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CONSUMER EXPECTATIONS: A RESIDUAL BASED APPROACH

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This paper presents an economy-wide consumer expectations indicator that reflects different degrees of optimism or pessimism with respect to consumers' confidence in their economy. The indicator provides a useful complement to traditional economic indicators that are frequently used to compare countries, such as gross domestic product (GDP) in purchasing power parity (PPP) terms. Our indicator may be seen as representing the influence of social wealth on economic behavior–that is, of effects left out of a standard economic analysis. We use a theoretical approach to integrate the expectations measure with the International Comparison Program's (ICP) PPP GDP statistics which produces a measure we term "effective GDP." Compared to the ICP's PPP figures, the measure of "effective GDP" differs from the ICP's PPP estimates by as much as four to five percent in the positive direction for apparently optimistic countries and as much as two percent downwards for pessimists.

JEL Codes: D12, D16, E10

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1. Introduction

There is a long-held view among economists that expectations play a critical role in economic behavior. Expectations are notoriously difficult to measure. However, if the long-held view is true one might expect an examination of consumer behavior to reveal something of their expectations. One way to develop this idea would be to consider a "residual-based" approach to the measurement of consumer expectations. Is there any justifiable precedent for this approach?

In the study of the economics of the firm, the issue of productivity has been of substantial importance for a number of years. Like expectations, productivity is something that is conceptually appealing but which is difficult to measure. Although there are many alternative and arguably more sophisticated approaches, yet—at its most basic—level of total factor productivity (TFP) can be viewed in very simple conceptual terms as the ratio of aggregate output to aggregate inputs. However, a vast range of production function literature provides estimates of the production function as a combination of parameter estimates and factor inputs.

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The estimated production function, if it does not additionally allow for productivity, can therefore be viewed really as an aggregate factor input index rather than as a direct measure of output. Taking the ratio of actual output to this factor input index then identifies productivity with the "residual" (in ratio form), viz. as the proportion of output that is not explained by the combination of factor inputs. Can a similar approach be applied in consumer studies to identify the conceptual variable "expectations"?

At first sight there is an obvious reason why the residual-based productivity approach cannot be directly taken over to the study of the consumer. This obvious reason is that consumer "output" (that is, "utility") is unobservable—viz. there is no "actual" to compare to a model-based estimate which is some function of inputs. However, on further thought, the same conceptual idea—to estimate a key contributing factor influencing behavior from a residual—is worthy of closer examination. While consumer utility is not observable, consumer expenditure patterns certainly are. Many models of consumer expenditure do not make room for the influence of expectations. However, if these models are reasonably correct in other respects, perhaps an approach akin to the "productivity-as-a-residual" paradigm can be employed to measure consumer expectations by reference to the relationship between actual and model-predicted consumer expenditure behavior.

This paper makes use of a model that explains the share of total consumer expenditure allocated to 12 broad consumption categories in a wide range of countries covered by the International Comparison Program (ICP). Differences in real income are accounted for within the model along with *non-homothetic* preferences. However, once income differences are controlled for, arguably a major reason for differences in consumer behavior across countries that cannot be explained by the model could be due to wide ranging variations in consumer expectations across countries. The model contains some simple features that ensure robustness of results as the number of low income countries in the sample is increased. In addition to the attention paid to modelling the consumption patterns of very poor countries, there is also a design feature that enhances the potential of the model to remain relevant as a predictive device as incomes increase. We illustrate this by providing estimates for a broad sample of 144 countries in 2005, with PPP per capita real incomes ranging from a low of 264 (Congo, Democratic Republic) to a high of 70,014 (Luxembourg).[1]

The model construction aims to minimize the effect of *a priori* assumptions and maximize the influence of well recognized empirical regularities such as Engel's Law and the implied *non-homotheticity* of preferences which underlies it. Engel's Law has been invoked to study many aspects of consumer behavior ranging from: international consumption comparisons (Theil and Chung 1989; Seale and Regmi 2006); a data-parsimonious way to determine the standard of living based simply on knowledge of the food budget share (Clements and Chen 2010); a foundation upon which to build a more detailed study of cross price elasticities

[1]These figures are GDP per capita in International PPP units. They are taken from the first column of the 2005 ICP Global Results Summary Table (ICP, 2008, pp. 23–27). Of the 146 countries represented in the table, we have had to exclude two countries, Burundi and Zimbabwe, because of lack of relevant data required for this study.

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(Regmi and Seale 2010); the study of properties of consumer demand systems (Barnett and Serlitis 2009); comparison of models and their implications for income elasticities (Gao 2012); and even to study bias in the construction of the CPI by studying the (in)stability of Engel Curves over time (Hamilton 2001).

Our particular interest is slightly different to the research objectives outlined in the above-mentioned studies. It is more narrowly focused than the broad international consumption comparison studies. Although we invoke Engel's Law, we use it not as a substitute for an income-based measure of the standard of living (although Clements and Chen show that it can in fact be used for this purpose) but rather to provide a complementary measure. While we recognize the importance of differences in relative prices, and note the many studies that have examined this, our immediate focus is on using the empirical regularity of Engel's Law to say more about real living standards than can be discerned from income and price data alone. We are also interested in utilizing a functional form which we know to be "integrable." This avoids problems of share bound violations which Gao (2012) notes is prominent in principle with many functional forms and which Regmi and Seale (2010) show can occur in situations where the analysis covers a large income range.

In our approach, the traditional economic measure of the standard of living (GDP per capita) and our confidence-based complementary measure—the latter reflecting aspects of the quality of life that are non-economic in the traditional view—are tied to the modelling in a rigorous manner which is both simple and transparent. Effectively, we collect certain information from the residuals in Engel Curve estimation and use this to enhance our understanding of the quality of life in the various countries. Since this involves an amalgam of influences that might reflect varying degrees of optimism or pessimism, we give this component the generic name "expectations" in this paper. Although aspects of this factor may reflect optimism or pessimism it also contains potentially many other explanators such as the degree to which the average person in a poor country may be able to rely on the informal sector and/or family networks to boost their sense of well-being or, in the case of more wealthy countries, the extent to which the existence of government supported social safety nets may serve a similar purpose. In short, our "expectations" measure also serves as a proxy for the influence of extra-economic public capital on consumer behavior. As such, it can be thought of as a measure of well-being that should complement a traditional economic measure of the standard of living.

Section 2 describes the underlying model. Section 3 outlines an extension to our model to incorporate expectations. Section 4 presents the basic data. Section 5 provides details of the estimation procedure and the preliminary statistical results. Our statistical procedure is a multi-step one, and section 6 explains our residual-based approach to the refinement of our initial expectations indicator. In the various steps, preliminary research findings are presented along the way. Our key research findings, related to the development of an "effective GDP" measure that combines expectations with official GDP, are presented in section 7. A brief conclusion is offered in section 8. An appendix provides more extensive tables of the basic data and of the statistical results.

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2. The Underlying Model

We employ the following parsimonious indirect utility function (IUF) specification:

$$V(c,p)=\left(\frac{c}{P_B}\right)\left[\ln\left(\frac{c}{P_A}\right)\right] \tag{1}$$

In (1), c denotes the average consumer's nominal total expenditure and P_A and P_B are two alternative price indexes to be described below. Effectively, the IUF is a product of two real total expenditure measures—a linear real total expenditure measure—c/P_B—and a logarithmic measure—$\ln(c/P_A)$. The presence of two real total expenditure measures, one in a linear form and one in a logarithmic form, allows for *non-homothetic* preferences to be represented in a reasonably flexible way.[2] This is the key specification that allows us to make use of Engel's Law.

In our empirical work, we distinguish 12 different categories of consumer expenditure. Consequently, the price indexes P_A and P_B may each be viewed as functions of 12 underlying individual commodity prices $p_1,\ldots,p_{12}$. It is convenient to define two sets of price index elasticities:

$$\begin{array}{l}\alpha_i=\partial\ln P_A/\partial\ln p_i\\ \beta_i=\partial\ln P_B/\partial\ln p_i\end{array},\quad i=1,\ldots 12. \tag{2}$$

Fully regular price indexes P_A and P_B should be non-decreasing and homogenous of degree 1 in prices. We impose this by specifying that $\sum_{i=1}^{12}\alpha_i=1$ and $\sum_{i=1}^{12}\beta_i=1$, and also that $\alpha_i\geq 0$ and $\beta_i\geq 0$ for all $i=1,\ldots,12$.[3] The i^{th} commodity budget share is defined as $s_i=p_iq_i/c$ where $c=\sum_{i=1}^{12}p_iq_i$ and of course it follows that $\sum_{i=1}^{12}s_i=1$.

