the review of income and wealth

Review of Income and Wealth
Series 58, Number 3, September 2012
DOI: 10.1111/j.1475-4991.2011.00482.x

ON THE "PRO-POORNESS" OF GROWTH IN A MULTIDIMENSIONAL CONTEXT

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This paper represents a first attempt to bring together the issues of multidimensional poverty and growth "pro-poorness" assessments. More specifically, we suggest the use of sequential dominance procedures to test the "pro-poorness" of observed growth spells when poverty is measured on the basis of income and another discrete well-being attribute. Sequential procedures are also used to obtain graphical tools that are consistent with the spirit of Ravallion and Chen's growth incidence curve and Son's poverty growth curve. Contrary to traditional unidimensional tests, our method makes it possible to take into account the importance of deprivation correlations at the individual level and thus may reverse results observed with the traditional tools used to check the "pro-poorness" of growth. An illustration of our approach is given using Turkish data for the period 2003–05.

JEL Codes: I32, C00

Keywords: pro-poor growth, growth incidence curve, sequential stochastic dominance

1. Introduction

The definition of the Millennium Development Goals in 2000 by the international community was a major break from the previous paradigm of the Washington consensus and its implicit reference to "trickle down" theories. One remarkable feature was the rehabilitation of Chenery *et al.*'s (1974) advocacy in favor of introducing redistributive concerns into growth policies in the developing world.[1] Indeed, since the late 1990s, many social researchers have forcefully argued in favor of assigning only an instrumental role to growth in respect to poverty issues. In other words, poverty alleviation should not be regarded as a desirable side effect of growth, but the ultimate goal to be reached in the spirit of former World Bank President Robert McNamara's desire to shift the focus toward targeted poverty reduction. However, how best this goal can be met is an open and

Note: The paper is part of the project PROPOORSUDS financed by the Agence Nationale de la Recherche. We would like to thank Conchita D'Ambrosio, Lars Osberg, and the two anonymous referees for their helpful comments. All remaining errors are our own. All estimations have been realized with R (R Development Core Team, 2010). The corresponding scripts are available upon request.

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[1]Today, such policies are generally called inclusive growth policies.

Published by Blackwell Publishing, 9600 Garsington Road, Oxford OX4 2DQ, UK and 350 Main St, Malden, MA, 02148, USA.

complex question, and researchers have spent a great deal of time in looking for lessons from the empirics of growth and poverty. In particular, many authors have tried to address the issue of the identification of "pro-poor" growth spells, i.e. growth spells that correspond to a marked improvement regarding the state of poverty. The nature of such a bias in favor of the poor has entailed many debates, notably concerning the desirabilty of observing a poverty alleviation effect of inequality reduction to tag a growth pattern as "pro-poor" (Kakwani and Pernia, 2000; Ravallion, 2004; Zepeda, 2004; Osmani, 2005), but the theoretical framework to be used for empirical assessment is now well defined (Duclos, 2009).

At the same time, the commitment of the international community to the achievement of the eight Millennium Development Goals has also been an official recognition of the multidimensional nature of poverty. It is well known (see, for instance, Sen, 1987, 1992; Streeten, 1994) that the linkages between income (or expenditure) and well-being are not straightforward and hinge on many determinants like idiosyncratic characteristics or market factors. As a result, the efficiency of poverty reducing policies should also be assessed on the basis of the satisfaction of non-income needs like health, education, or participation in social life. If poverty has to be reflected upon and measured using a multidimensional approach, it is then necessary to examine the "pro-poor" nature of growth beyond the sole monetary aspects of poverty. The recent abundant literature on these two concepts has evolved in a parallel way but, surprisingly, very few attempts have been carried out in order to include the additional information associated with other dimensions of well-being alongside the monetary one within the assessment of the "pro-poor" nature of growth. To our knowledge, the only studies that deal with this issue are Klasen (2008) and Grosse *et al.* (2008), who suggest making use of the tools developed for "pro-poor" growth tests to investigate the distribution of changes with respect to non-income attributes.

