

A GENERALIZED EXACTLY ADDITIVE DECOMPOSITION OF AGGREGATE LABOR PRODUCTIVITY GROWTH

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Aggregate labor productivity (ALP) growth—i.e., growth of output per unit of labor—may be decomposed into additive contributions due to *within-sector productivity growth effect*, *dynamic structural reallocation effect* (Baumol effect), and *static structural reallocation effect* (Denison effect) of cross-sectional components (e.g., industry or region) of output and labor. This paper implements ALP growth decomposition that is “generalized” to output in constant prices and to output in chained prices (i.e., chained volume measure or CVM) and “exactly additive” since with either output the sum of contributions exactly equals “actual” ALP growth. It compares this “generalized exactly additive” decomposition (GEAD) to the “traditional” (TRAD) ALP growth decomposition devised for output in constant prices. The results show GEAD and TRAD are exactly additive when output is in constant prices, but GEAD is exactly additive while TRAD is not when output is in CVM. Also, GEAD components are *empirically purer* than or *analytically superior* to those from TRAD. Moreover, considering that contributions to ALP growth can be classified by industry or region *each year over many years*, GEAD provides a more well-grounded picture over time of industrial or regional transformation than TRAD. Therefore, GEAD should replace TRAD in practice.

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1. INTRODUCTION

Aggregate labor productivity (ALP) growth or growth of output per unit of labor is a major factor in achieving the economy’s overall goals such as improving living standards, by increasing incomes, as well as enhancing market competitiveness, by improving efficiency. Thus, ALP growth analysis may be of interest to both technical researchers and policy makers.

However, this paper focuses on methodological issues of ALP growth decomposition in current practice and, hence, may be of interest mainly to technical practitioners. The objective is to compare two different ALP growth decomposition procedures in practice to determine which one is “better” empirically and analytically.

To pursue the above objective, this paper applies the ALP growth decomposition originally devised by Tang and Wang (2004) for output in chained volume

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measure (CVM) in Canada and the U.S. to output in constant prices in other countries. In this paper, the Tang–Wang ALP growth decomposition is called the “generalized exactly additive” decomposition (GEAD) since it is “generalized” here from CVM to output in constant prices and “exactly additive” because with either output measure the sum of contributions exactly equals “actual” ALP growth. For comparison, this paper applies the “traditional” (TRAD) ALP growth decomposition originally devised for output in constant prices (Denison, 1962) to output in CVM.

Section 2 of this paper presents GEAD and Section 3 presents TRAD. Section 4 compares them empirically and analytically. Three empirical illustrations show the differences between GEAD and TRAD. They cover current practices in measuring real output: in constant prices (Thailand) using *fixed-base* Laspeyres quantity and Paasche price indexes; in CVM (U.S.) based on *chained* Fisher quantity and Fisher price indexes; and also in CVM (Italy) based on *chained* Laspeyres quantity and Paasche price indexes. The results show GEAD and TRAD are exactly additive when output is in constant prices but GEAD is exactly additive while TRAD is not when output is in CVM. Moreover, GEAD yields empirically purer or analytically superior components than TRAD for measuring *within-sector productivity growth effect*, *dynamic structural reallocation effect* (Baumol effect), and *static structural reallocation effect* (Denison effect)—regardless of the measure of real output and of the behavior of relative prices.¹ However, GEAD and TRAD are identical when relative prices are constant. But considering that relative prices do change in reality, the findings support the recommendation in the concluding Section 5 that GEAD replace TRAD in practice. This recommendation motivates this paper given the widespread application and persistent use of TRAD in ALP growth decomposition, for example, in recent studies by ADB (2010) and IMF (2006).

2. “GEAD” DECOMPOSITION OF ALP GROWTH

Let the economy be subdivided into N cross-sections, e.g., industry sectors or regions, each indexed by j . Also, let Y_t be the economy’s *nominal* output (in current prices); and X_t be *real* output either in constant prices or in CVM. Output is net of intermediate inputs, and thus could be GNP, GDP, or value-added.² Moreover, let L_t be the economy’s labor employment which could be hours worked or number of “full-time equivalent” persons. Their corresponding sectoral values are given by Y_t^j , X_t^j , and L_t^j over a period, $t = 1, 2, \dots, T$.

