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TIME DUMMY HEDONIC AND QUALITY-ADJUSTED UNIT VALUE INDEXES: DO THEY REALLY DIFFER?

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One of the main approaches to constructing quality-adjusted price indexes is the time dummy hedonic method. An alternative but rather unconventional method is the estimation of quality-adjusted unit value indexes. An advantage of the latter method is the interpretation of the implicit quantity index as the simple ratio of quality-adjusted or standardized quantities. In this paper we compare the two methods. We show that the expenditure-share weighted time dummy price index and the quality-adjusted unit value index can be written as ratios of weighted geometric and harmonic means, respectively, of quality-adjusted prices. Next, we argue that the two indexes will have similar trends and volatility if the quality-adjusted prices in the quality-adjusted unit value index are based on the estimated time dummy model. Our theoretical findings are illustrated on New Zealand scanner data for seven consumer electronics products.

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1. INTRODUCTION

When the quality of a heterogeneous product changes, the price index of that product should be adjusted accordingly. Hedonic regression has become the default method for performing quality adjustment (ILO *et al.*, 2004), although implementation by statistical agencies is still at an early stage, perhaps due to the cost of collecting data on product characteristics. The literature distinguishes between three main hedonic approaches: the time dummy approach, the

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imputations approach, and the characteristics approach.[1] In this paper we focus on the first approach.

The time dummy model explains the log of price from a set of relevant product characteristics and dummy variables indicating the time periods. Quality-adjusted price index numbers are simply obtained by exponentiating the time dummy coefficients. In most applications, time dummy models are estimated by Ordinary Least Squares (OLS) regression, where each observation receives the same weight. A more general approach would be to weight the observations (according to some criterion), i.e. to use Weighted Least Squares (WLS).

Two important questions arise. What choice of weights is "best"? And what kind of aggregation is implicitly happening when a time dummy hedonic model is estimated by WLS regression? Diewert (2004) stresses that the observations should be weighted according to their economic importance and advocates the use of expenditure shares as weights in the regression; see Ivancic and Fox (2013) and de Haan and Krsinich (2014) for empirical applications. As we show in the present paper, the resulting time dummy indexes are ratios of expenditure-share weighted geometric means of quality-adjusted prices. Taking geometric means is just one way of averaging observations. For example, one could take harmonic means instead. It turns out that the ratio of expenditure-share weighted harmonic means of quality-adjusted prices can be viewed as what we refer to as a *quality-adjusted unit value index*.

The purpose of our paper is twofold. First, we want to explain the concept of a quality-adjusted unit value index because we think this concept, though unconventional, is very useful for measuring price change of broadly comparable items, such as different televisions. Its advantages are: *i*) if all items had the same characteristics, then the index simplifies to the ordinary unit value index, and *ii*) the implicit quantity index equals the ratio of quality-adjusted or standardized quantities, which is an appealing interpretation of quantity change at the product level. Our second aim is to show that the expenditure-share weighted time dummy index closely approximates the quality-adjusted unit value index if in the latter the quality-adjusted prices are also based on the WLS time dummy results (but not if they are based on OLS results). Thus, even if one prefers the concept of a quality-adjusted unit value index, the construction of an expenditure-share weighted time dummy index will suffice.

The paper is organized as follows. Section 2 outlines the log-linear time dummy model and provides an expression for the WLS time dummy index. Section 3 discusses the quality-adjusted unit value index. To explain the concept, we start off with a simple numerical example consisting of two items and three time periods and then turn to the general case with many different items and many periods. Section 4 compares the time dummy index and the quality-adjusted unit value index. Section 5 contains an empirical illustration using a scanner data set for consumer electronics purchased in New Zealand. Section 6 summarizes and concludes.

[1]The *Manual on Consumer Price Indices* (ILO *et al.*, 2004) provides an overview of the three methods. For a comparison of time dummy and hedonic imputation approaches, see e.g. Silver and Heravi (2007a), Diewert, Heravi and Silver (2009), and de Haan (2010).

2. Time Dummy Hedonic Indexes

Consider the following log-linear hedonic model for period t:

$$\ln p_i^t = \delta^t + \sum_{k=1}^{K} \beta_k z_{ik} + \varepsilon_i^t, \tag{1}$$

where p_i^t denotes the price of item i; z_{ik} is the (fixed) quantity of characteristic k for item i and β_k the corresponding parameter; δ^t is the intercept; by assumption, the errors ε_i^t are independently distributed with an expected value of zero. At this stage we do not make any assumptions about the variance of the errors. The parameters β_k in (1) are constant over time, which allows us to estimate the model on the pooled data of the item samples $S^0, S^1, \ldots, S^T$ (size $N^0, N^1, \ldots, N^T$) in periods $t=0, 1, \ldots, T$. The estimating equation for the pooled data is

$$\ln p_i^t = \delta^0 + \sum_{t=1}^{T} \delta^t D_i^t + \sum_{k=1}^{K} \beta_k z_{ik} + \varepsilon_i^t, \tag{2}$$

where the time dummy variable D_i^t has the value 1 if the observation pertains to period t and the value 0 otherwise; the time dummy parameters δ^t shift the hedonic surface upwards or downwards as compared to the base period value δ^0.

Suppose equation (2) is estimated by WLS regression using weights w_i^0 and w_i^t ($t=1, \ldots, T$), yielding parameter estimates $\hat{\delta}^0$, $\hat{\delta}^t$ and $\hat{\beta}_k$ ($k=1, \ldots, K$). We assume the weights are normalized such that they sum to unity in each period, i.e. $\sum_{i \in S^0} w_i^0 = 1$ and $\sum_{i \in S^t} w_i^t = 1$. Since changes in the item characteristics are controlled for, $\exp(\hat{\delta}^t)$ is an estimator of quality-adjusted aggregate price change going from the base period 0 to period t.

An explicit expression for the time dummy index can be derived as follows. The predicted prices for the base period and the comparison periods t ($t=1, \ldots, T$) are

$$\hat{p}_i^0 = \exp(\hat{\delta}^0) \exp\left[\sum_{k=1}^{K} \hat{\beta}_k z_{ik}\right]; \tag{3}$$

$$\hat{p}_i^t = \exp(\hat{\delta}^0) \exp(\hat{\delta}^t) \exp\left[\sum_{k=1}^{K} \hat{\beta}_k z_{ik}\right]. \tag{4}$$

Taking the weighted geometric average of the predicted prices for all items belonging to the samples S^0 and $S^1, \ldots, S^T$, using the weights w_i^0 and w_i^t, yields

$$\prod_{i \in S^0} (\hat{p}_i^0)^{w_i^0} = \exp(\hat{\delta}^0) \exp\left[\sum_{k=1}^{K} \hat{\beta}_k \sum_{i \in S^0} w_i^0 z_{ik}\right]; \tag{5}$$

$$\prod_{i \in S^t} (\hat{p}_i^t)^{w_i^t} = \exp(\hat{\delta}^0)\exp(\hat{\delta}^t)\exp\left[\sum_{k=1}^{K} \hat{\beta}_k \sum_{i \in S^t} w_i^t z_{ik}\right]. \tag{6}$$

