the review of income and wealth

Review of Income and Wealth
Series 65, Number 1, March 2019
DOI: 10.1111/roiw.12313

DIFFERENTIATING BETWEEN DIMENSIONALITY AND DURATION IN MULTIDIMENSIONAL MEASURES OF POVERTY: METHODOLOGY WITH AN APPLICATION TO CHINA

BY AARON NICHOLAS*

Deakin University

RANJAN RAY

Monash University

AND

KOMPAL SINHA

Macquarie University

We develop a multidimensional poverty measure that is sensitive to the within-individual distribution of deprivations across dimensions and time. Our measure combines features from a static multidimensional measure (Alkire and Foster, 2011a) and a time-dependent unidimensional measure (Foster, 2009). The proposed measure separately identifies—and can therefore be decomposed according to—the proportion of the poverty score attributable to: (i) the concentration of deprivations within periods; (ii) the concentration of deprivations within dimensions. In doing so it allows for a poverty ranking that is robust to assumptions about the trade-off between the two components. Previous measures have not allowed for the features proposed here due to the inability to calculate the exact contribution of each dimension to overall poverty. We overcome this by adapting to our measure the Shapley decomposition proposed in Shorrocks (2013) (based on Shapley, 1953). The measure is applied to data from China, 2000-2011.

JEL Codes: I31, I32

Keywords: multidimensional poverty, poverty in China, poverty duration, subgroup decomposability, Shapley decomposition

1. INTRODUCTION

Traditional income-based measures of poverty have been extended along two major directions: the broadening of the measures to incorporate a wider set of dimensions that jointly give a more accurate representation of welfare; and the lengthening of the measures to incorporate information that spans over several periods of observations. Extensions along the first direction, largely influenced by the writings of Sen (1985) and the popularity of the Human Development Index, focus on the multidimensional aspect of poverty: this is usually based on the

Note: We are grateful for comments from three anonymous referees, Gordon Anderson, François Bourguignon, Gaurav Datt, Birendra Rai and participants at several seminar/conference presentations. The usual disclaimer applies.

*Correspondence to: Aaron Nicholas, Department of Economics, Deakin University, Burwood, Victoria 3125, Australia (aaron.nicholas@deakin.edu.au)

individual's lack of access to a set of dimensions that include both market and non-market goods. Following Sen (1976)'s axiomatic approach to poverty measurement, there have been numerous axiomatic approaches to multidimensional poverty—examples include Tsui (2002), Atkinson (2003), Bourguignon and Chakravarty (2003), and Alkire and Foster (2011a) [henceforth AF].[1]

Extensions along the second direction consider the time aspect where repeated observations of deprivation are treated differently to cases where deprivation is infrequent. This is motivated by the need for differentiating chronic poverty from transient poverty: long spells of poverty may lead to social exclusion from which recovery may be very difficult; see, for example, Walker (1995). Examples of extensions to (income-based) poverty measures based on such a view include Foster (2009), Calvo and Dercon (2009), Hojman and Kast (2009), Duclos *et al*. (2010), Hoy and Zheng (2011), Bossert *et al*. (2012), Gradin *et al*. (2012) and Foster and Santos (2013).

Despite the usefulness provided by extensions along both directions, the literature has largely considered both extensions independent of each other, retaining either the unidimensional or static property of traditional measures. While still relatively rare, the increasing availability of nationally representative household and individual level *longitudinal* data on a wide range of dimensions of deprivation has prompted the question of how best to *jointly* make use of both extensions when making poverty assessments. Papers that have attempted to do so include Nicholas and Ray (2012), Bossert *et al*. (2014) and Alkire *et al*. (2014).[2]

While static poverty measures allow us to observe how aggregate level poverty changes over time (by calculating a poverty score for each period of observation), it is never clear if it is the same individuals who are becoming more (or less) deprived. A multidimensional measure of poverty that is sensitive to the length of deprivation would be able to account not only for whether the same individuals are getting more deprived over time, but also whether they are doing so in the same dimensions. This is particularly useful when the main intent of the policy maker is comparisons across groups of individuals over certain periods of time.

The principal motivation of this paper is to contribute to the relatively scant literature on time-dependent multidimensional measures of poverty by construction a measure that is:

(C1) sensitive to the distribution of deprivations *across* individuals, even when deprivations are only measured in an ordinal manner.

(C2) sensitive to the distribution of deprivations *within* individuals, thus allocating different weights to individuals with different distributions of deprivations across time and dimensions despite each individual having the same count of deprivations. Specifically, a higher weight is allotted to individuals

[1]See, also, Chakravarty and D'Ambrosio (2006), Bossert *et al*. (2007), Jayaraj and Subramanian (2010) and Permanyer (2014) for closely related work on the measurement of multidimensional deprivation. In contrast to measures such as the Human Development Index and Human Poverty Index, the class of subgroup decomposable measures aggregate first over dimensions for each individual prior to aggregating over individuals—Dutta *et al*. (2003) highlight the advantages of doing so.

[2]See also Merz and Rathzen (2014) where the time element is introduced into multidimensional poverty measurement by proposing a measure that "quantifies the shortest path to escape multidimensional poverty" Merz and Rathzen (2014, p. 555).

who experience deprivations across multiple dimensions within the same period ("dimensional convexity"), as well as to individuals who experience deprivations across multiple periods within the same dimension ("duration convexity").

(C3) decomposable into three components, notably, the component of poverty due to the count of deprivations; the component of poverty due to allowing for dimensional convexity; and the component of poverty due to allowing for duration convexity.

We provide an example to help elucidate these contributions in the next subsection.

It is important to note that, while desirable, the literature has shied away from measures satisfying the properties above due to the desirability of *dimensional decomposability*, a property where the contribution of each dimension to overall poverty can be additively decomposed. This is violated when poverty is a non-linear function of the count of dimensions of deprivation. We overcome this problem by applying the Shapley decomposition proposed in Shorrocks (2013, based on Shapley, 1953) specifically adapted to suit our proposed measure. To the best of our knowledge this method has not been applied to dimensional decomposition with the exception of Datt (2013) who has applied the technique to the static multidimensional case. In being able to differentiate between "dimensional convexity" and "duration convexity" the proposed measure also has the advantage of allotting different weights to both features and, consequently, is also able to provide a test of the robustness of a ranking of subgroups (such as provinces within the country) to assumptions about the trade-off between the two features.

We apply the proposed time-dependent multidimensional poverty measure to longitudinal data from China (2000–11). China is particularly useful to illustrate the application of our measure since while it is now well accepted that China has seen one of the largest poverty reductions over the past few decades, questions of how these differ across provinces and different subgroups is less established. While traditional static measures such as AF are well suited for characterizing changes in poverty over time for one specific group, they are less-suited for comparisons across groups, since different groups may improve or decline at different periods of time. To our knowledge, this is the first attempt at analyzing poverty in China on longitudinal data using a time-dependent multidimensional poverty measure.

As Lahoti *et al.* (2015) report, the reduction in Chinese poverty has been so dramatic that the headcount rate of world poverty alters sharply depending on the inclusion or exclusion of China from the calculations. Consequently, considerable attention has been paid by economists to studying poverty in China—see, for example, Bardhan (2010, Ch. 7), and the chapter by Park and Wang (2014) in the recent volume on China edited by Fan *et al.* (2014). Thanks to the increasing availability of data, there has been in recent years a significant literature on multidimensional poverty in China. Examples include Labar and Bresson (2011), Mishra and Ray (2012), Ray and Sinha (2015) who perform a static analysis of multidimensional poverty in China using the measure due to AF while You *et al.* (2014) examine the intertemporal aspect of multidimensional poverty using the measure due to Dutta *et al.* (2003). The present study contributes to this recent literature by providing the first time-dependent analysis of multidimensional poverty in China. We provide a ranking of subgroups according to provinces, gender and rural/urban residency, as well as a ranking of dimensional contributions. In addition, these rankings are assessed for robustness to the choice of the weight allotted to dimensional versus duration convexity.

1.1. *Examples for Contributions*

While straightforward in its interpretation and implementation, the popular "counting" approach to poverty measurement faces several limitations when attempting to fully utilize the wealth of information over multiple dimensions and periods contained in panel data. Consider below the deprivation profiles $\boldsymbol{D_n}$ of three individuals, $A,\ B$ and C. Each entry in the profile takes a value of 1 if an individual $n \in \{A,\ B,\ C\}$ is deprived in a particular dimension $j \in \{1, 2, 3\}$ at time $t \in \{1, 2, 3\}$, and a value of 0 otherwise. The rows represent the dimensions j and the columns, the periods t.

$$\boldsymbol{D_A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \boldsymbol{D_B} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \boldsymbol{D_C} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Consider a counting measure of poverty where deprivation is measured simply as the average count of deprivations. If we treat all three dimensions and time periods as equally valued and treat all three individuals as poor, then such a measure would score all three individuals as equally deprived. Such a measure would therefore be insensitive to: (i) the distribution of deprivations *across* the profiles; and (ii) the distribution of deprivations *within* the profiles.

