the review of income and wealth

Review of Income and Wealth
Series 61, Number 3, September 2015
DOI: 10.1111/roiw.12086

A PROPOSAL TO COMPARE CONSISTENTLY THE INEQUALITY AMONG THE POOR

BY OIHANA ARISTONDO

University of the Basque Country UPV/EHU and BRIDGE Research Group, Spain

FRANCISCO J. GOERLICH GISBERT

University of Valencia and Instituto Valenciano de Investigaciones Económicas IVIE, Spain

AND

CASILDA LASSO DE LA VEGA*

University of the Basque Country UPV/EHU and BRIDGE Research Group, Spain

A poverty index should be sensitive to the number of poor people, the extent of the shortfall of the poor, and the inequality among the poor. A difficulty arises when inequality among the poor needs to be assessed. The inequality may be analyzed in terms of either incomes or gaps. Depending on what side we focus on, the inequality level comparisons may be contradictory. This paper proposes a reinterpretation of the inequality component involved in the decompositions of well-known poverty indices. The alternative indices we introduce measure equally the income and gap inequality among the poor. The comparisons in inequality as measured by these indices are then independent of the viewpoint. An empirical application illustrates the proposal.

JEL Codes: D63, I30, I32

Keywords: income and gap inequality, inequality among the poor, poverty measurement

1. INTRODUCTION

Sen (1976) argued that a poverty index should take account of three aspects of poverty: the number of people below the poverty line, the extent of the shortfall of the income of the poor from the poverty line, and the inequality among the poor. Accordingly, any poverty measure should be expressed as a function of these three poverty indicators, showing the incidence, the intensity, and the inequality among the poor, respectively. These are the three "I"s of poverty according to Jenkins and Lambert's designation (Jenkins and Lambert, 1998). A number of decompositions have been proposed to explicitly identify these three underlying components.[1]

Note: Casilda Lasso de la Vega gratefully acknowledges the hospitality of the Brooks World Poverty Institute (BWPI) of the University of Manchester where she was visiting, supported by the Basque Government (program MV-2010-1-20), when the first version of this paper was prepared. We thank the Spanish Ministerio de Educación y Ciencia (projects ECO2011-23248 and ECO2012-31346) and the Gobierno Vasco (project IT568-13) for research support.

*Correspondence to: Casilda Lasso de la Vega, Facultad de Ciencias Económicas y Empresariales, Avenida Lehendakari Aguirre 83, 48015 Bilbao, Spain (casilda.lassodelavega@ehu.es).

[1]Besides Clark *et al.* (1981), Osberg and Xu (2000), Xu and Osberg (2002), and Aristondo *et al.* (2010), some of them may be found in Kakwani (1999).

As regards the inequality term, Sen (1976) points out that "a transfer of income from a person below the poverty line to anyone who is richer must increase the poverty measure." Consider two individuals below the poverty line. A transfer of income from the poorer to the richer entails a transfer of the gap from the richer one to the poorer one. Then the poverty measure is bound to increase if the inequality term involved in the index concentrates either on income or on gaps. In fact, in the mentioned decompositions this third component refers sometimes to income inequality and sometimes to gap inequality. For instance, whereas in the original proposals of Sen (1976) and Shorrocks (1995) the "Gini index of the poor income" takes part in the decompositions, Osberg and Xu (2000) and Xu and Osberg (2002) derive alternative decompositions in which the "Gini coefficient of the gaps" is involved. Similarly, the "inequality among the poor" is captured in terms of gaps in the TIP curves introduced by Jenkins and Lambert (1998) and in the decomposition for the FGT indices (Foster *et al.*, 1984) proposed by Aristondo *et al.* (2010).

However, the choice between income and gap inequality is not innocuous and different choices may lead to contradictory results. To illustrate this, let us consider two income distributions $\mathbf{y}^1 = \{4, 5, 25, 35, 37, 40\}$ and $\mathbf{y}^2 = \{3, 4, 22, 32, 38, 41, 42\}$. Let us assume that the poverty line is $z = 36$. Then the distributions of the poor are respectively $\mathbf{y}_p^1 = \{4, 5, 25, 35\}$ and $\mathbf{y}_p^2 = \{3, 4, 22, 32\}$. The corresponding normalized poverty gap distributions are $\mathbf{g}_1 = \left\{\frac{32}{36}, \frac{31}{36}, \frac{11}{36}, \frac{1}{36}\right\}$ and $\mathbf{g}_2 = \left\{\frac{33}{36}, \frac{32}{36}, \frac{14}{36}, \frac{4}{36}\right\}$. As $G(\mathbf{y}_p^1) = 0.409 < G(\mathbf{y}_p^2) = 0.430$, the Gini index of the income distributions concludes that the inequality among the poor is higher in the latter than in the former. The same result is obtained if the coefficient of variation is used since $C(\mathbf{y}_p^1) = 0.767 < C(\mathbf{y}_p^2) = 1.234$. Nevertheless this conclusion is reversed if we focus on the poverty gap distributions. In fact $G(\mathbf{g}_1) = 0.377 > G(\mathbf{g}_2) = 0.316$ and $C(\mathbf{g}_1) = 0.706 > C(\mathbf{g}_2) = 0.353$.

