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ON THE DECOMPOSITION OF THE FOSTER AND WOLFSON BI-POLARIZATION INDEX BY INCOME SOURCES

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This paper proposes a simple algorithm based on a matrix formulation to compute the Foster and Wolfson bipolarization index and then to decompose it by income sources. An empirical illustration based on EU-SILC data for the years 2007 and 2014 shows the usefulness of the proposed decomposition.

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1. Introduction

Many studies have emphasized the role played by the middle class in the process of economic development. It has in fact been argued that a sizable middle class is a pre-condition for a nation to develop. For Landes (1998), cited by Easterly (2001), "the ideal growth and development society" would have a "relatively large middle class". Easterly (2001) also cites a study by Adelman and Morris (1967) who wrote that "it is clear from many country studies that the growth of a robust middle class remains of crucial importance in contemporary low-income nations". Easterly (2001) in fact puts forth two determinants of economic development that are mentioned in two strands of literature: the first one emphasizes the role of resource endowments and the absence of high inequality; the second one links ethnic divisions to poor growth.

The first type of argument appears in papers by Engerman and Sokoloff (1997) and Sokoloff and Engerman (2000) in their studies of development in the New World. There they argue that in Latin America, economies of scale in tropical commodity factor endowments have led to an important concentration of wealth and hence to the existence of a small elite. To maintain its power this elite would

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then prevent any mass investment in human capital and, as a consequence, any significant development. The story was evidently completely different in North America where family farms were common in non-tropical land so that inequality was much smaller and investment in mass human capital more common.

The argument concerning the link between ethnic divisions and poor growth is that "an ethnically distinct group in power is reluctant to invest in public services for the other ethnic groups for fear that the other groups will be enabled to displace the first group from power" (Easterly, 2001, p. 319).

Such a combination of relative equality and ethnic homogeneity has been labeled "middle class consensus" by Easterly (2001) and it is this consensus that ultimately brings higher levels of income and growth.

Given the role played by the middle class, one may wonder whether there is any agreement about the way the relative importance of the middle class should be measured. There is however no common definition of the middle class in the literature. For Foster and Wolfson (1992; 2010) measuring the middle class implies four decisions: (i) choosing a "space" whether it is an "income-space" or a "people-space"; (ii) determining what should be the middle (e.g. the median or the mean income); (iii) selecting a range around the middle (defining a percentage interval above and below this "middle", whether it refers to the median or the mean); (iv) aggregating the data.

Thurow (1984), for example, included in the middle class those households whose income ranges from 75 percent to 125 percent of the median household income. Blackburn and Bloom (1985) recommended a wider interval, one which would comprise all those households whose income varies from 60 percent to 225 percentof the median. Horrigan and Haugen (1988) used several income ranges to define the middle class in the U.S. and concluded that the middle class was shrinking mainly because the declining proportion of families in the middle class had moved to the upper class, while the income share of the lower class declined. Davis and Huston (1992) defined the middle class as those families with incomes between 50 and 150 percent of the current-year median income while for Lawrence (1984) the middle class ranged from two-thirds to four-thirds of men's median weekly earnings. Using a kernel density approach Jenkins (1995) concluded that in the U.K. there was a shift in the concentration of people towards upper income ranges as well as an increase in the concentration at the very lowest incomes. Birdsall *et al*. (2000) agreed with Thurow's approach but Birdsall (2010) preferred to include in the middle class people at or above the equivalent of $10 day in 2005, and at or below the 90th percentile of the income distribution in their own country. Pressman (2007), using the LIS (Luxembourg Income Study) database, defined the "middle class" as those households receiving between 75 percent and 125 percent of median adjusted household income. Eisenhauer (2008) used the government's official poverty line as the demarcation between the poor and the middle class, and developed an equivalent distinction to separate the middle class from the wealthy. Rather than basing the definition of the middle class on the "income-space", Levy (1987a; 1987b) used a definition related to a "people-space", one where the "middle" ranges from the 20th to the 80th percentile but Beach (1989) was not convinced by Levy's (1987a; 1987b) assertion that the middle class is vanishing.

Gigliarano and Mosler (2009) suggested taking a multidimensional people's space approach. In their empirical illustration, based on German data, they used income and wealth as the relevant dimensions. Atkinson and Brandolini (2011) examined the identification of the "middle class", using data from the Luxembourg Income Study (LIS) as well as from the Luxembourg Wealth Study (LWS). They first considered definitions based purely on income. But they then argued that the concept of "class" requires the examination of other dimensions, such as the role of property and, drawing on the sociological literature, of occupations.

To understand why the middle class declined in several Western countries several explanations have been proposed. For the U.S., Davis and Huston (1992) mentioned the role of de-industrialization, the rising number of families headed by women, the increased polarization of incomes of black families, the aging of the baby boomers, the reduced number of average hours worked by family heads and the decline in union membership and in durable manufacturing jobs. More recently Piketty (2014), though emphasizing inequality more than the decline of the middle class, claimed that the share of capital in total income is an essential source of inequality in the Western world. In fact, whereas this share was low almost throughout the twentieth century, it has now returned to historical high levels, whether in France, the U.S. or the U.K. In addition, wage growth concentrates mainly in the upper tail of the wage distribution and these high wages have nothing to do with productivity but are rather the consequence of social norms. Piketty concludes that the present levels of inequality cannot be justified and, as a consequence, recommends important policy interventions, such as a global wealth tax.

Given that different income ranges have been suggested to measure the relative importance of the middle class, conclusions are likely to depend on the definition adopted. The same kind of problem is faced when trying to measure the extent of poverty as the latter depends on the poverty line which has been selected. In the field of poverty Atkinson (1987) and Foster and Shorrocks (1988) proposed to adopt an approach based on the concept of stochastic dominance when comparing the magnitude of poverty in two income distributions. Conclusions based on such an approach may then hold for a wide range of poverty lines and a large set of poverty measures (for more details on this method, see Ravallion, 1994; Jenkins and Lambert, 1997, and Fields, 2001).

Foster and Wolfson (1992; 2010) took a similar approach when analyzing bi-polarization but they evidently had to determine two cutoff points, the lower and upper levels of income. These thresholds may vary but still lead to the same conclusion, as far as the size of the middle class is concerned. Foster and Wolfson (1992; 2010) defined in fact two polarization curves, one related to the notion of "increasing spread", the other to that of "increased bipolarity".