Application of the logarithmic form of Roy's Identity to (1) leads to a system of optimal consumer demand equations for the budget shares which, given (2), can be written as:

$$s_i=\frac{\alpha_i+\beta_i\ln(c/P_A)}{1+\ln(c/P_A)},\quad i=1,\ldots,12. \tag{3}$$

It can be seen from (3) that when $c/P_A=1$, $s_i=\alpha_i$. If we were to normalize our data such that $c/P_A=1$ for an extremely poor country, we could identify α_i as the predicted expenditure share on commodity i for the very poor. On the other hand, as real income increases indefinitely, $s_i\rightarrow\beta_i$. Thus we can identify β_i as the

[2]Specification (1) is a slightly simplified version of the Modified Almost Ideal Demand System (MAIDS), introduced by Cooper and McLaren (1992).

[3]To simplify matters with little loss of generality we in fact impose strict positivity on the elasticities. The elasticities need not be constant. However, we will be dealing empirically with a case where there is little information on the actual levels of the 12 individual prices and in this circumstance it will be useful to consider the simplest case where the "true" elasticities are approximated by constants.

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asymptotic expenditure share on commodity i for the extremely wealthy. Given Engel's Law, we would expect $\alpha_i > \beta_i$ when i refers to food, so that the expenditure share of food declines as incomes rise. This of course applies to any necessity by definition. For luxuries, we have by definition that $\alpha_i < \beta_i$.

It is clear from the functional form of (3) and the properties of the parameters α_i and β_i (as elasticities derived from a positive and linearly homogenous function) that RHS (3) lies in the [0,1] interval, matching the property of the actual data shares s_i, $i=1, \ldots, 12$. For any of the 12 shares of primary interest modelled by (3), we can define its "complementary share" very simply as:[4]

$$1-s_i=\frac{(1-\alpha_i)+(1-\beta_i)\ln(c/P_A)}{1+\ln(c/P_A)} \tag{4}$$

Obviously, the complementary share also lies within the [0, 1] range. In order to develop a functional form for the model that accepts an additive random error, we proceed firstly to model the share ratios (or, more picturesquely, the "odds"):

$$\frac{s_i}{1-s_i}=\frac{\alpha_i+\beta_i\ln(c/P_A)}{(1-\alpha_i)+(1-\beta_i)\ln(c/P_A)} \tag{5}$$

In the model development, we need at some point to acknowledge the requirement for inclusion of a disturbance term. Modelling the "odds" can ultimately be accompanied by inclusion of a multiplicative disturbance term. As is standard, the notation $s_i/(1-s_i)$ on LHS (5) is used to denote the actual odds implicit in the data. However, RHS (5) is clearly an economic model meant to approximate these odds. It is convenient to work with the log-odds and to write the model with appended additive disturbance term as:

$$y_i \equiv \ln\left(\frac{s_i}{1-s_i}\right)=y_i^m+u_i \tag{6}$$

where

$$y_i^m=\ln\left(\frac{\alpha_i+\beta_i\ln(c/P_A)}{(1-\alpha_i)+(1-\beta_i)\ln(c/P_A)}\right) \tag{7}$$

An important design feature of our model is to allow it to be utilized with minimal data. In particular, we would like to proceed without utilizing any information on individual commodity prices and indeed without directly using total expenditure as an explanator. Further, since both price indexes P_A and P_B are relevant to our *non-homothetic* preference specification, we initially face the issue of which if any of these two indexes may be linked with the single GDP deflator that is usually available. We do note, however, that only one of the price indexes turns

[4]For notational simplicity, we drop reference to the range of commodities from this point. It is to be understood that $i=1, \ldots, 12$.

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up explicitly in (7).[5] We therefore propose to replace the real total expenditure term c/P_A (in a series of steps) with a specially scaled and normalized version of real per capita GDP in purchasing power parity units. The steps are as follows:

Step 1 (Scaling): Let G denote the data on real per capita GDP in PPP units, taken directly from ICP (2008, Summary Table). Let G_b denote a base level of real per capita GDP. We choose the real per capita GDP of the poorest country in the originally available sample (hence we set $G_b = 264$). Let G_m denote a level of real per capita GDP that would correspond to the income level of what would be the next ranked low income country in the sample if in fact the income levels were uniformly distributed between lowest and highest. Let G_h denote the real per capita GDP of the highest income country in the original sample and let n be the original number of countries. Then $G_m = G_b + (G_h - G_b)/n$, giving, for our original sample with $G_h = 70,014$ and $n = 144$, that $G_m = 748$.[6] These choices for G_b and G_m have been deliberately chosen to be held fixed in the case of re-estimation over subsamples of the data. In the case of a model exhibiting non-linearities, holding scale parameters fixed enhances interpretation of comparative results for alternative subsample selections (for example, a rich group versus a poor group) under controlled model specification conditions. It also acts as a model robustness control. That is, no attempt is made to find a different specification to fit the data for poor versus rich countries. The degree of *non-homotheticity* is reflected in the Engel Curves' (signed) distance measures $\delta_i \equiv \beta_i - \alpha_i$. These signed distance measures are positive for luxuries and negative for necessities.

Step 2 (Linking): We now link our model's theoretical but unobservable measure of real consumption capacity, c/P_A with the observable data on real per capita GDP by means of a simple non-stochastic aggregate consumption function:

$$c/P_A = 1 + (e-1)\frac{\ln G - \ln G_b}{\ln G_m - \ln G_b} \tag{8}$$

Specification (8) incorporates the normalization $c/P_A = 1$, and hence $\ln(c/P_A) = 0$, when $G = G_b$, allowing the α_i parameters to be interpreted as shares for the lowest income country. It also yields $c/P_A = e$, and hence $\ln(c/P_A) = 1$, when $G = G_m$.

Given our settings for G_b and G_m, the implied aggregate consumption function can be written in the linear-log form:[7]

[5]However, we emphasize that both price indexes play a role in influencing consumer behavior. In (7), both the elasticity of the P_A index with respect to the "own price" p_i (viz. α_i) and also the elasticity of the P_B index (β_i) have important roles. Nevertheless, (7) makes clear that, for the purposes of determining their relative importance, c/P_A has a special role, weighting β_i more strongly as c/P_A rises.

[6]Choice of G_m affects the degree of curvature of the Engel Curves, especially their initial slope. It also affects the long run asymptotic levels of the curves. Our choice has been made after experimentation, to ensure that the β_i parameters, to which the curves asymptote, remain within the [0,1] bounds. The choice of G_b has a somewhat similar influence on the α_i parameters.

[7]Note that we do not "estimate" the consumption function directly (because c/P_A is unobservable). Instead we embed the specification (9) within the overall model of consumer demand. Tests of the model would really be a joint test of the commodity allocation and aggregate consumption specifications. An advantage of this approach is that we do not require actual data on total consumption expenditure. We can rely on the arguably more accurate official GDP data in purchasing power parity terms. We do of course require data on commodity shares, but these can be assumed, by virtue of their construction, to be less prone to data error than actual expenditure levels would be.

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$$c/P_A = -8.195 + 1.649 \ln G \tag{9}$$

Step 3 (Normalization): A further data transformation is employed to produce an explanator that is bounded and which, as an added bonus, allows the budget share equation to be rendered in a (transformed) linear form. This transformation is:

$$Z = \frac{\ln(c/P_A)}{1 + \ln(c/P_A)} \tag{10}$$

The transformed real consumption variable Z lies in the [0, 1] interval. By combining (10) with (9), Z can be directly linked to the available data on real per capita GDP. Applying (10) to (7), the economic component of the model can be written as:

$$y_i^m(Z) = \ln[\alpha_i + \delta_i Z] - \ln[1 - \alpha_i - \delta_i Z] \quad \text{where} \quad \delta_i = \beta_i - \alpha_i \tag{11}$$

To provide an important interpretation of the model under this transformation, and to offer a comparison with somewhat related models, consider the application of (10) to the budget share equations (3). This gives:

$$s_i = \alpha_i + \delta_i Z \tag{12}$$

The transformed budget share equation (12) is a simple linear function of Z which is itself a transformed real total expenditure income measure and, in our development, is further linked back to real per capita GDP via (9) and (10). For estimation, we prefer (11) to (12) because of its facility for allowing an additive and untruncated random error. On the other hand, an understanding of the properties of the model is best learned through reference to (12) rather than (11). The form (12) is the basic building block for generation of a variety of systems of demand equations which are linear in transformed variables but which are nonetheless regular and potentially flexible in the following senses: First, it is a simple matter to induce any desired degree of flexibility in (12) by specifying the low income elasticity α_i and/or by specifying the high to low income elasticity difference δ_i to be functions of (exogenous) circumstances which may differentiate groups of consumers. This capability for "*effective flexibility*" is discussed in more detail in Rehman and Cooper (2014), where a generalization to allow for demographic flexibility is referred to as the Regular Effective Demand System (REDS). Second, (12) is fully consistent with regular consumer preferences, and in particular with the empirical evidence of Engel's Law, viz. *non-homotheticity* of preferences. The importance of "*effective non-homotheticity*" of preference specifications when the intention is to apply a model in examination of the welfare implications of exogenous income changes has been recently highlighted by Cooper, McLaren, Rehman and Szewczyk (2015).