We examine each of our proposed contributions in the context of the example:

***C1 (sensitivity to between-individual rearrangement)*:**

Existing approaches are insensitive to "rearrangements" across the deprivation profiles. If the deprivation of individual A at $j=2,\ t=2$ is switched with the equivalent for individual B, individual A would now have two counts of deprivations, and individual B would have four. While deprivation is now more concentrated in individual B, a poverty score that simply takes the average count of deprivations would remain the same. This insensitivity to the distribution of deprivations across individuals is relaxed to a lesser extent in AF's static measure through the use of the α parameter, which at $\alpha > 1$ gives increasing sensitivity to transfers across individuals, where "transfers" exclude rearrangements of the entries as just described. In addition, such sensitivity only arises when deprivations are cardinal.[3] Sensitivity to rearrangement, however, can be achieved by the exponentiation of each individual's deprivation count ratio by some parameter,

[3]In the context of panel data over multiple periods and multiple dimensions, Alkire *et al.* (2014) consider an approach where an additional cut-off (triple cut-off) is adopted to increase the sensitivity of the poverty measure to unequal distributions. We discuss this in more detail in Footnote 4.

which would be analogous to the $\alpha > 1$ parameter of the original Foster *et al.* (1984) measure. The reluctance to do so in the multidimensional literature has stemmed primarily from the desirability of dimensional decomposability, that is, the ability to identify the proportion of contribution of each dimension to the total poverty score, which is in principle violated when individual deprivation scores are not a linear function of the count of deprivations (for a discussion, see Alkire and Foster, 2016). We are able to overcome this through the use of the general Shapley decomposition method proposed in Shorrocks (2013), which, broadly, allows any output (in our case, the poverty index) to be allocated among the contributors (in our case, the dimensions of deprivation), even if the output is a non-linear function of the contributions.

***C2 (sensitivity to within-individual rearrangement)*:**

While the exponentiation of each individual's deprivation count ratio would make the poverty measure sensitive to the distribution of deprivations *across* individuals, the measure would continue to rank all three individuals A, B and C as equally deprived since it is not sensitive to the distribution *within* individuals.

The lack of such differentiation is problematic in many applications. Stiglitz *et al.* (2009), for example, highlight that "the consequences for quality of life of having multiple disadvantages [across different domains] far exceed the sum of their individual effects" (Stiglitz *et al.*, 2009, p. 15). The importance of recognizing the increasing cost of multiple deprivation (for any given period) is discussed in detail in Datt (2013). This is captured by the property "dimensional convexity," which scores individual B as more deprived than individual A.

There is also an underlying belief that *recurring* deprivations within the same dimension incur an increasing cost on the individual (see Sengupta, 2009; Hoy and Zheng, 2011, and Gradin *et al.*, 2012). This is captured by the property "duration convexity," which scores individual C as more deprived than individual A.

***C3 (within profile decomposability)*:**

While in many applications individuals B and C should be ranked as more deprived than individual A, it is not clear if individual B should be ranked as more, less or equally deprived as individual C. Our proposed measure allows one to weight the component due to dimensional convexity and duration convexity differently according to the analyst's priors, while simultaneously ensuring that individuals B and C are never ranked as less deprived than individual A. In doing so, the proposed measure becomes decomposable into three components, notably the component of poverty due to: the count of deprivations; dimensional convexity; and duration convexity.

Overall, our measure allows for a deeper look into the "black box" of the aggregate poverty score and allows us to differentiate subgroups that may have similar counts of deprivation, but a very different distribution of said deprivations. This allows us to identify subgroups of the population that contain individuals who not only have the most counts of deprivations, but who also experience them across the widest variety of dimensions in any given period, and/or who also experience them for the most periods in any given dimension. This also allows us, when doing dimensional decomposition, to allot more weight not only to dimensions which have longer average durations, but also to dimensions that occur within individuals who simultaneously suffer from the widest variety of deprivation.

The rest of the paper is organized as follows. Section 2 presents the analytical framework. Section 3 discusses the Shapley decomposition technique. Section 4 describes the dataset used for illustration of the proposed measures. The results from our empirical application are presented in Section 5. Conclusions are drawn in Section 6.

2. Analytical Framework

2.1. *Notation*

Consider a population of N individuals, J different dimensions of deprivation and T equally-spaced periods of time. x_{njt} is individual $n \in \{1, 2, \ldots, N\}$'s *achievement* in dimension $j \in \{1, 2, \ldots, J\}$ at time $t \in \{1, 2, \ldots, T\}$. Each n can be said to have an *individual achievement profile* $\boldsymbol{A_n} = \begin{pmatrix} x_{n11} & \ldots & x_{n1T} \\ . & \ldots & . \\ x_{nJ1} & \ldots & x_{nJT} \end{pmatrix}$. The *population achievement profile* is a vector $\boldsymbol{\rho} = (\boldsymbol{A}_1, \ldots, \boldsymbol{A}_N)$. Define the identification vector $\boldsymbol{v} = (c_1, \ldots, c_N)$ where c_n takes the value 1 if the individual is considered *poor*, and 0 otherwise. We return to the issue of whom to consider poor at the end of this section. A poverty index is a function $g(\boldsymbol{\rho}; \boldsymbol{v})$ that produces a single non-negative real number for any observed vector $\boldsymbol{\rho}$ and appropriately defined vector $\boldsymbol{v}$.

We say that n is *deprived* in dimension j at time t when $x_{njt} < F_j$, where F_j is a deprivation cut-off that determines whether or not an individual is considered deprived in a particular dimension at a particular time and $\boldsymbol{F}$ the vector of such cut-offs. For example, in the dimension "health," x may be the individual's Body Mass Index, in which case F_{health} would be some threshold below which the individual would be considered underweight and therefore deprived in the health dimension. For brevity, we assume these cut-offs do not vary across time, though the methodology allows for such an extension.

It is common for $\boldsymbol{\rho}$ to be transformed into the *population deprivation profile* $\boldsymbol{\delta} = (\boldsymbol{D}_1, \ldots, \boldsymbol{D}_N)$ where $\boldsymbol{D_n}$ is the *individual deprivation profile*, a $J \times T$ matrix where each element of $\boldsymbol{A_n}$ is transformed into *deprivations* defined as follows:

$$(1) \qquad d^{\alpha}_{njt} = \begin{cases} \left(1 - \dfrac{x_{njt}}{F_j}\right)^{\alpha} & \text{if } x_{njt} < F_j \\ 0 & \text{otherwise} \end{cases} \quad \forall j, \ t$$

where $\alpha \geq 0$ is a sensitivity parameter. When achievement levels are ordinal in at least one dimension, it is common to restrict $\alpha = 0$ such that $d^{\alpha}_{njt} \in \{0, 1\} \forall \ j, t$.

The function $h: \ \boldsymbol{D_n} \rightarrow R_+$ produces a deprivation score s_n for each individual. Following Bourguignon and Chakravarty (2003), AF, and the majority of axiom-based multidimensional measures, we restrict ourselves to the class of subgroup decomposable measures of the form:

$$m(\boldsymbol{\rho};\mathbf{v})=\frac{1}{N}\sum_{n=1}^{N}[h(\boldsymbol{D_n})\times c_n] \tag{2}$$

Our key contribution is therefore with regards to the form of $h(\boldsymbol{D_n})$ that yields contributions **C1, C2** and **C3**.

Following the "dual cut-off" method of the AF class of poverty measures, the poverty indicator function c_n takes the form:

$$c_n=\begin{cases}1 \text{ if } \sum_{t=1}^{T}\sum_{j=1}^{J}d_{njt}^{0}\geq z \\ 0 \qquad\qquad \text{otherwise}\end{cases} \tag{3}$$

where $(J\times T)\geq z\geq 1$. At $z=1$ we have the equivalent of the union method of identification, and at $z=(J\times T)$, the intersection method. Notice however, that unlike the AF method, deprivations are counted both across dimensions and time. This opens up the possibility of identifying the poor using an additional cut-off.[4] Clearly the choice of who to consider poor will affect the final poverty score. Yalonetzky (2014) for example, shows that in the static multidimensional case (e.g. AF), the idea of robustness to changes in parameter choices becomes exponentially demanding and unlikely to be satisfied when there are more than two dimensions. However, since the contribution of our proposed measure is the expansion of ways in which to quantify the *depth* of poverty among the poor $[h(\boldsymbol{D_n})]$, rather than whom to consider poor (c_n), we define our axioms independent of identification choices. For ease of exposition, the union method of identification is adopted in both the examples and empirical application. In the case of multidimensional measures, the union method of identification also has the added advantage of satisfying the strong transfer axiom (see for example, Datt, 2013).

2.2. *Dimensional and Durational Convexity—contribution C2*

Recall the three individuals from the introduction. Differentiating between them requires that the poverty measure assigns an increasingly higher weight to deprivations that share either the same period or the same dimension. This yields two properties that can be stated formally as follows:[5]

[4]Our measure can be extended to include the "triple cut-off" found in Alkire *et al.* (2014), though in this application we focus on properties associated with the more familiar dual cut-off framework of AF. An alternative approach in the static multidimensional framework can be found in Permanyer (2014); there the "Strong Focus" axiom maintained in the multidimensional poverty measurement literature is relaxed, allowing the welfare of non-poor households to enter the poverty measure. In Permanyer and Riffe (2015) the dual cut-off approach is generalised to allow the poverty cut-off to be defined by different combinations of dimensions.

[5]The following axioms are defined over deprivations, rather than achievements. The use of d_{njt}^{α} as the primitive in these axioms imply that equation (1) is not the only form of d_{njt}^{α} that satisfies Axioms 1 and 2. The literature has, for consistency, typically used the specification in equation (1). More importantly however, the conversion of achievements to deprivations is typically a function of data availability. In principle, achievements in every dimension are cardinal but in practice, are ordinal due to lack

Let the $m \in \{1 \ldots M\}$ where M is the number of poor individuals in the population.

Axiom 1: (*Dimensional Convexity*)

$$\frac{\partial^2 g}{\partial d^{\alpha}_{mjt} \partial d^{\alpha}_{mj't}} > 0 \ \forall \ j' \neq j$$

The effect of an increase in any of an individual's deprivation on the aggregate poverty score is a strictly positive function of the deprivations in *other dimensions* that share the *same period* as the deprivation in question.

