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INEQUALITY COMPARISONS IN A MULTI-PERIOD FRAMEWORK: THE ROLE OF ALTERNATIVE WELFARE METRICS

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This paper considers the use of alternative welfare metrics in evaluations of income inequality in a multi-period context. Using Norwegian longitudinal income data, it is found, as in many studies, that inequality is lower when each individual's annual average income is used as welfare metric, compared with the use of a single-period accounting framework. However, this result does not necessarily hold when aversion to income fluctuations is introduced. Furthermore, when actual incomes are replaced by expected incomes (conditional on an initial period), using a model of income dynamics, higher values of inequality over longer periods are typically found, although comparisons depend on inequality and variability aversion parameters. The results are strongly influenced by the observed high degree of systematic regression toward the (geometric) mean, combined with a large extent of individual unexpected effects.

JEL Codes: C23, D31, I31

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1. Introduction

Evaluations of changes in the distribution of income must begin by deciding on a number of fundamental ingredients, each of which involves value judgments. First, a choice of "welfare metric," concerning what is to be measured for each unit of analysis, must be made. Second, a decision is needed regarding the time period of analysis. Third, the unit of analysis itself must be chosen. Finally, the form of "social evaluation function," which encapsulates further explicit distributional value judgments, has to be specified. The present paper explores the use of alternative welfare metrics in a multi-period context, using the individual as the basic unit of analysis and an additive, individualistic Paretean social welfare function reflecting belief in the "principle of transfers" (whereby a transfer from relatively rich to poor individuals, leaving their rankings unchanged, is considered an improvement). The welfare metrics are based on alternative income concepts

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rather than, say, consumption or utility measures which allow for variations in the value of leisure time.

Consideration of a multi-period context necessarily introduces the role of income mobility. This implies that inequality of income measured over a longer period is lower than that in the highest single year.[1] A further argument concerns comparative static changes: if higher annual income inequality is associated with increased relative income mobility, it is possible that inequality of income measured over several years is lower. Hence, longer-period inequality may fall, and welfare might increase, despite the rise in annual income inequality: this is referred to as a "mobility offsetting" argument. However, the welfare metric could allow for other effects.[2] For example, if there is imperfect substitutability of incomes over time (Atkinson *et al.*, 1992) and individuals are averse to income variability, the offsetting argument is weakened; see Creedy and Wilhelm (2002).

The discussion is typically, as above, carried out in terms of *ex-post* income measures. An alternative approach, explored here, is to attempt to allow explicitly for the uncertainty associated with mobility by constructing a welfare metric based on an *ex-ante* income measure. This in turn requires the use of a model of expectations formation based on observed income dynamics. The association between mobility and *ex-ante* income uncertainty has also been stressed by Parker and Rougier (2001), Gottschalk and Spolaore (2002),[3] and Ben-Shahar and Sulganik (2008).[4]

This paper presents results where expected income is derived by estimating an autoregressive model of income dynamics. A closed-form expression for expected income, conditional on initial income, is obtained. Thus a "rational expectations" approach is used, whereby individuals are assumed to form expectations based on the dynamic model of incomes and associated parameter estimates. The model specifies the logarithm of individual income in a given period as a function of the relative distance from the geometric mean of a previous period's income, an individual fixed effect, and a stochastic component. The social welfare function, and hence distributional value judgments, examined are based on the Atkinson (1970) inequality index. To illustrate the framework, longitudinal data for individuals in Norway over the period 1993–2005 are used.

An alternative approach to measuring long-period inequality involves the use of a "utility-equivalent annuity," introduced by Nordhaus (1973) and defined as the constant value which gives the same lifetime utility as the actual time profile.[5] This can clearly allow for an aversion to variability as well as imperfect capital markets. This concept has recently been used by Aaberge and Mogstad (2010) and

[1]Conditions under which inequality is lower than in all years are examined by Creedy (1997a).

[2]The question of whether income mobility represents equality of opportunity, as in Bénabou and Ok (2001), is not considered here.

[3]They present a decomposition analysis where the extent to which future incomes depend on current income is separated from effects due to rank reversals. For other decompositions, see Ruiz-Castillo (2004), Van Kerm (2004), and Jenkins and Van Kerm (2006).

[4]Other studies involving mobility and long-term incomes include, for example, Shorrocks (1978a), Chakravarty *et al.* (1985), Fields (2010), and Hungerford (2011). For surveys of mobility, see Atkinson *et al.* (1992), Maasoumi (1998), and Fields and Ok (1999).

[5]Comparisons using this and alternative measures, using a lifetime simulation model, are reported in Creedy (1997b).

Aaberge *et al.* (2011) to examine equality of opportunity.[6] They distinguish two cases. First, equality of opportunity is reflected in equal outcomes for all those with the same "effort," so that emphasis is on within-group inequality of utility-equivalent annuity. Second, between-group comparisons are made where groups are instead defined by common "opportunity sets," and within-group inequality arising from different degrees of "effort" are considered irrelevant. The authors refer to the former as an *ex-post* approach and the latter as an *ex-ante* approach. This interesting perspective clearly differs from the *ex-ante* concept examined in the present paper.

In Section 2 the data and the Atkinson index are briefly described. Section 3 presents results using *ex-post* welfare metrics. Section 4 introduces *ex-ante* income uncertainty and presents a procedure for using expected future incomes in the welfare metric. Section 5 summarizes the main findings.