Dividing (6) by (5) and some rearranging gives

$$\exp(\hat{\delta}^t) = \frac{\prod_{i \in S^t} (\hat{p}_i^t)^{w_i^t}}{\prod_{i \in S^0} (\hat{p}_i^0)^{w_i^0}} \exp\left[\sum_{k=1}^{K} \hat{\beta}_k \left(\sum_{i \in S^0} w_i^0 z_{ik} - \sum_{i \in S^t} w_i^t z_{ik}\right)\right]. \tag{7}$$

Due to the inclusion of time dummies into equation (2), the weighted regression residuals sum to zero in each time period, yielding $\prod_{i \in S^0} (\hat{p}_i^0)^{w_i^0} = \prod_{i \in S^0} (p_i^0)^{w_i^0}$ and $\prod_{i \in S^t} (\hat{p}_i^t)^{w_i^t} = \prod_{i \in S^t} (p_i^t)^{w_i^t}$. Substitution into (7) gives

$$\hat{P}_{TD}^{0t} = \exp(\hat{\delta}^t) = \frac{\prod_{i \in S^t} (p_i^t)^{w_i^t}}{\prod_{i \in S^0} (p_i^0)^{w_i^0}} \exp\left[\sum_{k=1}^{K} \hat{\beta}_k \left(\sum_{i \in S^0} w_i^0 z_{ik} - \sum_{i \in S^t} w_i^t z_{ik}\right)\right]. \tag{8}$$

The bracketed exponential factor in (8) adjusts the ratio of weighted geometric average prices for changes in the weighted average characteristics. This factor is equal to 1 if the average characteristics are constant over time. The time dummy index then simplifies to the ratio of weighted geometric average prices. We will address the choice of regression weights in Section 4.

One disadvantage of the time dummy index is that it suffers from a discontinuity problem, although in Section 4 we will refine this point. Items with the same quantities for each of the characteristics are essentially the same from a consumer perspective, and if all items were essentially the same, then the time dummy index (8) would approach $\prod_{i \in S^t} (p_i^t)^{w_i^t} / \prod_{i \in S^0} (p_i^0)^{w_i^0}$. However, in this limiting situation the appropriate measure of price change is the unit value index (ILO *et al.*, 2004)[2]

$$P_{UV}^{0t} = \frac{\sum_{i \in S^t} p_i^t q_i^t \Big/ \sum_{i \in S^t} q_i^t}{\sum_{i \in S^0} p_i^0 q_i^0 \Big/ \sum_{i \in S^0} q_i^0} = \frac{\sum_{i \in S^t} p_i^t q_i^t \Big/ \sum_{i \in S^t} (p_i^t)^{-1} p_i^t q_i^t}{\sum_{i \in S^0} p_i^0 q_i^0 \Big/ \sum_{i \in S^0} (p_i^0)^{-1} p_i^0 q_i^0} = \frac{\left[\sum_{i \in S^t} s_i^t (p_i^t)^{-1}\right]^{-1}}{\left[\sum_{i \in S^0} s_i^0 (p_i^0)^{-1}\right]^{-1}}. \tag{9}$$

This reflects a more general problem: no standard index number formula will generate a unit value index when all items are essentially comparable. Silver (2011) speaks of an "index number formula problem". Notice that P_{UV}^{0t} can be written as

[2]Balk (1998) discusses the properties of the unit value index. Bradley (2005) and Silver (2010) address potential problems of using unit value indexes as measures of price change for heterogeneous products.

a ratio of weighted harmonic means of prices where the items' expenditure shares $s_i^0 = p_i^0 q_i^0 / \sum_{i \in S^0} p_i^0 q_i^0$ and $s_i^t = p_i^t q_i^t / \sum_{i \in S^t} p_i^t q_i^t$ serve as weights.

In Section 3 below we will discuss the *quality-adjusted unit value index*, which is an alternative to conventional index number formulae that does simplify to the ordinary unit value index if all items are essentially comparable. In Section 4 we will compare the quality-adjusted unit value index with the weighted time dummy index and argue that, under certain conditions, they are likely to be very similar.

3. Quality-Adjusted Unit Value Indexes

To the best of our knowledge, Dalén (2001) was the first to propose the use of a quality-adjusted unit value index rather than a conventional quality-adjusted price index when hedonic quality adjustment is performed. Here we draw heavily from de Haan (2004a), who provides a formal definition of a quality-adjusted unit value index. Before turning to the general case with multiple items and time periods, we will discuss an example with only two items and three time periods.

3.1. *The Two-Items, Three-Periods Case*

Suppose that in period 0 the statistical agency observes the price and quantity purchased of item 1. The item is also purchased in period 1 but it is no longer available in period 2. Thus, the quantities are $q_1^0 > 0$, $q_1^1 > 0$, and $q_1^2 = 0$. The agency selects a replacement item 2 which was not available in period 0 but which is purchased in periods 1 and 2. So we have $q_2^0 = 0$, $q_2^1 > 0$, and $q_2^2 > 0$.

Now assume that in each period t ($t = 0, 1, 2$), consumers are indifferent between buying one unit of item 2 and $\lambda_{2/1}^t$ units of item 1, hence between buying q_2^t units of item 2 and $\lambda_{2/1}^t q_2^t$ units of item 1. In other words, we assume that a quality-adjustment factor $\lambda_{2/1}^t$ exists which expresses the quantity purchased in period t of item 2 in terms of a quantity of item 1. A standardized average price or *quality-adjusted unit value* can be defined as

$$\frac{q_1^t p_1^t + \lambda_{2/1}^t q_2^t \tilde{p}_2^t}{q_1^t + \lambda_{2/1}^t q_2^t} = \frac{q_1^t p_1^t + q_2^t p_2^t}{q_1^t + \lambda_{2/1}^t q_2^t}, \tag{10}$$

where $\tilde{p}_2^t = p_2^t / \lambda_{2/1}^t$ is the quality-adjusted price of item 2 with respect to item 1.

The following hypothetical example illustrates the idea. Table 1 lists prices and quantities purchased in periods 0, 1 and 2 for two items: item 1 is a one liter package of a certain brand of orange juice, item 2 is a two liter package of the

TABLE 1
Hypothetical Example; Data

Item	Size (liters)	p^0	p^1	p^2	q^0	q^1	q^2
1	1	2.00	2.10	–	200	120	0
2	2	–	3.50	3.80	0	40	100

TABLE 2

Hypothetical Example; Results

	Period 0	Period 1	Period 2
Unit value	2.00	2.45	3.80
Quality-adjusted unit value; $\lambda_{2/1}=2$	2.00	1.96	1.90
Quality-adjusted unit value; $\lambda_{2/1}=1.67$	2.00	2.10	2.28
Unit value index	1.00	1.23	1.90
Quality-adjusted unit value index; $\lambda_{2/1}=2$	1.00	0.98	0.95
Quality-adjusted unit value index; $\lambda_{2/1}=1.67$	1.00	1.05	1.14

same juice. Note that the total volume of juice purchased is kept fixed at 200 liters.