Nominal output as well as labor are additive. Hence,

$$(2.1) \quad Y_t = \sum_j Y_t^j; L_t = \sum_j L_t^j; j = 1, 2, \dots, N; t = 1, 2, \dots, T.$$

However, additivity of real output is not necessary. The following analysis is valid either when X_t and X_t^j are in constant prices so that additivity holds $X_t = \sum_j X_t^j$

¹The names of the above effects in italics are adopted from the study by ADB (2010).

²ALP growth decomposition involves cross-sections of industries or regions. Hence, outputs should be net of intermediate inputs—which are inter-industry or inter-regional transactions—to avoid double counting.

or when they are CVMs, and hence non-additive $X_t \neq \sum_j X_t^j$ in present practice (Ehemann *et al.*, 2002; Schreyer, 2004).

For analytical purposes, define the following ratios:

$$(2.2) \quad X_t = \frac{Y_t}{P_t}; X_t^j = \frac{Y_t^j}{P_t^j}; p_t^j = \frac{P_t^j}{P_t}; Z_t = \frac{X_t}{L_t}; Z_t^j = \frac{X_t^j}{L_t^j}; l_t^j = \frac{L_t^j}{L_t}.$$

P_t^j and P_t are output price deflators. Their ratio p_t^j reflects relative price differences between a sector and the entire economy. Moreover, Z_t is *aggregate* labor productivity (ALP) while Z_t^j is *sectoral* labor productivity and l_t^j is a sector's share of total labor.

It follows from (2.1) and (2.2) that $Y_t = \sum_j Y_t^j = \sum_j P_t^j X_t^j$. Therefore, the relationship between Z_t and Z_t^j in any two adjoining periods t and $t - 1$ may be expressed as,

$$(2.3) \quad Z_t = \sum_j \frac{P_t^j}{P_t} \frac{L_t^j}{L_t} \frac{X_t^j}{L_t^j} = \sum_j p_t^j l_t^j Z_t^j; Z_{t-1} = \sum_j p_{t-1}^j l_{t-1}^j Z_{t-1}^j.$$

It may be emphasized that (2.3) is *generally valid* because $X_t = Y_t/P_t$ and $X_t^j = Y_t^j/P_t^j$ are true by definition of real output as a deflated value whatever the formula for the deflators P_t and P_t^j .³

Let G_t be the growth rate of Z_t and G_t^j be the growth rate of Z_t^j . That is,

$$(2.4) \quad G_t = \frac{Z_t - Z_{t-1}}{Z_{t-1}}; G_t^j = \frac{Z_t^j - Z_{t-1}^j}{Z_{t-1}^j}.$$

Mathematically, the aggregate growth rate G_t may be decomposed into the contributions of the sectoral growth rates G_t^j . However, it is important to recognize that there is no unique way for this decomposition. This being the case, this paper opts for the decomposition procedure that suits its objective to compare two different ALP growth decomposition procedures—namely, the TRAD and GEAD decompositions—to determine which one is “better” empirically and analytically. In effect, the choice of decomposition is in line with the analytic formulation implemented in current practice, specifically in the studies under examination.⁴

³In (2.3), aggregate and sectoral deflators should have the same base or reference period but they may differ in functional form. However, in the special case of constant prices, they are all *fixed-base* Paasche price indexes. But for CVMs, they could be *chained* Paasche, Fisher, or other price index formulas.

⁴Note in (2.3) that Z_t and Z_{t-1} are sums where each term is a product of three factors: sectoral relative price times sectoral labor share times sectoral labor productivity. The ratio Z_t/Z_{t-1} is itself the index of *aggregate* labor productivity change. In this case, Balk (2003) provides a method for decomposing Z_t/Z_{t-1} symmetrically into the product of three *sectoral* indexes, namely, an index of relative price change, an index of labor reallocation, and an index of labor productivity change. The decomposition is symmetric with respect to *time*, in contrast to this paper's decomposition in (2.4)—solved in more detail later by (2.7)—which is asymmetric in this regard. While (2.7) was chosen to permit comparisons with existing decompositions examined in this paper (e.g., TRAD), the author is grateful to an anonymous referee for apprising him and interested readers of symmetric decompositions in Balk (2003) that might be enlightening for other research purposes.

