smooth equal to the average of the first and third observations in the raw series (so that $P^2_{\mathrm{MEANS}} \equiv (1/2)[P^1_{\mathrm{MEAN}} + P^3_{\mathrm{MEAN}}]$.

TABLE 9
The Alternative Land-Price Series and the Price of Structures

Quarter t	PL^t_{MLIT}	PL^t_{L}	PL^t_{REIT}	PL^t_{OLP}	P_{St}
1	1.00000	1.00000	1.00000	1.00000	1.00000
2	1.55293	1.31711	1.02676	1.00000	0.99636
3	1.63422	1.42867	1.06369	1.00000	0.99211
4	1.53523	1.58159	1.10454	1.00000	0.98941
5	1.42096	1.70218	1.15004	1.07513	0.99184
6	1.76462	1.72654	1.19883	1.07513	0.99790
7	2.15588	1.87309	1.24744	1.07513	1.00351
8	1.75601	2.11368	1.29953	1.07513	1.00918
9	2.26798	2.25488	1.34499	1.16432	1.01241
10	2.62393	2.30843	1.38812	1.16432	1.01770
11	2.47061	2.45153	1.44194	1.16432	1.02208
12	2.42362	2.34178	1.49019	1.16432	1.02971
13	2.47153	2.15709	1.50818	1.33985	1.04572
14	1.71923	2.07466	1.49977	1.33985	1.09410
15	1.70045	1.88313	1.47635	1.33985	1.11708
16	2.05848	1.56540	1.44330	1.33985	1.09343
17	1.46597	1.35840	1.39905	1.25946	1.05114
18	0.88287	1.25719	1.34400	1.25946	1.02499
19	0.68422	1.04127	1.29130	1.25946	1.01141
20	1.19442	0.98237	1.24531	1.25946	0.99425
21	0.97889	1.05818	1.21368	1.18646	0.98011
22	1.17144	1.10914	1.18021	1.18646	1.00021
23	1.26194	1.02848	1.15830	1.18646	0.99197
24	0.93901	1.00673	1.14173	1.18646	0.98385
25	0.79111	1.01445	1.12141	1.14862	0.99586
26	0.87016	1.01155	1.10598	1.14862	1.00424
27	1.21003	1.08928	1.10225	1.14862	0.99826
28	1.24743	1.13287	1.09666	1.14862	0.99692
29	1.32764	1.18341	1.08618	1.13820	0.99776
30	1.00910	1.25811	1.07504	1.13820	1.00624
31	1.12286	1.24647	1.07151	1.13820	1.00058
32	1.58349	1.27629	1.06681	1.13820	1.00290
33	1.18925	1.35573	1.05778	1.11199	1.01027
34	1.47675	1.44159	1.04788	1.11199	1.02160
35	1.40632	1.46396	1.04320	1.11199	1.02960
36	1.55214	1.50250	1.03916	1.11199	1.05012
37	1.69536	1.54949	1.03814	1.09194	1.07326
38	1.38194	1.66709	1.04095	1.09194	1.08818
39	1.71167	1.63026	1.04657	1.09194	1.09886
40	1.99436	1.61806	1.05460	1.09194	1.11577
41	1.36798	1.64401	1.06887	1.11544	1.12204
42	1.63437	1.71076	1.08289	1.11544	1.12769
43	1.51167	1.73534	1.10053	1.11544	1.12651
44	2.04541	1.75991	1.12109	1.11544	1.11855

For the quarter-3 value of the smooth, we used the simple five-term centered moving average so that $P^3_{\mathrm{MEANS}} \equiv (1/5)[P^1_{\mathrm{MEAN}} + P^2_{\mathrm{MEAN}} + P^3_{\mathrm{MEAN}} + P^4_{\mathrm{MEAN}} + P^5_{\mathrm{MEAN}}]$ and we carried on using this five-term centered moving average until quarters 43 and 44, where we used rows four and five of the matrix $\mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T$ defined in Section 7 for our Henderson linear regression smoother. The resulting smoothed mean price series, P^t_{MEANS}, is listed in Table 10 and plotted in Figure 4. We note that the average value of the unsmoothed series P^t_{MEAN} is 1.1644, while the average value of the corresponding smoothed series P^t_{MEANS} is 1.1614.