Table 2 contains results for two values for $\lambda_{2/1}$. First, we assume that consumers are indifferent between buying one package of item 2 and two packages of item 1, i.e. we set $\lambda_{2/1}=2$ (independent of time). Due to compositional change, the ordinary unit value index increases by 90 percent between periods 0 and 2. When we standardize to liters, the result becomes much more reasonable with the quality-adjusted unit value index—the ratio of quality-adjusted unit values in the periods compared—measuring a price decline of 5 percent. Such linear quantity adjustments have been frequently used by statistical agencies but they can be problematic. The data suggest nonlinear pricing: the price per liter in period 1 for the two liter package (1.75) is lower than the price for the one liter package (2.10).[3]

Another way to estimate $\lambda_{2/1}$ is by overlap pricing. Since the items are available in period 1, we could assume that the price difference reflects the consumers' evaluation of the difference in quality. In this case we have $\lambda_{2/1}=p_2^1/p_1^1=3.50/2.10 \cong 1.67$, and the quality-adjusted unit value index now increases by 14 percent between periods 0 and 2.[4] A potential problem is random error: the law of one quality-adjusted price may not exactly hold, and a one-period overlap may be too short for evaluating quality differences.

We prefer using the predicted prices from a hedonic regression to estimate the quality-adjustment factors. This issue will be addressed in Section 4.

3.2. *The Many-Items, Many-Periods Case*

Let us now turn to the more realistic case when there are many different items available during periods $t=0,\ldots,T$. Not all items will be purchased in each time period. To define quality-adjusted unit values for the sets of items $S^0,\ldots,S^T$, a base item (or numeraire) b must be chosen. The base item does not necessarily have to be observable; it could also be an unobserved item described by, for example, sample average characteristics. We denote the quality-adjustment factor for item i in period t with respect to the base item b by $\lambda_{i/b}^t$ ($\lambda_{b/b}^t=1$) and define its quality-adjusted price as $\tilde{p}_i^t=p_i^t/\lambda_{i/b}^t$ ($\tilde{p}_b^t=p_b^t$). The quality-adjusted unit value is given by

[3]Fox and Melser (2014) use hedonic regression to investigate the effect of linear quantity adjustments on price indexes when nonlinear pricing is prevalent.

[4]Note that the chained matched-item index produces the same result: (2.10/2.00)(3.80/3.50)= 1.14.

$$(11) \qquad \frac{\sum_{i\in S^t}\lambda_{i/b}^t\tilde{p}_i^t q_i^t}{\sum_{i\in S^t}\lambda_{i/b}^t q_i^t}=\frac{\sum_{i\in S^t}p_i^t q_i^t}{\sum_{i\in S^t}\lambda_{i/b}^t q_i^t}=\frac{\sum_{i\in S^t}p_i^t q_i^t}{\sum_{i\in S^t}(\tilde{p}_i^t)^{-1}p_i^t q_i^t}=\left[\sum_{i\in S^t}s_i^t(\tilde{p}_i^t)^{-1}\right]^{-1},$$

which is a straightforward generalization of (10).

The ratio of the quality-adjusted unit values in period t $(t=1,\ldots,T)$ and period 0 defines the *quality-adjusted unit value index*[5]

$$(12) \qquad P_{QAUV}^{0t}=\frac{\sum_{i\in S^t}p_i^t q_i^t}{\sum_{i\in S^0}p_i^0 q_i^0}\left[\frac{\sum_{i\in S^t}\lambda_{i/b}^t q_i^t}{\sum_{i\in S^0}\lambda_{i/b}^0 q_i^0}\right]^{-1}=\frac{\left[\sum_{i\in S^t}s_i^t(\tilde{p}_i^t)^{-1}\right]^{-1}}{\left[\sum_{i\in S^0}s_i^0(\tilde{p}_i^0)^{-1}\right]^{-1}}.$$

In the last expression of (12), the quality-adjusted unit value index is written as a ratio of expenditure-share weighted harmonic means of quality-adjusted prices. The index is *transitive*, meaning that the quality-adjusted unit value index is invariant to the choice of base period and can be written as a period-on-period chained index. Transitivity is a useful property because the time series will be free from chain drift.

One of the main objectives of index number theory is to find ways to decompose the value change of an economic aggregate into a price change and a quantity change. It will therefore be useful to take a look at the quantity index implicitly defined by the quality-adjusted unit value index (12):

$$(13) \qquad Q^{0t}=\frac{\sum_{i\in S^t}p_i^t q_i^t}{\sum_{i\in S^0}p_i^0 q_i^0}\frac{1}{P_{QAUV}^{0t}}=\frac{\sum_{i\in S^t}\lambda_{i/b}^t q_i^t}{\sum_{i\in S^0}\lambda_{i/b}^0 q_i^0}.$$

In (13), the quantities purchased of the various items are added after expressing them in *constant-quality units*, i.e. in units of item b. As a matter of fact, this is the idea behind the present approach. To avoid arbitrary decisions, the quantity index, hence the quality-adjusted unit value index, should be invariant to the choice of base item.

In a matched-item context, a sufficient condition for the quantity index to satisfy the *identity test* is $\lambda_{i/b}^t=\lambda_{i/b}^0=\lambda_{i/b}$ for all time periods $t=1,\ldots,T$. That is, when there are no new or disappearing items and the quantities purchased of all items remain the same, the quantity index will be equal to 1 if the quality-adjustment factors are constant across time.

[5]Von Auer (2014) uses the term *generalized* unit value index. An important difference with our approach is that he assumes the set of items constant across time. So he addresses the issue of quality-mix change but not quality change in terms of new and disappearing items.

Note that the quality-adjusted unit value index does not satisfy the identity test, whether or not the quality-adjustment factors are constant; if the prices of all (matched) items are constant, the index will usually still differ from 1. The choice of index number formula involves a tradeoff of desirable properties, and which index is best depends on the context. Drift in weighted indexes is a particular concern in scanner data (see also Reinsdorf, 1999). Transitivity is therefore of great importance.

4. A Comparison of the Two Methods

In this section we show how the quality-adjusted prices and quality-adjustment factors can be estimated from the time dummy regression and then compare the weighted time dummy index with the quality-adjusted unit value index. We derive an exact expression for the difference between the two indexes and, for a particular choice of weights, an approximation that depends on the variance of the regression residuals. Several related issues are also discussed.