This difficulty arises not only in poverty measurement but also in different economic fields in which bounded variables are involved. Recent papers (e.g., Clarke *et al.*, 2002; Erreygers, 2009; Lambert and Zheng, 2011; Lasso de la Vega and Aristondo, 2012) deal with this issue in health measurement.

The results derived by Lambert and Zheng (2011) have a straightforward application to the measurement of the inequality among the poor. They introduce a property of consistency which requires that achievement and shortfall inequality rankings should not be reversed, and show that all relative and intermediate inequality indices fail their requirement. Accordingly, whenever a relative, as in the example above, or even an intermediate inequality index is involved in the decomposition of a poverty index, the inequality component is not consistent.

We think this is a serious drawback which may distort the conclusions in the analysis of the poverty trends, and consequently, the poverty decompositions are found wanting in displaying inequality among the poor, one of their main points.

In this paper we concentrate on three well-known poverty measures: the Sen index (Sen, 1976), hereafter the S index; the Sen index modified by Shorrocks (1995), *SST*; and the mean of square deprivation gaps, a distinguished member of the family of poverty measures introduced by Foster *et al.* (1984), which we will refer to as FGT_2. Taking as a basis the indicators introduced by Lasso de la Vega and Aristondo (2012), we propose alternative interpretations of the inequality term for their poverty decompositions. These inequality components will allow policy makers to determine, in a consistent way, if inequality among the poor has increased or decreased if they focus either on income or on gaps.

This paper is structured as follows. The basic notations and definitions are presented in Section 2. Section 3 introduces the proposal to incorporate inequality indicators in poverty decompositions for which the income inequality and the gap inequality are equal. Finally, a short illustration taking data from the European Union Survey on Income and Living Conditions (EU-SILC) shows the advantages of the inequality terms introduced.

2. Notation and Basic Definitions

We consider a population consisting of $n \geq 2$ individuals. Individual i's income is denoted by $y_i \in R_{++}$, $i = 1, 2, \ldots, n$. An income distribution is represented by a vector $\mathbf{y} = (y_1, y_2, \ldots, y_n) \in R^n_{++}$, that we assume throughout is nondecreasingly ordered, that is, $y_1 \leq y_2 \leq \ldots \leq y_n$. We let $D = \bigcup_{n=1}^{\infty} R^n_{++}$ represent the set of all finite dimensional income distributions. For any given poverty line $z \in R_{++}$ and distribution $\mathbf{y} \in D$ we define as poor all incomes $y_i \leq z$. We denote by $n = n(\mathbf{y})$ and $q = q(\mathbf{y}; z)$ the population size and the number of the poor, respectively, and by $\mu(\mathbf{y})$ the mean income of $\mathbf{y}$. Let $g_i = \max\{(z - y_i)/z, 0\}$ be the normalized poverty gap of the i-th individual and $\mathbf{g} = \{g_1, \ldots, g_n\}$ the normalized poverty gap vector. Let the set of poor people be denoted by Q. Then $\mathbf{y}_p = (y_1, y_2, \ldots, y_q)$ represents the income distribution of the poor and we denote $\mu_p = \mu(\mathbf{y}_p)$. In turn, $\mathbf{g}_p = (g_1, \ldots, g_q)$ is the normalized gap vector of the poor.

A number of poverty indices will be used in this paper. First, the *headcount ratio*, the archetypical measure of the incidence of poverty, is the proportion of the poor in the population, $H = H(\mathbf{y}; z) = q/n$. The intensity of poverty is usually measured by the *aggregate income gap ratio*,[2] which represents the mean among the poor of the normalized poverty gaps, $A = A(\mathbf{y}; z) = \mu(\mathbf{g}_p) = (1/q)\sum_{i \in Q} g_i$. The product of the headcount ratio and the aggregate income gap ratio, HA, is the *poverty gap ratio* and represents the mean among the whole population of the normalized poverty gaps.

The Sen index (Sen, 1976), denoted by S, is computed as follows:[3]

$$S(\mathbf{y}, z) = \frac{2}{qnz} \sum_{i=1}^{q} (z - y_i)(q + 0.5 - i). \tag{1}$$

[2]We follow Sen's proposal, although other authors (e.g., Jenkins and Lambert, 1998) use the poverty gap ratio to measure intensity of poverty.