The TRAD decomposition has a long history since Denison (1962, 1967) but is still in use, for example by Bloom *et al.* (1999), Dekle and Vandenbroucke (2006), IMF (2006), and ADB (2010). Fortunately for the purposes of this paper, both the TRAD and the Tang–Wang procedures decompose ALP growth into “three” distinct terms—presented in detail later—where the corresponding terms in the two procedures measure specific growth “effects” now recognized in the literature following the terminology by Nordhaus (2002). Moreover, this paper’s GEAD generalization of the original Tang–Wang procedure preserves the above decomposition into three distinct terms. The same three-term decomposition is employed by the more recent applications (IMF, 2006; ADB, 2010) of the TRAD procedure. Thus, the GEAD and TRAD decompositions of ALP growth presented below are comparable term by term empirically and analytically. Indeed, this paper shows that for each term, GEAD is better than TRAD in both senses, and therefore recommends that GEAD replace TRAD in practice.

To proceed with the GEAD decomposition of G_t into the contributions from G_t^j , substitute (2.3) into (2.4); add and subtract $p_t^j l_t^j Z_{t-1}^j$ to the result and then use G_t^j in (2.4) to obtain,

$$(2.5) \quad G_t = \sum_j \frac{Z_{t-1}^j}{Z_{t-1}} [p_t^j l_t^j G_t^j + (p_t^j l_t^j - p_{t-1}^j l_{t-1}^j)].$$

In expanding (2.5), use the fact from (2.1) and (2.2) that,

$$(2.6) \quad \frac{Z_{t-1}^j}{Z_{t-1}} p_{t-1}^j l_{t-1}^j = \frac{Y_{t-1}^j}{Y_{t-1}}.$$

Notice that (2.6) is a sector’s share of output in current prices. Using this share and adding and subtracting $\sum_j (Z_{t-1}^j / Z_{t-1}) p_{t-1}^j l_{t-1}^j G_t^j$ to (2.5) yields the GEAD ALP growth decomposition,

$$(2.7) \quad G_t = \sum_j \left[\frac{Y_{t-1}^j}{Y_{t-1}} G_t^j + \frac{Z_{t-1}^j}{Z_{t-1}} (p_t^j l_t^j - p_{t-1}^j l_{t-1}^j) G_t^j + \frac{Z_{t-1}^j}{Z_{t-1}} (p_t^j l_t^j - p_{t-1}^j l_{t-1}^j) \right].$$

Except for differences in notation, (2.7) is the decomposition formula devised by Tang and Wang (2004) for ALP growth in Canada and the U.S. where real outputs are CVMs.⁵ However, as shown later, (2.7) is also applicable to output in constant prices.

The first term in (2.7), $\sum_j (Y_{t-1}^j / Y_{t-1}) G_t^j$, is similar to the *pure productivity growth effect* (Nordhaus, 2002). To see why, suppose there are no changes in relative prices and in labor shares so that $(p_t^j l_t^j - p_{t-1}^j l_{t-1}^j) = 0$. In this case, the second

⁵A decomposition using similar data was also devised by Reinsdorf and Yuskavage (2010). However, they decomposed “value added” ALP growth indirectly, i.e., based on GDP by using gross output but netting out the contributions of intermediate inputs. Thus, the Tang and Wang ALP growth decomposition is more “direct” and requires “less” data than Reinsdorf and Yuskavage’s decomposition. Moreover, the former decomposition applies to the “arithmetic” rate of ALP growth while the latter applies to the “log-change” of ALP growth.

and third terms equal zero, and thus the first term measures the contribution to ALP growth from productivity growth *alone* of individual sectors, without interaction effects captured by the second and third terms.

However, in the presence of changes in relative prices and in labor shares, the second term, $\sum_j (Z_{t-1}^j / Z_{t-1})(p_t^j l_t^j - p_{t-1}^j l_{t-1}^j) G_t^j$, is non-zero. This is like the *Baumol effect* (Nordhaus, 2002) based on the finding (Baumol, 1967; Baumol *et al.*, 1985) that resources could be absorbed predominantly by stagnant industries. This is possible since industries with a low value of $(Z_{t-1}^j / Z_{t-1}) G_t^j$, i.e., stagnant, could have a high value of $(p_t^j l_t^j - p_{t-1}^j l_{t-1}^j)$.

Moreover, with the above changes, the third term, $\sum_j (Z_{t-1}^j / Z_{t-1})(p_t^j l_t^j - p_{t-1}^j l_{t-1}^j)$, is also non-zero. This term is similar to the *Denison effect* (Nordhaus, 2002) after Denison (1962) who pointed out that movement of resources from a low-productivity industry to a high-productivity industry could raise ALP growth even if the productivity growth rates of the two industries were the same. To illustrate, suppose there are two industries *a* and *b* with the same productivity growth rates or $G_t^a = G_t^b$ but *a* has a higher productivity, i.e., $Z_{t-1}^a > Z_{t-1}^b$. In this case, the third term above yields the Denison effect that the ALP growth rate G_t may rise if resources (e.g., labor) move from *b* to *a* when $(p_t^a l_t^a - p_{t-1}^a l_{t-1}^a) > (p_t^b l_t^b - p_{t-1}^b l_{t-1}^b)$.