In Rehman and Cooper (2014), the genealogy of REDS is linked to the Almost Ideal Demand System (AIDS) of Deaton and Muellbauer (1980) and the

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Modified Almost Ideal Demand System (MAIDS) of Cooper and McLaren (1992).[8] AIDS is a modern variant of the long-standing relationship known as Working's Model. REDS looks superficially similar to Working's Model, which could be written in very similar terms as:

$$s_i = \alpha_i + \delta_i \ln Y \tag{13}$$

where Y is either (variously) real total expenditure or real GDP. There is a critical difference, however. Working's Model (13) specifies a fractional expenditure share, s_i, as linear in the log of a real economic variable which in principle can grow without limit. This creates problems at high incomes because the dependent variable is naturally co-integrated of order zero whereas the single explanator in (13), ln Y, has the potential for unlimited growth. In contrast, the single explanator in the REDS model (12), that is, the variable Z defined as in (10), has by construction the same order of integration (zero) as the dependent variable s_i. As a result, a reasonable conjecture would be that REDS should perform better at the extremes than Working's Model does—notably, in terms of generation of share predictions that remain within the [0, 1] bounds. Clements and Chen (2010) exhibit a scatter diagram (Figure 2, p. 915) that suggests Working's model could fit a range of countries reasonably well. However, Clements and Chen note that they have eliminated a number of outliers from the dataset before presenting the figure.[9] All else being equal, elimination of outliers may help focus on commonalities among remaining countries. However, we are especially interested in producing a model that fits low income and mid-range countries where reliable price and total expenditure data may be scarce. Therefore, we will be particularly interested in the performance of our model with respect to the countries that Clements and Chen discarded.

3. The Estimating Form and Residual Speci.cation

For empirical work with the log-odds model (11), we distinguish both commodities i (ranging from 1 to 12) and countries k (ranging from 1 to 144). Now let ${Z_k}^0$ denote the observed value of Z for country k. By "observed" here we mean constructed from officially observed real per capita GDP data, the later denoted ${G_k}^0$, using a combination of (9) and (10), viz.

$${Z_k}^0 = \frac{\ln\left[-8.195 + 1.649 \ln {G_k}^0\right]}{1 + \ln\left[-8.195 + 1.649 \ln {G_k}^0\right]}, \quad \text{for country } k = 1, \ldots, K \tag{14}$$

[8]By choosing specific functional forms for the price indexes P_A and P_B, a variety of demand systems can be produced. These may all be classified in the MPIGLOG (Modified PIGLOG) class of "effectively globally regular" demand systems. More details of the various model relationships within the MPIGLOG class and, in particular, of the implications of alternative specifications on welfare evaluation, are provided in Cooper, McLaren, Rehman and Szewczyk (2015).

[9]In particular, our analysis includes coverage of the 12 countries that Clements and Chen discard: Azerbaijan, Armenia, Comoros, Ethiopia, Gabon, Liberia, Malawi, Mauritania, Tajikistan, Tanzania, Uganda and Zambia

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As a function of observable data and key parameters, the log-odds model (11) asserts:

$$y_{ik}{}^{m}(Z_k{}^0)=\ln\left[\alpha_i+\delta_i Z_k{}^0\right]-\ln\left[1-\alpha_i-\delta_i Z_k{}^0\right] \tag{15}$$

However, we now need to acknowledge that the typical consumer thinks of the standard of living as not given precisely by the official real GDP per capita for their country but by some variation on this, reflecting their degree of optimism or pessimism in the state of the economy and of the socio-economic conditions they face. To account for this difference in perception, define the "expectations" component of the standard of living for country k as the difference between the typical consumer's perceived and the official standard of living. In normalized terms, using our "Z" variable as the standard of living indicator, this "expectations adjustment" is:

$$\gamma_k \equiv Z_k - Z_k{}^0 \tag{16}$$

Now consider a Taylor series approximation of the model from the typical consumer's perspective. That is, consider an expansion of $y_{ik}{}^{m}(Z_k)$ around the "observable" (albeit, constructed) data $Z_k{}^0$. The first order expansion is:[10]

$$y_{ik}{}^{m}(Z_k)=y_{ik}{}^{m}(Z_k{}^0)+\frac{\delta_i}{[\alpha_i+\delta_i Z_k{}^0][1-\alpha_i-\delta_i Z_k{}^0]}\gamma_k+O(\gamma_k{}^2) \tag{17}$$

where the "intercept" $y_{ik}{}^{m}(Z_k{}^0)$ is constructed as given in (15). The model with appended disturbance term can now be written by combining (15) and (17) with the basic structure of (6) to give:

$$y_{ik}=\ln\left[\alpha_i+\delta_i Z_k{}^0\right]-\ln\left[1-\alpha_i-\delta_i Z_k{}^0\right]+\frac{\delta_i\gamma_k}{[\alpha_i+\delta_i Z_k{}^0][1-\alpha_i-\delta_i Z_k{}^0]}+v_{ik} \tag{18}$$

where the combined disturbance/error term is:

$$v_{ik}=O(\gamma_k{}^2)+u_{ik} \tag{19}$$

We assume v_{ik} yields realizations distributed randomly over the entire real interval. However, we do not assume normality. Ideally, we would propose to use nonlinear least squares estimation on (18) and calculate robust standard errors. However, experimentation with the software at our disposal (Stata13) led us to conclude that (18) with 12 commodities and 144 countries

[10]The expression $O(\gamma^2)$ denotes a term that is proportional to a quantity smaller in absolute value than γ^2. Recall from (16) that $\gamma_k \equiv Z_k - Z_k{}^0$ and from (10) that both Z_k and $Z_k{}^0$ will lie in the [0,1] range. Hence, we expect $\gamma_k{}^2$ to be quite small. We experimented with a second order Taylor series expansion, with $O(\gamma_k{}^3)$ remainder, but found that this did not offer any substantive improvement over what was achieved with the first order expansion.

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suffered from the curse of dimensionality. Since the variety of commodity demands is important to us in generating the information required to distinguish between pure random disturbances and expectations effects, and since the context of our research interest involves comparison over as large a range of countries as possible, we estimate (18) in a multi-step procedure which we now describe.[11]

In Step 1, we treat the expectations component as absorbed into the error term. For this purpose we work with the following variant of (18):

$$y_{ik} = \ln\left[\alpha_i + \delta_i Z_k{}^0\right] - \ln\left[1 - \alpha_i - \delta_i Z_k{}^0\right] + \omega_{ik} \tag{20}$$

where the expanded disturbance term is:

$$\omega_{ik} = \frac{\delta_i \gamma_k}{[\alpha_i + \delta_i Z_k{}^0][1 - \alpha_i - \delta_i Z_k{}^0]} + v_{ik} = \phi_{ik}\gamma_k + v_{ik} \tag{21}$$

To simplify (21) we have defined:

$$\phi_{ik} = \frac{\delta_i}{[\alpha_i + \delta_i Z_k{}^0][1 - \alpha_i - \delta_i Z_k{}^0]} \tag{22}$$

Having estimated (20), we use the Step 1 estimates $\hat{\alpha}_i$, $\hat{\delta}_i$ to define the Step 1 log-odds predictions:

$$\hat{y}_{ik} = \ln\left[\hat{\alpha}_i + \hat{\delta}_i Z_k{}^0\right] - \ln\left[1 - \hat{\alpha}_i - \hat{\delta}_i Z_k{}^0\right] \tag{23}$$

In addition, after Step 1, (22) can be constructed from data and parameter estimates as:

$$\hat{\phi}_{ik} = \frac{\hat{\delta}_i}{\left[\hat{\alpha}_i + \hat{\delta}_i Z_k{}^0\right]\left[1 - \hat{\alpha}_i - \hat{\delta}_i Z_k{}^0\right]} \tag{24}$$

Further, the Step 1 log-odds residuals may be constructed as:

$$\hat{\omega}_{ik} = y_{ik} - \hat{y}_{ik} \tag{25}$$

Combining these, we can now construct a Step 2 linear regression model which aims to extract an expectations component from the Step 1 residuals:[12]

[11]We acknowledge that the statistical properties of the estimator could be improved by using an appropriate single-step procedure. This is relegated to future research of a more technical nature. Our goal here is to provide economically meaningful results with some degree of attention to the technical issues. However, the emphasis of our research is not predominantly statistical beyond attention to a robust specification and reasonable care in the estimation procedure.

[12]The disturbance term in (26), viz.η_{ik}, is of course closely related to v_{ik} in (21). We could relate these further but this degree of further analysis is not necessary for our current purposes.