[3]Although this is not Sen's original proposal, it is common to refer to this modified expression as the Sen index. The drawbacks of this index, among them the failure of continuity and the transfer sensitive axiom, are well-known (for instance, Shorrocks, 1995).

Several proposals have been made in order to overcome the drawbacks of the Sen index.[4] Shorrocks (1995) proposes a measure which is identical to the limit of Thon's modified Sen index (Thon, 1979) and it is usually referred to as the Sen–Shorrocks–Thon index, *SST*.[5]

(2) $$SST(\mathbf{y}, z) = \frac{1}{n^2 z}\sum_{i=1}^{q}(z - y_i)(2n + 1 - 2i).$$

Finally, the mean of square deprivation gaps, FGT_2, belonging to the family introduced by Foster *et al.* (1984), is given by

(3) $$FGT_2(\mathbf{y}; z) = (1/n)\sum\nolimits_{i\in Q}(g_i)^2.$$

In the decompositions of a poverty measure into incidence, intensity, and inequality, an inequality measure is usually involved. In the standard income literature, an inequality index I is a real valued function $I : D \to \mathbb{R}$ which fulfils the following properties.

Pigou–Dalton Transfer Principle (TP). $I(\mathbf{y}') < I(\mathbf{y})$ whenever $\mathbf{y}'$ is obtained from $\mathbf{y}$ by a progressive transfer, that is, there exist two individuals $i, j \in \{1, \ldots, n\}$ and $h > 0$ such that $y_i' = y_i + h \le y_j - h = y_j'$ and $y_k' = y_k$ for every $k \in \{1, \ldots, n\}\setminus\{i, j\}$.

Normalization (NOR). $I(\lambda, \ldots, \lambda) = 0$ for all $\lambda > 0$.

Symmetry (SYM). $I(\mathbf{y}) = I(\mathbf{y}')$ whenever $\mathbf{y} = \mathbf{y}'\,\Pi$ for some permutation matrix Π.

Replication Invariance (RI). $I(\mathbf{y}) = I(\mathbf{y}')$ whenever $\mathbf{y}' = (\mathbf{y}, \mathbf{y}, \ldots, \mathbf{y})$ with $n(\mathbf{y}') = m\,n(\mathbf{y})$ for some positive integer m.

The crucial axiom in inequality measurement is the *Pigou–Dalton TP* which requires that a transfer from a richer person to a poorer one decreases inequality. *NOR*, *SYM*, and *RI* are standard assumptions for an indicator. Another usual requirement for an inequality measure is to demand *Scale Invariance*, that is, the inequality level should remain unchanged under proportional changes in all the values.

Scale Invariance. $I(\lambda\mathbf{y}) = I(\mathbf{y})$ for all $\lambda > 0$.

Relative measures are those which are scale invariant.

3. The Inequality Among the Poor

Since Sen's 1976 paper any poverty index should be expressed as $P = \phi(H, A, I)$, where I captures inequality among the poor, and ϕ is a non-decreasing function in its arguments. The inequality among the poor can be equivalently measured in terms of income or gaps. In fact, alternative decompositions in these terms have been identified for a number of poverty indices. Specifically the Sen index given

[4]Among them, the Takayama index (Takayama, 1979), which fails monotonicity, and the Thon index (Thon, 1979), which is not replication invariant.

[5]This index shares with the Sen modified index, symmetry, replication invariance, monotonicity, homogeneity of degree zero in $\mathbf{y}$ and z, and normalization in the range [0, 1]. Unlike the Sen index, it is also continuous and consistent with the transfer axioms.

in equation (1) permits the following two decompositions proposed by Sen (1976) and Xu and Osberg (2002), respectively,

$$S(\mathbf{y},z)=H\left(A+(1-A)G\left(\mathbf{y}_p\right)\right)=H\left(A+AG\left(\mathbf{g}_p\right)\right), \tag{4}$$

where $G(\mathbf{y}_p)$ and $G(\mathbf{g}_p)$ are the Gini coefficient of the poor income and the normalized gaps of the poor, respectively.

Similarly the *SST* index, equation (2), can also be decomposed taking into account the income inequality among the poor (Shorrocks, 1995) or the gap inequality (Xu and Osberg, 2002) as follows:

$$SST(\mathbf{y},z)=H\left((2-H)A+H(1-A)G\left(\mathbf{y}_p\right)\right)=H\left((2-H)A+HAG\left(\mathbf{g}_p\right)\right), \tag{5}$$

where $G(\mathbf{y}_p)$ and $G(\mathbf{g}_p)$ are the same as above.