4.1. *An Exact Expression and an Approximation*

Using the predicted prices (3) and (4) from the time dummy model, quality-adjusted prices in period 0 and period t with respect to an arbitrary base item b can be estimated in the following way:

$$\hat{\tilde{p}}_i^{\,0}=p_i^0(\hat{p}_b^0/\hat{p}_i^0)=p_i^0\exp\left[\sum_{k=1}^{K}\hat{\beta}_k(z_{bk}-z_{ik})\right]; \tag{14}$$

$$\hat{\tilde{p}}_i^{\,t}=p_i^t(\hat{p}_b^t/\hat{p}_i^t)=p_i^t\exp\left[\sum_{k=1}^{K}\hat{\beta}_k(z_{bk}-z_{ik})\right]. \tag{15}$$

The quality-adjustment factors in (12) and (13) are estimated as

$$\hat{\lambda}_{i/b}=\hat{p}_i^t/\hat{p}_b^t=\hat{p}_i^0/\hat{p}_b^0=\exp\left[\sum_{k=1}^{K}\hat{\beta}_k(z_{ik}-z_{bk})\right]. \tag{16}$$

Note that they are constant across time, hence the superscript for time has been omitted. The estimator for the quantity index (13) is

$$\hat{Q}^{0t}=\frac{\sum_{i\in S^t}\hat{\lambda}_{i/b}q_i^t}{\sum_{i\in S^0}\hat{\lambda}_{i/b}q_i^0}, \tag{17}$$

with $\hat{\lambda}_{i/b}$ given by (16).

The quality-adjusted unit value index can be indirectly estimated by dividing the value index by (17). Alternatively, it can be directly estimated from the first expression of equation (12) using the estimated quality-adjustment factors $\hat{\lambda}_{i/b}$. We will, however, start from the second expression of (12) and use $\hat{\tilde{p}}_i^0$ and $\hat{\tilde{p}}_i^t$, given by (14) and (15), as estimates of the quality-adjusted prices. Recalling that $\hat{p}_b^t/\hat{p}_b^0=\hat{P}_{TD}^{0t}$, we find

$$(18)\qquad \hat{P}_{QAUV}^{0t}=\frac{\left[\sum_{i\in S^t}s_i^t(\hat{\tilde{p}}_i^t)^{-1}\right]^{-1}}{\left[\sum_{i\in S^0}s_i^0(\hat{\tilde{p}}_i^0)^{-1}\right]^{-1}}=\frac{\left[\sum_{i\in S^t}s_i^t(p_i^t\hat{p}_b^t/\hat{p}_i^t)^{-1}\right]^{-1}}{\left[\sum_{i\in S^0}s_i^0(p_i^0\hat{p}_b^0/\hat{p}_i^0)^{-1}\right]^{-1}}=\hat{P}_{TD}^{0t}\left[\frac{\sum_{i\in S^0}s_i^0(\hat{p}_i^0/p_i^0)}{\sum_{i\in S^t}s_i^t(\hat{p}_i^t/p_i^t)}\right]$$
$$=\hat{P}_{TD}^{0t}\left[\frac{\sum_{i\in S^0}s_i^0\exp(u_i^0)}{\sum_{i\in S^t}s_i^t\exp(u_i^t)}\right].$$

Equation (18) says that the estimated quality-adjusted unit value index (which is based on quality-adjusted prices derived from the time dummy regression) can be expressed as the time dummy index multiplied by the ratio of weighted means of the exponentiated residuals $u_i^0=\ln(\hat{p}_i^0/p_i^0)$ and $u_i^t=\ln(\hat{p}_i^t/p_i^t)$ in periods 0 and t from the time dummy regression.

If $u_i^0=u_i^t=0$ $(t=1,\ldots,T)$ for all i, hence $R^2=1$, $\hat{P}_{TD}^{0t}$ and $\hat{P}_{QAUV}^{0t}$ will coincide in each period (as expenditure shares sum to unity). We have $\hat{p}_i^0=p_i^0$, $\hat{p}_b^0=p_b^0$, $\hat{p}_i^t=p_i^t$, and $\hat{p}_b^t=p_b^t$, and so the estimated quality-adjusted prices in periods 0 and t given by (14) and (15) are equal to p_b^0 and p_b^t for all items. The 'law of one quality-adjusted price' now holds true, and both the time dummy index and the quality-adjusted unit value index simplify to the price relative p_b^t/p_b^0. In reality we will of course never find a perfect fit to the data.

Importantly, relation (18) is independent of the choice of regression weights. But which choice is "best"? This question can be looked at from different angles. To start with, notice that the time dummy index (8) can be written as the ratio of the weighted geometric sample means of the estimated quality-adjusted prices in periods 0 and t:

$$(19)\qquad \hat{P}_{TD}^{0t}=\frac{\prod_{i\in S^t}(p_i^t)^{w_i^t}}{\prod_{i\in S^0}(p_i^0)^{w_i^0}}\frac{\prod_{i\in S^t}\left(\exp\left[\sum_{k=1}^{K}\hat{\beta}_k(z_{bk}-z_{ik})\right]\right)^{w_i^t}}{\prod_{i\in S^0}\left(\exp\left[\sum_{k=1}^{K}\hat{\beta}_k(z_{bk}-z_{ik})\right]\right)^{w_i^0}}=\frac{\prod_{i\in S^t}(\hat{\tilde{p}}_i^t)^{w_i^t}}{\prod_{i\in S^0}(\hat{\tilde{p}}_i^0)^{w_i^0}}.$$

Assuming the quality-adjusted unit value and time dummy methods are equally valid, and taking into account that the same (estimated) quality-adjusted prices are used, it seems natural to require that both methods yield approximately equal results.

As we argue below, this will be the case if expenditure shares are chosen as regression weights. By setting $w_i^0 = s_i^0$ and $w_i^t = s_i^t$ in (19), the time dummy index becomes

$$\hat{P}_{TD}^{0t} = \frac{\prod_{i \in S^t} (\hat{\tilde{p}}_i^t)^{s_i^t}}{\prod_{i \in S^0} (\hat{\tilde{p}}_i^0)^{s_i^0}}. \tag{20}$$

There is a striking similarity between equation (20) and the first expression in (18). The only difference is that the quality-adjusted unit value index is the ratio of expenditure-share weighted *harmonic* rather than geometric means of the estimated quality-adjusted prices.

From Jensen's inequality we know that weighted harmonic means are smaller than the corresponding weighted geometric means unless there is no variability in the data and the means coincide. This points towards the dispersion of the quality-adjusted prices or, equivalently, the dispersion of the regression residuals, as the main driver of the difference between the two indexes.[6] To gain more insight, we will approximate the bracketed factor in the last expression of (18) in terms of the variance of the residuals.