3. “TRAD” DECOMPOSITION OF ALP GROWTH

Let X_t^* stand for aggregate real output and X_t^{*j} for sectoral real output measured in constant prices. In this case, the sums in (2.1) still hold but the ratios in (2.2) become,

$$(3.1) \quad X_t^* = \frac{Y_t}{P_t^*}; X_t^{*j} = \frac{Y_t^j}{P_t^{*j}}; p_t^{*j} = \frac{P_t^{*j}}{P_t^*}; Z_t^* = \frac{X_t^*}{L_t}; Z_t^{*j} = \frac{X_t^{*j}}{L_t^j}; l_t^j = \frac{L_t^j}{L_t}.$$

Under constant prices, additivity of real output holds. That is,

$$(3.2) \quad X_t^* = \sum_j X_t^{*j}.$$

Using (3.2), let overall ALP level and ALP growth be Z_t^* and G_t^* . For a sector, let labor productivity level and growth be Z_t^{*j} and G_t^{*j} . Hence, (2.3) and (2.4) become,

$$(3.3) \quad Z_t^* = \sum_j \frac{L_t^j}{L_t} \frac{X_t^{*j}}{L_t^j} = \sum_j l_t^j Z_t^{*j}; Z_{t-1}^* = \sum_j l_{t-1}^j Z_{t-1}^{*j},$$

$$(3.4) \quad G_t^* = \frac{Z_t^* - Z_{t-1}^*}{Z_{t-1}^*}; G_t^{*j} = \frac{Z_t^{*j} - Z_{t-1}^{*j}}{Z_{t-1}^{*j}}.$$

Combining (3.1) to (3.4) yields the TRAD ALP growth decomposition given by,

$$(3.5) \quad G_t^* = \sum_j \left[\frac{Y_{t-1}^j P_{t-1}^*}{Y_{t-1} P_{t-1}^*} G_t^{*j} + \frac{Z_{t-1}^{*j}}{Z_{t-1}^*} (l_t^j - l_{t-1}^j) G_t^{*j} + \frac{Z_{t-1}^{*j}}{Z_{t-1}^*} (l_t^j - l_{t-1}^j) \right].$$

Formula (3.5) can be shown to be equivalent to the formulas in IMF (2006, p. 98), and in ADB (2010 p. 5), although they look different. In the ADB study, the first term of (3.5) is the *within-sector productivity growth effect* (WSPGE); the second term is the *dynamic structural reallocation effect* (DSRE); and the third term is the *static structural reallocation effect* (SSRE). Recalling the names for similar “effects” by Nordhaus (2002) in the GEAD ALP growth decomposition in (2.7), WSPGE corresponds to *pure productivity growth effect*; DSRE corresponds to *Baumol effect*; and SSRE corresponds to *Denison effect*.

4. COMPARING “GEAD” AND “TRAD”

To compare GEAD in (2.7) and TRAD in (3.5), these ALP growth decomposition formulas are applied to the agriculture sectors in Thailand where real output is GNP in constant prices; in the U.S. where it is value-added in CVM; and in Italy where it is also value-added in CVM but based on different chained indexes than in the U.S. These applications were chosen for convenience, given that the output and employment data for the agriculture sector in each country are disaggregated into only two subsectors, which will suffice for illustrative purposes.

Table 1 presents GNP and employment data (2008–09) in the agriculture sector of Thailand. The “actual” ALP growth of 1.3788 percent in 2009 is decomposed in Table 2.

The results from (2.7) are in the columns under GEAD while those from (3.5) are in the columns under TRAD in Table 2. This table shows that the GEAD and TRAD components could be different in size and in sign. For example, WSPGE by GEAD is positive, 0.4222, while that by TRAD is negative, -0.1821 . Moreover,

TABLE 1
AGRICULTURE SECTOR GNP AND EMPLOYMENT IN THAILAND, 2008–09

	GNP in Current Prices		GNP in Constant Prices		Employed Persons	
	(Million Baht)		(Million 1988 Baht)		(Thousand)	
	2008	2009	2008	2009	2008	2009
Agriculture, hunting, and forestry	955,710.0	931,907.0	320,058.0	322,342.0	14,283.3	14,228.3
Fishing	94,033.0	104,679.0	65,167.0	68,020.0	415.9	464.2
Total	1,049,743.0	1,036,586.0	385,225.0	390,362.0	14,699.1	14,692.5

Source: Data on GNP in current prices and in constant prices are from the National Economic and Social Development Board, Office of the Prime Minister. Data on employed persons are from the Report of the Labor Force Survey, National Statistical Office, Ministry of Information and Communication Technology.