Axiom 2: (*Durational Convexity*)

$$\frac{\partial^2 g}{\partial d^{\alpha}_{mjt} \partial d^{\alpha}_{mjt'}} > 0 \ \forall \ t' \neq t$$

The effect of an increase in any of an individual's deprivation on the aggregate poverty score is a strictly positive function of the deprivations in *other periods* that share the *same dimension* as the deprivation in question.

Notice that in the three person example, we have suggested that B and C are more deprived than A, but it is unclear if B is equally or more deprived than C, or vice versa. We may, for example, consider C to be more deprived than B from a simple policy perspective: C's long-lasting deprivation in a specific dimension can be targeted for future poverty reduction and should therefore be given more weight. One may instead take the opposite stance and suggest that while B's poverty was relatively more transient than C, it was "broader" at the time it occurred and may reflect a vulnerability to shocks. We seek a measure that allows the analyst such flexibility in deciding how important *dimensional convexity* should be relative to *durational convexity.* In addition, even in situations where the choice of whom to weight more heavily is not clear, our measure provides a means of checking for robustness by considering the entire range of such trade-offs.

Any super additive form for $h(\boldsymbol{D_n})$ will satisfy both axioms. However, we also require that $h(\boldsymbol{D_n})$ differentiates between the effects of *dimensional convexity* versus *durational convexity* on the aggregate poverty score. Our poverty measure therefore consists of two delineated components, the first of which only satisfies *dimensional convexity*, and the second of which only satisfies *durational convexity.*

Consider then the following measure of poverty that satisfies only *dimensional convexity*:

of data. Defining the axioms over the deprivations therefore describe properties that hold regardless of whether the actual data is cardinal or ordinal, and allow the properties to be stated in terms of strict convexity.

$$\Omega^{dimension} = \frac{1}{N}\sum_{n=1}^{N}\left(\frac{1}{T}\sum_{t=1}^{T}\left(\frac{1}{J}\sum_{j=1}^{J}d_{njt}\right)^{\beta}\right)\times c_n \tag{4}$$

Notice that $\Omega^{dimension}$ is a modification of the AF index—instead of using α raised over each d_{njt}, β is raised over the average count of deprivation over dimensions, thus allowing *dimensional convexity* as is done in Chakravarty and D'Ambrosio (2006), Jayaraj and Subramaniam (2010) and Datt (2013). The measure $\Omega^{dimension}$ is then simply a modified AF index calculated for each period separately, and then averaged over these periods. Because it is only averaged over periods it fails to satisfy *durational convexity*, just as we had required.

Similarly, the following measure satisfies only *durational convexity*:

$$\Omega^{duration} = \frac{1}{N}\sum_{n=1}^{N}\left(\frac{1}{J}\sum_{j=1}^{J}\left(\frac{1}{T}\sum_{t=1}^{T}d_{njt}\right)^{\beta}\right)\times c_n \tag{5}$$

$\Omega^{duration}$ is a modification of the individual-level Foster (2009) chronic measure, and a special case of the more general Gradin *et al.* (2012) measure.[6] The measure $\Omega^{duration}$ is calculated for each dimension, and then averaged over the dimensions, thus failing to satisfy *dimensional convexity.*

The final poverty measure is then a convex combination of both measures:

$$\Omega = \frac{1}{N}\sum_{n=1}^{N}\left(\delta\frac{1}{T}\sum_{t=1}^{T}\left(\frac{1}{J}\sum_{j=1}^{J}d_{njt}\right)^{\beta} + (1-\delta)\frac{1}{J}\sum_{j=1}^{J}\left(\frac{1}{T}\sum_{t=1}^{T}d_{njt}\right)^{\beta}\right)\times c_n \tag{6}$$

where $1 \geq \delta \geq 0$ and $\beta > 0$. Setting $\beta > 1$ and $\delta > 0$ ensures *dimensional convexity* while $\beta > 1$ and $\delta < 1$ ensures *durational convexity.* At $\beta = 1$ the measure collapses into a simple double-sum of the count of deprivations. β is analogous to the FGT α parameter: following the literature a reasonable value would be 2. As is common in this class of measures, each dimension can be assigned a different weight. We detail such a generalization in Online Appendix A1.

Ω is a simple combination of existing measures from two independent extensions in the poverty measurement literature. While *dimensional convexity* and *durational convexity* are satisfied (independently) by those measures, they are rarely explicitly stated as a desirable property since the focus is usually on the "transfer" or inequality-sensitivity properties of the measure. Unlike many other applications, the α parameter in Ω no longer has to be chosen but is instead set to 1 in the cardinal case, and 0 in the ordinal case, which means that there is effectively a net increase of only one parameter relative to the AF model. This is

[6]The extension to incorporate persistence as is done in Gradin *et al.* (2012) is straightforward but will change the interpretation of the three-part decomposition in Section 2.3. We present the simpler duration-only case in this paper. Also, unlike Foster (2009), we focus only on the aggregation step: while the measure allots different weights to chronic versus transient poverty it does not require cutting-off those who are transiently poor.

because *dimensional convexity* and *durational convexity* are in fact sufficient conditions for certain standard "transfer-type" properties, which we discuss in detail in subsection 2.4.

Because there are two additively separable components in equation (6), the parameter δ allows a clear linear weighting choice between the two. Of course, it is never clear a priori what the value of δ should be. When ranking, for example, two groups of individuals, one useful criteria is simply that both terms $\Omega^{dimension}$ and $\Omega^{duration}$ have higher scores in one group relative to the other, in which case the poverty ranking would be robust to any choice of δ. Nonetheless, in cases where such robustness does not hold, it is useful to understand how *dimensional convexity* and *durational convexity* are affecting the score. The next section therefore details how Ω can be decomposed into three components to reveal more information about the poverty profile.

2.3. *A Three-Part Decomposition—Contribution C3*

Recall the example of the three individuals in the introduction, who, while having the exact same count of deprivations, have a different *distribution* of said deprivations *within* their respective deprivation profiles. Ω can be decomposed to yield:

1) The proportion of the poverty score due to the count of deprivation and the distribution of the count of deprivations *across* individuals
2) The proportion of the poverty score due to the distribution of "breadth" within individuals, with the breadth of individual n at period t defined as $\frac{1}{J}\sum_{j=1}^{J} d_{njt}$
3) The proportion of the poverty score due to the distribution of "length" within individuals, with the length of individual n at dimension j defined as $\frac{1}{T}\sum_{t=1}^{T} d_{njt}$

For any given count of deprivation, a higher variation of breadth across periods necessarily implies a greater concentration of deprivation within specific periods. Equally, a higher variation of length across dimensions implies a greater concentration of deprivations within specific dimensions.

The decomposition is achieved by recognizing that $\Omega^{dimension}$ and $\Omega^{duration}$ share a "common" component; that is, the component of Ω that is invariant to the distribution of deprivations within the individual. Let $\bar{\Omega}$ be the deprivation score when for each individual, deprivations are distributed equally across each element of $\boldsymbol{D_n}$:

$$(7) \qquad \bar{\Omega}=\frac{1}{N}\sum_{n=1}^{N}\left(\frac{\sum_{t=1}^{T}\sum_{j=1}^{J} d_{njt}}{J*T}\right)^{\beta}\times c_n$$

The score $\bar{\Omega}$ therefore never changes when an individual with the same average count of deprivations is added. This is, in fact, the deprivation measure found in Nicholas and Ray (2012). Since $\bar{\Omega}=\delta\bar{\Omega}+(1-\delta)\bar{\Omega}$, we can add the left-hand-side term and deduct the right-hand-side term of this equation from equation (6), yielding the following three-part decomposition:

TABLE 1

Description of the Three-Part Decomposition of Poverty Measure Ω

Component	Title	Description
(1) $\bar{\Omega}$	Distribution-insensitive component (Ω^{I})	The proportion of the poverty score due to the count of deprivations, and when $\beta > 1$, due to how unequally distributed those counts are *across* individuals. A change in the distribution of deprivations *within* any individual has no effect on this component.
(2) $\left(\Omega^{dimension} - \bar{\Omega}\right)$	Distribution of breadth component (Ω^{II})	This component takes the value of zero when the *breadth* of deprivation is the same across each period for all individuals. Formally, it is zero when $\frac{1}{J}\sum_{j=1}^{J} d_{njt} = \frac{1}{J}\sum_{j=1}^{J} d_{njt'}\ \forall\ t, t', n$. When $\beta=2$ it yields the *individual-averaged variance of breadth*, with variance referring to variation over time
(3) $\left(\Omega^{duration} - \bar{\Omega}\right)$	Distribution of length component (Ω^{III})	This component takes the value of zero when the *length* of deprivation is the same across each dimension for all individuals. Formally, it is zero when $\frac{1}{T}\sum_{t=1}^{T} d_{njt} = \frac{1}{T}\sum_{t=1}^{T} d_{njt'}\ \forall\ j, j',\ n$. When $\beta=2$ it yields the *individual-averaged variance of length*, with variance referring to variation over dimensions

$$\Omega = \left(\bar{\Omega} + \delta\left(\Omega^{dimension} - \bar{\Omega}\right) + (1-\delta)\left(\Omega^{duration} - \bar{\Omega}\right)\right) * c_n \tag{8}$$

Table 1 describes each of these three components.