Using second-order Taylor expansion, for small residuals we can approximate $\exp(u_i^t)$ by

$$\exp(u_i^t) \cong 1 + u_i^t + \frac{1}{2}(u_i^t)^2. \tag{21}$$

The expenditure-share weighted sample mean of the exponentiated residuals in period t can therefore be approximated by

$$\sum_{i \in S^t} s_i^t \exp(u_i^t) \cong 1 + \sum_{i \in S^t} s_i^t u_i^t + \frac{1}{2}\sum_{i \in S^t} s_i^t (u_i^t)^2. \tag{22}$$

When running a WLS time dummy regression with expenditure shares as weights, the weighted sample mean of the residuals, $\sum_{i \in S^t} s_i^t u_i^t$, is equal to zero in each period t; we have used this orthogonality property earlier. In this case, (22) simplifies to

$$\sum_{i \in S^t} s_i^t \exp(u_i^t) \cong 1 + \frac{1}{2}\sum_{i \in S^t} s_i^t (u_i^t)^2 = 1 + \frac{1}{2}(\sigma^t)^2, \tag{23}$$

The term $\sum_{i \in S^t} s_i^t (u_i^t)^2$ in (23) equals the expenditure-share weighted variance of the regression residuals, denoted by $(\sigma^t)^2$ for short. Similarly, the weighted variance of the residuals in period 0 is denoted by $(\sigma^0)^2$, and we have

[6]For a discussion on the difference between various unweighted price indexes at the elementary level in terms of price dispersion and product heterogeneity, see Silver and Heravi (2007b).

(24)
$$\sum_{i \in S^0} s_i^0 \exp\left(u_i^0\right) \cong 1 + \frac{1}{2}\left(\sigma^0\right)^2.$$

Using (23) and (24), we obtain the following approximation of equation (18) in the case of expenditure-share weighted least squares regression:

(25)
$$\hat{P}^{0t}_{QAUV} \cong \hat{P}^{0t}_{TD}\left[\frac{1+\frac{1}{2}\left(\sigma^0\right)^2}{1+\frac{1}{2}\left(\sigma^t\right)^2}\right].$$

Expression (25) indicates that the quality-adjusted unit value index will sit below (above) the time dummy index when the variance of the regression residuals increases (decreases) over time. However, this type of heteroskedasticity is unlikely to be present because of the logarithmic functional form for the hedonic model. In a linear hedonic model with price rather than log of price as the dependent variable, the absolute errors tend to get bigger over time when there is inflation. As pointed out by Diewert (2004), the logarithmic transformation neutralizes this tendency. Thus, although the two indexes may exhibit slightly different period-on-period changes, they are expected to have equal trends and to be equally volatile. This is the first underpinning of expenditure shares as regression weights.

4.2. *Index Number Theory and the Choice of Weights*

Standard index number theory also supports our choice of regression weights. Suppose there are no new or disappearing items during the sample period. With $S^t = S^0 = S$ and $N^t = N^0 = N$, the last expression of equation (19) for the (matched-item) time dummy index becomes

(26)
$$\hat{P}^{0t}_{TD(M)} = \frac{\prod_{i \in S}\left(\hat{\tilde{p}}^{\,t}_i\right)^{w_i^t}}{\prod_{i \in S}\left(\hat{\tilde{p}}^{\,0}_i\right)^{w_i^0}}.$$

Equation (26) can be written as

(27)
$$\hat{P}^{0t}_{TD(M)} = \frac{\prod_{i \in S}\left(\hat{\tilde{p}}^{\,t}_i\right)^{w_i^t}}{\prod_{i \in S}\left(\hat{\tilde{p}}^{\,0}_i\right)^{w_i^0}} = \frac{\prod_{i \in S}\left(\hat{\tilde{p}}^{\,t}_i\right)^{w_i^0}}{\prod_{i \in S}\left(\hat{\tilde{p}}^{\,0}_i\right)^{w_i^0}} \frac{\prod_{i \in S}\left(\hat{\tilde{p}}^{\,t}_i\right)^{w_i^t}}{\prod_{i \in S}\left(\hat{\tilde{p}}^{\,t}_i\right)^{w_i^0}} = \prod_{i \in S}\left(\frac{p_i^t}{p_i^0}\right)^{w_i^0} \prod_{i \in S}\left(\hat{\tilde{p}}^{\,t}_i\right)^{w_i^t - w_i^0},$$

using the fact that for each (matched) item the ratio of quality-adjusted prices is equal to the ratio of observed prices: $\hat{\tilde{p}}^{\,t}_i / \hat{\tilde{p}}^{\,0}_i = p_i^t / p_i^0$. An alternative decomposition of (26) is

(28) $$\hat{P}^{0t}_{TD(M)} = \frac{\prod_{i\in S} (\hat{\hat{p}}^{t}_{i})^{w^t_i}}{\prod_{i\in S} (\hat{\hat{p}}^{0}_{i})^{w^0_i}} = \frac{\prod_{i\in S} (\hat{\hat{p}}^{t}_{i})^{w^t_i}}{\prod_{i\in S} (\hat{\hat{p}}^{0}_{i})^{w^t_i}} \frac{\prod_{i\in S} (\hat{\hat{p}}^{0}_{i})^{w^t_i}}{\prod_{i\in S} (\hat{\hat{p}}^{0}_{i})^{w^0_i}} = \prod_{i\in S} \left(\frac{p^t_i}{p^0_i}\right)^{w^t_i} \prod_{i\in S} (\hat{\hat{p}}^{0}_{i})^{w^t_i - w^0_i}.$$

A third decomposition is obtained by taking the geometric mean of (27) and (28):

(29) $$\hat{P}^{0t}_{TD(M)} = \prod_{i\in S} \left(\frac{p^t_i}{p^0_i}\right)^{\frac{w^0_i + w^t_i}{2}} \prod_{i\in S} (\hat{\hat{p}}^{0}_{i} \hat{\hat{p}}^{t}_{i})^{\frac{w^t_i - w^0_i}{2}}.$$

Suppose we run OLS instead of WLS regressions. Econometric textbooks tell us that if the errors have constant variance, i.e. when they are homoskedastic, OLS is most efficient. With $w^0_i = w^t_i = 1/N$, the second factor in equation (29) is equal to 1, and the time dummy index equals the (matched-item) Jevons index $\prod_{i\in S}(p^t_i/p^0_i)^{1/N}$. From an index number point of view this is problematic; items should be weighted according to their economic importance.[7] There is ample evidence that weighting matters: weighted price indexes can differ substantially from unweighted ones.

But what does weighting according to economic importance imply for the choice of weights in a time dummy regression? At first glance, quantities purchased may seem a good choice. This was proposed by Silver and Heravi (2005). The regression is then run on a 'weighted data set' where each item *i* counts q^0_i and q^t_i times in periods 0 and *t*. Since we want the weights to sum to unity in every time period, quantity shares would be more appropriate. On the other hand, adding up (unadjusted) quantities across heterogeneous items to calculate the shares is difficult to justify.