Finally, for the FGT_2 index, equation (3), the expression below holds (Foster *et al.*, 1984; Aristondo *et al.*, 2010)

$$FGT_2(\mathbf{y};z)=H\left(A^2+(1-A)^2C^2\left(\mathbf{y}_p\right)\right)=H\left(A^2+A^2C^2\left(\mathbf{g}_p\right)\right), \tag{6}$$

where $C^2(\mathbf{y}_p)$ and $C^2(\mathbf{g}_p)$ are the squared coefficient of variation of income and of normalized gaps among the poor, respectively.

In some empirical applications any of these decompositions have been used to analyze how much of a change in poverty is due to more people becoming poor, or increasing deprivation of the poor, or because the inequality among the poor has changed, or some combination of the above (e.g., Kakwani, 1980; Xu and Osberg, 2002; Aristondo *et al.*, 2010). Nevertheless, the interpretation of the inequality component may mislead policy makers because income or gap inequality may display contradictory trends.

The aim of this paper is to propose a reinterpretation of the inequality component in poverty comparisons.

Although we focus on the three poverty indices mentioned above and their corresponding decompositions, the results of this paper may also be applied to other existing decompositions. Specifically, the generalization is straightforward to those indices in which the inequality among the poor is captured by the Gini index or the coefficient of variation.

We begin with a general result. Given an inequality index I, we define the *consistent index associated with I* as the measure that, for any $\alpha > 0$, and for any distribution $\mathbf{y}$ upper bounded by α, takes the following value:

$$I_\alpha(\mathbf{y})=\frac{I(\mathbf{y})+I(\alpha\mathbf{1}-\mathbf{y})}{2}. \tag{7}$$

In other words, $I_\alpha(\mathbf{y})$ is just the average value of $I(\mathbf{y})$ and $I(\alpha\mathbf{1}-\mathbf{y})$.[6] Note that, by definition $I_\alpha(\mathbf{y}) = I_\alpha(\alpha\mathbf{1}-\mathbf{y})$. The next proposition establishes that for any $\alpha > 0$

[6]This is a specific member of the family introduced in Lasso de la Vega and Aristondo (2012). In demographic research, when the results of a given decomposition depend on the order in which the decomposition is performed, it is standard, without any theoretical justification, to perform the calculations in all possible ways and to average the results (Kitagawa, 1964; Andreev *et al.*, 2002).

the consistent index I_α inherits from I the standard properties of an inequality measure.

Proposition 1. *The consistent index I_α associated with an inequality measure I satisfies TP, NOR, SYM, and RI.*

Proof. See Lasso de la Vega and Aristondo (2012).

I_α is not a standard inequality measure since it depends on α. Nevertheless, it satisfies *TP*, considered as the basic axiom in the inequality field, so it is able to capture the distribution inequality. We propose two particular specifications of this indicator in order to capture the inequality among the poor.

On the one hand, since the income distribution of the poor, $\mathbf{y}_p$, is upper-bounded by the poverty line $z > 0$, we obtain

$$I_z(\mathbf{y}_p) = \frac{I(\mathbf{y}_p) + I(z\mathbf{1} - \mathbf{y}_p)}{2}. \tag{8}$$

A second specification of equation (7) is obtained taking $\alpha = 1$ and the distribution of the poor normalized gaps as follows:

$$I_1(\mathbf{g}_p) = \frac{I(\mathbf{g}_p) + I(\mathbf{1} - \mathbf{g}_p)}{2}. \tag{9}$$

After Proposition 1, $I_z(\mathbf{y}_p)$ is able to capture the inequality among the income of the poor whereas $I_1(\mathbf{g}_p)$ measures the inequality of the poverty gaps. However, note that whenever inequality measure I is relative we find that

$$I_z(\mathbf{y}_p) = I_1(\mathbf{g}_p). \tag{10}$$

In fact, from equation (8) and since I is relative we get

$$\begin{aligned} I_z(\mathbf{y}_p) &= \frac{I(\mathbf{y}_p) + I(z\mathbf{1} - \mathbf{y}_p)}{2} && \text{by definition} \\ &= \frac{I(\mathbf{y}_p/z) + I((z\mathbf{1} - \mathbf{y}_p)/z)}{2} && \text{since } I \text{ is a relative measure} \\ &= \frac{I(\mathbf{y}_p/z) + I(\mathbf{1} - (\mathbf{y}_p/z))}{2} = I_1(\mathbf{g}_p) && \text{operating.} \end{aligned}$$

Hence these equivalent indicators measure the income and the gap inequality among the poor equally. We aim to identify this term in the existing decompositions of the poverty indices and propose them as the inequality component. First, Proposition 2 below derives the consistent poverty indicators related to the Gini coefficient and to the coefficient of variation.