2. Data and Inequality Measurement

The data used below come from Income Statistics for Persons and Families in Norway 1993–2005 (Statistics Norway, 2006). These data contain register-based information on the whole population, derived primarily from information retrieved from all income tax returns in the Directorate of Taxes' Register of Personal Tax-Payers. The choice of time period, 1993–2005, is conditioned on the register being established in 1993, and the desirability of avoiding the tax reform of 2006, as a reform normally involves measurement challenges. Nevertheless, as data primarily are used to illustrate the alternative metrics, the choice of time period mainly follows from the desire to explore data from a sufficiently long time period to be divided into two periods of equal length.

The income measure is annual income after tax. Thus income is defined as labor income, plus positive capital income, plus net capital gains, plus transfers minus direct taxes. This is the definition used in all official income statistics in Norway. Negative capital income (interest paid on mortgages) is not included in the definition because there is no corresponding income from housing in the statistics. Estimates of income mobility are typically sensitive to persons entering and leaving the labor market. Hence, persons under age 26 and above 65 are excluded, and those with zero or negative income in any year are excluded.[7] The effects of inflation have been removed by deflating all incomes to the 1998 level using the consumer price index.

Results are presented using the well-known Atkinson (1970) inequality measure. Let individual *i*'s income (the welfare metric, ignoring time for now) be denoted y_i, for $i = 1, \ldots, n$. The Atkinson measure is based on the additive social welfare function, $W = W(y_1, \ldots, y_n)$ of the form $W = \sum_{i=1}^{n} U(y_i)$, where $U(y_i)$ is the weight attached to y_i, and is specified, for $\varepsilon \neq 1$, as:[8]

[6]They also measure mobility in terms of the reduction in this longer-period income concept, following Shorrocks (1978a).

[7]Zero incomes are excluded since many are due to measurement error rather than being true observations of zero income.

[8]If $\varepsilon = 1$, $y_i^{1-\varepsilon}/(1-\varepsilon)$ is replaced by $\log y$.

$$U(y_i|\varepsilon) = \frac{y_i^{1-\varepsilon}}{1-\varepsilon}. \tag{1}$$

Hence $\varepsilon \geq 0$ captures the concavity of U, corresponding to the aversion to relative inequality. Let y_{EDE} denote the equally distributed equivalent income, that is, the income which, if obtained by each person, gives the same social welfare as the actual distribution. Hence, for $\varepsilon \neq 1$:

$$y_{EDE} = \left(\frac{1}{n}\sum_{i=1}^{n} y_i^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}. \tag{2}$$

Atkinson's index of inequality, I, is the proportional difference between the arithmetic mean, $\bar{y}$, and y_{EDE}, so that:

$$I = \frac{\bar{y} - y_{EDE}}{\bar{y}}, \tag{3}$$

and I reflects the "wastefulness of inequality."

3. Alternative *Ex-Post* Evaluations

Figures 1 and 2 show, for the period 1993–2005 and for two inequality aversion parameters, the time profiles of inequality and the equally distributed

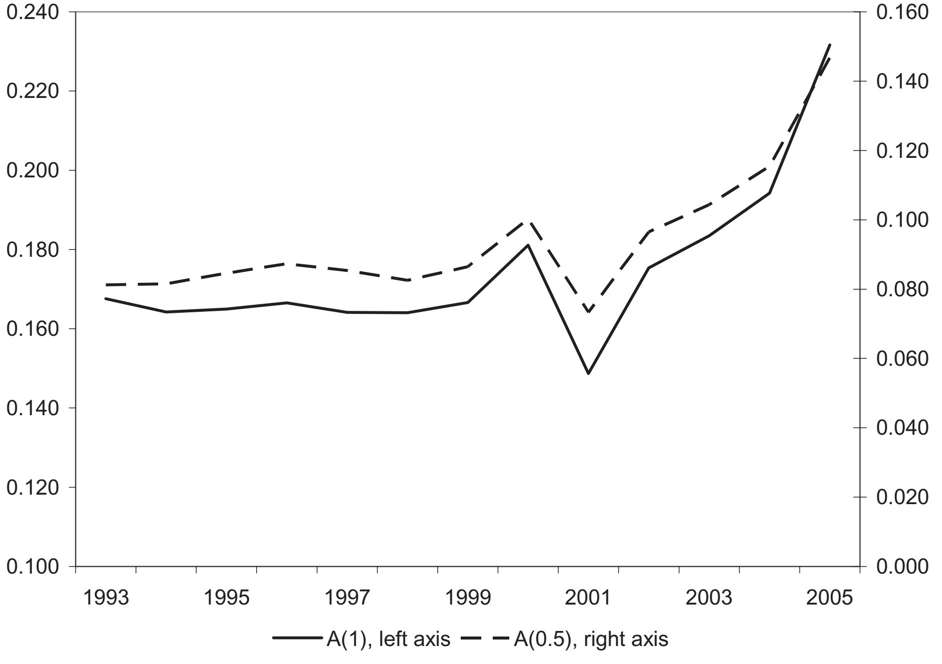

Figure 1. Atkinson Inequality Measures, Annual Income 1993–2005, $\varepsilon = 0.5$ and $\varepsilon = 1$