TABLE 10
THE ALTERNATIVE OVERALL COMMERCIAL PROPERTY PRICE INDEXES

Quarter t	P^t_{MEAN}	P^t_{MEANS}	P^t_{FMLIT}	P^t_{FMLITS}	P^t_{FREIT}	P^t_{LPHED}	P^t_{LPHEDS}
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2	1.18211	1.11644	1.25260	1.15058	1.01858	1.12578	1.12971
3	1.23289	1.16886	1.26945	1.18790	1.04447	1.25942	1.16157
4	1.22061	1.21225	1.21852	1.23585	1.07364	1.24404	1.20967
5	1.20868	1.24444	1.18141	1.28993	1.10761	1.17860	1.24030
6	1.21694	1.28112	1.33886	1.32389	1.14501	1.24052	1.27364
7	1.34307	1.30365	1.51703	1.40608	1.18219	1.27892	1.32785
8	1.41632	1.43708	1.33632	1.46794	1.22201	1.42612	1.42052
9	1.33326	1.43627	1.56545	1.56057	1.25638	1.51511	1.45011
10	1.87582	1.46519	1.76501	1.63832	1.28961	1.64193	1.51268
11	1.21289	1.47075	1.64041	1.63363	1.33051	1.38846	1.54175
12	1.48766	1.44451	1.64183	1.61144	1.36814	1.59179	1.49237
13	1.44413	1.34393	1.66991	1.55087	1.38555	1.57148	1.43715
14	1.20203	1.36900	1.35804	1.48591	1.39181	1.26820	1.43887
15	1.37291	1.33842	1.37911	1.45152	1.38040	1.36580	1.35784
16	1.33824	1.25163	1.52799	1.32192	1.34982	1.39708	1.27652
17	1.33477	1.18789	1.23700	1.19171	1.30617	1.18664	1.21455
18	1.01018	1.10527	0.95988	1.12128	1.25869	1.16489	1.16196
19	0.88336	1.01512	0.86424	1.02333	1.21607	0.95832	1.08358
20	0.95981	0.92967	1.08210	0.98875	1.17747	1.10288	1.05599
21	0.88749	0.90702	0.97954	1.01588	1.15025	1.00516	1.03546
22	0.90749	0.90880	1.07696	1.04978	1.13033	1.04872	1.04410
23	0.89697	0.88131	1.11188	1.00911	1.11178	1.06220	1.02744
24	0.89223	0.88767	0.96497	0.99329	1.09728	1.00156	1.03771
25	0.82235	0.91494	0.90797	1.00329	1.08506	1.01955	1.06638
26	0.91929	0.93288	0.94799	1.00717	1.07552	1.05652	1.06417
27	1.04386	1.01361	1.09552	1.04126	1.07103	1.19205	1.11638
28	0.98669	1.06931	1.10571	1.05758	1.06634	1.05119	1.15239
29	1.29586	1.08824	1.14663	1.08420	1.05854	1.26258	1.16835
30	1.10084	1.08174	1.00751	1.11102	1.05216	1.19961	1.15112
31	1.01394	1.10295	1.05406	1.10498	1.04790	1.13634	1.17367
32	1.01135	1.04434	1.27150	1.13668	1.04475	1.10587	1.16224
33	1.09274	1.05850	1.08869	1.15609	1.03958	1.16397	1.16253
34	1.00285	1.08167	1.21316	1.19959	1.03469	1.20540	1.18833
35	1.17163	1.13272	1.19967	1.22353	1.03293	1.20105	1.23764
36	1.12980	1.14552	1.26047	1.24145	1.03474	1.26535	1.25125
37	1.26657	1.17903	1.36487	1.30179	1.03940	1.35244	1.27450
38	1.15674	1.19541	1.22411	1.34261	1.04489	1.23201	1.30450
39	1.17042	1.20649	1.36521	1.33301	1.05141	1.32164	1.32267
40	1.25353	1.21222	1.51213	1.35536	1.06127	1.35104	1.35608
41	1.18520	1.22743	1.23270	1.34508	1.07319	1.35622	1.40102
42	1.29522	1.23749	1.35747	1.38881	1.08475	1.51951	1.44634
43	1.23277	1.23569	1.31046	1.40686	1.09736	1.45668	1.49583
44	1.22073	1.23388	1.55012	1.42906	1.11051	1.54825	1.54532