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TABLE 1
Extract from Appendix Table A1

Data Source		ICP (2008)	Equation (9)	Equation (10)	ICP (2008)	ICP (2008)
Country	ICP Rank	G	c/P_A	Z	s_1	s_7
Luxembourg	1	70014	10.20277	0.699036	0.068795	0.136743
Qatar	2	68696	10.17143	0.698757	0.136300	0.153128
Norway	3	47551	9.56475	0.693071	0.097311	0.10709
Brunei Darussalam	4	47465	9.56177	0.693042	0.183632	0.150923
Kuwait	5	44947	9.47187	0.692149	0.147707	0.123263
United States	6	41674	9.34719	0.690888	0.062357	0.106211
Gabon	47	12742	7.39306	0.666727	0.363636	0.063870
Azerbaijan	79	4648	5.73000	0.635796	0.578624	0.054054
Armenia	87	3903	5.44192	0.628823	0.651452	0.032365
Mauritania	113	1691	4.06258	0.583649	0.638528	0.056277
Tajikistan	119	1413	3.76639	0.570099	0.549645	0.085106
Zambia	124	1175	3.46222	0.553952	0.112264	0.129210
Comoros	127	1063	3.29703	0.544008	0.696180	0.008925
Tanzania	129	1018	3.22569	0.539414	0.686608	0.040789
Uganda	130	991	3.18137	0.536460	0.348592	0.063380
Malawi	138	691	2.58674	0.487285	0.234146	0.141463
Ethiopia	141	591	2.32895	0.458117	0.538642	0.023419
Guinea Bissau	142	569	2.26639	0.450002	0.523316	0.067358
Liberia	143	383	1.61361	0.323625	0.254386	0.026316
Congo Dem Rep	144	264	1	0	0.621951	0.036585

$$\hat{\omega}_{ik}=\hat{\phi}_{ik}\gamma_k+\eta_{ik} \tag{26}$$

In utilizing the residual equation (26), we treat both the Step 1 calculated residuals $\hat{\omega}_{ik}$ defined by (25) and the $\hat{\phi}_{ik}$ terms defined in (24) as data and use (26) to provide estimates $\hat{\gamma}_k$ of the country-specific expectations effects γ_k.

4. Data

Table 1 provides an extract from the more extensive Appendix Table A1. Real per capita GDP (PPP) data is denoted G in the table, directly corresponding to our model variable G and come from ICP (2008) as described in footnote 1.

Among our dataset of 144 countries, Table 1 shows the richest six countries, the poorest four, and a selected "middle group" of ten countries which have been specifically excluded from previous work with an otherwise similar dataset (Clements and Chen, 2010). Our constructed commodity share series, denoted s_i, $i=1, \ldots,$ 12 and fully labelled in appendix Table A1, comes from ICP Table 5.[13] The constructed food share is of course unit-less and corresponds to the food shares used in Gao (2012). One aspect of our data construction and subsequent empirical work that is different from Gao's is that we do not use the official ICP data on

[13]These are Nominal Expenditure Per Capita, US $(ICP, 2008). We have constructed the shares from the indicated expenditures divided by the sum of the 12 expenditures from "Food and non-alcoholic beverages" through to "Miscellaneous". In ICP (2008, Table 5, pp. 68–77), all the individual expenditure series are in US $millions.

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real total consumption expenditure. Rather we use our non-stochastic model of aggregate real consumption as described in the transformations running from equations (9) to (10). One advantage of this is that we do not need to find a price index in PPP terms that corresponds to total expenditure and we are therefore not faced with the problem of having an estimating form which implicitly allocates commodity shares from an index of real total expenditure that could in principle reflect a different ranking of countries to that given by the ICP GDP series. Instead we use the ICP defined measure of real income in PPP terms, though transformed (monotonically) via equations (9) and (10). Consequently, for purposes of estimation across countries and later comparison with our constructed expectations index, in our model of consumer choice we are able to work with a ranking of countries by real income that is fully consistent with the ranking implied by the real GDP per capita (PPP).

In Table 1, we have included two of the 12 constructed share series s_1 (Food) and s_7 (Transport). The variation in these shares across countries reasonably convincingly suggests that Food is a necessity and that Transport is a luxury. However, the expenditure share information for Qatar, Brunei-Darussalam and Kuwait suggests that these countries might be too highly ranked when their status is based on GDP alone.

Table 1 includes all 12 of the countries that Clements and Chen (2010) discarded. Ten of these make up the middle group in Table 1, while two (Ethiopia and Liberia) are in our poorest four group. Of these 12, it can be seen that six of them—Azerbaijan, Armenia, Mauritania, Tajikistan, Comoros and Tanzania—have food expenditure shares well above what might be expected by Engel's Law and by reference to other countries. This could have been the reason why Clements and Chen were forced to discard them. From our perspective, it is tempting to predict that they will turn up as pessimists after our residual correction for expectations. It is also clear that Zambia, Malawi and Liberia are stand-out cases where the opposite is true. Their very low food shares suggest optimism—or, what is equivalent in our modelling approach, that consumers in these countries are beneficiaries of the informal sector with respect to food consumption to a much greater degree than is the case in peer countries.

5. Statistical Results

Results from estimation firstly of (20), and then followed by estimation of (26) conditional on the results from (20), were obtained in both cases by using the NL routine with robust standard error option in Stata13, with estimation over the full sample of 144 countries.[14] The results are reported in appendix Table A2.

Because the additive disturbance term is applied to the log-odds form of the model, there are no adding up restrictions on the disturbances across equations, so we include all 12 equations in the estimation. However, to ensure adding up for the α_i and δ_i parameters, in Step 1 we estimate 11 of each freely and impose the

[14]Of course, (26) is linear. However, the sheer number of parameters (144) and the need to set these up with indicator variables to match the appropriate entries in the 1728 element "data" vector $\hat{\phi}_{ik}$ has meant that it is simpler in practice to use the nonlinear NL routine in Stata.

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TABLE 2

Extract from Table A2 – Statistical Results – Steps 1 and 2

Selected Step 1 results		R-squared	0.9430
		Coefficient	t-statistic
Commodity-specific Engel Curve intercept, viz. $\hat{\alpha}_i$ estimated via (20)			
		Coefficient	t-statistic
1	Food	0.797	17.55
10	Education	0.016	2.11
Commodity-specific Engel Curve slope, viz. $\hat{\delta}_i$ estimated via (20)			
		Coefficient	t-statistic
1	Food	−0.782	−10.63
10	Education	0.103	8.60
Selected Step 2 results		R-squared	0.3163
Country-specific expectations adjustment, viz. $\hat{\gamma}_k$ estimated via (26)			
	Very high incomes	Coefficient	t-statistic
1	Luxembourg	0.278	3.84
2	Qatar	0.114	1.48
3	Norway	0.250	4.54
4	Brunei Darussalam	0.122	1.80
5	Kuwait	0.098	1.53
6	U.S.	0.318	4.30
	Selected mid to low incomes		
40	Hungary	0.220	5.64
51	Russia	0.020	0.37
54	Argentina	0.113	1.97
64	Brazil	0.199	5.17
84	China	0.092	1.13
101	Pakistan	−0.281	−3.79
106	India	−0.058	−0.83
127	Comoros	−1.121	−2.86
	Very low incomes		
139	Central African Republic	−0.316	−4.62
140	Niger	−0.030	−0.43
141	Ethiopia	−0.352	−2.28
142	Guinea Bissau	−0.314	−2.27
143	Liberia	0.162	1.54
144	Congo Democratic Republic	0.117	9.29

required parameter restrictions in estimation, viz. $\alpha_{12}=1-\sum_{i=1}^{11}\alpha_i$ and $\delta_{12}=0-\sum_{i=1}^{11}\delta_i$. Parameter estimates also include 144 Step 2 estimates of the γ_k parameters.

Table 2 is an extract from the more detailed Table A2 in the appendix. As the table indicates, the goodness of fit at Step 1 is very high, with a R^2 statistic of 0.943. This is a remarkably high statistic for the estimation of 22 parameters cross-sectionally using a total of 1728 observations (12 observations per country x 144 countries). In Step 2, by contrast, the R^2 statistic is much lower, at 0.3163. This is to be expected since the regression at Step 2 is actually attempting to extract expectations information from previously estimated residuals. In fact, to find that over 30 percent of the residual variation from 1728 cross-sectional observations can be used for this purpose is heartening for our approach.

As Table 2 indicates, the parameter α_1 is extremely significant with a t-statistic of over 17. The actual coefficient value of 0.797 represents the estimated budget share of food in the budget of an extremely poor country—one with real income

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index $Z=0$, viz. that for Congo (Democratic Republic). The parameter $\hat{\delta}_1$, which gives the slope of the Engel Curve in its transformed linear format, viz. as depicted by (12), is significantly negative, at -0.782. As income rises indefinitely, the results imply that the food budget share asymptotes to $\hat{\beta}_1=0.797-0.782=0.015$. These results strongly corroborate Engel's Law. Table 2 also gives the results for a statistically significant luxury—commodity 10, Education.