"Increasing spread" refers to the fact that moving an individual from the middle position (the median) to the tails of the income distribution induces an increase in bipolarization, that is, a rank preserving increase (reduction) in incomes above (below) the median extends the gap between the two groups (those above and below the median) and hence increases the degree of bi-polarization.

"Increased bipolarity" refers to the case where the incomes below or above the median become closer to each other, so that the gaps between the incomes below (above) the median have been reduced, leading to an increase in bi-polarization.

These two concepts of "increasing spread" and "increased bipolarity" show clearly why the notions of "inequality" and "bi-polarization" are different. Whereas any transfer of income from a poor to a richer individual increases inequality, it increases bi-polarization only if money is transferred from an individual with an income below the median to one with an income above the median. But if money is transferred from an individual with an income below the median to a richer individual whose income remains below the median income or if money is transferred from an individual with an income above the median to another richer individual, bi-polarization will decrease.[1]

The approach of Foster and Wolfson (1992; 2010) has been expanded in several directions. Wang and Tsui (2000) extended the index of Foster and Wolfson, on the basis of the generalization of the Gini index originally proposed by Donaldson and Weymark (1980) and Weymark (1981). Chakravarty and Majumder (2001) derived a welfare theoretic measure of bi-polarization not from the Gini index, but from the Atkinson (1970)—Kolm (1969)—Sen (1973) general ethical inequality index. Chakravarty *et al.* (2007) defined an absolute concept of bipolarization. They scaled up the Foster-Wolfson (first) bi-polarization curve by the median in order to obtain what they call an “Absolute Polarization Curve” (APC). Chakravarty and D’Ambrosio (2010) derived an additional generalization and defined the concept of “Intermediate Polarization Curve” (*IPC*). Rodriguez and Salas (2003) extended the use of the Foster and Wolfson (1992; 2010) index to the case where an extended rather than the traditional Gini index is used. Though not limiting their analysis to the case of bipolarization, Zhang and Kanbur (2001) defined a polarization index as the ratio of the between- to that of the within-group inequality, inequality being measured via the Theil index. Silber *et al.* (2007) defined a bi-polarization index on the basis of the work of Berrebi and Silber (1989) on the measurement of the flatness of a distribution. They then showed that such an index had the two most desirable properties of a polarization index, namely the axioms of Non-Decreasing Spread and Non-Decreasing Bipolarity. Following the work of Zheng (2007a; 2007b; 2007c) on the concept of unit-consistency, which requires that inequality (poverty) rankings, rather than levels, should not be affected by the units in which incomes is expressed, Lasso de la Vega *et al.* (2010) extended Zheng’s ideas to the measurement of bipolarization.

Using the concept of concentration index, Apouey (2010) extended the concept of income bipolarization to that of health bipolarization. Apouey and Silber (2013) suggested two approaches to the quantification of inequality and bi-polarization in income and health. In the first approach, inequality and bi-polarization will be minimal when health and income are independent. In an alternative approach, they supposed that there is no inequality and bi-polarization when all individuals report the exact same health and income categories.

In this paper we introduce a simple algorithm derived from the matrix formulation suggested by Silber (1989) to compute the Gini index and obtain a decomposition of the Foster and Wolfson (1992, 2010) bipolarization index by

[1]As already stressed in Esteban and Ray (1994) as well as in Duclos *et al.* (2004), the concepts of “increasing spread” (polarization rises when “across-group” inequality increases) and “increased bipolarity” (polarization increases when “within-group” inequality is reduced) are also central notions in the vast literature dealing with the measurement of polarization. We believe however, as already stressed in Nissanov *et al.* (2010), that polarization and bipolarization are somehow different notions.

income sources. To the best of our knowledge the focus of almost all the empirical studies of bi-polarization has been on the distribution of total income, the study of Deutsch and Silber (2010) being the only attempt to decompose bipolarization indices by income sources. The procedure they proposed is however quite cumbersome.[2] By presenting a simple algorithm to compute the Foster and Wolfson (1992; 2010) bipolarization index and showing how such an algorithm leads to quite a straightforward decomposition of this index by income sources, the present paper is likely to be very useful to those investigating the extent of the bipolarization of incomes. Moreover, given the parallelism between the decomposition of income inequality by income sources and that of the variance of earnings in an income generating function (see, for example, Morduch and Sicular, 2002, and Fields, 2003), the decomposition presented in this paper could be extended, using a Mincerian earnings function, to a breakdown of the bipolarization of wages or earnings by its determinants.

The paper is organized as follows. Section 2 describes this simple algorithm allowing the computation of the Foster and Wolfson bipolarization index. Section 3 then shows how this algorithm can also be used to decompose bipolarization by income sources. Section 4 gives an empirical illustration based on EU-SILC data for the years 2007 and 2014. Concluding comments are given in Section 5.

2. On A Matrix Representation of the Foster and Wolfson Bi-Polarization Index

The Foster and Wolfson bi-polarization index, FW, may be expressed (see, Foster and Wolfson, 1992, 2010) as

$$FW=(G_B-G_W)\left(\frac{\bar{y}}{m}\right) \tag{1}$$

where G_B, G_W, $\bar{y}$ and m are respectively the between and within groups Gini indices, the arithmetic mean and the median of an income distribution. In this formulation it is assumed that there are only two groups, those having an income y_i higher than the median income and those with an income lower than the median income.

It is however known that in the case of two non-overlapping groups the overall Gini index I_G is equal to the sum of the between and within groups Gini indices. Moreover, it has been shown (see, Silber, 1989) that the Gini index I_G could be expressed as

$$I_G=e'G\ s \tag{2}$$

where e' is a row vector of n elements being all equal to $(1/n)$, s a column vector of

[2]Araar (2008) proposed a decomposition of the Duclos-Esteban-Ray (2004) *DER* index of polarization by income sources. Since in such an approach the number of poles is determined by the data, it cannot be applied to the decomposition of bipolarization, the latter being limited evidently to only two poles, and of equal population size.

the n income shares, the latter being ranked by decreasing income, G a n by n square matrix (called the G-matrix) whose typical element g_{ij} is equal to 0 if $i=j$, to -1 if $j>i$ and to $+1$ if $i>j$, n referring to the number of individuals in the population.