Table 9 in the previous section lists the land-price index PL^t_{MLIT} based on the builder's model using the MLIT transactions data. Table 9 also lists the quarter-t structure price indexes, P_{St}. We can use the predicted values from the Model 11 regression explained in Section 5 in order to construct the imputed value of land sold during quarter t. This quarter-t value of land is defined as follows:

$$V^t_L \equiv \alpha_t \sum_{n=1}^{N(t)} \left(\sum_{j=1}^{4} \omega_j D_{W,tnj} \right) \left(\sum_{m=1}^{5} \chi_m D_{EL,tnm} \right) \left(1+\mu(H_{tn}-3)\right) \left(1+\eta(DS_{tn}-0)\right) \times \left(1+\theta(TT_{tn}-1)\right) f_L(L_{tn}); \qquad t=1,\ldots,44. \tag{36}$$

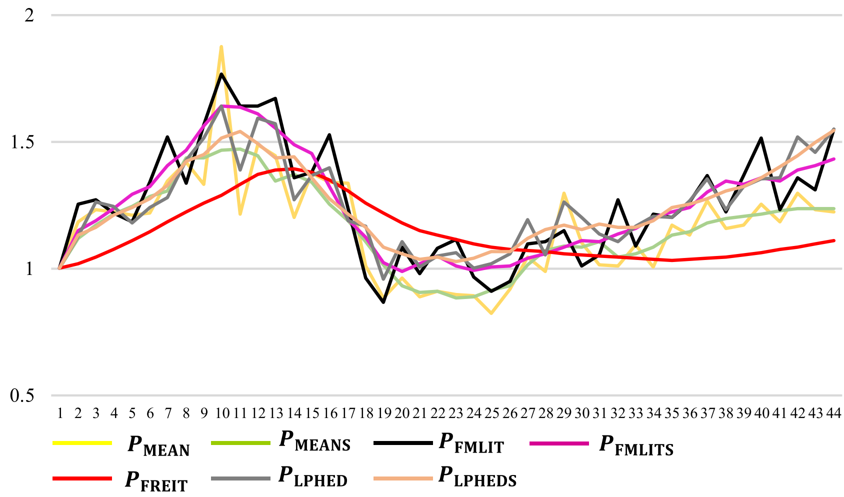

Figure 4. The Alternative Commercial Property Price Indexes Using MLIT and REIT Data [Colour figure can be viewed at wileyonlinelibrary.com]

In a similar fashion, we can use the predicted values from the Model 11 regression in order to define the imputed value of structures sold during quarter t, V_S^t, as follows:

$$V_S^t \equiv p_{St} \sum_{n=1}^{N(t)} g_A(A_{tn}) \left(\sum_{h=3}^{10} \phi_h D_{H,tnh} \right) S_{tn}; \qquad t=1,\ldots,44. \tag{37}$$

The *quality-adjusted quarter-t quantities of land and of structures*, Q_L^t and Q_S^t, are defined as follows:

$$Q_L^t \equiv V_L^t / PL_{\mathrm{MLIT}}^t ; Q_S^t \equiv V_S^t / P_{St}; \qquad t=1,\ldots,44. \tag{38}$$

With the prices and quantities of land and structures defined for each quarter, we calculated Fisher (1922) property price indexes, which are listed as P_{FMLIT}^t in Table 10 and plotted in Figure 4.[42]

From viewing Figure 4, it can be seen that the Fisher property price indexes using MLIT data, P_{FMLIT}^t, are quite volatile (due, of course, to the volatility of the MLIT land-price component indexes, P_{LMLIT}^t). The Henderson linear regression smooth of the unsmoothed land-price series P_{LMLIT}^t was listed as PL_{L}^t in Table 9. We use this smoothed land-price series along with the new land quantities defined as $Q_L^t \equiv V_L^t / PL_{\mathrm{L}}^t$ in order to define the smoothed Fisher property price index,

[42]The Laspeyres and Paasche indexes for quarter t are defined as $P_L^t \equiv [P_{\mathrm{LMLIT}}^t Q_L^1 + P_{St} Q_S^1]/[P_{\mathrm{LMLIT}}^1 Q_L^1 + P_{S1} Q_S^1]$ and $P_P^t \equiv [P_{\mathrm{LMLIT}}^t Q_L^t + P_{St} Q_S^t]/[P_{\mathrm{LMLIT}}^1 Q_L^t + P_{S1} Q_S^t]$, respectively. The quarter-t Fisher index is defined as $P_{\mathrm{FMLIT}}^t \equiv [P_L^t P_P^t]^{1/2}$, for $t = 1, \ldots, 44$. For additional materials on these indexes, see Fisher (1922). The Fisher index has strong economic and axiomatic justifications (see Diewert, 1976, 1992). We also calculated chained Fisher property price indexes using the same data, but these indexes were virtually the same as the Fisher fixed-base indexes listed in Table 10.