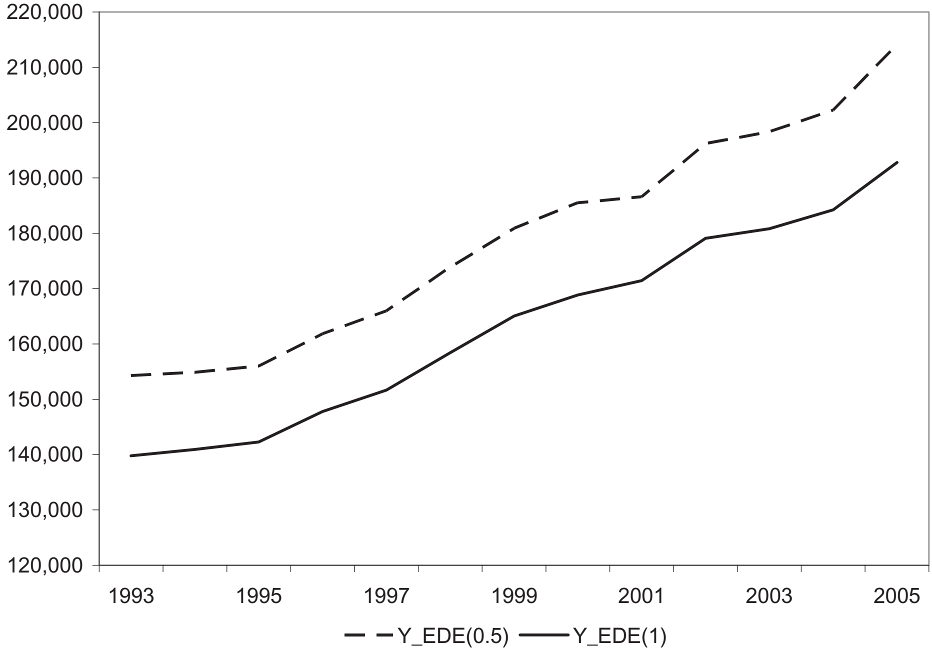

Figure 2. Equally Distributed Equivalent Annual Income: 1993–2005

equivalent. The period may be divided into two periods of equal length, 1994–99 and 2000–05.[9] The first period reflects a relatively stable degree of inequality while the second period displays more variability, initially decreasing and then increasing steadily. Both the general economic development and tax-payers' behavioral reactions to tax legislative changes have influenced the observed income patterns. The first part of the time period coincides with a period of high economic growth (GDP growth above 4 percent in the period 1994–97), then growth rates are lower and more variable in the period 1998–2005 (but above 2 percent in all years, except in 2003). However, from an income distribution perspective, it is suggested that the development of the personal income tax schedule is at least equally important.

The tax reform of 1992 introduced a dual income tax system, which combines a low proportional tax rate on capital income and progressive tax rates on labor income. These separate schedules for capital and labor income created obvious incentives for taxpayers to take advantage of the lower tax on capital. For example, Thoresen and Alstadsæter (2010) show that owners of small businesses were able to gain from this schedule by changing organizational form. Overall, there was a notably increase in (low taxed) dividend income transfers to households over the period, from less than 10 billion in 1994 to nearly NOK100 billion in 2005. Only 2001 does not fit into the steady upward trend, as there was a temporary tax on dividends for shareholders that year. Furthermore, corporations

[9]1993 is therefore not used in inequality comparisons, but as a base year when estimating *ex-ante* incomes.

TABLE 1

Atkinson Inequality Measures: Annual Average Income Inequality and Inequality with Annual Average as Welfare Metric

	Period 1994–99			Period 2000–05		
	$\varepsilon = 0.5$	$\varepsilon = 1.0$	$\varepsilon = 1.5$	$\varepsilon = 0.5$	$\varepsilon = 1.0$	$\varepsilon = 1.5$
Inequality: Annual average of single year values						
$\bar{I}_y$	0.084	0.165	0.263	0.108	0.188	0.307
Using each individual's annual average as welfare metric						
$I_{\bar{y}}$	0.071	0.134	0.183	0.082	0.147	0.228

brought forward their distribution of profits because of the pre-announced tax reform of 2006 (Alstadsæter and Fjærli, 2009), given that it was evident that the reform introduced taxation of dividends at both the corporate and individual level (in contrast to the 1992 reform, which had only corporate-level taxation), through a shareholder income tax; see Sørensen (2005) for further details. As dividend income is unequally dispersed—for example, 95 percent of dividends were received by individuals in decile 10 in 2004—this has resulted in a substantial increase in post-tax income inequality over the period (with an exception for 2001) and changes in the composition of income across income distributions; for more details see Lambert and Thoresen (2009, figure 3).

A time subscript must now be added to each individual's income. For convenience, the following ignores discounting. Consider first an *ex-post* evaluation over T periods which uses as welfare metric for each individual the average annual income, $\bar{y}_i = \frac{1}{T}\sum_{t=1}^{T} y_{it}$. Hence the welfare function is not actually concerned with the way in which any individual's income is distributed over the time period, and thus may be said to reflect a lack of concern for the nature of the mobility process. For the period 1994–2005, the use of the average annual income as welfare metric for each individual gives Atkinson inequality measures of 0.076, 0.134, and 0.210, respectively for values of ε of 0.5, 1.0, and 1.5. These each reflect value judgments which tolerate substantial leaks in making equalizing transfers.[10] These values may be compared with the annual average inequality measures of 0.099, 0.181, and 0.298, respectively. The use of a longer period whereby individual incomes are averaged is thus equalizing in this case.