Let us assume at this stage that n is even, say, $n=6$. The G-matrix is therefore written as

$$G=\begin{bmatrix} 0 & -1 & -1 & -1 & -1 & -1 \\ 1 & 0 & -1 & -1 & -1 & -1 \\ 1 & 1 & 0 & -1 & -1 & -1 \\ 1 & 1 & 1 & 0 & -1 & -1 \\ 1 & 1 & 1 & 1 & 0 & -1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

If we assume, as mentioned previously, two groups of equal size, those with an income higher and those with an income smaller than the median income, both of equal size, and if we divide the G-matrix into 4 sub-matrices of equal size (each of size (n/2) by (n/2)) called respectively A (northwest sub-matrix), B (northeast sub-matrix), C (southwest sub-matrix) and D (southeast sub-matrix) it can be proven (see, Silber, 1989) that the contribution G_W of the within groups inequality to the overall Gini index I_G will be written as

(3) $$G_W=e^{1'}As^1+e^{2'}Ds^2$$

where A and D are each G-matrices of size (n/2) by (n/2), $e^{1'}$ and $e^{2'}$ are row vectors of size (n/2) with each element equal to (1/n), s^1 a column vector of the (n/2) highest income shares and s^2 a column vector of the (n/2) lowest income shares, the shares being in each case ranked by decreasing values.

Similarly it is easy to prove (see, Silber, 1989) that the contribution G_B of the between groups inequality to the overall Gini index I_G will be expressed as

(4) $$G_B=e^{1'}Cs^1+e^{2'}Bs^2$$

Let us now call A^t and D^t matrices that are respectively the transpose of the matrices A and D. It then follows that $e^{1'}A^ts^1+e^{2'}D^ts^2$ will be equal to $(-G_w)$ so that

(5) $$e^{1'}Cs^1+e^{2'}Bs^2+e^{1'}A^ts^1+e^{2'}D^ts^2=G_B-G_w$$

Given the way we have defined the matrices A^t and D^t it is not difficult to derive that the difference (G_B-G_w) may be also written as

$$(G_B - G_w) = e' P s \tag{6}$$

where P (henceforth called the P-matrix) is a square n by n matrix, which, when $n = 6$, will be written as

$$P = \begin{bmatrix} 0 & 1 & 1 & -1 & -1 & -1 \\ -1 & 0 & 1 & -1 & -1 & -1 \\ -1 & -1 & 0 & -1 & -1 & -1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & -1 & 0 & 1 \\ 1 & 1 & 1 & -1 & -1 & 0 \end{bmatrix}$$

From this representation of the P-matrix we easily derive that the difference $(G_B - G_w)$ may be also written as

$$(G_B - G_w) = \sum_{i=1}^{n/2} \left(\frac{2i-1}{n}\right) s_i - \sum_{i=\left(\frac{n}{2}\right)+1}^{n} \left(\frac{2(n-i+1)-1}{n}\right) s_i \tag{7}$$

Since $s_i = (y_i/(n\bar{y}))$, we conclude, combining (1) and (7) that

$$FW = \sum_{i=1}^{n/2} \left[\left(\frac{2i-1}{n}\right)\left(\frac{y_i}{nm}\right)\right] - \sum_{i=\left(\frac{n}{2}\right)+1}^{n} \left[\left(\frac{2(n-i+1)-1}{n}\right)\left(\frac{y_i}{nm}\right)\right] = e' P \tilde{s} \tag{8}$$

where $\tilde{s}$ is a n by 1 vector whose typical element $\tilde{s}_i$ is equal to $\frac{y_i}{nm}$.

The case where n is odd would be written as:

$$FW = \sum_{i=1}^{(n-1)/2} \left[\left(\frac{2i-1}{n}\right)\left(\frac{y_i}{nm}\right)\right] - \sum_{i=\left(\frac{n+3}{2}\right)}^{n} \left[\left(\frac{2(n-i+1)-1}{n}\right)\left(\frac{y_i}{nm}\right)\right] \tag{9}$$

where it is assumed that the median income (corresponding to $i = \left(\frac{n+1}{2}\right)$) either belongs to both groups or to none of them (as in the previous expression) but if n is big enough the decomposition remains the same.[3] We will from now on assume that n is even.

Expression (8) may then be used to derive a decomposition of the Foster and Wolfson index by income sources.

[3]Rodríguez (2006) in a paper on bipolarization reached in fact a similar conclusion.

3. The Decomposition of the Foster and Wolfson Bi-Polarization Index by Income Sources

Let us now call y_{ij} the income that individual $\boldsymbol{i}$ receives from income source j and assume that there are J sources of income, and $y_i = \sum_{j=1}^{J} y_{ij}$.

We can now rewrite (8) as

$$FW = \sum_{j=1}^{J} \left\{ \sum_{i=1}^{n/2} \left[\left(\frac{2i-1}{n} \right) \left(\frac{y_{ij}}{nm} \right) \right] - \sum_{i=\left(\frac{n}{2}\right)+1}^{n} \left[\left(\frac{2(n-i+1)-1}{n} \right) \left(\frac{y_{ij}}{nm} \right) \right] \right\} \tag{10}$$

so that the contribution C_j of income source j to the Foster and Wolfson index FW will be

$$C_j = \sum_{i=1}^{n/2} \left[\left(\frac{2i-1}{n} \right) \left(\frac{y_{ij}}{nm} \right) \right] - \sum_{i=\left(\frac{n}{2}\right)+1}^{n} \left[\left(\frac{2(n-i+1)-1}{n} \right) \left(\frac{y_{ij}}{nm} \right) \right] \tag{11}$$

If we prefer to use the P-matrix to compute the index FW as in (8), we can define a column vector y whose typical elements are the incomes of all n individuals, these incomes being ranked by decreasing values of y_i . Let now $y_{.j}$ be a column vector of the incomes the various individuals receive from source j, the individuals being ranked by decreasing values of total income y_i.

Expression (10) may therefore be rewritten as:

$$FW = e'P\tilde{s} = \frac{1}{nm} e'Py = \sum_{j=1}^{J} \frac{1}{nm} e'Py_{.j} \tag{12}$$

so that the contribution of a given income source to the Foster and Wolfson bipolarization index of total income may be expressed as

$$C_j = \left(\frac{1}{nm} \right) (e'Py_{.j}) = \left(\frac{m_j}{m} \right) \left(\frac{e'Py_{.j}}{nm_j} \right) \tag{13}$$

where m_j is the median income for source j.