P^t_{FMLITS}, which is listed in Table 10 and plotted in Figure 4. This series is our preferred measure of overall commercial property prices for Tokyo.

Recall Model 3 in Section 8, which used REIT data to implement a version of the builder's model. We can use the predicted values from the Model 3 regression in order to construct the imputed value of land sold during quarter t. This quarter-t value of land is defined as follows:

$$V^t_L \equiv \sum_{n=1}^{41} \alpha_t \omega_n L_{tn}; \qquad t = 1, \ldots, 44. \tag{39}$$

In a similar fashion, we can use the predicted values from the Model 3 REIT regression in order to define the impute value of structures sold during quarter t, V^t_S, as follows:

$$V^t_S \equiv \sum_{n=1}^{41} p_{St}(1-\delta)^{A(t,n)} S_{tn} \qquad t = 1, \ldots, 44. \tag{40}$$

The (REIT data based) quality-adjusted land price for quarter t is the α_t that appears in equation (39) and is listed as PL^t_{REIT} in Table 9. The price of structures is $P^t_S = p_{St}$, where p_{St} is the official construction price index. The corresponding period-t quantities of land and structures are defined as follows:

$$Q^t_L \equiv V^t_L / PL^t_{\text{REIT}}; \quad Q^t_S \equiv V^t_S / P^t_S; \qquad t = 1, \ldots, 44. \tag{41}$$

The overall REIT-based property price index for quarter t is defined as the Fisher index P^t_{FREIT}, using the above prices and quantities for land and structures as the basic building blocks. The REIT-based overall property price series P^t_{FREIT} is listed in Table 10 and plotted in Figure 4. It can be seen that this series is not volatile and does not require any smoothing.

Our final property price index will be generated by a traditional log price time dummy hedonic regression using the MLIT data.[43]

We use the same notation and definitions of variables as was used in Section 4. Define the natural logarithms of V_{tn}, L_{tn} and S_{tn} as LV_{tn}, LL_{tn} and LS_{tn}, for t = 1, …, 44 and n = 1, …, $N(t)$. The log price time dummy hedonic regression model is the following linear regression model:

$$\begin{aligned} LV_{tn} = \beta_t + \sum_{j=2}^{4} \omega_j D_{W,tnj} + \gamma A_{tn} + \lambda LL_{tn} + \mu LS_{tn} + \sum_{h=4}^{10} \phi_h D_{H,tnh} \\ + \eta DS_{tn} + \theta TT_{tn} + \varepsilon_{tn}; \qquad t = 1, \ldots, 44, \quad n = 1, \ldots, N(t). \end{aligned} \tag{42}$$

The four combined-ward dummy variables $D_{W,tnj}$ were defined by equation (4) and the discussion around Model 2 in Section 3. The building-height dummy variables, $D_{H,tnh}$, were defined just above equation (19) in Section 3. However, due to the small number of observations in heights equal to 10–14 stories, all buildings in this range were aggregated into the ten-story height category. As usual, A_{tn} is the

[43]Recent developments in estimating traditional log price hedonic regression property models are reviewed by Hill *et al.* (2018) and Silver (2018).

age of building n sold in quarter t, and DS_{tn} and TT_{tn} are the two subway variables pertaining to building n in quarter t. The 44 time dummy-variable coefficients are $\beta_1, \ldots, \beta_{44}$. Note that the dummy variable for the first combined ward, $D_{W,tn1}$, is not included in the linear regression defined by equation (42) in order to prevent multicollinearity. Similarly, the dummy variable for a building height equal to 3 was also excluded from the regression to prevent multicollinearity. There are 59 unknown parameters in the regression. The R^2 value for this regression was 0.7593. This is higher than our Model 9 and Model 11 R^2 values using the same data, which were 0.7091 and 0.7143, respectively. The estimated coefficients and their t-statistics are listed in Table 11.