TABLE 2
DECOMPOSITION OF THAI AGRICULTURE SECTOR LABOR PRODUCTIVITY GROWTH OF 1.3788 PERCENT
IN 2009

	WSPGE		DSRE		SSRE		Total	
	(Pure Productivity Growth Effect)		(Baumol Effect)		(Denison Effect)		(Labor Productivity Growth)	
	GEAD	TRAD	GEAD	TRAD	GEAD	TRAD	GEAD	TRAD
Agriculture, hunting, and forestry	1.0035	0.9158	-0.0099	-0.0031	-0.8948	-0.2823	0.0989	0.6303
Fishing	-0.5813	-1.0979	-0.1292	-0.1281	1.9905	1.9745	1.2800	0.7485
Total	0.4222	-0.1821	-0.1390	-0.1313	1.0957	1.6922	1.3788	1.3788

Source: Author's calculations based on procedures (noted below) applied to data in Table 1.

Note: WSPGE is *within-sector productivity growth effect*; DSRE is *dynamic structural reallocation effect*; and SSRE is *static structural reallocation effect*. These terms are used in the ADB study (October 2010) and these terms correspond to *pure productivity growth effect*, *Baumol effect*, and *Denison effect* (Nordhaus, 2002). This paper's adaptation of the Tang and Wang (2004) decomposition formula in (2.7) yields the results reported in the columns headed by GEAD while ADB's (2010) and IMF's (2006) decomposition formulas, which are equivalent to (3.5), yield the results reported in the columns headed by TRAD.

GEAD and TRAD yield slightly different DSREs although these are both negative. Finally, while the SSREs are positive, GEAD yields 1.0957 while TRAD yields a larger value, 1.6922.

If output is in constant prices, the last two columns of Table 2 show that GEAD and TRAD are exact by yielding the "actual" overall labor productivity growth of 1.3788 percent. However, the components are different. "Agriculture, hunting and forestry" contributed only 0.0989 from GEAD but a larger 0.6303 from TRAD. "Fishing" contributed 1.2800 according to GEAD but a much smaller 0.7485 contribution according to TRAD.

Considering that the decompositions in Table 2 can be done *each year over many years*, the above results show that GEAD and TRAD could paint very different pictures of the economy's industrial transformation. In turn, GEAD and TRAD will have different implications for policy. Therefore, choosing one over the other needs to be analytically well-grounded.

The empirical differences between GEAD and TRAD components in Table 2 may be explained analytically by the way prices are incorporated in their respective formulas. To see this, substitute (2.2) and (2.4) into (2.7) as well as (3.1) to (3.4) into (3.5) to obtain,

$$(4.1) \quad G_t = \sum_j \left[\frac{Y_{t-1}^j}{Y_{t-1}} G_t^j + \frac{Z_t^j}{Z_{t-1}} (p_t^j l_t^j - p_{t-1}^j l_{t-1}^j) \right];$$

$$(4.2) \quad G_t^* = \sum_j \left[\frac{Y_{t-1}^j}{Y_{t-1}} \frac{P_{t-1}^*}{P_{t-1}^{*j}} G_t^{*j} + \frac{Z_t^{*j}}{Z_{t-1}^*} (l_t^j - l_{t-1}^j) \right].$$

The first terms of GEAD in (4.1) and TRAD in (4.2) measure WSPGE as before in (2.7) and (3.5). However, DSRE and SSRE are now combined in the second terms of (4.1) and (4.2).

GEAD's WSPGE given by the first term of (4.1) involves only sectoral deflators in the real growth term G_t^j , i.e., no overall deflator is involved. But TRAD's WSPGE given by the first term of (4.2) involves sectoral deflators in the real growth term G_t^{*j} and the overall deflator P_{t-1}^* . The presence of P_{t-1}^* implies that TRAD's WSPGE is not purely a "within sector" measure. In contrast, all deflators in GEAD's WSPGE are sector j 's own and, thus, a purely "within sector" measure. Thus, GEAD yields an *empirically purer* WSPGE than TRAD.