Further details for the two sub-periods are shown in Table 1.[11] For the second period, the absolute reduction in inequality, when using annual average income as the welfare metric compared with annual average inequality, is double the reduction obtained for the first period. The inequality-reducing effects of using a longer accounting period, mentioned above, therefore appear to be greater in a period when annual inequality is generally increasing.[12] However, the percentage reduc-

[10]For example, if 1 unit is taken from A to make a transfer to B, where A is twice as rich as B, a transfer of 0.5 units leaves social welfare unchanged if $\varepsilon = 1$. This falls to 0.25 units if $\varepsilon = 2$.

[11]Standard errors of the estimates, which can be obtained by bootstrapping procedures, are not reported as they in general are very small due to the sample size (more than 2,600,000 observations).

[12]This of course differs from the comparative static argument discussed in the introduction, and examined in detail by Creedy and Wilhelm (2002) in terms of inequality and social welfare.

TABLE 2
Atkinson Inequality Measures with Aversion to Income Fluctuations

	Period 1994–99			Period 2000–05		
	$\varepsilon = 0.5$	$\varepsilon = 1.0$	$\varepsilon = 1.5$	$\varepsilon = 0.5$	$\varepsilon = 1.0$	$\varepsilon = 1.5$
$\gamma = 0$	0.071	0.134	0.217	0.082	0.147	0.289
$\gamma = 0.5$	0.085	0.148	0.236	0.098	0.161	0.311
$\gamma = 2.0$	0.130	0.206	0.349	0.144	0.217	0.375
$\gamma = 3.0$	0.151	0.235	0.427	0.166	0.244	0.499

tion in inequality is larger in the first period for $\varepsilon = 1.5$. This arises because the very high degree of inequality aversion places more emphasis on the low end of the income distribution.

The income mobility which produces the increasing annual inequality is thus also responsible for reducing the inequality of a multi-period income measure (each person's annual average) below average annual inequality. However, mobility may not necessarily be seen as beneficial from an individual's point of view. It may also be seen as an undesirable source of economic instability. For example, individuals may for some reason be unable to smooth consumption over time when facing income fluctuations and they might be averse to such variability in income. Imperfections of capital markets or other constraints may prevent individuals from smoothing consumption over time; see Atkinson *et al.* (1992).[13]

It may therefore be desirable to allow, in the welfare metric, for an aversion to income variability, as suggested by Creedy and Wilhelm (2002); see also Jarvis and Jenkins (1998) on the disutility of income volatility. This can be done, in an *ex-post* context, by using instead of $\bar{y}_i$ a welfare metric, $\tilde{y}_i$, defined as:

$$\tilde{y}_i = \frac{1}{T}\sum_{t=1}^{T}\frac{y_{it}^{1-\gamma}}{1-\gamma}. \tag{4}$$

The parameter γ measures the degree of aversion to variability of income over time, and the same parameter is assumed to apply to all individuals. As *ex-post* values are used, the aversion coefficient, γ, is not interpreted in terms of risk aversion: this is discussed in Section 4. The relative values of ε and γ determine whether inequality aversion (of the judge whose value judgments are represented by the welfare function) is high enough to overcome the individuals' aversion to income variability over time. When aversion to income variability is high relative to inequality aversion, a more "static" society is preferred, in which income is more stable at the "cost" of higher inequality of multi-period income.

Table 2 shows the extent to which the values in Table 1 are increased when aversion to intertemporal fluctuations is introduced. In order to eliminate the effect of general income growth over time, incomes were adjusted so that average

[13]Shorrocks (1978a) argues that mobility is always desirable, whereas Chakravarty *et al.* (1985) establish a no-mobility hypothetical benchmark from which they can distinguish between desirable and undesirable mobility. Like King (1983), they make use of the equally distributed equivalent idea. The difficulty of establishing a reasonable social welfare understanding of income mobility is discussed by Atkinson (1981), Dardanoni (1993), and Fields (2010).

annual income is constant (and equal to the overall mean) in each period. Hence the inequality values in the first row of Table 2 differ slightly from those in the final row of Table 1. It is clear that inequality increases as individuals' aversion to income variability increases. The inequality differences between the first and second sub-periods are less influenced by aversion to income variability over time. The differences for $\gamma = 0$ and positive γ's are approximately similar in the two subperiods. As the last period involves a temporary tax on dividends for the shareholders in 2001, one may expect stronger effects from increases in the value of γ in that period, but no clear manifestation of such effects is evident in Table 2.

4. An *Ex-Ante* Perspective

The suggestion that relative income mobility is associated with uncertainty leads to the idea that an alternative evaluation may be based on an *ex-ante* measure, rather than *ex-post* incomes as in the previous section. For example, Shorrocks (1978b, p. 1016) argues that "interest in mobility is not only concerned with movement but also predictability." Furthermore, the uncertainty aspect of mobility is emphasized by contributions which see mobility in terms of future opportunities, as in Bénabou and Ok (2001), or account for origin independence, as in Gottschalk and Spolaore (2002). An *ex-ante* perspective is introduced in Section 4.1. The approach requires a model of income dynamics and this is described in Section 4.2. Results using the new metric are presented in Section 4.3.