Let us consider an illustration using the three individuals from the introduction, setting $\beta=2$, $\delta=0.5$ and, $z=1$. Though in practice it is rare to compare across individuals, *Dimensional Convexity* and *Durational Convexity* are properties associated with the distribution of deprivations within individuals; hence the gains from a measure satisfying the two properties are more clearly demonstrated by comparison

TABLE 2

Poverty Scores for Individuals *A, B, C*

	Ω_A	Ω_B	Ω_C	$\Omega = \frac{1}{N}\sum_{n=1}^{N} \Omega_n$
Ω^{I}	0.11	0.11	0.11	0.11
Ω^{II}	0	0.22	0	0.07
Ω^{III}	0	0	0.22	0.07
$\Omega^{I} + \left(0.5*\Omega^{I}\right) + \left(0.5*\Omega^{III}\right)$	0.11	0.22	0.22	0.18

Notes: Calculated using $\beta=2$, $\delta=0.5$, $z=1$; rounded to two decimal points; the first three columns depict poverty score calculated for each individual separately while the last column depicts the aggregate level score.

across individuals rather than groups. The scores are shown in Table 2, with Ω_n being the poverty score calculated using $N=1$.

All three individuals have the same score in the first component by virtue of having the same count of deprivation. Since individual A has her deprivations equally distributed across periods (one count in total for each period), and across dimensions (one count in total for each dimension), the second and third components have a score of zero. Both individual B and C have a poverty score that doubles that of individual A—for individual B, this score difference is attributable entirely to the uneven distribution of breadth across periods (all 3 counts in one period) while for individual C, this score difference is attributable entirely to the uneven distribution of length across dimensions (all 3 counts in one dimension).

Therefore while it is clear from the total scores that individuals B and C are more deprived than A, the decompositions quickly tell us that this ranking is driven by the distributions within their respective profiles rather than the actual count of deprivations. This information cannot be gleaned by looking at only $\Omega^{dimension}$ and $\Omega^{duration}$, since even when both terms have higher scores for one individual relative to another, differences in ranking could be driven by either Ω^{I} or the distribution sensitive terms Ω^{II} and Ω^{III}.

The three-part decomposition found in Table 1 is achieved because a single parameter β is used as the exponent for both poverty terms $\Omega^{dimension}$ and $\Omega^{duration}$. Consider an alternative specification where, say, $\Omega^{dimension}$ is unchanged and raised to the power β while $\Omega^{duration}$ is raised to the power γ. When $\beta \neq \gamma$, Ω^{II} and Ω^{III} can take on negative values and hence cannot be interpreted as we have in Table 1. This is because an increase in the exponent parameter typically reduces its average score (since the base of the exponent is a ratio that can never be greater than one). This makes comparability across the $\Omega^{dimension}$ and $\Omega^{duration}$ difficult, and effectively reduces them to being two different measures.

2.4. *The Transfer Axiom (Contribution C1) and Other Properties*

Given that Ω is constructed from existing measures, the following axiomatic properties are analogous to their static multidimensional counterparts, modified to take into account repeated observations over time. Following Pigou (1932), the transfer axiom requires that if inequality in income between two individuals were to decrease through a transfer from one party to the other, the aggregate poverty score should decrease. If we take ‘income’ to cover the broader notion of “achievements” from the multidimensional literature, then Pigou’s notion of “transfer” implies that taking some achievement x_{njt} from a “richer” individual and giving it to a second “poorer” individual should reduce overall poverty. What remains is to define “richer” and “poorer” over the multidimensional, multi-periodic space.

Let x'_{njt} be the post-transfer achievement.

Definition (progressive transfer)

$\boldsymbol{\rho}'$ is obtained from $\boldsymbol{\rho}$ by a progressive transfer if there is a transfer of achievement from individual Q to R along $x_{nj't'}$ such that:

(1) $x_{Qj't'} > x'_{Qj't'} > x_{Rj't'}$ (2) $x_{Qjt} \geq x_{Rjt}\ \forall\ j,\ t$

The first set of inequalities ensures that the transfer is a strictly positive amount and that after the transfer Q has more achievement than R did prior to the transfer. The second inequality combined with $x_{Qj't'} > x_{Rj't'}$ from the first ensure that the achievement profile $\boldsymbol{A}_Q$ strictly dominates $\boldsymbol{A}_R$ prior to the transfer.

Axiom 3 (weak transfer)
$g(\boldsymbol{\rho}, \boldsymbol{v}) \geq g(\boldsymbol{\rho}', \boldsymbol{v})$ if $\boldsymbol{\rho}'$ is obtained from $\boldsymbol{\rho}$ by a progressive transfer among two poor individuals.

Our weak transfer axiom is analogous to Bourguignon and Chakravarty's (2003) "One Dimensional Transfer Principle". However, several differences arise: first, it is more general in the sense that it allows achievements of the transferor that are not being transferred to not only be identical, but to dominate the corresponding achievements of the recipient. Second, it does not require that the achievements being transferred be below the respective deprivation cut-offs. Third, for the axiom to hold with the use of dual cut-offs, a reversal of the transfer conditions stipulated above (that is, a regressive transfer) requires that both individuals remain poor after the transfer.[7]

When the receiving achievement is below the dimensional cut-off prior to the transfer (that is, the person is deprived in the dimension), a stronger form of Axiom 3 emerges:

Axiom 4 (transfer with deprivation focus)
$g(\boldsymbol{\rho}, \boldsymbol{v}) > g(\boldsymbol{\rho}', \boldsymbol{v})$ if $\boldsymbol{\rho}'$ is obtained from $\boldsymbol{\rho}$ by a progressive transfer among two poor individuals and in addition $x_{Rj't'} < F_{j'}$.

Another class of inequality-sensitivity axioms are those based on the association/correlation-decreasing switch (AF, Atkinson and Bourguignon, 1982; Bourguignon and Chakravarty, 2003). Define an *association-decreasing switch* as a switch in achievement $x_{nj't'}$ between two individuals Q and R (i.e. $x'_{Qj't'} = x_{Rj't'}$) such that $\boldsymbol{A}_Q$ strictly dominates $\boldsymbol{A}_R$ prior to the switch but not after.

Axiom 5 (weak rearrangement)
$g(\boldsymbol{\rho}, \boldsymbol{v}) \geq g(\boldsymbol{\rho}', \boldsymbol{v})$ if $\boldsymbol{\rho}'$ is obtained from $\boldsymbol{\rho}$ by an association-decreasing switch among two poor individuals.

Axiom 6 (deprivation rearrangement)
$g(\boldsymbol{\rho}, \boldsymbol{v}) > g(\boldsymbol{\rho}', \boldsymbol{v})$ if $\boldsymbol{\rho}'$ is obtained from $\boldsymbol{\rho}$ by an association-decreasing switch among two poor individuals and: (i) $x_{Qj't'} \geq F_{j'}$; (ii) $x_{Rj't'} < F_{j'}$

Axiom 6 is the time-dependent multidimensional analogue to the "dimensional transfer" axiom found in Alkire *et al.* (2015, Chapter 2) and, together with Axioms 1 and 2, are the additional properties that are satisfied by Ω

[7]A progressive transfer from a non-poor individual from a non-deprived dimension may turn the transferor poor if the non-union identification method is adopted. Similarly, a regressive transfer to a poor individual in a deprived dimension may turn the recipient non-poor. Both of these cause a discrete "jump" in the poverty function which violate the basic property of the transfer axiom (see Datt, 2013). A union method of identification would allow the axioms to be satisfied without the restrictions relating to the "poor" status of the individuals as utilised here.

relative to the static multidimensional AF measure. Specifically, if we restrict $\boldsymbol{D_n}$ to contain only one column entry, then the AF measure can be applied to it: the AF measure would satisfy *weak arrangement* for $\alpha \geq 0$, *weak transfer* for $\alpha \geq 1$ and *transfer with deprivation focus* for $\alpha > 1$. Our measure Ω satisfies, at α=1 and $\beta > 1$ all those and, in addition, *deprivation rearrangement*. When α=0 and $\beta > 1$, *weak arrangement*, and *deprivation rearrangement* are satisfied.

Consistent with the AF class of measures, our measure also satisfies the following axioms:

Axiom 7 (subgroup decomposability)

$$g(N, \boldsymbol{\rho},\ \boldsymbol{v}) = \sum_s \frac{N_S}{N} g(N_S, \boldsymbol{\rho},\ \boldsymbol{v}).$$

Define a single replication of N as $A=(N, N)$; of $\boldsymbol{\rho}$ as $B=(\boldsymbol{\rho}, \boldsymbol{\rho})$; of $\boldsymbol{v}$ as $E=(\boldsymbol{v}, \boldsymbol{v})$.

Axiom 8 (replication invariance)
$g(\mathrm{N}, \boldsymbol{\rho}, \boldsymbol{v}) = g(A, B, E)$ for any number of replications of N, $\boldsymbol{\rho}$ and $\boldsymbol{v}$ so long as the number of replications are the same for N, $\boldsymbol{\rho}$ and $\boldsymbol{v}$.

Axiom 9 (symmetry/anonymity)
$g(\boldsymbol{\rho},\ \boldsymbol{v}) = g(\boldsymbol{\rho}',\ \boldsymbol{v})$ where $\boldsymbol{\rho}'$ is any permutation of the vector $\boldsymbol{\rho}$.

Axiom 10 (normalization and nontriviality)
$g(\boldsymbol{\rho},\ \boldsymbol{v})$ achieves at least two distinct values: a minimum value of 0 and a maximum value of 1.

Axiom 11 (poverty focus)
$g(\boldsymbol{\rho},\ \boldsymbol{v}) = g(\boldsymbol{\rho}',\ \boldsymbol{v})$ if $\boldsymbol{\rho}'$ is obtained from $\boldsymbol{\rho}$ by having a non-poor individual experience an achievement increase in any deprivation.

Axiom 12 (deprivation focus)
$g(\boldsymbol{\rho},\ \boldsymbol{v}) = g(\boldsymbol{\rho}',\ \boldsymbol{v})$ if $\boldsymbol{\rho}'$ is obtained from $\boldsymbol{\rho}$ by an increase in the achievement in a dimension/period where an individual is not considered deprived.