To show the relative empirical purity of the GEAD measure of WSPGE over that by TRAD, consider the following situation. Suppose that output is measured in constant prices as in Table 1. In this case, (4.1) and (4.2) yield $G_t^j = G_t^{*j}$ and $G_t = G_t^*$, as borne out by the ALP growth decomposition results in Table 2. However, since in this case relative prices are not constant—i.e., sectoral deflators differ from each other and also differ from the aggregate deflator—then $P_{t-1}^* \neq P_{t-1}^{*j}$.⁶ Therefore, the above results together with the definitions in (3.1) imply that the TRAD measure of WSPGE in (4.2) may be rewritten as,

$$(4.3) \quad \frac{Y_{t-1}^j}{Y_{t-1}} \frac{P_{t-1}^*}{P_{t-1}^{*j}} G_t^{*j} = \frac{Y_{t-1}^j / P_{t-1}^{*j}}{Y_{t-1} / P_{t-1}^*} = \frac{X_{t-1}^{*j}}{P_{t-1}^*} G_t^j.$$

Thus, when output is measured in constant prices, TRAD uses *real* shares (X_{t-1}^{*j} / X_{t-1}^*) while GEAD uses *nominal* shares (Y_{t-1}^j / Y_{t-1}) as weights to determine the sectoral ALP growth contribution to the WSPGE component of aggregate ALP growth. However, the use of real shares based on constant prices from an *outdated* fixed base period tends to *overestimate* the growth contribution of sectors with *falling* output prices since in this case $X_{t-1}^{*j} / X_{t-1}^* \geq Y_{t-1}^j / Y_{t-1}$ is likely. The reason is that in (4.3), P_{t-1}^{*j} and P_{t-1}^* are Paasche deflators with a fixed base period, for example b , that could be in the distant past relative to period $t - 1$. Thus, for a fast growing sector with falling output prices, the nominal share at $t - 1$ could be lower than the real share based on higher output prices in the old base period b .⁷ The converse holds that the use of real shares above tends to *underestimate* the growth contribution of sectors with *rising* output prices since in this case $X_{t-1}^{*j} / X_{t-1}^* \leq Y_{t-1}^j / Y_{t-1}$ is also likely. Thus, the GEAD measure of WSPGE is relatively free of the above upward and downward biases—due to real shares with outdated fixed base periods—that afflict the TRAD measure of WSPGE in the first term of (4.2).

Turning away from the above adverse effects of real shares, the use of nominal shares as weights to compute WSPGE in the first term of (4.1) illustrates the wisdom of the original Tang and Wang (2004) ALP growth decomposition under-

⁶Relative prices are not constant since deflators changed from 2008 to 2009. In Thailand (Table 1), for example, the deflator for "fishing" changed from $94,033/65,167 = 1.4430$ in 2008 to $104,679/68,020 = 1.5389$ in 2009. It can be verified that relative prices are also not constant in the U.S. (Table 3) and in Italy (Table 5).

⁷Indeed, this situation was a major motivation for the U.S. shift from GDP in constant dollars to GDP in chained dollars so as not to overestimate the growth contribution of the then relatively fast-growing *information and computer technology* sector (Landefeld and Parker, 1997).

lying GEAD by *isolating* in the second term of (4.1) the combined effects of changes in relative prices and in labor shares. In principle, this isolation separates ALP growth into pure efficiency and induced reallocation effects that may be visualized in a production–welfare theoretic framework as follows.

For simplicity, imagine an original equilibrium in a two-good case defined by the tangency between a (production) transformation curve and a (social welfare) indifference curve. Assume a technology improvement that shifts outward the transformation curve. Holding relative prices constant at the original equilibrium, WSPGE is the “pure efficiency” growth effect corresponding to the movement to a point on the new transformation curve. However, this point is not necessarily the new equilibrium unless it coincides with the new tangency between the higher transformation and also higher indifference curves. If they are not coincident, then the old relative prices will change to the relative prices in the new tangency, thus inducing labor reallocation or change in labor shares. These combined effects of changes in relative prices and labor shares are the “induced reallocation” growth effects measured by the combined DSRE and SSRE in the second term of GEAD in (4.1).