4.1. *The Welfare Metric*

The approach considered here is to replace the above welfare metric with one defined in terms of expected incomes, conditional on income in a specified period, $E(y_{it}|y_{i0})$, so that (4) is replaced by:

$$E(\tilde{y}_i) = \frac{1}{T}\sum_{t=1}^{T}\frac{E\left(y_{it}^{1-\gamma}|y_{i0}\right)}{1-\gamma}. \tag{5}$$

Here the parameter γ can be interpreted in terms of risk aversion. In a one-commodity setting and with indifference with respect to the timing of risk, risk aversion is the inverse of the elasticity of intertemporal substitution. Thus resistance to intertemporal substitution, or variability aversion, is closely related to risk aversion.

Application of this approach therefore requires knowledge of the conditional expectation of future incomes. The following subsection proposes a measure of expected income obtained by modeling the income process.

4.2. *Modeling Income Dynamics*

The aim of this section is to present and estimate a simple model of individuals' expectations of future incomes which can be used to produce *ex-ante* income measures. Consider a dynamic process containing both a stochastic component and a component in which changes depend on the position of individuals relative

TABLE 3
Parameter Estimates for the Income Mobility Process

Method and Parameter	All Years	1994–99	2000–05
LSDV			
β	0.452	0.279	0.227
	(0.001)	(0.001)	(0.001)
σ_η	0.276	0.280	0.282
AB-GMM			
β	0.492	0.477	0.351
	(0.002)	(0.001)	(0.002)
σ_η	0.415	0.308	0.316
GMMSYS			
β	0.476	0.486	0.387
	(0.001)	(0.001)	(0.001)
σ_η	0.419	0.309	0.313
GMM-MA(1)			
β	0.473	0.527	0.443
	(0.002)	(0.003)	(0.003)
σ_η	0.524	0.406	0.436

Note: Robust standard errors are in parentheses below parameter estimates.

to the geometric mean; see also Creedy (1985) and Creedy and Wilhelm (2002). As before, y_{it} denotes individual i's income in period t, and let μ_t denote the mean of logarithms in period t, with $m_t = \exp(\mu_t)$ as the geometric mean. The income process can be written as:

$$y_{it} = \left(\frac{y_{it-1}}{m_{t-1}}\right)^{\beta} \exp(\mu_t + v_i + \eta_{it}), \tag{6}$$

where the stochastic component consists of an individual-specific effect, v_i, and a random component, η_{it}, assumed to be independent of income, with zero mean and a variance in each period of σ_η^2. Equation (6) can be rewritten as:

$$(\log y_{it} - \mu_t) = \beta(\log y_{it-1} - \mu_{t-1}) + v_i + \eta_{it}. \tag{7}$$

The autoregressive parameter, β, captures variations in income which decline more slowly over time. In other words it reflects movements in income that, while not permanent, tend to persist for several years. Suppose also that in this simple income process, the autoregressive parameter and income variance is common for all individuals, and heterogeneity in the process is represented through the individual fixed effect (individual fixed level relative to the mean) and the error term.

Table 3 reports results of using several estimators. These include the least squares dummy variables (LSDV), and generalized method of moments (GMM) estimators as in Arellano and Bond (1991) and Blundell and Bond (1998). Because the lagged dependent variable is correlated with the error term, it has been shown that the use of LSDV results in biased estimates. Anderson and Hsiao (1981) suggested first eliminating the fixed effect by taking first differences, and then using

y_{t-2} as instrument for Δy_t. However, this does not exploit all the relevant moment conditions so it is not the efficient GMM estimator. Arellano and Bond (1991) derived other moment conditions to be used in GMM estimation. This estimator is known as the Arellano–Bond GMM estimator. Other instruments have been suggested by a succession of researchers, such as the Arellano and Bover/Blundell and Bond system estimator (Arellano and Bover, 1995; Blundell and Bond, 1998), which uses moment conditions in which lagged first differences of the dependent variable are instruments for the level equation. In practice, it is difficult to find good instruments for the first-differenced lagged dependent variable, which can itself create problems for the estimation. Kiviet (1995) shows that panel data models using instrumental variable estimation often lead to poor finite sample efficiency and bias. Also, tests show that none of the methods reject the assumption of no autocorrelation in first differenced errors. Thus, a specification based on a GMM model assuming moving-average serial correlation in the residuals is also included.[14]

Table 3 shows that a common result for all specifications is that the estimated value of β is higher in the first sub-period than in the second sub-period, while the estimate of σ_η^2 is higher in the second sub-period. Since the standard errors are low, it may be inferred that the estimated β's in the two periods are also significantly different from each other. The lower degree of regression toward the mean and lower variance in the first period implies lower income mobility, and therefore higher predictability of future incomes. Conversely, the parameter values imply higher mobility and less predictability in the second sub-period. In the following subsection, reported results are based on the Arellano–Bond GMM estimator.

The model specified in (7) is simple compared to a number other approaches used. A less parsimonious model, such as the error component model, which is now a standard model in the income process literature,[15] would probably provide more reliable estimates for the income process. Adding heterogeneity in the model parameters as in Baker and Solon (2003) and Browning *et al.* (2010) would improve the model even more.

However, the basic model used here has been chosen for two reasons. First, it is helpful to keep the perspective of the social planner. An income process is specified that the planner is assumed to use for prediction of future incomes, based on observations of current incomes. All individual characteristics, observable or unobservable, that may explain individual income levels are captured by the individual fixed effect. We assume that the planner, possibly with the help of an econometrician, has estimated the distribution of these fixed effects on historical data, and furthermore, assumes that these are constant over time. Second, the simple autoregressive income process is in line with the Markov models often used in the income mobility index literature, and therefore provides a link between the income mobility literature and the more econometrically orientated income process literature.