Axiom 13 (deprivation monotonicity)
$g(\boldsymbol{\rho},\ \boldsymbol{v}) > g(\boldsymbol{\rho}',\ \boldsymbol{v})$ if $\boldsymbol{\rho}'$ is obtained from $\boldsymbol{\rho}$ by a deprivation decrement among the poor.

Proposition 1. Let $\alpha \epsilon \{0, 1\}$. Ω satisfies *subgroup decomposability, replication invariance, symmetry, nontriviality, normalization, poverty focus, deprivation focus and deprivation monotonicity for* $\beta > 0$; *weak rearrangement* for $\beta \geq 1$;*deprivation rearrangement* for $\beta > 1$; *weak transfer* for $\beta \geq 1$ and α=1; *transfer with deprivation focus* for $\beta > 1$ and α=1; *dimensional convexity* for $\beta > 1$ and $\delta > 0$; *durational convexity* for $\beta > 1$ and $\delta < 1$.

Proof. Online Appendix B. ■

3. Dimensional Decomposition

The decomposability of a poverty measure according to the contribution of each dimension to overall poverty is a property that is desirable for multidimensional measures.[8] An additional feature of the three-part decomposition detailed in Section 2.3 is that the dimensional contributions can also be further decomposed into three components and interpreted as the contribution of each dimension to each respective component. A dimension's contribution to Ω^I tells us the dimension's contribution to the count and across-individual distribution component of deprivation, while a dimension's contribution to Ω^{II} tells us the dimension's contribution to the within-individual distribution of *breadth*; likewise, a dimension's contribution to Ω^{III} tells us the dimension's contribution to the within-individual distribution of *length*. The contribution of a dimension to Ω^{II} is particularly interesting, since it highlights our measure's ability to assign more weight not only to dimensions that have the most periods of deprivation, but also those that tend to occur with other dimensions of deprivation. Consider an individual with the following deprivation profile:

$$\boldsymbol{D_D} = \begin{pmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

Notice that if we were to do simple "count-based" dimensional decomposition as is prevalent in the literature, we would conclude that the first and second dimensions are equally important (they both have the longest duration at two periods each), while the third dimension is clearly the least important. But this ignores that deprivation in the second dimension occurs jointly with the third dimension in the first period, while the first dimension never occurs jointly with any other dimension. Dimension two should then be considered more important than dimension one because an individual who could retrospectively choose to remove an entire dimension of deprivation would (in most cases) choose to remove dimension two over one by virtue of the *breadth* of deprivation that dimension two contributes to in period one.

3.1. *Shapley Decomposition*

Notice from equation (6) that the contribution of every dimension to overall poverty is a function of the other dimensions that occur jointly with it through the term $\Omega^{dimension}$, meaning that the measure is not directly decomposable according to dimensions. However, the Shapley decomposition procedure found in Shorrocks (2013) can be applied here to yield an exact (additive) decomposition. Effectively, since every dimension's contribution to the overall score is a function of the other dimensions that are present, the Shapley decomposition reports the

[8]Also called "dimensional breakdown" (Alkire *et al*., 2015, Chapter 2); "factor decomposability" (Chakravarty *et al*., 1998) and "additive decomposability in attributes" (Bossert *et al*., 2013).

dimension's contribution averaged over every possible combination of other dimensions. We describe the general procedure next but also provide a detailed example with three dimensions in Online Appendix C. For an application in the static multidimensional case, see Datt (2013).

Notice from equation (6) that because the second term $\Omega^{duration}$ is simply an average over all dimensions, it is *directly* additively decomposable according to dimensions. The first term $\Omega^{dimension}$, however, is not *directly* additively decomposable when $\beta \neq 1$, and will require a Shapley decomposition to yield an additive decomposition.

Define $\Omega_{j|D=0}$ as the overall poverty score when all deprivations associated with dimension j are set to 0. The dimension score $\Omega_j=\Omega-\Omega_{j|D=0}$ is therefore the marginal contribution of dimension j to Ω. Because Ω has two components, we have:

$$\Omega_j=\delta\left(\Omega^{dimension}-\Omega^{dimension}_{j|D=0}\right)+(1-\delta)\left(\Omega^{duration}-\Omega^{duration}_{j|D=0}\right) \quad (9)$$

For any $\beta>0$, the second bracketed term is simply: $(1-\delta)\frac{1}{N}\sum_{n=1}^{N}\frac{1}{J}\left[\frac{\sum_t^T d_{nj't}}{T}\right]^{\beta}\times c_n$. When $\beta\neq 1$, the first term is a function of the order in which dimension j' is removed.[9] For any J number of dimensions there are a total of $K=2^{J-1}$ orders in which j' can be removed. Let $\Omega^{dimension}[k]$ be the score $\Omega^{dimension}$ at order $k\in\{1\ldots K\}$ and let $\Omega^{dimension}_{j|D=0}[k]$ be the score $\Omega^{dimension}$ when all deprivations associated with dimension j is set to 0 at order k. Shapley's decomposition therefore yields:

$$\Omega_j=\delta\frac{1}{K}\sum_{k=1}^{K}\left(\Omega^{dimension}[k]\ -\Omega^{dimension}_{j|D=0}[k]\right)+(1-\delta)\frac{1}{N}\sum_{n=1}^{N}\frac{1}{J}\left[\frac{\sum_t^T d_{nj't}}{T}\right]^{\beta}\times c_n \quad (10)$$

A dimension's proportion contribution to overall poverty is then simply $\gamma_j=\frac{\Omega_j}{\Omega}$ where $\sum_{j=1}^{J}\gamma_j=1$. One may find it strange that the dimensional decomposition in equation (10) uses the Shapley rule for only the first-half of the equation; this is done simply for brevity: when the Shapley rule is applied to the second-half of equation (10), it yields exactly the same score as a *direct* additive decomposition would. In this sense, the direct additive decomposition common in the literature can be seen as a special case of the Shapley decomposition method.[10] The approach detailed above yields two terms for each dimension. The Shapley decomposition can also be applied to the three-part decomposition in equation (8) to yield three terms for each dimension instead. While the three-part decomposition is more informative, the two terms from equation (10) are sufficient to yield a ranking of dimensions that are robust to the choice of δ.

[9] For example, when $J=2$, dimension j' can be removed when j'' is present or when j'' is also removed.

[10] A similar decomposition as that found in equation (9) can also be made for each period; however such a decomposition would be less useful as there is less reason for durational convexity if the interest was in understanding how poverty changes across each period.

4. Data Set and Summary Features

To apply the multidimensional poverty measure proposed in this paper we require a sufficiently "long" longitudinal dataset to take advantage of the time-dependent aspect of the measure. While there are several longitudinal datasets from developing countries[11] the China Health and Nutrition Survey (CHNS) is the longest: we use data spanning 2000–11. The present study follows Labar and Bresson (2011) in conducting the analysis of multidimensional poverty in China on the CHNS data base.

The CHNS is an ongoing international project between the Carolina Population Center at the University of North Carolina at Chapel Hill and the National Institute of Nutrition and Food Safety at the Chinese Centre for Disease Control and Prevention. This project was designed to examine the effects of health, nutrition and family planning policies and programs implemented by the national and local governments and to see how the social and economic transformation is affecting the health and nutritional status of the population. A detailed description of the CHNS database has been presented in Popkin *et al.* (2010). The surveys took place over a three-day period using a multi-stage, random cluster process to draw a sample of over 4000 households in nine provinces that vary substantially in geography, economic development, public resources and health indicators. We converted household level information to the individual level by assuming that the household's access to a facility such as drinking water or electricity is the same for all individuals in that household. Only individuals aged 18 years and above in the first year of the panel were included in construction of the balanced panel.

The CHNS data does not provide information on all the provinces in China. The present study was conducted on the nine provinces on which panel information was available (the provinces are listed in the results in Section 5). While the CHNS data providers do not claim that the data set is fully representative of the population, they do point out an attractive feature of this data, especially for the purpose of this study: "Data have been collected in a way that enables the team to answer China's policy-relevant questions concerning the design and impact of programs and policies affecting … health and nutritional status of its population" (http://www.cpc.unc.edu/projects/china/about/proj_desc).

In the present analysis we convert all variables to discrete "1", "0" variables to be consistent (we therefore set $\alpha=0$). A description of these dimensions is presented in Online Appendix D where the deprivation cut-offs used in the quantitative dimensions: years of education, body mass index (BMI) and blood pressure (BP) are explained. The summary statistics, year and dimension-wise, of the deprivation rates in China are presented in Table 3.

Following the nature of the available information in the CHNS data set, the chosen dimensions contained a mix of some at the household level and others at the level of the individual. Deprivation at the household level is converted to individual level deprivation by assuming that if a household is deprived in a particular dimension (e.g. fuel or electricity), so are all members of that household. This is similar to

[11]Examples include the Indonesian Family Life Survey; the Vietnam Household Living Standards Survey; the Mexican Family Life Survey; and the China Health and Nutrition Survey (see Ray and Sinha, 2015 for a static multidimensional application).