Proposition 2. *The consistent indicator associated with*

(i) the Gini coefficient satisfies that

(11) $$G_z(\mathbf{y}_p) = \frac{G(\mathbf{y}_p)}{2A} = G_1(\mathbf{g}_p) = \frac{G(\mathbf{g}_p)}{2(1-A)}.$$

(ii) the coefficient of variation satisfies that

(12) $$C_z(\mathbf{y}_p) = \frac{C(\mathbf{y}_p)}{2A} = C_1(\mathbf{g}_p) = \frac{C(\mathbf{g}_p)}{2(1-A)},$$

where z is the poverty line and $\mathbf{y}_p$ *and* $\mathbf{g}_p$ *represent the income and the normalized gap distributions of the poor, respectively.*

Proof. See Appendix.

The following proposition shows how to incorporate the consistent inequality term in the existing decompositions.

Proposition 3. *The following expressions hold:*

(13) (*i*) $$S(\mathbf{y}, z) = HA\left(1 + 2(1-A)G_z(\mathbf{y}_p)\right) = HA\left(1 + 2(1-A)G_1(\mathbf{g}_p)\right),$$

(14) (*ii*) $$SST(\mathbf{y}, z) = HA\left((2-H) + 2H(1-A)G_z(\mathbf{y}_p)\right) = HA\left((2-H) + 2H(1-A)G_1(\mathbf{g}_p)\right),$$

(15) (*iii*) $$FGT_2(\mathbf{y}; z) = HA^2\left[1 + 4(1-A)^2\left(C_z(\mathbf{y}_p)\right)^2\right] = HA^2\left[1 + 4(1-A)^2\left(C_1(\mathbf{g}_p)\right)^2\right].$$

Proof. It is clear from Proposition 2 and equations (4), (5), and (6).

According to equations (13), (14), and (15), the inequality among the poor may be assessed by an indicator whose value does not change if we focus on gaps or on incomes.

4. Empirical Application

This section shows the difficulties encountered when determining the evolution of the inequality among the poor, and illustrates how to apply the methodology proposed. For this purpose we use the European Union Survey on Income and Living Conditions (EU-SILC). For comparability reasons, we use the same distributional assumptions used by Eurostat in the calculation of the headline indicator of poverty: population living "at-risk-of-poverty," which is in fact the *headcount ratio*. Hence, we consider household disposable income as the income variable,[7] even if the recipient unit is the individual; to account for differences in household size and composition we use the modified OECD equivalence scale, and consider the poverty line as 60 percent of the median national equivalized household income in the current year.[8] A person is poor if he lives in a household below this threshold. Calculations take into account cross-sectional sample weights and within-household non-respondent inflation factors.

First, we perform a cross-section comparison for 2009. In this case monetary values are converted to Purchasing Power Standards (PPS), to account for differences in the purchasing power of different national currencies, including those countries that share a common currency (the Euro). Note however that this only has an effect on the poverty line and the mean income of the poor, since all of the inequality and poverty indices remain unchanged given that we live in a scale invariant world. Table 1 shows the basic results for this year for the 27 EU countries plus Iceland and Norway.

This table shows a great deal of heterogeneity among European countries, with at one extreme the Czech Republic, with a *headcount ratio* not reaching 10 percent, and at the other extreme Latvia with over 25 percent. This heterogeneity can also be seen in the poverty line, which reflects different levels of development among the countries considered. From our point of view, two cases are of special interest. If we compare Finland and Austria, the *poverty gap ratio*, A, and the poverty indices considered show a higher level of poverty in Austria, despite its enjoying a lower *headcount ratio*, H. Using TIP curves (Jenkins and Lambert, 1998), we find poverty dominance of Austria over Finland for the poverty lines considered, since the TIP curve for Austria lies everywhere above the Finland curve for each population share, p, considered. Hence, we find more poverty in Austria than in Finland for all poverty indices defined over normalized poverty gaps, but note that this question has no particular relevance for the problem of measuring the inequality among the poor.[9] If we are interested in the inequality among the poor, then contradictory results are obtained whether we consider their incomes or their gaps. Using the income of the poor, Austria shows higher inequality, either with the Gini index or with the coefficient of variation, whereas using gaps, Finland appears to have a more unequal distribution for poor people. Eventually, the consistent indices introduced in this paper reconcile results, showing higher inequality among the poor in Austria, with this result being independent of the particular index used.

[7]The definition of income used excludes imputed rent and is neither top-coded nor bottom-coded, but negative values were excluded from the analysis. It also excludes non-cash transfers, such as education and healthcare provided free or subsidized by the government, non-monetary income components, and pensions from private plans.