[14]Lags three or higher are used as valid instruments for the differenced equation.

[15]This literature is represented by the works of Lillard and Weiss (1979), MaCurdy (1982), Abowd and Card (1989), Baker (1997), Carroll and Samwick (1997), and Ramos (2003).

Including other explanatory variables, such as age, family composition and education would substantially complicate the prediction of future incomes. While age is straightforward to predict, prediction of future family composition is rather demanding. Education is challenging too, as the specification already accounts for a fixed effect. Thus, fixed effects soak up much of the explanatory power of variables that are either time-invariant or close to time-invariant. We have explored the effects of using other explanatory variables, such as age, family composition, and education. The estimated autoregression coefficients became somewhat lower, but the overall result did not change. This suggests that the main difference between income mobility in the two periods is due to genuine income dynamics rather than, for instance, substantial differences in family dynamics. Comparisons were also made using alternative income definitions. Labor income yields similar estimates for the autoregression coefficient, but exhibits a much larger variance. For gross income there is less regression toward the mean (that is, higher β) than for the two other income definitions, and the difference between the two periods is larger. Also, as expected, the standard deviation of gross income is higher than for income after tax.

4.3. *Inequality Using the Ex-Ante Welfare Metric*

In order to obtain measures for the contribution of the estimated income process to the overall *ex-ante* welfare evaluation, a closed-form expression for expected income as a function of income in the initial period, $E(y_{it}|y_{i0})$, is required. It is shown in the online Appendix that:

$$E(y_{it} \mid y_{i0}, v_i) = \exp\left\{\mu_t + \beta^t(\log y_{i0} - \mu_0) + v_i\left(\frac{1-\beta^t}{1-\beta}\right) + \frac{1-\beta^{2t}}{2(1-\beta^2)}\sigma_\eta^2\right\}, \tag{8}$$

where estimates of individual fixed-effects, v_i, are obtained using their sample counterparts. The corresponding equally distributed equivalent in terms of expected income, $EDE_{E(y|y_0)}$, can be expressed as:

$$EDE_{E(y|y_0)} = \left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{T}\sum_{t=1}^{T}E(y_{it} \mid y_{i0}, v_i)^{1-\gamma}\right)^{\frac{1-\varepsilon}{1-\gamma}}\right]^{\frac{1}{1-\varepsilon}}, \tag{9}$$

from which, given the arithmetic mean, the Atkinson measure can be obtained in the usual way. In this case it depends on the degree of regression toward the mean, the income variance, the degree of aversion to inequality, and the degree of aversion to fluctuations in income. When $\beta < 1$, the initial (relative) position is given less weight over time, while the role of the individual-specific position is increasing over time. Expected income is also increasing over time.

The inequality measures for the *ex-ante* welfare metric are shown in Table 4, where again any effects of income growth are eliminated by maintaining the arithmetic mean constant. These may be compared with Table 2. For the sub-period 1994–99, inequality is lower for all values of ε examined and for the

TABLE 4
ATKINSON INEQUALITY MEASURES FOR EXPECTED INCOME

	Period 1994–99			Period 2000–05		
	$\varepsilon = 0.5$	$\varepsilon = 1.0$	$\varepsilon = 1.5$	$\varepsilon = 0.5$	$\varepsilon = 1.0$	$\varepsilon = 1.5$
$\gamma = 0$	0.004	0.068	0.135	0.109	0.165	0.218
$\gamma = 0.5$	0.037	0.095	0.165	0.150	0.208	0.246
$\gamma = 2.0$	0.132	0.221	0.417	0.250	0.367	0.492
$\gamma = 3.0$	0.166	0.394	0.513	0.290	0.481	0.603

variability aversion coefficients of 0 and 0.5. For the very high values of γ of 2.0 and 3.0, inequality is higher when the *ex-ante* measure is used, particularly for the high inequality aversion coefficient. The estimated value of β is rather low while that of σ_η^2 is high compared with values reported in earlier studies; see Creedy (1985). The considerable variability implied by the high σ_η^2 would produce increasing annual inequality over time, without the low value of β, implying considerable regression toward the mean. In the expression for $E(y_{it}|y_{i0})$, the effects of terms involving powers of β rapidly become insignificant. Expected incomes are dominated by the high σ_η^2 which, for the high mobility-aversion cases, implies a higher measured inequality. From (8), setting all terms involving β^t and β^{2t} to zero[16] and rearranging gives:

$$\log E(y_{it}) - \mu_t = \frac{v_i}{1-\beta} + \frac{\sigma_\eta^2}{2(1-\beta^2)}. \tag{10}$$

Hence the variance of logarithms of expected income soon becomes $\sigma_v^2/(1-\beta)^2$, where σ_v^2 is the variance of the fixed effect in the autoregressive income-generation equation. Therefore for higher σ_v^2 and lower β, as in the second sub-period considered, inequality of expected values is quickly increasing toward a relatively high value.

In the *ex-post* case, there is less inequality than anticipated as a result of the regression toward the mean. For the second sub-period, the role of unanticipated, but systematically equalizing, mobility is even greater and β is lower. Hence the *ex-ante* welfare metric produces higher inequality, for nearly all combinations of variability aversion and inequality aversion parameters, than for the *ex-post* metric. The exceptions are for the combination of low variability aversion with very high inequality aversion. Also, the inequality differences between the two sub-periods are maintained or increased when moving from the *ex-post* to the *ex-ante* perspective.