The non-income growth incidence curve proposed in these studies allows for widening the scope of "pro-poor" growth analyses and may highlight potential discrepancies between progress in the monetary and non-monetary dimensions. However, these graphical tools only focus on the marginal distributions of well-being attributes, and thus do not take into account the additional information provided by the joint distribution of these attributes. Many authors (Atkinson and Bourguignon, 1982; Bourguignon and Chakravarty, 2002; Tsui, 2002) have stressed the importance of correlations between the distributions of the different attributes in multidimensional poverty measurement. Indeed, if poverty indices are based on individualistic welfare functions that are not separable with respect to the different attributes (Kolm, 1977), poverty may increase or decrease without any changes occurring in the marginal distribution of the attributes if some attributes are substitutes or complements with respect to well-being.

However, the literature on sequential stochastic dominance offers a promising way to address this issue without requiring strong assumptions about how dimensions of poverty should specifically be related, an assumption about which there may not be a wide agreement. Originally, sequential stochastic dominance was introduced in order to address comparisons of income distributions with households of differing composition and size. Though the use of equivalence scales makes it possible to obtain homogeneous distributions of equivalent incomes, it entails several problems as it relies on strong normative assumptions. Consequently, the distributional income comparisons may be highly sensitive to the choice of the equivalence scale. Sequential stochastic dominance techniques (Bourguignon, 1989; Atkinson, 1992; Jenkins and Lambert, 1993; Chambaz and Maurin, 1998; Duclos and Makdissi, 2005) address this kind of issue, since they highlight the conditions to be met so that the results of poverty comparisons are robust as regards the choice of this equivalence scale. As noted in recent studies by the likes of Duclos and Échevin (2009), the method can naturally be extended to cases where household size can be replaced by non-income poverty dimensions.

In the present paper, we go a step further and, using this sequential stochastic dominance framework, suggest a new way of testing the "pro-poor" nature of growth for poverty measures based on both income and other characteristics that can be summed up using any ordinal index. It is worth noting that the proposed dominance criteria are related to classes of poverty measures complying with axioms upon which it is reasonable to think that agreement should be met unambiguously. Our different propositions rely in particular on two crucial assumptions. The first one is that the income poverty line may vary with the level reached by the ordinal variable and can be set to zero above some values for that index. Thus, the approach is compatible with different rival approaches of poverty identification. The second assumption is that the marginal contribution of income to well-being decreases with the level of the non-income attribute. Using this minimal set of assumptions, we define criteria that are robust with respect to choices in both the value of the poverty lines and the functional form of the bidimensional poverty measure when performing "pro-poor" growth checks.

The paper is organized as follows. In Section 2, we describe the traditional tools used for "pro-poor" growth tests; we extend their use in Section 3 to multidimensional approaches to poverty using sequential stochastic dominance criteria. In Section 4, the method is illustrated by the case of Turkey, taking into account, alongside the income dimension, the educational level reached by each individual. Section 5 concludes.

2. "Pro-Poorness" with Unidimensional Poverty

As stated earlier, we first begin with a general overview of the traditional definition of growth "pro-poorness" as well as of the tools used in empirical studies. Our aim is mostly to introduce notations and the general intuition of growth "pro-poorness" checks so as to highlight the linkage between these procedures and the one proposed in Section 3.

Let $y_i \in \mathbb{R}_+$ be the level of some monetary variable, like income or expenditure, for the i-th person of a given population of size $n \in \mathbb{N}^*$.[2] The distribution of income among the population can then be described by the n-vector $\boldsymbol{y} := \{y_1, \ldots, y_n\}$. In order to ease the comparisons between distributions of different sizes, it is often preferable to use the univariate cumulative distribution function (*cdf*) $F(z; \boldsymbol{y})$. The *cdf* returns the probability $p \in [0, 1]$ of picking out of $\boldsymbol{y}$ an income

[2]For the sake of simplicity, we will consider that y denotes the income level, but this choice does not preclude using any other concepts that would be relevant in the assessment of monetary poverty.

whose value is less than the threshold z. It is worth noting that, in the context of poverty analysis, the *cdf* corresponds to the widely used poverty measure known as the headcount index.