[8]We follow the usual procedure to determine the poverty line as a percentage of the median income despite the clash between this method and the theoretical framework for the poverty indices. In fact, whenever the poverty line is established depending on the distribution, the focus axiom is violated.

[9]There is also an issue to be resolved when making dominance comparisons, and that concerns the degree of detail to which the dominance comparisons should be carried out. This is a question related to issues of statistical inference (Davidson and Duclos, 2000). Our TIP dominance is checked at each p from 0 to 1, in increments of 0.001.

TABLE 1

Poverty Statistics: EU-SILC 2009. Monetary variable in PPS

				Poverty Indices			Inequality among the Poor				Consistent Indices	
	Poverty Line (z)	Incidence % (H)	Intensity (A)	Sen	SST	FGT_2	$G(\mathbf{y}_p)$	$C(\mathbf{y}_p)$	$G(\mathbf{g}_p)$	$C(\mathbf{g}_p)$	$\frac{G_1(\mathbf{y}_p/z)}{G_1(\mathbf{g}_p)}$	$\frac{C_1(\mathbf{y}_p/z)}{C_1(\mathbf{g}_p)}$
France	10,594	12.81	0.2176	0.0402	0.0538	0.0100	0.1227	0.2248	0.4413	0.8084	0.2820	0.5166
Belgium	10,509	14.31	0.2160	0.0444	0.0593	0.0112	0.1206	0.2271	0.4378	0.8244	0.2792	0.5257
Luxembourg	16,230	14.83	0.2203	0.0466	0.0625	0.0118	0.1210	0.2260	0.4282	0.8002	0.2746	0.5131
Netherlands	11,551	10.79	0.2421	0.0385	0.0508	0.0116	0.1518	0.2929	0.4750	0.9166	0.3134	0.6047
Germany	10,785	15.44	0.2609	0.0575	0.0770	0.0171	0.1511	0.2793	0.4279	0.7911	0.2895	0.5352
Italy	9,154	18.21	0.2901	0.0765	0.1003	0.0257	0.1834	0.3361	0.4489	0.8225	0.3162	0.5793
United Kingdom	10,284	17.08	0.2599	0.0642	0.0846	0.0196	0.1565	0.2933	0.4457	0.8354	0.3011	0.5644
Ireland	10,559	15.11	0.2214	0.0493	0.0642	0.0137	0.1345	0.2625	0.4729	0.9230	0.3037	0.5927
Denmark	10,780	12.30	0.2420	0.0444	0.0577	0.0137	0.1569	0.3034	0.4914	0.9504	0.3241	0.6269
Greece	7,621	19.20	0.2738	0.0742	0.0992	0.0224	0.1546	0.2815	0.4101	0.7466	0.2824	0.5140
Portugal	5,646	17.86	0.2794	0.0710	0.0947	0.0220	0.1640	0.2950	0.4231	0.7608	0.2936	0.5279
Spain	8,488	18.98	0.3149	0.0867	0.1133	0.0317	0.2072	0.3794	0.4506	0.8252	0.3289	0.6023
Sweden	11,289	13.02	0.2621	0.0505	0.0660	0.0163	0.1702	0.3211	0.4792	0.9040	0.3247	0.6126
Finland	10,370	13.80	0.2013	0.0404	0.0535	0.0099	0.1141	0.2202	0.4527	0.8736	0.2834	0.5469
Austria	11,318	12.00	0.2343	0.0408	0.0544	0.0114	0.1377	0.2623	0.4501	0.8572	0.2939	0.5598
Malta	7,717	15.12	0.2123	0.0475	0.0617	0.0128	0.1290	0.2520	0.4785	0.9350	0.3037	0.5935
Hungary	4,104	12.41	0.2027	0.0356	0.0485	0.0082	0.1060	0.1979	0.4168	0.7786	0.2614	0.4883
Estonia	4,800	19.53	0.2367	0.0679	0.0877	0.0195	0.1457	0.2748	0.4697	0.8860	0.3077	0.5804
Czech Republic	6,065	8.58	0.2344	0.0290	0.0393	0.0078	0.1359	0.2480	0.4438	0.8100	0.2898	0.5290
Latvia	4,418	25.58	0.3196	0.1129	0.1506	0.0383	0.1789	0.3209	0.3808	0.6831	0.2798	0.5020
Poland	4,428	17.12	0.2630	0.0636	0.0855	0.0184	0.1474	0.2660	0.4131	0.7455	0.2803	0.5057
Cyprus	11,784	16.17	0.2110	0.0474	0.0648	0.0107	0.1043	0.1854	0.3900	0.6935	0.2471	0.4395
Lithuania	4,410	20.42	0.2893	0.0852	0.1114	0.0285	0.1798	0.3326	0.4418	0.8171	0.3108	0.5748
Slovenia	8,649	11.36	0.2335	0.0374	0.0513	0.0096	0.1250	0.2254	0.4103	0.7400	0.2676	0.4827
Slovakia	4,713	10.89	0.2822	0.0439	0.0596	0.0140	0.1684	0.3082	0.4282	0.7837	0.2983	0.5460
Bulgaria	3,452	21.77	0.3100	0.0942	0.1261	0.0314	0.1778	0.3176	0.3958	0.7070	0.2868	0.5123
Romania	2,068	22.29	0.3568	0.1099	0.1481	0.0412	0.2120	0.3731	0.3820	0.6725	0.2970	0.5228
Iceland	12,951	10.04	0.2276	0.0338	0.0445	0.0095	0.1416	0.2683	0.4807	0.9107	0.3112	0.5895
Norway	14,490	11.57	0.2831	0.0478	0.0635	0.0159	0.1812	0.3340	0.4588	0.8458	0.3200	0.5899