Discounting has been ignored here for ease of exposition. Introducing discounting of time periods would imply that greater weight is placed on the first period. In other words, the initial distribution would play a relatively larger role than without discounting. As long as time preference is homogeneous across individuals and constant over time, introducing discounting would not change the

[16]This is the same as replacing $E(y_{it}|y_{i0})$ by $E(y_{it})$, that is the unconditional expectation.

qualitative difference between the two sub-periods considered.[17] But, as the initial distributions become more dominant under discounting, the inequality estimates in Table 4 would be modestly increased.

It is therefore necessary to consider the role of the initial distributions. It may be argued that it is difficult to interpret the results for the two periods in Table 4 in terms of different income processes because they begin with different initial distributions. For this reason two sensitivity analyses were carried out. First, it was assumed simply that $(\log y_{i0} - \mu_0) = 0$ for all individuals (so that the fixed effect is the only individual variation). Second, the same initial distribution was used in both periods (hence, the second period process was estimated using the initial 1993 distribution). Unreported results show that the results are not sensitive to the choice of initial distributions. This lack of sensitivity is likely to arise because of the high degree of regression toward the mean.

5. Conclusions

The aim of this paper has been to consider the use of alternative welfare metrics in evaluations of income inequality when a multi-period income measure is used, and hence relative income mobility plays a crucial role in influencing the relationship between short- and long-period inequality. One basic approach, most commonly adopted in income distribution studies, is to base measures on *ex-post* magnitudes. Using Norwegian longitudinal income data, it was found, as in many studies, that income inequality is lower when each individual's annual average income is used as welfare metric, compared with the use of a single-period accounting framework. However, the longer accounting period can produce both lower and higher inequality than annual measures, depending on the assumed degree of aversion to income fluctuations over time.

The second approach took as its starting point the argument that relative income mobility introduces uncertainty about future incomes, so that it may be desired to evaluate inequality using an *ex-ante* approach. To this end, a regression model of income dynamics was used in order to generate individuals' expected values of future income, conditional on actual income in a specified initial period. The use of expected incomes was found generally to produce higher values of inequality over longer periods, although again comparisons depend on the assumption regarding the aversion to income inequality of the social welfare function, and aversion to income fluctuations on the part of individuals. The results were strongly influenced by the observed high degree of systematic regression toward the (geometric) mean, combined with a large extent of random proportional income changes. The distinction between expected and unexpected mobility was thus found to be important.

In the choice of welfare metric there is of course no single "correct" approach, and the contribution of the economist is to investigate the implications of adopting alternative value judgments. The present paper is therefore in this spirit of extending the range of value judgments which can be examined.

[17]However, if individuals were not indifferent to the timing of risk, then introducing discounting would lead to a preference for early resolution of risk.