Weighting by quantity shares has another, more severe problem. Suppose for a moment that we are comparing two periods, 0 and 1, and run a time dummy regression on the pooled (and matched) data of the two periods. Taking the average of the shares in periods 0 and 1 as regression weights has an advantage: if the weights stay the same, the second factor in (29) is equal to 1 and the resulting matched-item index is not model-dependent. The problem is that the price relatives will be weighted by average quantity shares. Price relatives should be weighted by expenditure shares, not quantity shares. As noted by Diewert (2004), the natural choice of regression weights in the bilateral case is the average expenditure shares in the two periods, $w^0_i = w^1_i = (s^0_i + s^1_i)/2$, because the resulting time dummy index then equals the superlative Törnqvist index.[8]

The above does not carry over to the multilateral case; it is not possible to obtain a multilateral time dummy index that exactly equals the matched-item

[7]This may be different in a housing context though. Houses are unique, and so the quantity sold is 1 by definition. Hill and Melser (2008, footnote 9) make a strong case for giving equal weight to each house, as in the Jevons index, rather than giving more expensive houses more weight.

[8]De Haan (2004b) proposes using half the expenditure shares for the unmatched items when there are new and disappearing items because this will produce an imputation Törnqvist index.

Törnqvist index. To understand why, recall that any time dummy index is transitive while the Törnqvist is not. Our preferred choice of weights is the expenditure shares in the periods the items are observed, i.e. $w_i^0 = s_i^0$ and $w_i^t = s_i^t$ for all i. As can be seen from equation (29), the time dummy index will then be equal to the matched-item Törnqvist price index times a factor that makes the result transitive and therefore free from chain drift. This factor is dependent on the model specification, but we expect it to be relatively small. Just as the quality-adjusted unit value index, the weighted time dummy index fails the identity test in a matched-item context.

The proposed set of regression weights can be readily extended to the situation with new and disappearing items. Each item, whether matched or unmatched, receives a weight that is equal to its expenditure share (reflecting its economic importance) in the period of observation.

4.3. *Further issues*

When all items have the same quantities of characteristics, the hedonic model, hence the quality-adjusted prices, cannot be estimated. This limiting case does not pose a problem for calculating a quality-adjusted unit value index; since there are no quality differences, we can simply set $\lambda_{i/b}^t = \lambda_{i/b}^0 = \lambda_{i/b} = 1$ for all i in (12) to obtain the ordinary unit value index. But what can we say about the discontinuity problem of the time dummy index raised earlier?

The answer is that a problem only occurs when items are defined more narrowly than necessary. Suppose for a moment that we define items by barcode. This means that goods with the same (quantities of) characteristics but with different barcodes would be identified as different items. However, as they are essentially similar, it seems better to treat such goods as a single composite item and calculate unit values across the different barcodes. This is what we do in Section 5. The time dummy index is then automatically equal to the unit value index if all items are essentially comparable.

The time dummy approach assumes constancy across time of the characteristics parameters. If the quality-adjustment factors in the quality-adjusted unit value index are based on predicted prices from the time dummy hedonic model, as proposed above, this assumption implies constancy across time of the quality-adjustment factors. Consumers' evaluations of quality differences may of course change over time. More generally, the fixed-parameters assumption is restrictive (Pakes, 2003), and it might therefore be better to use hedonic imputation methods, where models are estimated in each time period separately. However, the aim of our paper is not to come up with best-practice methods to estimate hedonic price indexes but merely to explain the similarities and differences between the time dummy index and the quality-adjusted unit value index.

Finally, we will elaborate on a few econometric issues. The estimated quality-adjusted prices (14) and (15)—to be substituted into the second expression of (12) for the quality-adjusted unit value index—are biased as taking exponentials is a

TABLE 3
Average Monthly Number of Distinct Items for Each Product

Category	Products
Desktop computers	150
Laptop computers	432
Portable media players	161
DVD players/recorders	202
Digital cameras	289
Camcorders	88
Televisions	341

non-linear transformation. The time dummy index is similarly biased.[9] It is questionable whether bias adjustments would be appropriate, though, at least from an index number point of view. For instance, recall the two-period case with only matched items, where Diewert's (2004) choice of regression weights ensures that the time dummy index is equal to the superlative Törnqvist price index index. Correcting for the "bias" would mean that this useful property does no longer hold, and so there is a tension between econometrics and index number theory.

Our preference for the index number perspective can have implications for the efficiency of the estimators. It may be that our choice of weights introduces or amplifies heteroskedasticity and leads to estimators that are less efficient than strictly necessary. But a loss of efficiency is not as bad as ending up with indexes that are not grounded in index number theory.

5. Illustration on Scanner Data for Consumer Electronics

In this section we will illustrate our theoretical findings using a New Zealand scanner data set from market research company GfK for seven consumer electronics products: desktop computers, laptop computers, portable media players, DVD players/recorders, digital cameras, camcorders, and televisions.[10] We have monthly data from mid-2008 to mid-2011. The data is close to full-coverage of the New Zealand consumer market. For a more comprehensive description, see de Haan and Krsinich (2014).

There are no barcode numbers provided in the data, as such. Instead, we define an item as the unique combination of brand, model and the full set of physical characteristics available in the data, around 40 in total for each product. In other words, we identify items by the set of characteristics included in the hedonic model rather than by barcode. Table 3 shows the average monthly number of distinct items for each product.

The data is aggregated across outlets. This means we are unable to control for changes in the composition of the sample in terms of outlets. A key feature of the data is a high degree of churn; there are many new product specifications becoming

[9]Bias correction terms for the time dummy index, depending on the standard errors of the time dummy coefficients, can be found in the literature; see e.g. Kennedy (1981) and van Garderen and Shah (2002). In practice the bias correction term is usually small enough to be ignored.

[10]The original data set also includes microwaves. We decided to leave these out because microwaves are not really "consumer electronics" and because there is much less rapid technological and quality change going on.

TABLE 4

Average Monthly Rates of New, Disappearing and Matched Items

Product	new	disappearing	matched
Desktop computers	29%	29%	42%
Laptop computers	29%	29%	43%
Portable media players	24%	25%	52%
DVD players/recorders	25%	25%	50%
Digital cameras	25%	25%	49%
Camcorders	27%	27%	46%
Televisions	24%	23%	53%

available on the market and, conversely, old specifications dropping out of the market as they become obsolete. Table 4 shows the average monthly rates of new, disappearing and matched items; note that these are not weighted by expenditure shares.