Note: Own elaboration from EU-SILC data.

A similar circumstance occurs if we compare poverty in Slovakia and Bulgaria. The latter country shows unambiguously more poverty that the former for every poverty index considered, as well as for the *poverty gap ratio*, A, and the *headcount ratio*, H. In fact, we find Bulgaria TIP poverty dominates Slovakia, with the exception of a single $p = 0.1$ percent, the first one considered. But if our interest lies in the inequality among the poor, then again contradictory results are obtained. If we look at incomes, Bulgaria shows unambiguously more inequality, while if we look at gaps, Slovakia displays higher inequality among the poor. The consistent indices reconcile results since both the Gini index and the coefficient of variation show higher inequality among the poor for Slovakia, despite this country getting a much lower poverty level.

Second, we perform a time series exercise for the maximum period available, 2004–09, and show that similar contradictory results can be obtained for the income distribution of the poor. In this case monetary values are deflated by the Harmonized Index of Consumer Prices (HICP, base 2005) to account for inflation. Again this only has an effect on the poverty line and the mean income of the poor and not on relative inequality or poverty indices.

Focusing on the initial and final period, Table 2 shows the four countries for which opposite results are obtained when comparing the inequality among the poor if we use either income or gaps, that is, Italy, Estonia, Iceland, and Norway. Poverty experiences a decreasing tendency in all these countries with the exception of Norway, which shows higher values of all indicators in 2009 than in 2004. Indeed, for Norway we find that the 2009 TIP curve dominates the one for 2004; the opposite is true for Estonia, whereas for Italy and Iceland we find no dominance. Focusing on the inequality among the poor we get the same trends using income as the variable of interest, so inequality in the income distribution of the poor falls for Italy, Estonia, and Iceland, but increases for Norway. However, the trend is reversed if we switch to gaps as the variable of interest; Italy, Estonia, and Iceland show an increasing trend in inequality, whereas Norway shows a decreasing trend. This is robust to the indicator used, either Gini or the coefficient of variation.

Looking at the consistent indices, we reach the conclusion that inequality among the poor decreases in Italy and Estonia, but increases in Iceland. For Norway results are dependent upon the indicator used, an increase for the Gini and a decrease for the coefficient of variation, so unambiguous results cannot be obtained in this case.

5. Conclusion

This paper draws attention to the difficulty that arises when the inequality among the poor needs to be examined. According to Sen (1976), it makes sense to measure the income or gap inequality. However, the analysis of the inequality trends or the comparisons among different situations may not lead to conclusive results. This paper proposes an alternative indicator that measures income and gap inequality equally. This component is easily accommodated in the existing decompositions. We hope this paper will help policy makers to better understand and explain poverty trends.

TABLE 2

Poverty Statistics: EU-SILC 2004–2009. Monetary Variable Deflated by HICP (2005 = 100)