References

Aaberge, R. and M. Mogstad, "On the Measurement of Long-Term Income Inequality and Income Mobility," IZA Discussion Paper No. 4699, Institute for the Study of Labor, Bonn, January 2010.
Aaberge, R., M. Mogstad, and V. Peragine, "Measuring Long-Term Inequality of Opportunity," *Journal of Public Economics*, 95, 193–204, 2011.
Abowd, J. M. and D. Card, "On the Covariance Structure of Earnings and Hours Changes," *Econometrica*, 57, 411–45, 1989.
Alstadsæter, A. and E. Fjærli, "Neutral Taxation of Shareholder Income? Corporate Responses to an Announced Dividend Tax," *International Tax and Public Finance*, 16, 571–604, 2009.
Anderson, T. W. and C. Hsiao, "Estimation of Dynamic Models with Error Components," *Journal of the American Statistical Association*, 76, 589–606, 1981.
Arellano, M. and S. Bond, "Some Tests of Specification for Panel Data: Monte Carlo Evidence and an Application to Employment Equations," *Review of Economic Studies*, 58, 277–97, 1991.
Arellano, M. and O. Bover, "Another Look at the Instrumental-Variable Estimation of Error-Components Models," *Journal of Econometrics*, 68, 29–51, 1995.
Atkinson, A. B., "On the Measurement of Inequality," *Journal of Economic Theory*, 2, 244–63, 1970.
———, "The Measurement of Economic Mobility," in P. J. Eggelshaven and L. J. van Gemerden (eds), *Inkommens verdeling en Openbare Financien*, Het Spectrum, Leiden, 1981 (reprinted in A. B. Atkinson, *Social Justice and Public Policy*, Harvester Wheatsheaf, New York, 61–75, 1983).
Atkinson, A. B., F. Bourguignon, and C. Morrisson, "Empirical Studies of Earnings Mobility," in J. Lesourne and H. Sonnenschein (eds), *Fundamentals of Pure and Applied Economics*, Vol. 52, Harwood Academic Publishers, Philadelphia, PA, 1992.
Baker, M., "Growth Rate Heterogeneity and the Covariance Structure of Life-Cycle Earnings," *Journal of Labor Economics*, 15, 338–75, 1997.
Baker, M. and G. Solon, "Earnings Dynamics and Inequality among Canadian Men, 1976–1992: Evidence from Longitudinal Income Tax Records," *Journal of Labor Economics*, 21, 289–321, 2003.
Bénabou, R. J. and E. A. Ok, "Mobility as Progressivity: Ranking Income Processes according to Equality of Opportunity," NBER Working Paper No. 8431, National Bureau of Economic Research, Cambridge, MA, August 2001.
Ben-Shahar, D. and E. Sulganik, "Partial Ordering of Unpredictable Mobility with Welfare Implications," *Economica*, 75, 592–604, 2008.
Blundell, R. and S. Bond, "Initial Conditions and Moment Restrictions in Dynamic Panel Data Models," *Journal of Econometrics*, 87, 115–43, 1998.
Browning, M., M. Ejrnaes, and J. Alvarez, "Modelling Income Processes with Lots of Heterogeneity," *Review of Economic Studies*, 77, 1353–81, 2010.
Carroll, C. D. and A. A. Samwick, "The Nature of Precautionary Wealth," *Journal of Monetary Economics*, 40, 41–71, 1997.
Chakravarty, S. R., B. Dutta, and J. A. Weymark, "Ethical Indices of Income Mobility," *Social Choice and Welfare*, 2, 1–21, 1985.
Creedy, J., *Dynamics of Income Distribution*, Basil Blackwell, Oxford, 1985.
———, "Inequality, Mobility and Income Distribution Comparisons," *Fiscal Studies,* 18, 293–302, 1997a.
———, "Lifetime Inequality and Tax Progressivity with Alternative Income Concepts," *Review of Income and Wealth*, 43, 283–95, 1997b.
Creedy, J. and M. Wilhelm, "Income Mobility, Inequality and Social Welfare," *Australian Economic Papers*, 41, 140–50, 2002.
Dardanoni, V., "Measuring Social Mobility," *Journal of Economic Theory*, 61, 372–94, 1993.
Fields, G. S., "Does Income Inequality Equalize Longer-Term Incomes? New Measures of an Old Concept," *Journal of Economic Inequality*, 8, 409–27, 2010.
Fields, G. S. and E. A. Ok, "The Measurement of Income Mobility: An Introduction to the Literature," in J. Silber (ed.), *Handbook of Inequality Measurement*, Kluwer Academic Publishers, Norwell, MA, 557–96, 1999.
Gottschalk, P. and E. Spolaore, "On the Evaluation of Economic Mobility," *Review of Economic Studies*, 69, 191–208, 2002.
Hungerford, T. L., "How Income Mobility Affects Income Inequality: US Evidence in the 1980s and the 1990s," *Journal of Income Distribution*, 20, 83–103, 2011.
Jarvis, S. and S. P. Jenkins, "How Much Income Mobility is There in Britain?" *Economic Journal*, 108, 428–43, 1998.
Jenkins, S. P. and P. Van Kerm, "Trends in Income Inequality, Pro-Poor Income Growth, and Income Mobility," *Oxford Economic Papers*, 58, 531–48, 2006.

King, M. A., "An Index of Inequality: With Applications to Horizontal Inequity and Social Mobility," *Econometrica*, 51, 99–115, 1983.
Kiviet, J. F., "On Bias, Inconsistency, and Efficiency of Various Estimators in Dynamic Panel Data Models," *Journal of Econometrics*, 68, 53–78, 1995.
Lambert, P. J. and T. O. Thoresen, "Base Independence in the Analysis of Tax Policy Effects: With an Application to Norway 1992–2004," *International Tax and Public Finance*, 16, 219–52, 2009.
Lillard, L. A. and Y. Weiss, "Components of Variation in Panel Earnings Data: American Scientists, 1960–70," *Econometrica*, 47, 437–54, 1979.
Maasoumi, E., "On Mobility," in D. E. Giles and A. Ullah (eds), *The Handbook of Applied Economic Statistics*, Marcel Dekker, New York, 119–76, 1998.
MaCurdy, T. E., "The Use of Time Series Processes to Model the Error Structure of Earnings in Longitudinal Data Analysis," *Journal of Econometrics*, 18, 83–114, 1982.
Nordhaus, W. D., "The Effects of Inflation on the Distribution of Economic Welfare," *Journal of Money, Credit and Banking*, 5, 465–508, 1973.
Parker, S. C. and J. Rougier, "Measuring Social Mobility as Unpredictability," *Economica*, 68, 63–76, 2001.
Ramos, X., "The Covariance Structure of Earnings in Great Britain, 1991–1999," *Economica*, 70, 353–74, 2003.
Ruiz-Castillo, J., "The Measurement of Structural and Exchange Income Mobility," *Journal of Economic Inequality*, 2, 219–28, 2004.
Shorrocks, A. F., "Income Inequality and Income Mobility," *Journal of Economic Theory*, 19, 376–93, 1978a.
———, "The Measurement of Mobility," *Econometrica*, 46, 1013–24, 1978b.
Statistics Norway, "Income Statistics for Persons and Families" (http://www.ssb.no/english/subjects/05/01/inntpf_en/), 2006.
Sørensen, P. B., "Neutral Taxation and Shareholder Income," *International Tax and Public Finance*, 12, 777–801, 2005.
Thoresen, T. O. and A. Alstadsæter, "Shifts in Organizational Form under a Dual Income Tax System," *FinanzArchiv/Public Finance Analysis*, 66, 384–418, 2010.
Van Kerm, P., "What Lies Behind Income Mobility? Reranking and Distributional Change in Belgium, Western Germany and the USA," *Economica*, 71, 223–39, 2004.

Supporting Information

Additional Supporting Information may be found in the online version of this article:

Appendix: Expected Income