Detailed results, in the more extensive appendix Table A2, show that food is in fact the only significant necessity. While the Clothing Engel Curve does have a negative slope as a point estimate, the result is statistically insignificant. Alcohol/Tobacco has an insignificant positive slope. An objective view of these results would be that the Engel Curves for both of these products are essentially flat. The remaining nine statistically estimated slope parameters register all the corresponding commodities statistically significant as luxuries.[15] While food is the stand-out commodity from a consumer interest perspective, we need to use the information on residual patterns among a number of the luxuries, together with the information on the necessity food, in order to distinguish between random disturbances and expectational effects as reasons for consumption off the Engel Curves.

Table 2 also contains selected Step 2 results. We use the selected results, backed up by the details in Appendix Table A2, to offer a preliminary view of expectations in the various countries. We note before proceeding that the results, inasmuch as they may reflect differing degrees of optimism or pessimism, apply to the situation that prevailed in 2005. We refine our expectations indicator below and merely note some general trends at this stage.

First, the majority of wealthy countries appear to be optimists. However, looking at the six very wealthy countries, it can be seen that three of these, Qatar, Brunei-Darussalum and Kuwait, have insignificant expectations coefficients. The conclusion would seem to be that, if expectations were taken into account, the ranking of these three countries would be downgraded relative to that of other wealthy countries.

The majority of poor countries appear to be pessimistic and, moreover, most of the pessimistic results are statistically significant. Although we have not specifically highlighted them in Table 2, the countries that were discarded in the Clements-Chen analysis are predominantly pessimists, and significantly so. The list of countries that appear to be strongly pessimistic from a numerical perspective (with $\hat{\gamma}_k$ coefficients less than -0.3) include Azerbaijan, Syria, Armenia, Bhutan, Sudan, Nigeria, Chad, Mauritania, Tajikistan, Ghana, Comoros, Tanzania, Mozambique, Central African Republic, Ethiopia and Guinea-Bissau. These results have a certain ring of authenticity, although the reasons for the apparent pessimism may vary—ranging from internal strife, famine, poor public services, skewed distribution of income relative to the norm in "comparison" countries, or simply substantial poverty relative to neighbors. Of course we do not have the data to attribute reasons but, on the face of it, we see a reasonably understandable pattern in the results.

Only a sprinkling (four) of the optimistic countries appear to be optimistic to a degree commensurate to the strongly pessimistic cases noted above. That is, we

[15]To obtain measures of statistical significance on α_i and δ_i parameters that were left out due to the adding up constraints, we re-ran the regressions with alternative parameters excluded. This makes no numerical difference to the estimates of included parameters and enables us to present all relevant measures of significance in the extended Table A2.

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find $\hat{\gamma}_k > 0.3$ only for the U.S., Singapore, Ireland and the U.K. In fact, based on the sum of the $\hat{\gamma}_k$ coefficients, which comes in at -1.817, the prevailing mood across the countries appears to be one of pessimism.

There are some interesting contrasts that should be mentioned. One of the most strongly optimistic countries among the Eastern Europeans is Hungary. This result is highly significant and it is interesting to compare it with Russia, which shows a decidedly insignificant result. Among the Latin Americans, Argentina and Brazil show an interesting contrast. While Argentina is a little wealthier than Brazil on the 2005 figures, it is nowhere near as optimistic, either from the numeric strength of the effect or based on its statistical significance. We propose to refine the expectations indicator below and then attempt to combine it with the GDP figures to provide an overall integrated measure of well-being. It will be interesting to see if this leads to a reversal of ranking in the Argentina/Brazil case. In a somewhat similar situation is the comparison between results for Pakistan and India. On a strict per capita GDP basis Pakistan outranks India slightly. However, the average consumer in Pakistan seems to be both numerically and statistically much more pessimistic than is the case for the Indian consumer. Finally, we note that, on the results as they stand, the average consumer in China does not appear to be statistically significantly optimistic.

What is missing from Table A2 is a deeper sense of the relative importance that can be placed upon the estimated degree of optimism or pessimism. While one might hope that the numerical size and statistical significance of the γ_k coefficients would be a useful indicator, it is also important to note that the estimates of the γ_k parameters, which represent the difference between the official economic and perceived average standard of living in each country, put all the emphasis on the appropriateness or otherwise of the GDP data. Of course, the commodity share data may also be missing valuable information, especially to the extent of importance of the informal sector, which may lead to unrecorded consumption, especially for example in the case of food. We turn to an extension of our approach to address this problem.

6. Expectations Refinement via Residual Adjustment

The underlying economic model specification (11) may be combined with (16) and the additive error log-odds model as described by (6), and then recast as the (multiplicative) odds model:[16]

$$\frac{s_{ik}}{1-s_{ik}} = \frac{\left[\alpha_i+\delta_i(Z_k{}^0+\gamma_k)\right]}{\left[1-\alpha_i-\delta_i(Z_k{}^0+\gamma_k)\right]}\exp\left(u_{ik}\right) \tag{27}$$

and thence further to the share form:

[16]The discerning reader may note that, given $y_i \equiv \ln\left[s_i/(1-s_i)\right]$, the multiplicative model (27) could be written in log-linear form as $y_{ik}=\ln\left[\alpha_i+\delta_i(Z_k{}^0+\gamma_k)\right]-\ln\left[1-\alpha_i-\delta_i(Z_k{}^0+\gamma_k)\right]+u_{ik}$. This could have been used as a single-step non-linear estimation specification instead of the two-step variant involving the Taylor series approximation that we actually employed and described in (20)–(26). Of course, a package non-linear estimation routine itself uses approximations that are typically of a Taylor series type, so it is not obvious *a priori* which formulation would work better in a non-linear estimation routine that is something of a "black box." Of these two alternatives, we found through experimentation using the NL routine in Stata that estimation of (20) and (26) led to much better convergence properties.

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$$s_{ik}=\frac{\frac{[\alpha_i+\delta_i(Z_k{}^0+\gamma_k)]}{[1-\alpha_i-\delta_i(Z_k{}^0+\gamma_k)]}\exp(u_{ik})}{1+\frac{[\alpha_i+\delta_i(Z_k{}^0+\gamma_k)]}{[1-\alpha_i-\delta_i(Z_k{}^0+\gamma_k)]}\exp(u_{ik})} \tag{28}$$

Consequently, from the two-step estimates taken from the log-odds model, setting u_{ik} to zero in (28) we can construct the predicted shares with expectations adjustment to GDP. In fact, with u_{ik} set at zero, (28) reduces to the simple predictive equation:

$$\hat{s}_{ik}=\hat{\alpha}_i+\hat{\delta}_i(Z_k{}^0+\hat{\gamma}_k) \tag{29}$$

Now consider a "neutral expectations" Engel Curve that is consistent with (29) but constructed for the case $\hat{\gamma}_k=0$. This curve is:

$$\hat{\hat{s}}_{ik}=\hat{\alpha}_i+\hat{\delta}_iZ_k{}^0 \tag{30}$$

In (30), use of the "neutral expectations" Engel Curve generates the predicted commodity share under the circumstance where the original GDP data are taken at face value. Put another way, the difference:

$$r_{ik}{}^s=s_{ik}-\hat{\hat{s}}_{ik}=(s_{ik}-\hat{s}_{ik})+(\hat{s}_{ik}-\hat{\hat{s}}_{ik}) \tag{31}$$

contains a combination of estimates of expectations and pure random error. On the other hand, the difference:

$$e_{ik}{}^s=\hat{s}_{ik}-\hat{\hat{s}}_{ik}=\hat{\delta}_i\hat{\gamma}_k \tag{32}$$

is an estimate of the component of the error that would arise due to ignoring expectations. It needs to be noted that $r_{ik}{}^s$ and $e_{ik}{}^s$ may not have the same sign. This is one reason why it has been important to estimate a set of equations rather than a single commodity share equation to enable the distinction between the "off-the-Engel-Curve" expectations term (32) and the combined source of error (31) to be made.[17] With a single commodity equation there is no (non-arbitrary) way to split $r_{ik}{}^s$ into an expectations term and pure random errors.

As noted above, the distance from the expectations neutral Engel Curve to the best estimate of the share prediction given by $\hat{s}_{ik}$ from (29), is evaluated as $\hat{\delta}_i\hat{\gamma}_k$. It is also the case that we can use (29) to give a standard Engel Curve

[17]An estimate of $\hat{\gamma}_k$ is critical to construction of both r_{ik} and e_{ik}, but a single commodity share equation estimation across countries would give zero degrees of freedom for this purpose. It should also be noted that with the log-odds model a two-equation system is also not useful. This is because an additive error in a two-equation variant of the log odds model adds to zero, one equation is statistically redundant and must be eliminated from estimation, and we are back to estimation with only one observation per country. With only one observation per country we cannot estimate country-specific γ_k parameters. With $\gamma_k=0$, (29) and (30) show that $\hat{s}_{ik}=\hat{\hat{s}}_{ik}$ and hence $e_{ik}=0$. However, our 12-equation model gives 11 degrees of freedom to estimate the γ_k parameter for each country k. Moreover in the log-odds model with more than two equations, there is no redundancy in an additive disturbance and all equations can be estimated as a system.