Let us similarly define a vector $z_{.j}$ whose typical elements are also equal to the incomes from source j, these incomes being now ranked by decreasing values. Using expressions (6) to (8), we may then define the Foster and Wolfson bipolarization index FW_j for income source j as

$$FW_j = \left(\frac{e'Pz_{.j}}{nm_j} \right) \tag{14}$$

Given that the elements in the vector $y_{.j}$ are the same as those in the vector $z_{.j}$ but are generally ranked in a different order, we will similarly define a Pseudo-bipolarization index PFW_j for income source j as

$$PFW_j = \left(\frac{e'Py_{.j}}{nm_j}\right) \tag{15}$$

Expression (12) may then be also written in terms of the FW measure of bipolarization of each income source, that is, as

$$\begin{aligned} FW &= \sum_{j=1}^{J}\left(\frac{1}{nm}\right)\left(e'Py_{.j}\right) = \sum_{j=1}^{J}\left(\frac{m_j}{m}\right)\frac{\left(e'Pz_{.j}\right)}{nm_j}\left(\frac{\left(e'Py_{.j}/nm_j\right)}{\left(e'Pz_{.j}/nm_j\right)}\right) \\ &= \sum_{j=1}^{J}\left(\frac{m_j}{m}\right)\left(FW_j\right)\left(\frac{PFW_j}{FW_j}\right) \end{aligned} \tag{16}$$

The contribution C_j of a given income source to the Foster and Wolfson bipolarization index may therefore be also written as

$$C_j = \frac{m_j}{m}\frac{\left(e'Pz_{.j}\right)}{nm_j}\left(\frac{\left(e'Py_{.j}\right)}{\left(e'Pz_{.j}\right)}\right) = \left(\frac{m_j}{m}\right)\left(FW_j\right)\left(\frac{PFW_j}{FW_j}\right) = \left(\frac{m_j}{m}\right)PFW_j \tag{17}$$

In expression (16) this contribution C_j is expressed as a function of three elements:[4]

- The ratio $\left(\frac{m_j}{m}\right)$ of the median income from source j over the median value of total income. This ratio is a measure of the relative importance of income source j in total income.[5] Therefore, the higher this ratio, the higher, ceteris paribus, the degree of bipolarization of the distribution of total income.
- The ratio $\left(\frac{(e'Pz_{.j})}{nm_j}\right)$ which is in fact equal to the index of bi-polarization FW_j for the distribution of income source j, since the incomes from source j that are the elements of $z_{.j}$ are ranked by decreasing values. Therefore, the higher this index, the higher, ceteris paribus, the degree of bipolarization of the distribution of total income.
- The ratio $\left(\frac{\left(e'Py_{.j}\right)}{\left(e'Pz_{.j}\right)}\right) = \left\{\frac{\left[\left(e'Py_{.j}\right)/\left(nm_j\right)\right]}{\left[\left(e'Pz_{.j}\right)/\left(nm_j\right)\right]}\right\} = \frac{PFW_j}{FW_j}$ which can be positive or negative. The higher in absolute value the value of this ratio, the higher,

[4]Such a distinction between three determinants is similar to that made when examining the determinants of the contribution of an income source to the Gini index of total income. In the latter case the determinants are respectively the ratio of the average income of the source over the average total income, the Gini index of the source and what Lerman and Yitzhaki (1985) called the Gini correlation coefficient.

[5]The ratio of the average income from source j to the average total income would be a better indicator, as the sum of the average incomes from all sources is equal to the average total income. Such a ratio appears indeed in the decomposition of the Gini index by income sources. The median is nevertheless one of the indicators of the location of a distribution and this is why we can still consider the ratio of the median income from source j to the overall median income as a measure of the relative importance of this income source.

ceteris paribus, its impact in absolute value on the bipolarization of total income.[6]

On the other hand in expression (17), the contribution of income source j to the bipolarization of total income may be considered as a function of only two determinants: the ratio $\left(\frac{m_j}{m}\right)$ of the median income from source j over the median value of total income, and what was previously called the Pseudo Foster and Wolfson bipolarization index for income source j.

Let now G_{Bj} and G_{Wj} refer respectively to the between and within groups Gini indices for income source j. The between groups Gini index G_{Bj} is derived by giving to each "income source j rich" individual (each individual whose income from source j is higher than the median income from source j) the average income $\bar{y}_{rj}$ from source j among the "income source j rich" individuals, and to each "income source j poor" individual (each individual whose income from source j is lower than the median income from source j) the average income $\bar{y}_{pj}$ from source j among the "income source j poor" individuals. Note that, following Berrebi and Silber (1989) we may also express this between groups Gini index as

$$G_{Bj}=\left(\frac{1}{4\bar{y}_j}\right)\left(\bar{y}_{rj}-\bar{y}_{pj}\right) \tag{18}$$

The within groups Gini index G_{Wj} is equal (see, for example, Silber, 1989) to the sum of two expressions. The first expression is the product of three elements respectively equal to the share (one half) in the total population of the "income source j rich" individuals, the share s_{rj} of the "income source j rich" individuals (as defined previously) in the total income from source j and the "within income source j rich" Gini index G_{Wrj}. Similarly the second expression is the product of three elements respectively equal to the share (one half) in the total population of the "income source j poor" individuals, the share s_{pj} of the "income source j poor" individuals (as defined previously) in the total income from source j and the "within income source j poor" Gini index G_{Wpj}.

In other words we may write that

$$G_{Wj}=\frac{1}{2}s_{rj}G_{Wrj}+\frac{1}{2}s_{pj}G_{Wpj} \tag{19}$$

Using (1) we easily derive, limiting ourselves to the distribution of the incomes from source j, that

[6]The same kind of ratio appears in the decomposition of the Gini index by income sources (see, Lerman and Yitzhaki, 1985) but there the G-matrix rather than the P-matrix is used and the product $\left(n\bar{y}_j\right)$ rather than the product $\left(nm_j\right)$ is introduced. Lerman and Yitzhaki (1985) have called "Gini correlation coefficient" the ratio $\left\{\frac{\left[\left(e'Gy_{.j}\right)/\left(n\bar{y}_j\right)\right]}{\left[\left(e'Gz_{.j}\right)/\left(n\bar{y}_j\right)\right]}\right\}$. In fact Lerman and Yitzhaki (1985) did not make use of the G-matrix but rather gave a formulation of the Gini index in terms of the covariances between the income sources and total income.