We estimated time dummy models both by expenditure-share weighted and OLS regression.[11] Table 5 lists unadjusted and adjusted R^2 values. For the WLS regressions, the adjusted values range from 0.964 (DVD players/recorders) to 0.989 (portable media players). The OLS R^2 values are lower, especially for DVD players/recorders, but still quite satisfactory. The relatively high R^2 values are partly due to the aggregation over goods with identical characteristics. If we had instead used the individual barcodes as items in the regressions, then the fit of the models would not be as good.

Figure 1 shows the resulting WLS and OLS time dummy and quality-adjusted unit value indexes for each product. As expected, the two types of index are very similar when using WLS; the two lines can hardly be distinguished. OLS yields quite different results. The OLS-based indexes tend to be more volatile, and they sit well below the WLS-based indexes for all products except televisions and camcorders, for which they sit higher.

As explained in Section 4, the fact that the quality-adjusted unit value index (i.e.

TABLE 5

R Squared Values for Time Dummy Regressions

	WLS		OLS	
	unadjusted	adjusted	unadjusted	adjusted
Desktop computers	0.983	0.978	0.899	0.870
Laptop computers	0.980	0.977	0.894	0.878
Portable media players	0.991	0.989	0.881	0.859
DVD players/recorders	0.967	0.964	0.765	0.746
Digital cameras	0.987	0.985	0.927	0.913
Camcorders	0.973	0.967	0.908	0.888
Televisions	0.988	0.986	0.918	0.907

the ratio of harmonic means of quality-adjusted prices) and the time dummy index (the ratio of geometric means) almost coincide when expenditure-share weighted regression is used, is due to the weighted variance of the regression residuals being

[11]The regression results can be obtained from the authors upon request.

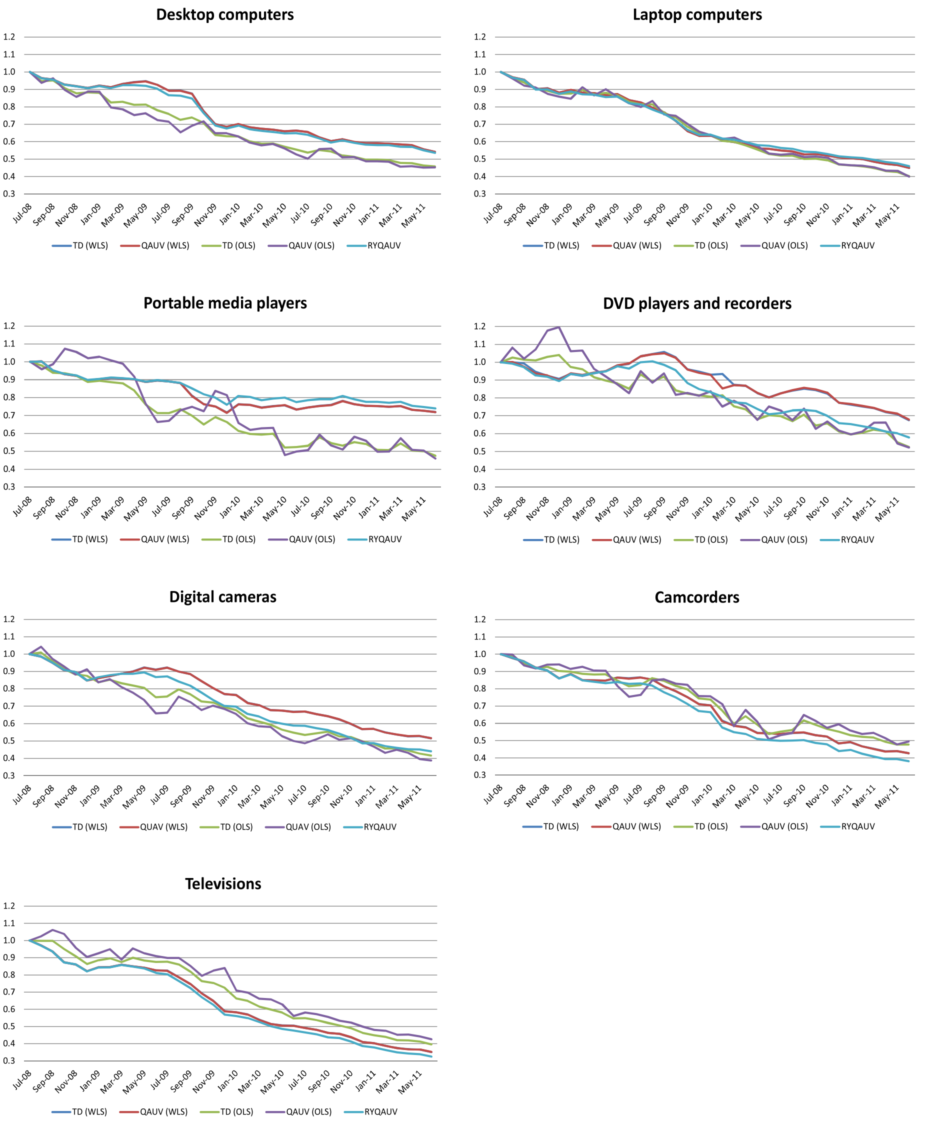

Figure 1. Time dummy and quality-adjusted unit value indexes [Colour figure can be viewed at wileyonlinelibrary.com]

approximately constant over time.[12] The graphs in the Appendix confirm that there are no trends in the weighted variance of the WLS residuals for the various products.

[12]The Taylor approximations of the sample means of weighted exponentiated residuals given by (23) and (24) work quite well. The correlation coefficients between the actual and approximated values across all months are 0.996, 0.991, 0.990, 0.940, 0.995, 0.997, and 0.983 for desktop computers, laptop computers, portable media players, DVD players/recorders, digital cameras, camcorders and televisions, respectively. We have also estimated quality-adjusted unit value indexes using approximation (25). For all products, the indexes virtually coincide with the original indexes.