					Poverty Indices			Inequality among the Poor				Consistent Indices	
	Year	Poverty Line (z)	Incidence % (H)	Intensity (A)	Sen	SST	FGT_2	$G(\mathbf{y}_p)$	$C(\mathbf{y}_p)$	$G(\mathbf{g}_p)$	$C(\mathbf{g}_p)$	$G_1(\mathbf{y}_p/z)$ $G_1(\mathbf{g}_p)$	$C_1(\mathbf{y}_p/z)$ $C_1(\mathbf{g}_p)$
Italy	2004	8.524	18.78	0.3009	0.0818	0.1071	0.0283	0.1929	0.3503	0.4481	0.8138	0.3205	0.5820
	2009	8.718	18.21	0.2901	0.0765	0.1003	0.0257	0.1834	0.3361	0.4489	0.8225	0.3162	0.5793
Estonia	2004	1.661	19.89	0.3120	0.0899	0.1173	0.0321	0.2030	0.3671	0.4475	0.8093	0.3252	0.5882
	2009	3.024	19.53	0.2367	0.0679	0.0877	0.0195	0.1457	0.2748	0.4697	0.8860	0.3077	0.5804
Iceland	2004	13.921	9.98	0.2432	0.0354	0.0472	0.0102	0.1471	0.2735	0.4576	0.8509	0.3024	0.5622
	2009	10.976	10.04	0.2276	0.0338	0.0445	0.0095	0.1416	0.2683	0.4807	0.9107	0.3112	0.5895
Norway	2004	16.116	10.75	0.2473	0.0392	0.0517	0.0120	0.1563	0.2977	0.4756	0.9058	0.3159	0.6017
	2009	18.977	11.57	0.2831	0.0478	0.0635	0.0159	0.1812	0.3340	0.4588	0.8458	0.3200	0.5899

Note: Own elaboration from EU-SILC data and HICP from Eurostat web page.

References

Andreev, E. M., V. M. Shkolnikov, and A. Z. Begun, "Algorithm for Decomposition of Differences Between Aggregate Demographic Measures and Its Application to Life Expectancies, Healthy Life Expectancies, Parity-Progression Ratios and Total Fertility Rates," *Demographic Research*, 7, 499–522, 2002.

Aristondo, O., C. Lasso de la Vega, and A. Urrutia, "A New Multiplicative Decomposition for the Foster–Greer–Thorbecke Poverty Indices," *Bulletin of Economic Research*, 18, 153–72, 2010.

Clark, S., R. Hemming, and D. Ulph, "On Indices for the Measurement of Poverty," *Economic Journal*, 91, 515–26, 1981.

Clarke, P. M., U. G. Gerdtham, M. Johannesson, K. Bingefors, and L. Smith, "On the Measurement of Relative and Absolute Income-Related Health Inequality," *Social Science and Medicine*, 55, 1923–28, 2002.

Davidson, R. and J. Y. Duclos, "Statistical Inference for Stochastic Dominance and for the Measurement of Poverty and Inequality," *Econometrica*, 68, 1435–64, 2000.

Erreygers, G., "Can a Single Indicator Measure Both Attainment and Shortfall Inequality?" *Journal of Health Economics*, 28, 885–93, 2009.

Foster, J., J. Greer, and E. Thorbecke, "A Class of Decomposable Poverty Measures," *Econometrica*, 52, 761–6, 1984.

Jenkins, S. and P. Lambert, "Three 'I's of Poverty Curves and Poverty Dominance: TIPs for Poverty Analysis," *Research on Economic Inequality*, 8, 39–56, 1998.

Kakwani, N., "On a Class of Poverty Measures," *Econometrica*, 48, 437–46, 1980.

———, "Inequality, Welfare and Poverty," in J. Silber (ed.), *Handbook of Income Inequality Measurement*, Kluwer Academic Publishers, Norwell, MA, 599–628, 1999.

Kitagawa, E., "Standardized Comparisons in Population Research," *Demography*, 1, 296–315, 1964.

Lambert, P. and B. Zheng, "On the Consistent Measurement of Attainment and Shortfall Inequality," *Journal of Health Economics,* 30, 214–19, 2011.

Lasso de la Vega, C. and O. Aristondo, "Proposing Indicators to Measure Achievement and Shortfall Inequality Consistently," *Journal of Health Economics*, 31, 578–83, 2012.

Osberg, L. and K. Xu, "International Comparisons of Poverty Intensity: Index Decomposition and Bootstrap Inference," *Journal of Human Resources*, 35, 1–81, 2000.

Sen, A. K., "Poverty: An Ordinal Approach to Measurement," *Econometrica*, 44, 219–31, 1976.

Shorrocks, A., "Revisiting the Sen Poverty Index," *Econometrica*, 63, 1225–30, 1995.

Takayama, N., "Poverty, Income Inequality, and Their Measures: Professor Sen's Axiomatic Approach Reconsidered," *Econometrica*, 47, 747–59, 1979.

Thon, D., "On Measuring Poverty," *Review of Income and Wealth*, 25, 429–40, 1979.

Xu, K. and L. Osberg, "The Social Welfare Implications, Decomposability, and Geometry of the Sen Family of Poverty Indices," *Canadian Journal of Economics*, 35, 138–52, 2002.

Supporting Information

Additional Supporting Information may be found in the online version of this article at the publisher's web-site:

Appendix: Proof of Proposition 2