In the present section, monetary poverty is first assessed using the following class Π_1 of additive poverty measures:

$$\Theta(\boldsymbol{y}, z) := \int_0^z \theta(y, z)\, dF(y; \boldsymbol{y}) \tag{1}$$

with $\theta(z, z) = 0$, $\partial\theta/\partial z \geq 0$, $\partial\theta/\partial y \leq 0$ if $y < z$, $\partial\theta/\partial y = 0$ if $y \geq z$ so that the measure complies with the traditional axioms of focus, weak monotonicity, continuity, anonymity, population, non-decreasingness with respect to the poverty line, and subgroup additivity.[3] This class of subgroup additive poverty measures (Foster and Shorrocks, 1991) is very general and includes the most widely used poverty measures like the one suggested by Watts (1968) and Foster *et al.* (1984).[4] Here we would like to stress the particular importance of the anonymity axiom that states that income is the sole relevant variable to be used to make a distinction between people for poverty analysis. In equation (1), the adherence to the anonymity axiom then entails that the individual poverty function θ is the same for each individual. In a sense, this crucial assumption will be partially slackened in Section 3 with the inclusion of additional information regarding individual attributes for the estimation of poverty at the individual level.

As stressed in the literature (Kakwani and Pernia, 2000; Ravallion and Chen, 2003; Kraay, 2006), whether an observed growth pattern is "pro-poor" or not crucially depends on the social evaluator's definition of what a "pro-poor" growth may be. In particular, it relies on the way any additional income should be shared between the different members of the population so as to obtain a growth pattern that is neither "pro-poor" nor "anti-poor," but ethically "neutral." In the present paper, the proposed method is orthogonal with respect to this specific point as it is consistent with many definitions of a "neutral" growth pattern. Consequently, we will refer here to the general definition of "pro-poor" growth proposed by Duclos (2009). For Duclos, the assessment of the "pro-poorness" of growth between years t and $t + 1$ always implies the comparison of the poverty level in $t + 1$ with the level that would have been observed for some counterfactual distribution defined by $\boldsymbol{y}_t$ and some real-valued function γ that relates to the social evaluator's definition of "pro-poorness." In other words, growth is deemed "pro-poor" with respect to some given poverty measure Θ and some poverty line z if and only if:

$$\Theta(\boldsymbol{y}_{t+1}, z) - \Theta(\gamma(\boldsymbol{y}_t), z) \leq 0. \tag{2}$$

Duclos (2009) argues that the definition of γ may be determined by ethical, statistical, or administrative arguments, and that this diversity explains the heterogeneity of feelings with respect to what could be a "pro-poor" growth. As noted by Kakwani and Son (2008), empirical investigations generally focus on three rival

[3]See Zheng (1997) for a comprehensive review of the axiomatic framework used for unidimensional poverty analysis.

[4]In the first case, the individual poverty function is defined by $\theta(x, z) := \log x - \log z$. With Foster *et al.*'s (1984) class of poverty measures, the function becomes $\theta(x, z) := (1 - x/z)^\alpha$, $\alpha \geq 0$.

definitions of γ, hereafter called the "poverty reducing," the "relative," and the "absolute" approaches of "pro-poor" growth. In the first case, growth is deemed "pro-poor" if poverty has decreased over the corresponding period, so that $\gamma^p(\boldsymbol{y}_t) := \boldsymbol{y}_t$. On the contrary, with the two remaining approaches it is supposed that growth should be associated with a decrease in inequality for the benefit of the poor, in order to observe "pro-poor" growth. With the "relative" approach (Baulch and McCulloch, 1998; Kakwani and Pernia, 2000), income inequalities are considered in relative terms and the counterfactual distribution is simply $\gamma^r(\boldsymbol{y}_t) := \boldsymbol{y}_t \frac{\mu_{t+1}}{\mu_t}$ where μ_t is the mean value of $\boldsymbol{y}_t$. On the other hand, using the "absolute" view means that income inequalities are considered on the basis of absolute income differences among the population, hence $\gamma^a(\boldsymbol{y}_t) := \boldsymbol{y}_t + (\mu_{t+1} - \mu_t)$. While the first approach does not impose any restriction on Θ, it is important to stress that the use of the "relative" and "absolute" approaches should confine the analysis to poverty measures that are respectively scale-invariant and translation-invariant.[5]