(20) $$FW_j = \left(G_{Bj} - G_{Wj}\right)\left(\frac{\bar{y}_j}{m_j}\right)$$

Let us now call PG_{Bj} and PG_{Wj} the between and within groups Pseudo Gini indices[7] for income source *j,* the two groups including respectively those individuals with a total income above median income *m,* and those with a total income below the median income *m*. The Pseudo-between groups Gini index PG_{Bj} is derived by giving to each "total income rich" individual (each individual whose total income is higher than the median total income) the average income $\ddot{y}_{rj}$ from source *j* among the "total income rich" individuals, and to each "total income poor" individual (each individual whose total income is lower than the median total income) the average income $\ddot{y}_{pj}$ from source *j* among the "total income poor" individuals. By analogy with expression (15) we may also write that

(21) $$PG_{Bj} = \left(\frac{1}{4\bar{y}_j}\right)\left(\dddot{y}_{rj} - \dddot{y}_{pj}\right)$$

Similarly, on the basis of the decomposition of the within groups Gini index, we may express the pseudo-within groups Gini index PG_{Wj} as the sum of two expressions. The first expression is the product of three elements respectively equal to the share (one half) in the total population of the "total income rich" individuals, the share $\dddot{s}_{rj}$ of the "total income rich" individuals (as defined previously) in the total income from source *j* and the "pseudo-within income source *j* rich" Gini index, $\dddot{P}G_{Wrj}$, which is computed by assuming that the incomes from source *j* among the "total income rich" individuals are ranked by decreasing total income. Similarly the second expression is the product of three elements respectively equal to the share (one half) in the total population of the "total income poor" individuals, the share $\dddot{s}_{pj}$ of the "total income poor" individuals (as defined previously) in the total income from source *j* and the "pseudo-within income source *j* poor" Gini index $\dddot{P}G_{Wpj}$ which is computed by assuming that the incomes from source *j* among the "total income poor" individuals are ranked by decreasing total income. In other words

(22) $$PG_{Wj} = \frac{1}{2}\dddot{s}_{rj}\dddot{P}G_{Wrj} + \frac{1}{2}\dddot{s}_{pj}\dddot{P}G_{Wpj}$$

Similarly to what appears in expression (20) we may then write that

(23) $$PFW_j = \left(PG_{Bj} - PG_{Wj}\right)\left(\frac{\bar{y}_j}{m_j}\right)$$

[7]See Silber (1989) for a definition of the Pseudo-Gini index. The Gini index, as explained before, is $I_G = e'Gs$, while the Pseudo-Gini index (concentration ratio) is $PG = e'Gv$ where the typical elements of *v* are the same incomes shares as in *s*, the only difference being that these incomes shares are this time ranked attending to the value of another variable.

where PFW_j, as mentioned previously, refers to the degree of "pseudo-bipolarization" of the distribution of income source j with respect to total income.

Going back to the last part of expression (17) it should be clear, given that the ratio $\left(\frac{m_j}{m}\right)$ is always positive[8], that a necessary condition for the contribution to the bipolarization of total income of income source j to be negative is that the difference between the between and within groups Pseudo-Gini indices is negative. In other words this contribution of income source j will be negative if the dispersion of the incomes from source j within the two groups of "total income rich" and "total income poor" is greater than that existing between these two groups. This is a very intuitive result given that, as mentioned in the introduction, the bipolarization of total income is an increasing function of the between groups (the "total income rich" and the total income poor") and a negative function of the within groups income dispersion. In Appendix A we give a simple empirical illustration based on the incomes of only four individuals and showing how each of the determinants mentioned previously affect the bipolarization of total income groups.[9]

4. An Empirical Illustration Based on Eu-Silc Data

The empirical investigation is based on the European Union Statistics on Income and Living Conditions (EU-SILC) data set for the 2007 and 2014 waves. We focus on disposable income. Disposable money income includes net income from work, other private income not related to work, pensions and other social transfers. Net money income includes all income sources received by the household and by each of its current members in the year preceding the survey. Since a given level of household income will correspond to a different standard of living, depending on the size and composition of the household, we adjust for these differences using the modified-OECD equivalence scale.

The decomposition of the Foster and Wolfson bi-polarization index by income sources will be based on four income sources:

1. Old-age and survivor's benefits (old-age and survivors benefits)
2. Benefits other than old-age and survivor's benefits (other benefits)
3. Income from rental of a property or land and interest, dividends, profit from capital investments in unincorporated business (property and interest)
4. Income available before including sources 1 to 3 (income before)

In this empirical analysis we first compute the contribution of each income source to the value of the Foster and Wolfson index in 2007 and 2014. Then we compute the contribution of the variation of each income source between 2007 and 2014 to the change in the Foster and Wolfson index for total income during this same period.

[8]We assume that incomes, whatever their sources, are always positive or nil.

[9]We observe, for income source 1 in the simple illustration given in Table A-2 in Appendix A, that the contribution C_j of income source 1 is negative and we can check that, as expected in such a case, the within groups Pseudo Gini index is equal to -0.03 while the between groups Pseudo Gini index for this same source 1 is smaller and equal to -0.3.

TABLE 1a

Decomposition of the Foster-Wolfson Index by Income Sources in 2007 with 95% Bootstrap Confidence Intervals