While GEAD in (4.1) recognizes the role of relative prices in reallocation by the presence of the price index ratios, p_{t-1}^j and p_t^j , TRAD in (4.2) gives no reallocation role to relative prices by the absence of these ratios. If labor shares are constant, the second term of TRAD equals zero, implying no reallocation effects. In contrast, the second term of GEAD could still be non-zero given that relative prices change, implying non-zero reallocation effects. This is possible because changes in relative prices could change the output mix, and hence induce reallocation effects from more intensive use of other inputs (e.g., capital equipments) even though labor may be immobile so that labor shares are constant. These considerations make GEAD *analytically superior* to TRAD in measuring DSRE and SSRE when relative prices change.

If relative prices are constant, all price indexes are equal to a positive constant α . Hence, $P_t^j = P_t = P_t^{*j} = P_t^* = \alpha$ so that $p_t^j = P_t^j / P_t = p_t^* = P_t^{*j} / P_t^* = 1$, all (j, t) . Therefore, GEAD in (2.7) or (4.1) and TRAD in (3.5) or (4.2) become identical. But if relative prices are not constant *over time* while real output is measured in constant prices of a *fixed* base year (Table 1), GEAD and TRAD are equal but not identical (Table 2). That is, GEAD and TRAD components add up to the same (i.e., equal) “actual” overall ALP growth but the corresponding components are different. However, GEAD is superior to TRAD, as shown above.

For another contrasting feature, GEAD is exact but TRAD is not when real output is measured in CVM. For illustration, these formulas were applied to value added and FTE employment data (2008–09) in the agriculture sector of the U.S. (Table 3).

In Table 4, the sum of GEAD components exactly equals “actual” 2009 U.S. agricultural sector labor productivity growth of 7.6529 percent. In contrast, the sum of TRAD components is 7.6349 percent, which is different. This is not surprising because TRAD applies only to real output in constant prices but U.S. real output is a CVM based on the chained Fisher quantity–Fisher price index framework. In this case, given that TRAD is “inexact” while GEAD is “exact,” the latter is analytically superior to the former.

TABLE 3
AGRICULTURE SECTOR VALUE ADDED AND EMPLOYMENT IN THE US, 2008–09

	Value Added in Current Prices		Value Added in CVM		Full-Time Equivalent	
	(Million Dollars)		(Million Chained 2005 Dollars)		(Thousand)	
	2008	2009	2008	2009	2008	2009
Farms	131,142.0	103,964.0	102,346.0	108,544.0	627.0	634.0
Forestry, fishing, and related activities	28,989.0	29,174.0	26,219.0	26,830.0	456.0	425.0
Total	160,131.0	133,138.0	129,366.0	136,180.0	1,083.0	1,059.0

Source: Data on value added in current prices and in chained prices (CVM) and FTE employment are from the US Bureau of Economic Analysis. Note that value added in CVM is not additive. Thus, in CVM valued in chained 2005 dollars, the sum of the value added of farms, forestry, fishing, and related activities is not equal to the sector total value added in US agriculture in 2008 and in 2009 above.

TABLE 4
DECOMPOSITION OF US AGRICULTURE SECTOR LABOR PRODUCTIVITY GROWTH OF 7.6529 PERCENT IN 2009

	WSPGE		DSRE		SSRE		Total	
	(Pure Productivity Growth Effect)		(Baumol Effect)		(Denison Effect)		(Labor Productivity Growth)	
	GEAD	TRAD	GEAD	TRAD	GEAD	TRAD	GEAD	TRAD
Farms	4.0006	3.8647	-0.0854	0.1317	-1.7486	2.6962	2.1666	6.6926
Forestry, fishing, and related activities	1.7731	1.9851	0.3312	-0.0930	3.3819	-0.9497	5.4862	0.9423
Total	5.7737	5.8497	0.2458	0.0387	1.6333	1.7465	7.6529	7.6349

Source: Author's calculations based on procedures (noted below) applied to data in Table 3.

Note: WSPGE is *within-sector productivity growth effect*; DSRE is *dynamic structural reallocation effect*; and SSRE is *static structural reallocation effect*. These terms are used in the ADB study (October 2010) and these terms correspond to *pure productivity growth effect*, *Baumol effect*, and *Denison effect* (Nordhaus, 2002). This paper's adaptation of the Tang and Wang (2004) decomposition formula in (2.7) yields the results reported in the columns headed by GEAD while ADB's (2010) and IMF's (2006) decomposition formulas, which are equivalent to (3.5), yield the results reported in the columns headed by TRAD.