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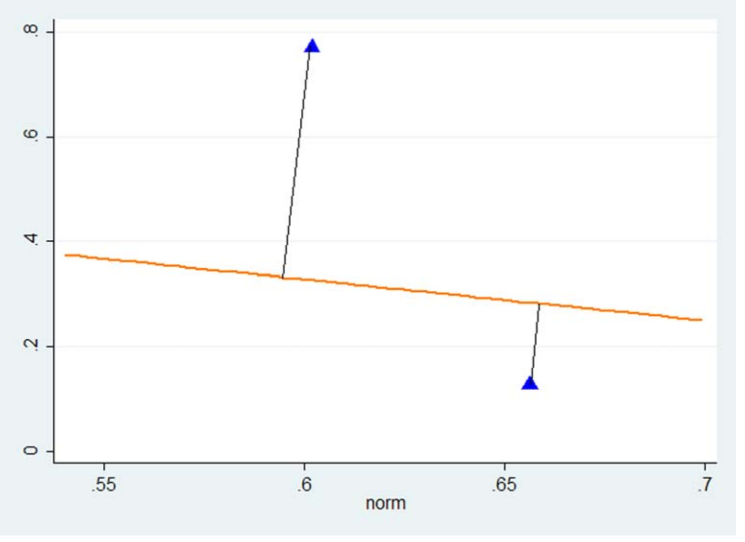

Figure 1. Engel Curve (33) and two Country-Specific Minimum-Distance Lines (34) [Colour figure can be viewed at wileyonlinelibrary.com]

prediction simply by interpreting the actual (normalized) standard of living in country k as best represented by $Z_k \equiv Z_k{}^0 + \hat{\gamma}_k$. However, this procedure allocates all of the adjustment due to poor measurement of data in the presence of optimism or pessimism to the official GDP data. No consideration is given to the possibility that the commodity share data may also be poorly measured. We now propose a compromise approach that allocates the adjustment equally to the GDP and the commodity share data.

Figure 1 shows an expectations-neutral Engel Curve for an illustrative necessity. The curve plots a commodity share against the normalized form of real income, denote "norm" on the figure. This corresponds to the variable Z in the equations of the model. The figure also shows share predictions (triangles) for two example countries. Because the share predictions include an expectations adjustment for each country, these predicted shares are obviously off the expectations-neutral Engel Curve. For the two example countries the points of minimum distance from their predicted share to the expectations neutral Engel Curve is the point of intersection of the expectations-neutral Engel Curve with their respective minimum distance lines. These points are the solution to two linear equations. One of these is obviously the expectations neutral Engel Curve itself, viz. (30), since the points lie on this curve. Generically, this commodity-specific equation is:

$$s_i = \hat{\alpha}_i + \hat{\delta}_i Z \tag{33}$$

For each country, the second equation required to find the minimum-distance adjustment is a straight line drawn from the predicted share point to the

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Engel curve, with a slope perpendicular to the Engel Curve. Although both commodity- and country-specific, this equation may also be represented generically as:

$$s_i = \hat{s}_{ik} + (-1/\hat{\delta}_i)\left[Z - Z_k{}^0\right] \tag{34}$$

The point $\hat{s}_{ik}$ in (34), which is critical to the validity of the "point-slope" formula (34), corresponds to the small triangles in Figure 1. This point is calculated from (29) for the two illustrated countries and the example commodity.

In Figure 1, the lower income country, with a Z score of about 0.61, is shown as a pessimist, so that "effective Z" is lower than the recorded Z. The diagram illustrates a case where the major part of the adjustment is taken up by a lowering of the commodity share. Effectively, there is only a small degree of evidence of pessimism, as measured by the income reduction required to rationalize the adjusted share allocation. The major adjustment in the case illustrated is deemed in this case to be due to inaccurate commodity share measurement. The wealthier country in the illustration, with a Z score of about 0.66, would be classified as an optimist.

The minimum distance expectations-adjusted solutions for s_{ik} and Z_k correspond to the solutions of (33) and (34), with the aid of (29), to give:

$$Z_{ik}{}^e = Z_k{}^0 + \left(\frac{\hat{\delta}_i{}^2}{1+\hat{\delta}_i{}^2}\right)\hat{\gamma}_k = Z_k{}^0 + \hat{\psi}_{ik} \tag{35}$$

$$s_{ik}{}^e = \hat{\alpha}_i + \hat{\delta}_i Z_{ik}{}^e = \hat{\alpha}_i + \hat{\delta}_i(Z_k{}^0 + \hat{\psi}_{ik}) \tag{36}$$

Note that in (35) and (36) we have defined

$$\hat{\psi}_{ik} \equiv \left(\frac{\hat{\delta}_i{}^2}{1+\hat{\delta}_i{}^2}\right)\hat{\gamma}_k \tag{37}$$

While the expectations adjustments to (normalized) income, $\hat{\psi}_{ik}$, are commodity-specific, they are necessarily always positive for optimistic countries and negative for pessimistic countries. Given that $\hat{\alpha}_i$, $\hat{\delta}_i$ and $\hat{\psi}_{ik}$ are all available from our two-step estimation procedure, we simply use (35) and (36) to obtain $s_{ik}{}^e$ and $Z_{ik}{}^e$.

As described above, the ordered pairs $(Z_{ik}{}^e,\ s_{ik}{}^e)$, calculated from (35)–(36) for our two country/one commodity example, are represented by Engel Curve and minimum-distance line intersections in Figure 1. As the figure shows, this approach offers the advantage of distributing the data adjustment. The allocation between commodity share and income adjustment is dictated by the slope of the Engel Curve, as the figure illustrates. If the Engel Curve were flat, there would be no information to make an income adjustment. Strongly sloped Engel Curves, whether for necessities or for luxuries, play a more prominent role in assessing off-Engel-curve data as an indicator of expectations that may be suggestive of the need for a re-assessment of "true" income.

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While the approach of "neutral" apportionment of the adjustment (by moving to the closest point on the expectations-neutral Engel Curve using an unweighted distance measure) seems justified by the unit-less and similarly bounded nature of the measures on both axes of Figure 1, a difficulty with the above-mentioned approach is that it is commodity specific. In fact, if we were to consider Figure 1 applied to each of 12 commodities, each with their own expectations-neutral Engel Curve, then we would have 12 potentially different expectations-adjusted income suggestions for each country. To provide a unique income-oriented measure of expectations for each country, an obvious thing to do is to average all 12 possible suggestions. We seek to do this while at the same time inflicting minimal damage on the role of the individual expectations-neutral Engel Curves. Effectively, we seek a country-specific parameter which we denote θ_k, representing an adjustment to $Z_k{}^0$ which will take us as close as possible to the set of 12 $Z_k{}^e$ estimates and still act as a good predictor of $s_{ik}{}^e$ when applied as the explanator in (36). That is, for each country k, a single θ_k needs to serve as an appropriate average of the $\hat{\psi}_{ik}$ across commodities $i=1,\ldots,12$. As before, to allow for possible random errors a log-odds formulation initially suggests itself. We therefore consider the specification:

$$(38) \qquad y_{ik}{}^e \equiv \ln\left(\frac{s_{ik}{}^e}{1-s_{ik}{}^e}\right)=\ln\left(\frac{\hat{\alpha}_i+\hat{\delta}_i(Z_k{}^0+\theta_k)}{1-\hat{\alpha}_i-\hat{\delta}_i(Z_k{}^0+\theta_k)}\right)+\omega_{ik}{}^e$$

To estimate the parameters θ_k in (38) we again use a Taylor series expansion and consider the estimating equation:[18]

$$(39) \qquad \ln\left(\frac{s_{ik}{}^e}{1-s_{ik}{}^e}\right)=\ln\left(\frac{\hat{\alpha}_i+\hat{\delta}_i Z_k{}^0}{1-\hat{\alpha}_i-\hat{\delta}_i Z_k{}^0}\right)+\frac{\hat{\delta}_i}{\left[\hat{\alpha}_i+\hat{\delta}_i Z_k{}^0\right]\left[1-\hat{\alpha}_i-\hat{\delta}_i Z_k{}^0\right]}\theta_k+\eta_{ik}{}^e$$

Next, given the results from Steps 1 and 2, define:

$$(40) \qquad \hat{\omega}_{ik}{}^e=\ln\left(\frac{s_{ik}{}^e}{1-s_{ik}{}^e}\right)-\ln\left(\frac{\hat{\alpha}_i+\hat{\delta}_i Z_k{}^0}{1-\hat{\alpha}_i-\hat{\delta}_i Z_k{}^0}\right)$$

Using (40), (39) can be written analogously to (26) as:

$$(41) \qquad \hat{\omega}_{ik}{}^e=\hat{\phi}_{ik}\theta_k+\eta_{ik}{}^e$$

Results from estimation of the θ_k parameters in (41) are referred to as Step 3 estimates and are presented in appendix Table A3. A condensed version is presented as Table 3:

[18]As a first order expansion, the remainder term $\eta_{ik}{}^e$ is of order $\theta_k{}^2$.