The standard errors for the time coefficients β_t were fairly large (in the 0.13–0.15 range). Define the unnormalized land price for quarter t, α_t, as the exponential of β_t; that is, $\alpha_t \equiv \exp(\beta_t)$, for $t = 1, \ldots, 44$. The log price hedonic regression property price for quarter t, P^t_{LPHED}, is defined as α_t/α_1, for $t = 1, \ldots, 44$. This traditional hedonic regression model property price index P^t_{LPHED} is listed in Table 10 and graphed in Figure 4.

It is interesting to note that our estimated λ and μ parameters almost sum to unity. Thus a generic commercial property sold in quarter t at price P with land and structure areas L and S, respectively, has a price that is approximately proportional to the Cobb–Douglas function $\alpha_t L^\lambda S^\mu$, which has returns to scale that are approximately equal to one. Note also that the estimated ω_k follow the same pattern that we saw in Sections 3–5 for land prices; that is, the composite ward 1 is the most expensive ward, ward 2 is the next most expensive, ward 3 is less expensive again, and ward 4 has the lowest level of property prices. The height dummy variables exhibit the same trends that we saw in our MLIT builder's models: the higher the height of the structure, the higher is the price of the property. Finally, the parameter for the distance from the nearest subway station, η, is significantly negative, indicating that the property value falls as the distance increases. The subway travel-time parameter θ has an unexpected positive sign but is not significantly different from 0. Finally, it is possible to convert the estimated age coefficient γ into an estimate for a geometric rate of structure depreciation, δ. The formula for this conversion is $\delta \equiv 1 - e^{\gamma/\beta}$.[44] When this conversion formula is utilized, we find that our estimated δ is 0.01945; that is, the traditional hedonic regression model generates an implied annual geometric depreciation rate equal to 1.945 percent per year, which is a reasonable estimate.

Viewing Table 10 or Figure 4, it can be seen that the time dummy hedonic regression model implied property price index P^t_{LPHED} is just as volatile as the corresponding builder's model property price index P^t_{FMLIT}. Thus we apply our modified Henderson linear smoothing operator to P^t_{LPHED} which produces the smoothed series P^t_{LPHEDS}, which is also listed in Table 10 and plotted in Figure 4.

The two top jagged lines are the Fisher property price index using the builder's model, P^t_{FMLIT}, and the log price time dummy hedonic regression property price index, P^t_{LPHED}. Both of these series use the MLIT sales-transaction data. Their linear smooths are P^t_{FMLITS} and P^t_{LPHEDS}. It can be seen that these two smoothed

[44]For derivations of this formula, see McMillen 2003, pp. 289–90), Shimizu *et al.* (2010), and Diewert *et al.* 2017, p. 24).

TABLE 11
The Estimated Coefficients for the Log Price Time Dummy Hedonic Regression Model

Coefficient	Estimate	*t*-Statistic	Coefficient	Estimate	*t*-Statistic	Coefficient	Estimate	*t*-Statistic
β_1	−0.20529	−1.542	β_{21}	−0.20015	−1.393	β_{41}	0.09941	0.718
β_2	−0.08681	−0.607	β_{22}	−0.15772	−1.076	β_{42}	0.21310	1.472
β_3	0.02537	0.194	β_{23}	−0.14495	−1.024	β_{43}	0.17087	1.205
β_4	0.01308	0.100	β_{24}	−0.20373	−1.462	β_{44}	0.23184	1.554
β_5	−0.04096	−0.299	β_{25}	−0.18593	−1.248	γ	−0.00970	−10.490
β_6	0.01025	0.075	β_{26}	−0.15031	−0.981	μ	0.49390	12.890
β_7	0.04072	0.301	β_{27}	−0.02962	−0.203	λ	0.52956	14.200
β_8	0.14967	1.077	β_{28}	−0.15537	−1.098	ω_2	−0.32900	−13.990
β_9	0.21020	1.524	β_{29}	0.02786	0.195	ω_3	−0.49677	−16.620
β_{10}	0.29059	2.097	β_{30}	−0.02329	−0.160	ω_4	−0.73092	−21.530
β_{11}	0.12291	0.844	β_{31}	−0.07748	−0.531	ϕ_4	0.02949	0.868
β_{12}	0.25957	1.811	β_{32}	−0.10465	−0.698	ϕ_5	0.12691	3.384
β_{13}	0.24673	1.738	β_{33}	−0.05345	−0.378	ϕ_6	0.12419	2.766
β_{14}	0.03231	0.227	β_{34}	−0.01848	−0.132	ϕ_7	0.18867	3.748
β_{15}	0.10645	0.751	β_{35}	−0.02209	−0.160	ϕ_8	0.31515	5.874
β_{16}	0.12910	0.861	β_{36}	0.03006	0.214	ϕ_9	0.45593	7.579
β_{17}	−0.03416	−0.239	β_{37}	0.09662	0.712	ϕ_{10}	0.55094	8.261
β_{18}	−0.05266	−0.323	β_{38}	0.00336	0.023	η	−0.00018	−4.234
β_{19}	−0.24786	−1.715	β_{39}	0.07358	0.522	θ	0.00067	0.458
β_{20}	−0.10737	−0.737	β_{40}	0.09558	0.680			