For another illustration of the exactness of GEAD but inexactness of TRAD in ALP growth decomposition, consider Tables 5 and 6 for the agriculture sector of Italy where real output is also a CVM based on the chained Laspeyres quantity–Paasche price index framework adopted by EU countries (Schreyer, 2004).

In similar manner to Table 4, Table 6 shows that the sum of GEAD components exactly equals “actual” 2009 Italian agricultural sector labor productivity growth of 1.3160 percent. Moreover, GEAD components differ from TRAD components and the latter sum to 1.1461 percent, which is different from the above actual labor productivity growth.

TABLE 5
AGRICULTURE SECTOR VALUE ADDED AND EMPLOYMENT IN ITALY, 2008–09

	Value Added in Current Prices		Value Added in CVM		Full-Time Equivalent	
	(Million Euros)		(Million Chained 2000 Euros)		(Thousand)	
	2008	2009	2008	2009	2008	2009
Agriculture, hunting, and forestry	27,313.5	24,536.7	28,447.9	27,663.0	454.6	435.8
Fishing	1,203.6	1,349.0	759.3	817.8	33.4	34.7
Total	28,517.1	25,885.6	29,052.0	28,378.7	488.0	470.5

Source: Data on value added in current prices and in chained prices (CVM) and FTE employment are from Istat - Istituto Nazionale di Statistica. Note that value added in CVM is not additive. Thus, in CVM valued in chained 2000 euros, the sum of the value added of agriculture, hunting and forestry and fishing is not equal to the sector total value added in Italian agriculture in 2008 and in 2009 above.

TABLE 6
DECOMPOSITION OF ITALIAN AGRICULTURE SECTOR LABOR PRODUCTIVITY GROWTH OF 1.3160
PERCENT IN 2009

	WSPGE		DSRE		SSRE		Total	
	(Pure Productivity Growth Effect)		(Baumol Effect)		(Denison Effect)		(Labor Productivity Growth)	
	GEAD	TRAD	GEAD	TRAD	GEAD	TRAD	GEAD	TRAD
Agriculture, hunting, and forestry	1.3752	1.4060	-0.0158	-0.0080	-1.1026	-0.5580	0.2568	0.8400
Fishing	0.1550	0.0960	0.0320	0.0074	0.8721	0.2027	1.0592	0.3062
Total	1.5303	1.5020	0.0162	-0.0006	-0.2305	-0.3553	1.3160	1.1461

Source: Author's calculations based on procedures (noted below) applied to data in Table 5.

Note: WSPGE is *within-sector productivity growth effect*; DSRE is *dynamic structural reallocation effect*; and SSRE is *static structural reallocation effect*. These terms are used in the ADB study (October 2010) and these terms correspond to *pure productivity growth effect*, *Baumol effect*, and *Denison effect* (Nordhaus, 2002). This paper's adaptation of the Tang and Wang (2004) decomposition formula in (2.7) yields the results reported in the columns headed by GEAD while ADB's (2010) and IMF's (2006) decomposition formulas, which are equivalent to (3.5), yield the results reported in the columns headed by TRAD.

Tables 4 and 6 complete the illustration of the exactness of GEAD and the inexactness of TRAD in ALP growth decomposition in the CVM framework in current practice.⁸

⁸Presumably, the same results will follow from the CVM framework based on chained Paasche quantity and Laspeyres price indexes. However, no country appears to have adopted this framework. The reason could be that fixed-base Laspeyres quantity and Paasche price indexes underlie real output in constant prices. Thus, it seems natural or logical for countries converting output from constant prices to CVM—like the countries in the EU—to adopt the chained Laspeyres quantity and Paasche price indexes.

5. CONCLUSION

This paper showed that in ALP growth decomposition, GEAD is exact if real (net) output—e.g., GNP, GDP, or value-added—is measured either in constant prices or in CVM. In contrast, TRAD is exact only if real output is in constant prices. In the latter case, with changing relative prices, GEAD and TRAD are both exact but their components (i.e., WSPGE, DSRE, and SSRE) are different. However, the components from GEAD were shown empirically purer than or analytically superior to those from TRAD. On the above grounds, considering that the contributions to ALP growth can be classified by industry or region *each year over many years*, GEAD provides a more analytically well-grounded picture over time of the economy's industrial or regional transformation than TRAD. The overall implication is that GEAD should replace TRAD in practice. This finding motivates this paper given the widespread application and persistent use of TRAD in ALP growth decomposition, for example, in recent studies by ADB (2010) and IMF (2006).

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