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TABLE 3

Extract From Table A3 – Statistical Results – Step 3

	Selected Step 3 results	R-squared	0.4661	Comparison (Table 2)	
	Country-specific income adjustment, viz. $\hat{\theta}_k$ estimated via (41)			$\hat{\gamma}_k$	
	Very high incomes	Coefficient	t-statistic	Coefficient	t-statistic
1	Luxembourg	0.067	5.01	0.278	3.84
2	Qatar	0.025	1.87	0.114	1.48
3	Norway	0.058	4.32	0.250	4.54
4	Brunei Darussalam	0.026	1.97	0.122	1.80
5	Kuwait	0.021	1.57	0.098	1.53
6	U.S.	0.076	5.65	0.318	4.30
	Selected mid to low incomes				
40	Hungary	0.047	3.47	0.220	5.64
51	Russia	0.004	0.29	0.020	0.37
54	Argentina	0.022	1.64	0.113	1.97
64	Brazil	0.040	2.94	0.199	5.17
84	China	0.016	1.20	0.092	1.13
101	Pakistan	−0.043	−3.18	−0.281	−3.79
106	India	−0.009	−0.67	−0.058	−0.83
127	Comoros	−0.151	−11.37	−1.121	−2.86
	Very low incomes				
139	Central African Republic	−0.036	−2.86	−0.316	−4.62
140	Niger	−0.003	−0.27	−0.030	−0.43
141	Ethiopia	−0.038	−3.08	−0.352	−2.28
142	Guinea Bissau	−0.033	−2.72	−0.314	−2.27
143	Liberia	0.012	1.21	0.162	1.54
144	Congo Democratic Republic	0.003	0.81	0.117	9.29

It is interesting to compare the estimated $\hat{\theta}_k$ coefficients, which are the main subject of Table 3, with the previously estimated $\hat{\gamma}_k$ coefficients, previously presented in Table 2 and appended to Table 3 for ease of comparison. As would be expected, the allocation of expectations corrections to both commodity shares and to income has led to a substantial reduction in the income adjustment. This has also led typically to an improvement in statistical significance for wealthy countries and a reduction for poor countries. There are some exceptions that are notable. Comoros, which on the basis of full adjustment to income was showing an unreasonably large pessimism estimate is now showing a much more reasonable figure which has, moreover, become much more highly statistically significant. This actually brings into line the one outstandingly problematic result obtained for the 144 countries after Step 2.[19] Another major improvement is the result for the lowest income country, Congo (Democratic Republic). In Step 2 this country shows evidence of extremely statistically significant optimism, although the effect was not particularly substantial numerically. After Step 3, this effect is insignificant both statistically and numerically. The other cross-country comparisons that were made in discussing Table 2 remain valid. For example, Pakistan remains more pessimistic than India, while Brazil remains more optimistic than Argentina.

[19]The Step 2 result for Comoros was greater than unity in absolute value, implying a blow-out in the remainder term for the Taylor series expansion. After Step 3, this problem has been resolved.

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Given these refined expectations estimates, the expectations-adjusted commodity shares and normalized income measures are calculated by use of expressions similar to (35) and (36) except that the combined country-specific and commodity-specific $\hat{\phi}_{ik}$ parameters are replaced by the country-specific approximations $\hat{\theta}_k$, obtained from estimation of (39), viz.

$$\hat{Z}_k{}^e = Z_k{}^0 + \hat{\theta}_k \tag{42}$$

$$\hat{s}_{ik}{}^e = \hat{\alpha}_i + \hat{\delta}_i (Z_k{}^0 + \hat{\theta}_k) \tag{43}$$

7. Research Findings

On the reasonable proposition that optimistic expectations reflect a higher quality of life than pessimistic expectations, we propose to interpret the expectations indicator $\hat{\theta}$ as a complementary "quality-of-life" index to the more standard economic real income index G(simply, GDP in PPP units).

In addition to the development of an expectations indicator θ that may be indicative of optimism or pessimism within an economy, our model suggests a natural way to combine this with our normalized measure of GDP per capita, denoted Z in the formal model development. The relationship is simply:[20]

$$Z^e = Z^0 + \theta \tag{44}$$

In appendix Table A4 we report the results for the expectations-adjusted standard of living, converted to GDP in the ICP's PPP units. For this conversion from the ICP's official GDP figures in PPP terms to what we might call a measure of "effective GDP" or "expectations-adjusted GDP," we again use a Taylor series expansion, in this case:[21]

$$\begin{aligned} \ln G^e &\equiv \ln GDP(Z^e) = \ln GDP(Z^0) + \left.\frac{\partial \ln GDP}{\partial Z}\right|_{Z^0} \theta + O\left(\theta^2\right) \\ &\approx \ln G^0 + (1/1.649)\left[-8.195 + 1.649 \ln G^0\right]\left\{1 + \ln\left[-8.195 + 1.649 \ln G^0\right]\right\}^2 \theta \end{aligned} \tag{45}$$

We also present, as the final column in Table A4 in the appendix, the implied percentage change in GDP, calculated directly from (45) as $\ln G^e - \ln G^0$. Table 4 summarizes appendix Table A4 for very high and very low income countries as well as the same selection of mid-range countries considered in Tables 2 and 3.

In Table 4 we have presented the results in the same ordered ranking as the ICP data. In fact, there are very few rank changes in the entire list of countries, and in fact none are evident in Table 4. In Table 5 we list the only countries for which the rankings have changed.

[20]For notational simplicity we now drop reference to the notation for parameter estimates and to the subscript k that was used to denote countries. We retain superscript ^{0} for original data and superscript ^{e} for our constructed expectations-adjusted data.

[21]The second line in (45) makes use of our constructions (9)–(10).

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TABLE 4

Extract From Table A4 – Selected Countries

ICP Rank	Very high incomes	Official GDP	Effective GDP	Percentage Change
1	Luxembourg	70014	73217	4.57
2	Qatar	68696	69865	1.70
3	Norway	47551	49248	3.57
4	Brunei Darussalam	47465	48239	1.63
5	Kuwait	44947	45521	1.28
6	U.S.	41674	43552	4.51
	Selected mid to low incomes			
40	Hungary	17014	17372	2.10
51	Russia	11861	11879	0.15
54	Argentina	11063	11157	0.85
64	Brazil	8596	8715	1.38
84	China	4091	4108	0.40
101	Pakistan	2396	2377	−0.78
106	India	2126	2123	−0.15
127	Comoros	1063	1048	−1.45
	Very low incomes			
139	Central African Rep	675	674	−0.21
140	Niger	613	613	−0.02
141	Ethiopia	591	590	−0.18
142	Guinea Bissau	569	568	−0.15
143	Liberia	383	383	0.03
144	Congo Dem Rep	264	264	0.00

TABLE 5

Countries With Changed Rankings After GDP Adjustment

ICP Rank	Revised Rank		Official GDP	Effective GDP	Percentage Change
11	10	Iceland	35630	36937	3.67
10	11	Hong Kong China	35680	36871	3.34
19	18	Sweden	31995	33099	3.45
18	19	Belgium	32077	32997	2.87
38	37	Czech Republic	20281	20711	2.12
39	38	Portugal	20006	20454	2.24
37	39	Oman	20334	20395	0.30
51	50	Russia	11861	11879	0.15
50	51	Equatorial Guinea	11999	11870	−1.08
60	59	Bulgaria	9353	9447	1.00
59	60	Romania	9374	9392	0.19
64	63	Brazil	8596	8715	1.38
63	64	Serbia	8609	8625	0.19
66	65	South Africa	8477	8550	0.86
65	66	Belarus	8541	8498	−0.50
68	67	Turkey	7786	7803	0.21
67	68	Montenegro	7833	7788	−0.57
86	85	Maldives	4017	4017	0.00
85	86	Syria	4059	3996	−1.54
88	87	Paraguay	3900	3898	−0.04
87	88	Armenia	3903	3826	−1.98
91	90	Bolivia	3618	3620	0.05
90	91	Congo Republic	3621	3614	−0.20

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Interestingly, only one ranking change involved a move of more than one position. Also, none of the 50 lowest income countries changed their ranking at all. Since the original rankings used by ICP are mostly preserved, this is a helpful indicator that the approach we have taken is not too extreme, honoring the degree of care that went into the ICP PPP adjusted GDP estimates in the first place.

Although the specific country expectations adjustments do not lead to any major changes in the country rankings, there are some substantial differences in the sizes of adjustments that are worth discussing. Table 6 ranks the countries by the percentage size of the income revision. We have split the table into four groups—optimists, positive/neutral, negative/neutral and pessimists. There are 40 countries in our optimistic group and 34 countries in our pessimistic group. The remaining 70 countries, who may be thought of as fence-sitters, have been characterized as "positive/neutral" or "negative/neutral" in the table, effectively dividing them into those leaning slightly to optimism (40 countries) and those leaning slightly to pessimism (30 countries).