	Old-age and survivor's benefits			Other Benefits			Property and Interest			"Income before"		
	Value	2.5%	97.5%	Value	2.5%	97.5%	Value	2.5%	97.5%	Value	2.5%	97.5%
Austria	0.0180	0.0086	0.0255	−0.0086	−0.0120	−0.0050	0.0032	0.0024	0.0046	0.0944	0.0827	0.1083
Belgium	−0.0237	−0.0320	−0.0183	−0.0151	−0.0184	−0.0099	0.0062	0.0043	0.0083	0.1471	0.1346	0.1608
Bulgaria	−0.0186	−0.0269	−0.0094	−0.0026	−0.0053	0.0011	0.0012	0.0002	0.0022	0.1717	0.1534	0.1913
Cyprus	−0.0076	−0.0135	−0.0004	−0.0018	−0.0044	0.0001	0.0047	0.0023	0.0073	0.1316	0.1150	0.1471
Czech Republic	−0.0476	−0.0541	−0.0408	−0.0032	−0.0057	−0.0006	0.0023	0.0016	0.0031	0.1504	0.1353	0.1664
Germany	−0.0008	−0.0081	0.0059	−0.0106	−0.0131	−0.0077	0.0080	0.0064	0.0097	0.1271	0.1154	0.1428
Denmark	−0.0302	−0.0373	−0.0228	−0.0219	−0.0293	−0.0176	0.0011	−0.0019	0.0037	0.1508	0.1365	0.1684
Estonia	−0.0460	−0.0597	−0.0375	0.0021	0.0002	0.0043	0.0005	−0.0001	0.0010	0.2000	0.1785	0.2272
Greece	0.0212	0.0119	0.0337	−0.0010	−0.0032	0.0007	0.0116	0.0090	0.0145	0.1124	0.0944	0.1295
Spain	−0.0081	−0.0131	−0.0022	0.0017	−0.0001	0.0040	0.0042	0.0028	0.0055	0.1432	0.1268	0.1590
Finland	−0.0228	−0.0288	−0.0174	−0.0178	−0.0210	−0.0127	0.0061	0.0046	0.0076	0.1469	0.1309	0.1618
France	0.0175	0.0096	0.0250	−0.0065	−0.0091	−0.0036	0.0087	0.0070	0.0102	0.0875	0.0765	0.0987
Iceland	−0.0230	−0.0288	−0.0118	−0.0058	−0.0120	−0.0015	0.0088	0.0056	0.0136	0.1256	0.1100	0.1379
Italy	0.0089	0.0022	0.0132	0.0025	0.0014	0.0037	0.0064	0.0053	0.0077	0.1168	0.1061	0.1316
Lithuania	−0.0409	−0.0480	−0.0329	0.0026	−0.0004	0.0066	0.0015	0.0006	0.0023	0.1947	0.1738	0.2170
Luxembourg	0.0061	−0.0044	0.0210	−0.0012	−0.0060	0.0031	0.0092	0.0066	0.0131	0.1001	0.0801	0.1150
Netherlands	−0.0050	−0.0099	0.0005	−0.0173	−0.0216	−0.0136	0.0072	0.0055	0.0091	0.1214	0.1094	0.1350
Norway	0.0000	0.0000	0.0000	−0.0126	−0.0159	−0.0069	0.0021	0.0007	0.0036	0.1079	0.0957	0.1180
Poland	0.0362	0.0285	0.0434	−0.0070	−0.0089	−0.0045	0.0016	0.0011	0.0021	0.0996	0.0869	0.1151
Portugal	0.0180	0.0069	0.0279	0.0018	−0.0028	0.0056	0.0040	0.0022	0.0057	0.1434	0.1261	0.1645
Sweden	−0.0355	−0.0406	−0.0263	−0.0099	−0.0137	−0.0065	0.0027	0.0015	0.0046	0.1422	0.1270	0.1530
Slovenia	0.0016	−0.0053	0.0069	−0.0047	−0.0072	−0.0020	0.0018	0.0011	0.0026	0.1030	0.0927	0.1166
Slovak Republic	−0.0401	−0.0483	−0.0318	−0.0016	−0.0038	0.0007	0.0005	0.0002	0.0008	0.1363	0.1225	0.1509
U.K.	−0.0114	−0.0162	−0.0065	−0.0165	−0.0194	−0.0128	0.0088	0.0069	0.0107	0.1605	0.1453	0.1779

Notes: 400 quantiles for each country have been considered. The bootstrap confidence intervals are based on 1000 simulations.

Other Benefits refer to benefits other than old-age and survivor's benefits. Property and Interest refer to Income from rental of a property or land + Interest, dividends, profit from capital investments in unincorporated business. "Income before" refer to Income available before including the three other income sources.

TABLE 1b

Decomposition of the Foster-Wolfson Index by Income Sources in 2014 with 95% Bootstrap Confidence Intervals

	Old-age and survivor's benefits			Other Benefits			Property and Interest			"Income before"		
	Value	2.5%	97.5%	Value	2.5%	97.5%	Value	2.5%	97.5%	Value	2.5%	97.5%
Austria	0.0201	0.0107	0.0261	−0.0113	−0.0140	−0.0084	0.0042	0.0031	0.0058	0.0992	0.0889	0.1140
Belgium	−0.0235	−0.0301	−0.0164	−0.0158	−0.0206	−0.0121	0.0064	0.0050	0.0083	0.1487	0.1349	0.1642
Bulgaria	0.0024	−0.0067	0.0082	0.0151	0.0119	0.0183	0.0033	0.0019	0.0048	0.1218	0.1082	0.1412
Cyprus	0.0096	−0.0008	0.0185	−0.0014	−0.0045	0.0036	0.0050	0.0026	0.0077	0.1344	0.1198	0.1511
Czech Republic	−0.0383	−0.0461	−0.0313	0.0017	−0.0015	0.0035	0.0019	0.0011	0.0028	0.1310	0.1183	0.1468
Germany	0.0125	0.0048	0.0210	−0.0108	−0.0141	−0.0080	0.0065	0.0052	0.0077	0.1219	0.1098	0.1360
Denmark	−0.0244	−0.0319	−0.0153	−0.0217	−0.0276	−0.0150	0.0015	−0.0009	0.0035	0.1534	0.1376	0.1684
Estonia	−0.0529	−0.0627	−0.0452	0.0099	0.0066	0.0136	0.0018	0.0006	0.0032	0.2228	0.2000	0.2511
Greece	0.0396	0.0283	0.0547	−0.0003	−0.0021	0.0014	0.0091	0.0064	0.0114	0.0872	0.0752	0.1012
Spain	0.0242	0.0178	0.0330	0.0010	−0.0025	0.0037	0.0111	0.0088	0.0128	0.1153	0.1019	0.1304
Finland	−0.0225	−0.0295	−0.0166	−0.0111	−0.0147	−0.0077	0.0054	0.0040	0.0071	0.1410	0.1274	0.1557
France	0.0228	0.0151	0.0314	−0.0067	−0.0098	−0.0041	0.0194	0.0157	0.0242	0.0756	0.0668	0.0854
Iceland	−0.0088	−0.0191	0.0007	−0.0145	−0.0204	−0.0077	0.0055	0.0019	0.0097	0.1111	0.0974	0.1279
Italy	0.0208	0.0127	0.0309	0.0022	0.0005	0.0033	0.0096	0.0076	0.0117	0.0985	0.0878	0.1092
Lithuania	−0.0165	−0.0291	−0.0078	0.0071	0.0033	0.0129	0.0023	0.0013	0.0036	0.1662	0.1465	0.1897
Luxembourg	0.0329	0.0157	0.0517	−0.0097	−0.0142	−0.0048	0.0101	0.0072	0.0121	0.0854	0.0700	0.1001
Netherlands	0.0045	−0.0026	0.0120	−0.0178	−0.0224	−0.0133	0.0058	0.0039	0.0074	0.1141	0.1019	0.1262
Norway	−0.0080	−0.0123	−0.0012	−0.0093	−0.0126	−0.0059	0.0044	0.0031	0.0058	0.1093	0.0971	0.1204
Poland	0.0251	0.0171	0.0312	−0.0055	−0.0071	−0.0041	0.0016	0.0010	0.0025	0.1070	0.0932	0.1240
Portugal	0.0357	0.0253	0.0448	−0.0005	−0.0040	0.0017	0.0081	0.0063	0.0107	0.1026	0.0896	0.1186
Sweden	−0.0207	−0.0316	−0.0142	−0.0081	−0.0113	−0.0035	0.0054	0.0043	0.0073	0.1355	0.1214	0.1523
Slovenia	0.0073	0.0008	0.0127	−0.0053	−0.0086	−0.0023	0.0053	0.0040	0.0066	0.1018	0.0918	0.1151
Slovak Republic	−0.0210	−0.0312	−0.0087	−0.0025	−0.0049	−0.0002	0.0003	0.0000	0.0006	0.1196	0.1031	0.1345
U.K.	0.0124	0.0062	0.0187	−0.0210	−0.0252	−0.0181	0.0063	0.0046	0.0078	0.1363	0.1228	0.1540