TABLE 3

Summary Deprivation Rates by Dimension and Time for China

Year	Toilet	Fuel	Electricity	Drink Water	Vehicle	Radio/TV	BMI	Illness	Blood Pressure	Compulsory Education	Average	Household Income < 0.5* Median
2000	0.573	0.642	0.0048	0.569	0.214	0.274	0.086	0.129	0.653	0.635	0.378	0.694
2004	0.519	0.648	0.0038	0.546	0.284	0.143	0.089	0.223	0.682	0.553	0.369	0.602
2006	0.481	0.586	0.0041	0.502	0.308	0.083	0.097	0.207	0.702	0.531	0.350	0.547
2009	0.427	0.424	0.0025	0.464	0.307	0.020	0.098	0.226	0.773	0.553	0.329	0.360
2011	0.412	0.384	0.0086	0.433	0.322	0.023	0.112	0.206	0.771	0.560	0.323	0.297
Average	0.482	0.537	0.0048	0.503	0.287	0.109	0.096	0.198	0.716	0.565	0.378	0.499

Notes: Income used as a benchmark and not an actual dimension.

the treatment of the expenditure dimension in AF though a wider range of dimensions is considered in the present study. This is also consistent with the unidimensional poverty literature where if a household is observed to be living below the poverty line, it is assumed that is true of all individuals in that household. The household level dimensions used here are those access to which are essential for a household to be classed as a "non poor" household and, hence, for all the household members to be classed as "non poor individuals". Besides, these are also dimensions for which the above mentioned assumption of household access being synonymous with individual access can be reasonably expected to hold.

At the individual level, the present study shares with AF the use of BMI and education as dimensions. Illness and blood pressure are chosen in the same spirit as BMI, since all three reflect underlying morbidity that in turn can be due to an unobserved range of deprivations faced by the individual. For example, there are now well documented linkages between blood pressure and poverty. As Ibrahim and Damasceno (2012, p. 611) note, "High illiteracy rates, poor access to health facilities, bad dietary habits, poverty, and high costs of drugs contribute to poor blood pressure control". Further evidence of a link between racial/ethnic based socioeconomic disparities and systolic and diastolic blood pressure is contained in Morenoff *et al.* (2007). One should therefore view poor performance in these chosen health dimensions as picking up deprivation on a range of living indicators that need to be addressed.

While some dimensions such as access to Fuel, Toilet and Radio/TV recorded large improvements over the period, the opposite is true for other dimensions such as abnormal Blood Pressure and BMI, which highlights the importance of taking into account the time-aspect of deprivation at the dimensional level since this would not be picked up by comparing static multidimensional measures over time. Interestingly, the average deprivation score across dimensions show very minor improvements over time (from 0.378 in 2000 to 0.323 in 2011) while the comparison household half-median income shows large improvements, once again highlighting the differences in information conveyed by these measures.

To motivate the use of a multidimensional approach, Online Appendix E compares the rankings of the nine provinces in the CHNS based on the multidimensional poverty measure proposed in this study ($\Omega, \beta=2, \ \delta=0.5$) with their per capita GDP numbers (averaged over the period 2000–12) taken from the National Bureau of Statistics of China, with a higher rank indicating a higher level of deprivation. While both the GDP per capita and our measure rank the Jiangsu province as the least deprived (lowest rank), a large discrepancy arises for the province of Guangxi which is ranked second according to per capita GDP, but eight according to our poverty measure. Liaoning, Shandong and Hunan also experience moderate differences in rankings, switching in three positions across the two poverty indicators. Online Appendix E therefore provides some support for the recent literature on multidimensional poverty in China by showing that in resource allocation between provinces it may be misleading to be guided by measures such as per capita GDP alone (as also noted in Labar and Bresson, 2011).

TABLE 4
Five Specific Measures of Poverty, Based on Ω

Measure	Description
i. Ω $(\beta=1)$	baseline: sum the count of deprivations and average them over individuals
ii. Ω^{I} $(\beta=2)$	baseline measure (i), but with each individual deprivation profile squared prior to averaging over individuals, thus allowing sensitivity to across-individual distribution
iii. $\Omega^{dimension}$ $(\beta=2)$	measure (ii) with the addition of Ω^{II}
iv. $\Omega^{duration}$ $(\beta=2)$	measure (ii) with the addition of Ω^{III}
v. $\Omega(\beta=2,\ \delta=0.5)$	measure (ii) with the addition of an equally weighted combination of Ω^{II} and Ω^{III}

5. Results

We illustrate the usefulness of Ω by first comparing across subgroups of the CNHS. The comparisons are: (1) between females and males; (2) between individuals in rural and urban regions; (3) between the nine provinces chosen for the CNHS.

For each of these comparisons, we first consider the poverty ranking provided by five specifications (Table 4), all of which are special cases or components of Ω, starting with the simplest measure Ω $(\beta=1)$ and adding additional elements until we reach Ω $(\beta=2)$. By introducing small changes to each measure we can ascertain the source of any changes in rankings. To allow all observations to input their variation into the measure, the union approach is adopted in these calculations (i.e. $z=1$).

We say that a pairwise ranking is *robust to the choice of* δ if $\Omega^{dimension}$ and $\Omega^{duration}$ both score higher for one group relative to the other.

In addition to these five measures, we explore the three-part decomposition to examine which component(s) explains differences in ranking. Finally, we do a dimensional decomposition for the most deprived subgroups to identify the most effective dimension-targeted policy measures.[12]

5.1. *Subgroup Comparisons*

Table 5 shows us the results for Female/Male, Rural/Urban and Provincial subgroup comparisons. For both the Female/Male and Rural/Urban comparisons we have all five specifications agree. This means the rankings are highly robust to changes in the measure's sensitivity to the between and within individual distribution of deprivations. Notice that for the score Ω $(\beta=2,\ \delta=0.5)$, the gap between Females and Males is relatively small (0.2324 versus 0.2302) relative to the gap between Rural and Urban residents (0.2609 versus 0.1364). This suggests that the size of the deprivation gap between groups are in themselves insufficient to tell us how robust differences in the groups will be to changes in weights allotted to the breadth (Ω^{II}) and length $\left(\Omega^{III}\right)$ sensitive components.

Looking at the three components, Ω^{I}, Ω^{II} and Ω^{III}, we can see that Females are more deprived in Ω^{I} and Ω^{II} relative to Males, whereas rural residents are

[12]The results presented here can be subject to a variety of additional robustness tests (e.g. by changing the poverty cut-off z, adopting a different weighting scheme, or choosing higher values of β). Since these are standard amongst AF-type multidimensional measures, interested readers are referred to Alkire *et al.* (2015, Chapter 8). We present the formula for the standard errors of Ω in Online Appendix A2.

TABLE 5

Poverty Scores for Female/Male; Rural/Urban and Subgroup Comparisons

Measure	Female	Male	Rural	Urban	Henan	Guizhou	Heilong-jiang	Shandong	Liaoning	Hubei	Hunan	Guangxi	Jiangsu
Ω (β=2, δ=0.5)	0.2324	0.2302	0.2609	0.1364	0.3019	0.2725	0.2554	0.2461	0.2228	0.2111	0.2107	0.1905	0.1846
$\Omega^{dimension}$ (β=2)	0.1668	0.1633	0.1912	0.0819	0.2222	0.2092	0.1801	0.1735	0.1550	0.1505	0.1498	0.1336	0.1239
$\Omega^{duration}$ (β=2)	0.2979	0.2970	0.3306	0.1910	0.3815	0.3358	0.3306	0.3188	0.2906	0.2718	0.2716	0.2474	0.2453
Ω^{I} (β=2)	0.1583	0.1550	0.1823	0.0749	0.2146	0.1991	0.1738	0.1643	0.1476	0.1411	0.1415	0.1241	0.1158
Ω^{II} (β=2)	0.0085	0.0083	0.0089	0.0070	0.0076	0.0101	0.0063	0.0092	0.0074	0.0094	0.0082	0.0094	0.0080
Ω^{III} (β=2)	0.1396	0.1420	0.1483	0.1161	0.1669	0.1367	0.1568	0.1545	0.1430	0.1306	0.1301	0.1232	0.1294
Ranking by Specification													
(i) Ω (β=1)	1	2	1	2	1	2	3	4	5	7	6	8	9
(ii) Ω^{I} (β=2)	1	2	1	2	1	2	3	4	5	7	6	8	9
(iii)$\Omega^{dimension}$ (β=2)	1	2	1	2	1	2	3	4	5	6	7	8	9
(iv) $\Omega^{duration}$ (β=2)	1	2	1	2	1	2	3	4	5	6	7	8	9
(v) Ω (β=2, δ=0.5)	1	2	1	2	1	2	3	4	5	6	7	8	9

Notes: A higher rank indicates more deprived. a=0; z=1 in all specifications.

more deprived than their urban counterparts in all three components, with the Ω^{I} component nearly triple that of the Urban residents.

The rankings at the provincial level are once again largely robust: in fact, the ranking are all robust to the choice of δ. However, the Hubei—Hunan comparison yields some interesting information: while Hubei has a smaller average count of deprivations (apparent from it ranking lower at Ω, $\beta=1$), it has both a larger concentration of deprivations in dimensions $\left(\Omega^{II}\right)$ and in specific periods of time (Ω^{III}). As we move from specification (i) to (ii), rankings do not change, suggesting that introducing FGT-type between-individual distribution sensitivity makes little difference for Hubei—Hunan. However, the rankings change when we take into account within-individual distributions of deprivations (specifications (iii)–(v)).

One thing to notice in all these subgroup comparisons is that size of the Ω^{III} component is relatively larger than Ω^{II}, meaning that deprivations are more likely to be repeated across time, rather than spread out across dimensions. While the choice of δ is never obvious since the notion of whether additional periods in the same dimension should be weighted more heavily than additional dimensions in the same period is highly subjective, we have shown that rankings robust to δ with respect to groups of provinces can be established using $\Omega^{dimension}$ and $\Omega^{duration}$.[13]

The scores associated with each of the three components are useful for comparisons across subgroups. For example, Henan is the province that has both the highest average count of deprivations, $(\Omega,\ \beta=1)$ and the largest concentration of these deprivations across time in specific dimensions (Ω^{III}). However, in terms of Ω^{II}, only Heilongjiang and Liaoning are ranked lower than Henan. Instead, Guizhou, with the second highest average count of deprivations, $(\Omega,\ \beta=1)$ has the largest concentration of deprivations across dimensions in specific periods $\left(\Omega^{II}\right)$ but has a Ω^{III} component that is surpassed by Liaoning, Shandong, Heilongjiang and Henan. In the next section we explore in greater detail the heterogeneity in deprivation across Henan and Guizhou through the use of dimensional decomposition.