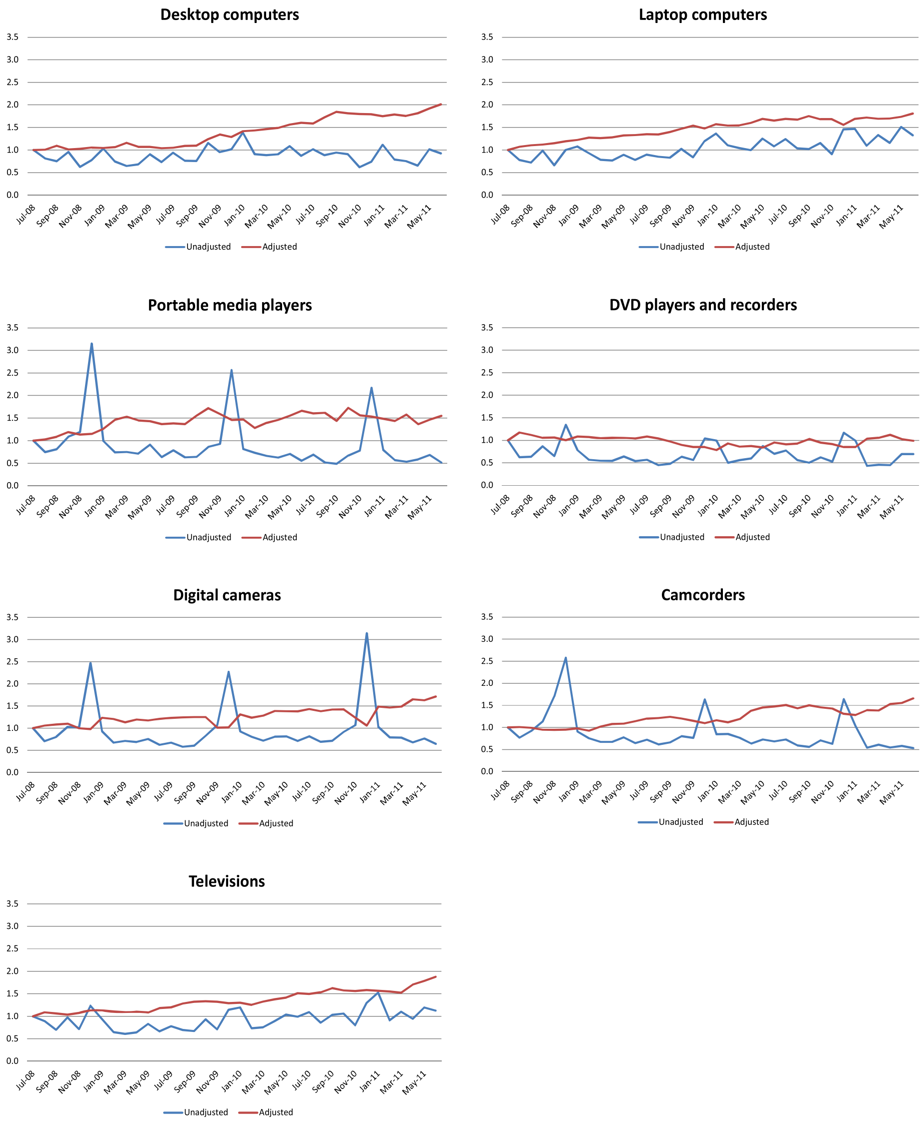

Figure 2. Indexes of unadjusted and quality-adjusted quantities [Colour figure can be viewed at wileyonlinelibrary.com]

An approximation of equation (18) for the OLS case is derived in the Appendix. While the graphs in the Appendix do not reveal any trends in the variance of the OLS residuals, the two indexes can differ considerably. In particular, the OLS-based quality-adjusted unit value indexes are much more volatile than the OLS time dummy indexes. This is in accordance with our expectations. Equation (A.4) in the Appendix shows that a simple (approximate) relation between the variance of the residuals and the difference between the OLS-based quality-adjusted unit value index and the OLS time dummy index no longer exists. Although in the

OLS case the quality-adjusted unit value index will be driven in the long run by the time dummy index, the "second terms" in (A.4) can add significantly to the volatility of the index if the expenditure shares of the various items differ.

A disadvantage of multilateral methods such as the time dummy hedonic method is that when data for the next period is added and the model re-estimated, the results for all previous periods will change. Statistical agencies generally do not accept continuous revisions. A rolling window approach can be used to overcome the revisions problem. The time dummy model is estimated on the data of a window with a fixed length, which is shifted forward each time period. The most recent period-on-period index movement is then repeatedly spliced onto the existing time series. For a monthly CPI a rolling year approach seems appropriate because a window of 13 months is the shortest one that can deal with seasonal goods.[13]

As can be seen from Figure 1, for digital cameras and DVD players/recorders, the rolling-year WLS time dummy indexes are closer to the pooled OLS indexes than to the pooled WLS indexes. Figure 1 also includes rolling year versions of the WLS-based quality-adjusted unit value indexes. For DVD players/recorders and digital cameras the rolling year indexes differ a lot from the original WLS-based quality-adjusted unit value indexes. This result shows that the choice of window length is an important issue when it comes to implementation in official statistics. We leave this for future work.

Figure 2 plots indexes of quality-adjusted quantities and unadjusted quantities. The quality-adjusted indexes, given by equation (17), have been calculated by dividing the value index for each product by the WLS-based quality-adjusted unit value index. The indexes of unadjusted quantities are simple ratios of the quantities purchased, i.e. the number of sales, and follow from (17) by setting $\hat{\lambda}_{i/b}=1$ for all i. The unadjusted quantities are quite volatile with sales typically spiking in December/January because of Christmas season sales, though not so much for computers. After adjusting for quality differences, the picture changes radically. Due to improvements in average quality of items sold, all consumer electronics products except DVD players/recorders exhibit a strong increase in quality-adjusted quantities. As we would expect, the adjusted quantity indexes are smoother than the unadjusted indexes. The December/January peaks vanish, and sometimes even turn into troughs, when we adjust quantities for quality differences; Christmas season sales apparently consist of items that tend to be of lesser quality than items sold in other months.

6. Conclusions

Our findings can be summarized as follows. The use of expenditure shares as weights in a time dummy regression is supported by standard index number theory: this produces a quality-adjusted price index which is transitive, hence drift free, and where items are weighted according to their economic importance. The

[13]De Haan and Krsinich (2014), using the same scanner data set as we do, apply a rolling year approach to estimating various quality-adjusted price indexes, including weighted time dummy indexes. Ivancic, Diewert and Fox (2011) and de Haan and van der Grient (2011) use a rolling window approach in the context of GEKS indexes for supermarket goods. For an alternative splicing method, see Krsinich (2016).

weighted time dummy index can alternatively be expressed as the ratio of expenditure-share weighted *geometric* means of quality-adjusted prices. The ratio of expenditure-share weighted *harmonic* means of quality-adjusted prices defines a quality-adjusted unit value index, which is transitive as well. Put differently, the expenditure-share weighted time dummy index can be viewed as the geometric counterpart to a quality-adjusted unit value index, something which has been overlooked in the literature.

In our opinion, the quality-adjusted unit value index is a very useful concept for measuring aggregate price change of broadly comparable items. We have shown that, if the quality-adjusted prices in the quality-adjusted unit value index are the same as those in the weighted time dummy index, the two indexes are expected to have the same trend and volatility. This was confirmed by empirical work on New Zealand scanner data for consumer electronics: for all seven products examined, the weighted time dummy and quality-adjusted unit value indexes were virtually identical. That is, even if one prefers the quality-adjusted unit value index, the easier-to-construct expenditure-share weighted time dummy index will suffice.

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Supporting Information

Additional supporting information may be found in the online version of this article at the publisher's web-site:

Appendix: Components of approximation of equation (18), OLS
Figure A.1: Components of equations (25) and (A.4)
Table A.1: Monthly summary measures for regression residuals