Notes: 400 quantiles for each country have been considered. The bootstrap confidence intervals are based on 1000 simulations.

Other Benefits refer to benefits other than old-age and survivor's benefits. Property and Interest refer to Income from rental of a property or land plus Interest, dividends, profit from capital investments in unincorporated business. "Income before" refer to Income available before including the three other income sources.

TABLE 2
Value of the Foster-Wolfson (FW) Index in 2007 and 2014 and its Change Between 2007 and 2014, with 95% Bootstrap Confidence Intervals

	FW index in 2014			FW index in 2007			Change in FW index		
	Value	2.5%	97.5%	Value	2.5%	97.5%	Value	2.5%	97.5%
Austria	0.1123	0.0994	0.1245	0.1071	0.0961	0.1199	0.0052	−0.0120	0.0222
Belgium	0.1159	0.1038	0.1276	0.1145	0.1032	0.1281	0.0014	−0.0165	0.0185
Bulgaria	0.1427	0.1255	0.1585	0.1517	0.1364	0.1705	−0.009	−0.0350	0.0142
Cyprus	0.1475	0.1302	0.1656	0.1269	0.1137	0.1424	0.0206	−0.0012	0.0434
Czech Republic	0.0962	0.0853	0.1076	0.1018	0.0913	0.1144	−0.0056	−0.0216	0.0099
Germany	0.1301	0.1156	0.1440	0.1237	0.1111	0.1390	0.0064	−0.0143	0.0260
Denmark	0.1088	0.0969	0.1202	0.0998	0.0903	0.1112	0.009	−0.0068	0.0246
Estonia	0.1816	0.1606	0.2022	0.1566	0.1403	0.1768	0.025	−0.0029	0.0507
Greece	0.1356	0.1202	0.1509	0.1442	0.1292	0.1626	−0.0086	−0.0321	0.0141
Spain	0.1516	0.1347	0.1678	0.1410	0.1267	0.1586	0.0106	−0.0133	0.0333
Finland	0.1128	0.1006	0.1239	0.1123	0.1012	0.1254	0.0005	−0.0172	0.0173
France	0.1111	0.0983	0.1245	0.1073	0.0965	0.1206	0.0038	−0.0134	0.0215
Iceland	0.0934	0.0831	0.1030	0.1056	0.0950	0.1192	−0.0122	−0.0291	0.0027
Italy	0.1311	0.1164	0.1450	0.1347	0.1206	0.1513	−0.0036	−0.0258	0.0177
Lithuania	0.1592	0.1399	0.1776	0.1578	0.1411	0.1783	0.0014	−0.0267	0.0259
Luxembourg	0.1186	0.1052	0.1314	0.1142	0.1023	0.1275	0.0044	−0.0137	0.0221
Netherlands	0.1066	0.0945	0.1183	0.1063	0.0957	0.1192	0.0003	−0.0168	0.0165
Norway	0.0964	0.0860	0.1065	0.0974	0.0876	0.1092	−0.001	−0.0164	0.0141
Poland	0.1282	0.1133	0.1423	0.1303	0.1166	0.1470	−0.0021	−0.0244	0.0184
Portugal	0.1460	0.1283	0.1628	0.1671	0.1492	0.1913	−0.0211	−0.0502	0.0036
Sweden	0.1121	0.1000	0.1236	0.0996	0.0899	0.1111	0.0125	−0.0036	0.0281
Slovenia	0.1092	0.0967	0.1204	0.1018	0.0917	0.1139	0.0074	−0.0094	0.0233
Slovak Rep.	0.0965	0.0854	0.1073	0.0951	0.0859	0.1067	0.0014	−0.0140	0.0160
U.K.	0.1340	0.1193	0.1482	0.1413	0.1268	0.1597	−0.0073	−0.0307	0.0139

Notes: 400 quantiles for each country have been considered. The bootstrap confidence intervals are based on 1000 simulations.

Regarding the contribution of each income source to the value of the Foster and Wolfson index in 2007 and 2014, Tables 1a and 1b show that "income before" is the source that has the highest contribution to bi-polarization, this being true for all countries. Old-age and survivor's benefits as well as "other benefits" are the sources that contribute the least to bi-polarization and in several cases their contribution is even of opposite sign to that of "income before" (e.g. Bulgaria and U.K.). We hence conclude that benefits are often a counterweight to the impact of "income before" on bipolarization in 2007 as well as in 2014, that is, before as well as after the financial crisis.

If we look at the change in bipolarization between 2007 and 2014, Table 2 shows that the change in bipolarization has not been the same across countries. We observe that there are countries in which bi-polarization increased, like in Cyprus and Estonia, while in other countries, like Portugal, bi-polarization decreased, but none of these changes was in fact statistically significant.[10]

Such non-significant changes in bi-polarization may however hide significant changes in the contribution of the different income sources, which act in opposite directions. Table 3 shows, for example, that in Bulgaria, the change in the

[10]We test the significance of the estimates through bootstrap confidence intervals.