5.2. *Dimensional Decomposition*

Dimensional decomposition allows policies to target specific dimensions that are the key "culprits" in overall deprivation. Results for dimensional decomposition using a Shapley decomposition are presented in Tables F1–F4 of the Online Appendix F, while Table F5 shows us the rankings of dimensional contributions to overall poverty that are robust to δ. We have reported these results for only the most deprived groups; i.e. Henan and Guizhou for the province comparison, females for the gender comparison and rural residents for the rural-urban comparison.

The dimension rankings are robust for the Rural and Henan subgroups, while they vary across the Guizhou and Female subgroups. Interestingly, the Female and Rural subgroups (Tables F1 and F2) exhibit a striking degree of similarity in their dimensional contributions: specification (v) ranks all dimensional contributions identically, with Compulsory Education being the largest (25 percent and 23 percent contribution to overall poverty for specification (v) for

[13]Consensus weighting (e.g. Bossert *et al.*, 2013) is one way to obtain an estimate of society's willingness to trade-off the two.

Female and Rural subgroups) and Electricity being the smallest (<1% for both subgroups) contributor to deprivation. For Females, the rank of access to Fuel, Drink Water and a Toilet are not robust to δ: Fuel ranks higher if $\left(\Omega^{II}\right)$ is given a higher weight, while Drink Water ranks higher if $\left(\Omega^{III}\right)$ is given a higher weight, suggesting that lack of access to fuel is relatively more likely to occur jointly with other problems, while lack of access to drink water tends to be persistent. Similarly, lack of access to radio/TV and BMI are also not robust, with the latter favoring a weight on $\left(\Omega^{III}\right)$.

Our calculations suggest some heterogeneity across the provinces of Henan and Guizhou (Tables F3 and F4). Notably, the robust rankings (Table F5) are considerably different at the higher ranks. While Compulsory Education, Blood Pressure and Fuel are important for both provinces, Vehicle is an additional important factor for Guizhou, and Drink Water is an additional important factor for Henan. Guizhou is also relatively more sensitive to the choice of δ: Fuel ranks higher if Ω^{II} is given more weight, while Vehicles ranks higher if Ω^{III} is given more weight.

For all subgroups, (lack of) Compulsory Education is characterized by the highest Ω^{I} and Ω^{III} components relative to the other dimensions: the result on Ω^{III} is not surprising since lack of education is likely to persist over time. What is equally interesting is that Compulsory Education also contributes very little to overall Ω^{II}: for all four subgroups, only Electricity has a lower contribution. This means that while Compulsory Education tends to be persistent, it does not tend to occur with other dimensions of deprivation. On the other hand, Fuel and Illness are characterized by large contributions to Ω^{II}, suggesting that they tend to occur jointly with other dimensions of deprivation. In the case of Illness this suggests that illnesses may themselves be a function of other deprivations (e.g. lack of access to material resources leads to illness), though these illnesses do not seem to persist (<1% contribution to overall Ω^{III} component.[14])

Overall, given the data available to us and assuming each dimension is equally valued, our policy advice would be that an increase in the access to nine-year compulsory education would be the most "efficient" dimension to target since it greatly affects all the disadvantaged groups. If this cannot be achieved, then policies concerned with the rural residents or females should target the issue of high blood pressure, while policies concerned with provincial inequality should target access to fuel in Henan and Guizhou. If one recalls our earlier discussion citing evidence linking high blood pressure with adverse living conditions, individuals showing symptoms of hypertensions could be identified as those experiencing severe deprivation. For effective policy intervention, one needs further examination of the specific causes of high blood pressure in the targeted individuals.

6. Conclusion

The contribution of this paper is methodological, analytical and empirical with the objective of better utilizing longitudinal data for informing policy. The

[14]It is possible for the Shapley contribution of the Ω^{II} and Ω^{III} component to be negative, since the presence/absence of certain dimensions can actually reduce rather than increase the purely distributional aspects that these components capture.

methodological contribution rests on the fact that it proposes a time-dependent multidimensional poverty measure that is sensitive to the distribution of deprivations both across and within individuals; notably, it is able to differentiate the contribution of deprivations that are concentrated in specific dimensions versus those that are concentrated in specific periods of time. The paper shows analytically that the proposed measure satisfies a set of axiomatic properties that have been shown to be desirable from a welfare perspective and that are also commonly utilized in the literature. In addition, the proposed poverty measure is useful for policy makers in allowing for a three-part decomposition that distinguishes between the distribution of *breadth* and *length* components and, in turn, both of them from a distribution insensitive component. Furthermore, it allows a (partial) ranking of subgroups and dimensional contributions that are robust the choice of weights across these components. The use of the Shapley decomposition procedure allows for dimensional decomposition while preserving the aforementioned properties of the measure. The application to Chinese data heightens the empirical interest of this study since it is able to supplement well known information on China's large poverty reductions with relative differences across subgroups.

It is important to recognize that multidimensional poverty measures have not displaced traditional income based approaches but are rather complementary to them. Any analyst of poverty needs to balance the usefulness of a single aggregated number versus disaggregated but more precise information. The advantage of poverty measures that are generalizations of other measures, and that also yield decomposability across a range of different factors such as subgroups and dimensions of deprivations, is that they offer *both* features (a single aggregated number and various decomposed ones) to the analyst within a consistent axiomatic framework that then makes explicit the assumptions used in making various trade-offs.[15]

These advantages naturally extend to the measure proposed in this paper: by virtue of being a generalization of existing measures in the time-dependent unidimensional literature and the static multidimensional literature, the measure can be decomposed into these components, while at the same time allowing for aggregation to a single number where the trade-off between duration-sensitivity and dimension-sensitivity is captured by the choice of the parameter, δ. Of course, making the trade-off assumption explicit through a single parameter does not necessarily make the decision easier: our measure also allows (partial) rankings that are robust to any such parametric choice. This has the advantage that even if δ cannot be chosen objectively, attention to disaggregated-level deprivation characteristics can be restricted to groups that fail to satisfy the robustness criteria, which obviates the need to look at all the disaggregated deprivation characteristics simultaneously.

In our Chinese application, for example, we find that Liaoning unambiguously ranks above Hubei and Hunan—for purely ranking purposes, there is no need to scrutinize the disaggregated measures and worry about the within-individual distribution of deprivations when comparing across, say, Liaoning and Hubei. When

[15]See for example, Alkire *et al.*'s (2011) summary of their exchange with Martin Ravallion, and Alkire and Foster (2011b) for the usefulness and desirability of aggregation into a single number.

comparing Hubei and Hunan (whose pairwise rankings are ambiguous), our measure allows us to say that: (1) Hubei is poorer when we allow for sensitivity of the measure to the concentration of deprivations within specific periods and dimensions; (2) Hunan is poorer when such within-individual distributions are deemed unimportant.

Of course, our Chinese application has also shown that where large gaps of poverty exist (e.g. between rural/urban areas), the extensions considered here are unlikely to change the rankings produced by a simpler counting approach.[16] Nonetheless, in such cases the decomposition properties of the measure remain important: in the income versus multidimensional measure comparison, multidimensional measure allow a disaggregation according to dimensions, thus allowing a deeper peek into the "black box" of the aggregate poverty score. Our measure goes further, by allowing not only decomposition according to dimensions, but according to how deprivations are distributed, on average, within individuals.

In our Chinese application, for example, dimensional decomposition allows us to conclude that high blood pressure issues, lack of education, and lack of access to drinking water are key dimensions of deprivation; in addition, our three-part decomposition tells us that a large proportion of this can be attributed to these deprivations recurring over time. In contrast, while no access to fuel is also a large contributor to overall poverty, it is also characterized by being relatively more concentrated in specific periods of time across many dimensions; this suggests that lack of access to fuel is a result of individual or household *vulnerability*. The fact that the dimension of "being ill" exhibits a similar characteristic supports this, since those who suffer from multiple deprivations are theoretically *causally* more likely to suffer from health issues. It is exactly this type of vulnerability—i.e. the most deprived being more likely to suffer specific additional deprivations—that our measure is able to capture, which may then lead the analyst to subsequent causal investigation.

This study has also drawn attention to the need for, and advantages of, having more balanced panel data sets at the level of individual or household containing information on deprivations by dimensions and covering a long enough period of time to allow dynamic multidimensional poverty estimation of the sort that is attempted here. The CHNS data set is one of the very few such data sets available for a developing country. Such information also needs to be gathered at the regional level, especially in the case of large and heterogeneous countries such as China and India. The large heterogeneity in the provincial poverty estimates in China points to the importance of a coordinated global program for calculating and monitoring subnational multidimensional poverty. With global poverty reduction featuring as an important goal in both the Millennium Development Goals (MDG) and Sustainable Development Goals (SDG), and the Human Development Reports moving from the earlier use of HDIs to the more sophisticated MPIs in assessing progress, the need for both methodological advances in the poverty measures and for disaggregated information in the data that is available cannot be overstated.

[16]Because the measure is ultimately a poverty measure, an increase in deprivation must increase the poverty score even if it reduces inequality (deprivation monotonicity).