The income-adjustment rankings given in Table 6 largely corroborate findings discussed earlier with respect to cross-country comparisons. Based on our categorization, Brazil is optimistic while Argentina is a (positive) fence-sitter. Pakistan is pessimistic while India is a (negative) fence-sitter. Hungary is among the group of optimists, well above Russia, which finds itself at best a positive fence-sitter. Unfortunately there are no African countries among the clear optimists. Although there are some fence-sitting African countries, many fall in the clearly pessimistic group, where in fact they make up the majority of entries.

What may be surprising, however, is that the average income adjustment across countries is in the positive direction, whereas the initial findings suggested the opposite. This revision in outlook arises after some of the initial off-Engel-Curve results are reconciled via an adjustment in presumed actual commodity shares as distinct from the official information. This adjustment has obviated the need to put all of the burden of adjustment in expectations on income. It reflects the fact, for example, that an observation on a low official budget share spent on luxuries may not entirely be reflecting pessimism but could be an indicator of greater informal sector activity. For example lower non-measured family involvement in raising children can compensate to some extent for less spending on formal education. We do not have the data to investigate these possibilities, but the results suggest that this possibility is worthy of further research. Despite this possibility, however, the results clearly show a link between official poverty measures and pessimism, suggesting that the true impact of poverty is underestimated.

Finally, we look as a group at the list of 12 countries that were excluded from the Clements and Chen analysis. Of the 12, seven have been catalogued in our clearly pessimistic group. These are Azerbaijan, Armenia, Comoros, Gabon, Mauritania, Tajikistan and Tanzania. The need to adjust income downward in order to fit these countries adequately could be a possible reason why Clements and Chen decided to discard them. While the other five countries—Ethiopia, Liberia, Malawi, Uganda and Zambia—do not require a major revision of income in order to fit them adequately, there has of course been adjustment to their perceived commodity shares under our approach and this could explain why

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TABLE 6

Optimism/pessimism (O/P) Rankings Based on Percentage Income Adjustment

O/P Rank	ICP Rank		% Income adjustment	O/P Rank	ICP Rank		% Income adjustment
Optimistic Group (40)				Optimistic Group (continued)			
1	8	Ireland	5.72	21	26	Spain	2.68
2	1	Luxembourg	4.57	22	30	New Zealand	2.68
3	6	U.S.	4.51	23	9	Macao China	2.63
4	7	Singapore	4.22	24	33	Slovenia	2.55
5	20	U.K.	3.99	25	32	Israel	2.46
6	13	Canada	3.92	26	25	Italy	2.45
7	17	Australia	3.83	27	34	Korea Rep	2.36
8	14	Netherlands	3.75	28	36	Malta	2.33
9	15	Austria	3.72	29	28	Taiwan	2.33
10	11	Iceland	3.67	30	39	Portugal	2.24
11	3	Norway	3.57	31	38	Czech Republic	2.12
12	19	Sweden	3.45	32	40	Hungary	2.10
13	12	Switzerland	3.43	33	31	Cyprus	2.01
14	16	Denmark	3.42	34	29	Greece	1.71
15	10	Hong Kong China	3.34	35	2	Qatar	1.70
16	22	Finland	3.23	36	4	Brunei Darussalam	1.63
17	21	Germany	3.21	37	42	Slovak Republic	1.59
18	24	France	2.90	38	41	Estonia	1.54
19	18	Belgium	2.87	39	52	Malaysia	1.45
20	23	Japan	2.68	40	64	Brazil	1.38
Positive/Neutral Group (40)				Positive/Neutral Group (continued)			
41	5	Kuwait	1.28	61	37	Oman	0.30
42	45	Croatia	1.21	62	102	Moldova	0.28
43	48	Chile	1.19	63	57	Mauritius	0.25
44	60	Bulgaria	1.00	64	68	Turkey	0.21
45	44	Poland	1.00	65	73	Peru	0.21
46	70	Thailand	0.98	66	59	Romania	0.19
47	46	Latvia	0.90	67	63	Serbia	0.19
48	61	Uruguay	0.88	68	138	Malawi	0.16
49	66	South Africa	0.86	69	51	Russia	0.15
50	54	Argentina	0.85	70	121	Kenya	0.14
51	35	Saudi Arabia	0.76	71	72	Bosnia Herzegovina	0.13
52	53	Mexico	0.68	72	92	Morocco	0.13
53	27	Bahrain	0.67	73	55	Iran	0.09
54	58	Venezuela	0.55	74	80	Namibia	0.07
55	75	Colombia	0.48	75	91	Bolivia	0.05
56	62	Kazakhstan	0.41	76	105	Vietnam	0.04
57	84	China	0.40	77	74	Tunisia	0.03
58	43	Lithuania	0.38	78	143	Liberia	0.03
59	77	Albania	0.37	79	124	Zambia	0.00
60	71	Ecuador	0.33	80	144	Congo Dem Rep	0.00
Negative/Neutral Group (30)				Negative/Neutral Group (continued)			
81	86	Maldives	0.00	96	69	Macedonia	−0.16
82	140	Niger	−0.02	97	133	Togo	−0.18
83	99	Cape Verde	−0.02	98	134	Rwanda	−0.18
84	82	Jordan	−0.02	99	132	Guinea	−0.18
85	130	Uganda	−0.02	100	141	Ethiopia	−0.18
86	94	Georgia	−0.03	101	128	Mali	−0.19

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Table 6 *Continued*

Negative/Neutral Group (30)				Negative/Neutral Group (continued)			
87	88	Paraguay	−0.04	102	90	Congo Rep	−0.20
88	76	Ukraine	−0.04	103	139	Central African Rep	−0.21
89	137	Gambia	−0.07	104	115	Côte d'Ivoire	−0.21
90	56	Lebanon	−0.08	105	120	Benin	−0.24
91	135	Sierra Leone	−0.08	106	112	Kyrgyz Republic	−0.27
92	83	Fiji	−0.13	107	118	Lesotho	−0.33
93	142	Guinea Bissau	−0.15	108	126	Nepal	−0.36
94	106	India	−0.15	109	117	Cambodia	−0.37
95	125	Burkina Faso	−0.16	110	136	Mozambique	−0.38
Pessimistic Group (34)				Pessimistic Group (continued)			
111	114	Senegal	−0.43	128	49	Botswana	−0.63
112	122	Bangladesh	−0.46	129	123	Ghana	−0.69
113	131	Madagascar	−0.47	130	78	Egypt	−0.76
114	108	Djibouti	−0.49	131	47	Gabon	−0.78
115	65	Belarus	−0.50	132	101	Pakistan	−0.78
116	107	Cameroon	−0.53	133	111	Chad	−0.85
117	110	Lao	−0.54	134	113	Mauritania	−0.90
118	98	Philippines	−0.54	135	81	Swaziland	−0.95
119	100	Mongolia	−0.54	136	109	Nigeria	−1.01
120	116	São Tomé Principe	−0.55	137	129	Tanzania	−1.05
121	96	Indonesia	−0.56	138	50	Equatorial Guinea	−1.08
122	97	Iraq	−0.57	139	89	Bhutan	−1.41
123	67	Montenegro	−0.57	140	127	Comoros	−1.45
124	103	Yemen	−0.57	141	104	Sudan	−1.46
125	119	Tajikistan	−0.59	142	85	Syria	−1.54
126	95	Sri Lanka	−0.61	143	79	Azerbaijan	−1.61
127	93	Angola	−0.62	144	87	Armenia	−1.98

we have been able to maintain all of these countries in our analysis without untoward effects.

8. Conclusion

This paper derives information from Engel Curve estimation residuals which allows construction of an expectations indicator capable of interpretation as optimism or pessimism. The significant factors which may contribute to residuals in such an economic model could be historical background, institutional capacity, productivity differences, welfare statism and probably many other specific factors. Together these might be thought of as generating differences in social wealth. Our approach offers some prospect for measuring the impact of this social wealth, given that differences in income only go so far in explaining differences in economic behavior. These explorations merely scratch the surface, but they do highlight the potential differences in expectations among citizens of different countries.

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From a sample of 144 countries worldwide, we have found that optimism could be reflective of up to a four to five percent effective addition to income while pessimism could be leading to as much as a two percent reduction in effective income. These effects are coincident with countries apparently making more or less use of the informal sector to modify commodity shares in ways not reflected in official expenditure statistics. Matching the expectational differences that this research has found to some of the extra-economic differences between countries would appear to be a fruitful area of research worth investigating in future. This could contribute to a better understanding of the differences between countries that may be hindering consumer confidence and reducing opportunities for greater socio-economic development in countries where it is sorely needed.

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Supporting Information

Additional Supporting Information may be found in the online version of this article:

Appendix: Tables A1-A6 are expanded versions of Tables 1-6.

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