TABLE 3

Contribution of Income Sources to the Change (Between 2007 and 2014) in the Foster and Wolfson Index, with 95% Bootstrap Confidence Intervals

	Change in Old-age and survivor's benefits			Change in Other Benefits			Change in Property and Interest			Change in "Income before"		
	Value	2.5%	97.5%	Value	2.5%	97.5%	Value	2.5%	97.5%	Value	2.5%	97.5%
Austria	0.0021	−0.0103	0.0120	−0.0027	−0.0071	0.0021	0.001	−0.0009	0.0026	0.0048	−0.0130	0.0244
Belgium	0.0002	−0.0077	0.0119	0.0007	−0.0082	0.0038	0.0002	−0.0023	0.0029	0.0016	−0.0199	0.0220
Bulgaria	0.021	0.0071	0.0297	0.0177	0.0128	0.0218	0.0021	0.0004	0.0039	−0.0499	−0.0751	−0.0231
Cyprus	0.0172	0.0038	0.0276	0.0004	−0.0032	0.0066	0.0003	−0.0036	0.0035	0.0028	−0.0214	0.0258
Czech Rep.	0.0093	−0.0013	0.0189	0.0049	0.0007	0.0077	−0.0004	−0.0015	0.0006	−0.0194	−0.0395	0.0016
Germany	0.0133	0.0031	0.0242	−0.0002	−0.0047	0.0036	−0.0015	−0.0035	0.0004	−0.0052	−0.0253	0.0126
Denmark	0.0058	−0.0046	0.0177	0.0002	−0.0073	0.0103	0.0004	−0.0031	0.0037	0.0026	−0.0210	0.0237
Estonia	−0.0069	−0.0198	0.0086	0.0078	0.0039	0.0120	0.0013	−0.0001	0.0028	0.0228	−0.0150	0.0556
Greece	0.0184	0.0008	0.0342	0.0007	−0.0019	0.0038	−0.0025	−0.0066	0.0010	−0.0252	−0.0467	−0.0051
Spain	0.0323	0.0239	0.0421	−0.0007	−0.0055	0.0024	0.0069	0.0043	0.0091	−0.0279	−0.0503	−0.0069
Finland	0.0003	−0.0086	0.0091	0.0067	0.0006	0.0114	−0.0007	−0.0027	0.0016	−0.0059	−0.0275	0.0150
France	0.0053	−0.0051	0.0168	−0.0002	−0.0048	0.0035	0.0107	0.0069	0.0159	−0.0119	−0.0274	0.0031
Iceland	0.0142	−0.0035	0.0232	−0.0087	−0.0162	0.0010	−0.0033	−0.0098	0.0015	−0.0145	−0.0336	0.0098
Italy	0.0119	0.0024	0.0250	−0.0003	−0.0026	0.0012	0.0032	0.0007	0.0055	−0.0183	−0.0372	−0.0028
Lithuania	0.0244	0.0100	0.0358	0.0045	−0.0007	0.0106	0.0008	−0.0004	0.0023	−0.0285	−0.0582	0.0035
Luxembourg	0.0268	0.0043	0.0465	−0.0085	−0.0147	−0.0019	0.0009	−0.0044	0.0039	−0.0147	−0.0365	0.0100
Netherlands	0.0095	0.0004	0.0188	−0.0005	−0.0068	0.0065	−0.0014	−0.0043	0.0009	−0.0073	−0.0259	0.0116
Norway	−0.008	−0.0124	−0.0017	0.0033	−0.0034	0.0081	0.0023	0.0006	0.0044	0.0014	−0.0155	0.0167
Poland	−0.0111	−0.0230	−0.0009	0.0015	−0.0015	0.0037	0.0000	−0.0008	0.0011	0.0074	−0.0122	0.0278
Portugal	0.0177	0.0023	0.0325	−0.0023	−0.0073	0.0026	0.0041	0.0014	0.0071	−0.0408	−0.0675	−0.0195
Sweden	0.0148	−0.0003	0.0227	0.0018	−0.0028	0.0079	0.0027	0.0007	0.0049	−0.0067	−0.0257	0.0172
Slovenia	0.0057	−0.0026	0.0149	−0.0006	−0.0053	0.0032	0.0035	0.0020	0.0050	−0.0012	−0.0172	0.0153
Slovak Rep.	0.0191	0.0063	0.0344	−0.0009	−0.0041	0.0021	−0.0002	−0.0006	0.0003	−0.0167	−0.0406	0.0044
U.K.	0.0238	0.0159	0.0323	−0.0045	−0.0106	−0.0006	−0.0025	−0.0053	0.0000	−0.0242	−0.0467	−0.0012

Notes: 400 quantiles for each country have been considered. The bootstrap confidence intervals are based on 1000 simulations.

Other Benefits refer to benefits other than old-age and survivor's benefits. Property and Interest refer to Income from rental of a property or land plus Interest, dividends, profit from capital investments in unincorporated business. "Income before" refer to Income available before including the three other income sources.

contribution of old-age and survivor benefits, other benefits and property and interest, would per se have led to a significant increase in bi-polarization, while the change in the contribution of "income before" would per se have led to a significant decrease in bi-polarization. The net result of these changes was a non-significant change in bipolarization. We also observe that in all countries where the change in the contribution of old-age and survivor benefits to bi-polarization was significant, it would have led to an increase in bi-polarization, except in Poland and Norway. The same is true for "other benefits", the exceptions being here Luxembourg and the U.K., but the number of countries with a significant change in the contribution to bi-polarization is here much smaller. Finally, all the significant changes in the contribution of property and interest to bi-polarization would per se have led to an increase in bi-polarization while the opposite is true for the changes in the contribution of "income before", when they were significant.

It thus appears that even though there were no significant changes in the degree of bi-polarization, between 2007 and 2014, in total income, there were some significant changes in the contribution of the different income sources to overall bi-polarization, mainly in the case of "old-age and survivor" benefits and "income before".

5. Concluding Comments

This paper has shown that by expressing the Foster and Wolfson (1992; 2010) bi-polarization index in matrix form the decomposition of the Foster and Wolfson bi-polarization index by income sources becomes relatively simple. An empirical illustration based on EU-SILC data for the years 2007 and 2014 has shown the usefulness of the proposed approach since we were able to show that, although there was no country in which the change in bi-polarization was significant, this was for some countries due to the fact that the changes in the contribution of the income sources "old-age and survivor benefits" and "other benefits" had an impact opposite in sign to that of "income before".

The proposed breakdown could naturally be also applied to an analysis of the bi-polarization of the distribution of wages or earnings. Using then a traditional earnings function it becomes possible to derive the contribution to the bi-polarization of wages of the explanatory variables of such a function. We indeed intend to explore these issues in future empirical work.

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Supporting Information

Additional Supporting Information may be found in the online version of this article at the publisher's website:

Appendix A: A simple empirical illustration

Table A-1: The original hypothetical data
Table A-2: Computation of various indicators