References

Alkire, S., M. Apablaza, S. Chakravarty, and G. Yalonetzky, "Measuring Chronic Multidimensional Poverty: A Counting Approach," *Oxford Poverty & Human Development Initiative working paper*, 75, 2014.
Alkire, S. and J. Foster, "Counting and Multidimensional Poverty Measurement," *Journal of Public Economics*, 95, 476–87, 2011a.
———, "Understandings and Misunderstandings of Multidimensional Poverty Measurement," *Journal of Economic Inequality*, 9, 289–314, 2011b.
———, "Dimensional and Distributional Contributions to Multidimensional Poverty," OPHI Working Paper 100, University of Oxford, 2016.
Alkire, S., J. Foster, and M. Santos, "The Value-Added of Multidimensional Poverty measures," Accessed online at http://www.ophi.org.uk/wp-content/uploads/Summary-of-Debate-with-Martin-Ravallion.pdf, April 2011.
Alkire, S., J. Foster, S. Seth, M. Santos, J. Roche, and P. Ballon, *Multidimensional Poverty Measurement and Analysis: A Counting Approach*, Oxford University Press, Oxford, 2015.
Atkinson, A., "Multidimensional Deprivation: Contrasting Social Welfare and Counting Approaches," *Journal of Economic Inequality*, 1, 51–65, 2003.
Atkinson, A. B. and F. Bourguignon, "The Comparison of Multi-Dimensioned Distributions of Economic Status," *Review of Economic Studies*, 49, 183–201, 1982.
Bardhan, P., *Awakening Giants, Feet of Clay*, Princeton University Press, Princeton, NJ, 2010.
Bossert, W., L. Ceriani, S. Chakravarty, and C. D'Ambrosio, "Intertemporal Material Deprivation" in G. Betti and A. Lemmi (eds), *Poverty and Social Exclusion*, 128–42, Routledge, Oxford, 2014.
Bossert, W., S. Chakravarty, and C. D'Ambrosio, "Poverty and Time," *Journal of Economic Inequality*, 10, 145–62, 2012.
———, "Multidimensional Poverty and Material Deprivation with Discrete Data," *Review of Income and Wealth*, 59, 29–43, 2013.
Bossert, W., C. D'Ambrosio, and V. Peragine, "Deprivation and Social Exclusion," *Economica*, 74, 777–803, 2007.
Bourguignon, F. and S. Chakravarty, "The Measurement of Multidimensional Poverty," *Journal of Economic Inequality*, 1, 25–49, 2003.
Calvo, C. and S. Dercon, "Chronic Poverty and All That: The Measurement of Poverty Over Time," in T. Addison, D. Hulme, and R. Kanbur (eds), *Poverty Dynamics: interdisciplinary perspectives*, 29–58. Oxford University Press, Oxford, 2009.
Chakravarty, S. and C. D'Ambrosio, "The Measurement of Social Exclusion," *Review of Income and Wealth*, 52, 377–98, 2006.
Chakravarty, S., D. Mukherjee, and R. Ranade, "On the Family of Subgroup and Factor Decomposability Measures of Multidimensional Poverty," in D. J. Slottje (ed.), *Research on Economic Inequality* 8, JAI Press, 175–94, 1998.
Datt, G., "Making Every Dimension Count: Multidimensional Poverty without the Dual Cut Off," *Monash Economics Working Papers 32–13*, Monash University, 2013.
Duclos. J, A. Araar, and J. Giles, "Chronic and Transient Poverty: Measurement and Estimation, with Evidence from China," *Journal of Development Economics*, 91, 266–77, 2010.
Dutta, I., P. Pattanaik, and Y. Xu, "On Measuring Deprivation and the Standard of Living in a Multidimensional Framework on the Basis of Aggregate Data," *Economica*, 70, 197–221, 2003.
Fan, S., R. Kanbur, S. Wei, and X. Zhang, (eds), *Oxford Companion to the Economics of China*, OUP, Oxford, 2014.
Foster, J., "A Class of Chronic Poverty Measures," in T. Addison, D. Hulme, and R. Kanbur (eds), *Poverty Dynamics: Interdisciplinary Perspectives*, 59–76, Oxford University Press, Oxford, 2009.
Foster, J. and M. Santos, "Measuring Chronic Poverty", in Betti, G. and Achille, L (eds), *Poverty and Social Exclusion. New Methods of Analysis*, pp. 143–65, Routledge, Oxford, 2013.
Foster, J., J. Greer, and E. Thorbecke, "A Class of Decomposable Poverty Indices," *Econometrica*, 52, 761–66, 1984.
Gradin, C., C. del Rio and O. Canto, "Measuring Poverty Accounting for Time," *Review of Income and Wealth*, 58, 330–54, 2012.
Hojman, D. and F. Kast, "On the Measurement of Poverty Dynamics," *Faculty Research Working Paper Series, RWP 09–35, Harvard Kennedy School*, 2009.
Hoy, M. and B. Zheng, "Measuring Lifetime Poverty," *Journal of Economic Theory*, 146, 2544–62, 2011.
Ibrahim, M. and A. Damasceno, "Hypertension in Developing Countries," *The Lancet*, 380, 611–9, 2012.
Jayaraj, D. and S. Subramanian, "A Chakravarty-D'Ambrosio view of Multidimensional Deprivation: Some Estimates for India," *Economic and Political Weekly*, XLV, 53–65, 2010.
Labar, K. and F. Bresson, "A Multidimensional Analysis of Poverty in China from 1991 to 2006," *China Economic Review*, 22, 646–68, 2011.

Lahoti, R, A. Jayadev, and S. Reddy, "The Global Consumption and Income Project (GCIP): An Introduction and Preliminary Findings," *UN/DESA Working Paper No. 140*, 2015.
Merz, J. and T. Rathzen, "Multidimensional Time and Income Poverty: Well-Being Gap and Minimum 2DGAP Poverty Intensity—German Evidence," *Journal of Economic Inequality*, 12, 555–80, 2014.
Mishra, A. and R. Ray, "Multidimensional Deprivation in the Awakening Giants: A Comparison of China and India on Micro Data," *Journal of Asian Economics*, 23, 454–65, 2012.
Morenoff, J., J. House, B. Hansen, D. Williams, G. Kaplan, and H. Hunte, "Understanding Social Disparities in Hypertension Prevalence, Awareness, Treatment, and Control: The Role of Neighbourhood Context," *Social Science & Medicine*, 65, 1853–66, 2007.
Nicholas, A. and R. Ray, "Duration and Persistence in Multidimensional Deprivation: Methodology and Australian Application," *Economic Record*, 88, 106–26, 2012.
Park, A. and S. Wang, "Poverty," in Fan, S., Kanbur, R., Wei, S., and X. Zhang (eds), *Oxford Companion to the Economics of China*, OUP, Oxford, 2014.
Permanyer, I., "Assessing Individuals' Deprivation in a Multidimensional Framework," *Journal of Development Economics*, 109, 1–16, 2014.
Permanyer, I. and T. Riffe, "Multidimensional Poverty Measurement: Making the Identification of the Poor Count," Paper presented at the 6th meeting of the Society for the Study of Economic Inequality, 2015.
Pigou, A. F, *The Economics of Welfare*, 4th Edition, London, Macmillan, 1932.
Popkin, B. M., S. Du, F. Zhai, and B. Zhang, "Cohort Profile: The China Health and Nutrition Survey-Monitoring and Understanding Socio-Economic and Health Change in China, 1989–2011," *International Journal of Epidemiology*, 39, 1435–40, 2010.
Ray, R. and K. Sinha, "Multidimensional Deprivation in China, India and Vietnam: A Comparative Study on Micro Data," *Journal of Human Development and Capabilities*, 16, 69–93, 2015.
Sen, A., "Poverty: An Ordinal Approach to Measurement," *Econometrica*, 44, 291–31, 1976.
———, *Commodities and Capabilities*, North Holland, Amsterdam, 1985.
Sengupta, M., "Unemployment Duration and the Measure of Unemployment," *Journal of Economic Inequality*, 7, 273–94, 2009.
Shapley, L., "A Value for n-person Games," in *Contributions to the Theory of Games*, volume II, by H.W. Kuhn and A.W. Tucker (eds), *Annals of Mathematical Studies* v. 28, pp. 307–17, Princeton University Press, Princeton, NJ 1953.
Shorrocks, A. F., "Decomposition Procedures for Distributional Analysis: A Unified Framework Based on the Shapley Value," *Journal of Economic Inequality*, 1, 99–126, 2013.
Stiglitz, J. E., A. Sen, and J. P. Fitoussi, "Report by the Commission on the Measurement of Economic Performance and Social Progress," Available at http://www.insee.fr/fr/publications-et-services/dossiers_web/stiglitz/doc-commission/RAPPORT_anglais.pdf, 2009.
Tsui, K., "Multidimensional Poverty Indices," *Social Choice and Welfare*, 19, 69–93, 2002.
Walker, R., "The Dynamics of Poverty and Social Indices," in Room, G. (ed.), *Beyond the Threshold*, The Policy Press, Bristol, 1995.
Yalonetzky, G., "Conditions for the Most Robust Multidimensional Poverty Comparisons Using Counting Measures and Ordinal Variables," *Social Choice and Welfare*, 43, 773–807, 2014.
You, J., S. Wang, and L. Roope, 'Multi-Dimensional Intertemporal Poverty in Rural China,' *University of Oxford Centre for the Study of African Economies Working Paper Series No. WPS/2014–3*, 2014.

Supporting Information

Additional Supporting Information may be found in the online version of this article at the publisher's web-site:

Appendix A1: Dimension Weights
Appendix A2: Standard Errors
Appendix B: Proofs
Appendix C: Shapley Decomposition Example
Appendix D: Description of Dimensions used for analysis
Appendix E: China province rankings according to GDP/capita and Ω
Appendix F: Result Tables
Table F1: Dimensional Decomposition for Females
Table F2: Dimensional Decomposition for Rural residents
Table F3: Dimensional Decomposition for Henan
Table F4: Dimensional Decomposition for Guizhou
Table F5: Rankings of Dimensional Contributions to overall poverty, robust to δ