series approximate each other reasonably well.[45] What is somewhat surprising is that the smoothed mean index P^t_{MEANS} (which uses the same transactions data) approximates the two smoothed hedonic indexes to some degree, but the series gradually diverge due to the fact that an index based on average prices per square meter cannot take depreciation into account.[46] The hills and valleys in the P^t_{MEANS} series are less pronounced than the corresponding fluctuations in the P^t_{FMLITS} and P^t_{LPHEDS} series but the turning points are the same. Finally, it can be seen that the Fisher property price series that is based on appraised values of properties, P^t_{FREIT} , does not provide a good approximation to the two smoothed series based on transactions, the P^t_{FMLITS} and P^t_{LPHEDS} series. The fluctuations in P^t_{FREIT} are too small and the turning points in this series lag well behind our preferred series.

12. Conclusion

Our main conclusions are as follows:

- It is possible to construct a quarterly transactions-based commercial property price index that can be decomposed into land and structure components.
- The main characteristics of the properties that are required in order to implement our approach are: (i) the property location (or neighborhood); (ii) the floor-space area of the structure on the property; (iii) the area of the land plot; (iv) the age of the structure; and (v) the height of the building. We also require an appropriate exogenous commercial property construction cost index that gives the average cost of construction per square meter for each period in the sample.
- The land-price index that our hedonic regression model generates may be too volatile and hence may need to be smoothed. We found that a slightly modified five-quarter moving average of the raw land-price indexes did an adequate job of smoothing. This means that the final land-price index could be produced with a two-quarter lag.
- We found that a smoothed version of a traditional log price time dummy hedonic regression model produced an acceptable approximation to our preferred smoothed builder's model overall price index.
- We also found that a very simple overall price index that is proportional to the quarterly arithmetic average of each property price divided by the corresponding structure area provided a rough approximation to our preferred price index. This model cannot take depreciation into account and hence will in general have an downward bias, but it has the advantage of requiring information on only a single property characteristic (the structure floor-space area) in order to be implemented.

[45]Diewert (2010) noticed that the Fisher property price index generated by the builder's model frequently approximated the traditional log price time dummy property price index using the same data. However, the key to a successful approximation is that the time dummy model must generate a reasonable implied structure depreciation rate, which is the case for our particular dataset.

[46]If the age structure of the quarterly sales of properties remains reasonably constant, then this neglect of depreciation will not be a factor.

6. The price indexes that were based on appraisal and assessed value information were not satisfactory approximations to the transactions-based indexes. The turning points in these series lagged our preferred series and the appraisal-based series smoothed the data-based series to an unacceptable degree.[47]
7. The two versions of the builder's model that estimated multiple (net) depreciation rates produced virtually the same indexes and virtually identical depreciation schedules. These rates of depreciation changed materially as the structure aged and the depreciation rates became appreciation rates for structures over age 40.

Our overall conclusion is that it should be possible for national income accountants to construct acceptable commercial land-price series using transactions data on the sales of commercial properties. The required information on the characteristics of the properties is being collected by some private-sector businesses. It should be possible for government statisticians to collect the same information using building permit, land registry, and property assessment data.

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[47]These points are well known in the real estate literature; (see Geltner et al., 2014, Ch. 25).

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