A traditional problem with the criterion defined in equation (2) is that using any other poverty measure Θ or changing the poverty line z may reverse the statement made about the "pro-poorness" of growth over a given period. It is then necessary to assess the robustness of the results using stochastic dominance criteria (Atkinson, 1987; Foster and Shorrocks, 1988).[6] As shown by Duclos (2009), the use of first-order dominance properties for these issues leads to the following result:

Proposition 1. *For a given counterfactual scenario γ and a given maximum value z^+ for the poverty line, the statement that the growth pattern observed between years t and $t+1$ is "pro-poor" is weakly robust with respect to the choice of the poverty measure among the family Π_1 and the value of the poverty line z if and only if:*

$$(3) \qquad F(z; \boldsymbol{y}_{t+1}) - F(z; \gamma(\boldsymbol{y}_t)) \leq 0 \quad \forall z \leq z^+,$$

with at least one value $z^* \in [0, z^+]$ such that:

$$(4) \qquad F(z^*; \boldsymbol{y}_{t+1}) - F(z^*; \gamma(\boldsymbol{y}_t)) < 0.$$

Ravallion and Chen (2003) have also demonstrated that first-order stochastic dominance can also be easily assessed using a single graph of the observed growth rates for each percentile of the population over the corresponding period. Instead of the *cdf*, these authors then prefer working with the quantile function, or Pen's parade, F^{-1} that, using Gastwirth (1971) definition, is simply:

$$(5) \qquad F^{-1}(p; \boldsymbol{y}_t) := \min\{y_{it} \in \boldsymbol{y}_t | F(y_{it}; \boldsymbol{y}_t) \geq p\}.$$

[5]Regarding the family of poverty measures defined in equation (1), Θ is scale-invariant if and only if $\theta(\lambda y, \lambda z) = \theta(y, z)$, $\forall \lambda \in \mathbb{R}_{++}$, and translation-invariant if and only if $\theta(y + \varepsilon, z + \varepsilon) = \theta(y, z)$, $\forall \varepsilon \in \mathbb{R}$. On this issue, see notably Bresson and Labar (2007).

[6]We should be cautious with the use of the term "robustness" since this type of robustness check does not take into account the issues of sampling errors.

Using equation (5), the growth incidence curve (GIC) proposed by Ravallion and Chen (2003) is obtained by plotting for each $p \in [0, 1]$ the value of the function:[7]

$$g_1(p; \boldsymbol{y}_t, \boldsymbol{y}_{t+1}) := \frac{F^{-1}(p; \boldsymbol{y}_{t+1})}{F^{-1}(p; \boldsymbol{y}_t)} - 1. \tag{6}$$

Then, the evaluation of "pro-poor" growth can be performed by comparing the function $g_1(p; \boldsymbol{y}_t, \boldsymbol{y}_{t+1})$, that is the observed growth rate at each quantile p, with the function $g_1(p; \boldsymbol{y}_t, \gamma(\boldsymbol{y}_t))$, that is the counterfactual growth rate at each quantile, up to the percentile that corresponds to the highest admissible value z^+ for the poverty line, i.e. $F(z^+, \boldsymbol{y}_t)$. The following corollary sums up this idea in a concise formal manner:

Corollary 1. *For a given criterion γ and a given maximum value z^+ for the poverty line, the statement that the growth pattern observed between years t and $t + 1$ is "pro-poor" is weakly robust with respect to the choices of the poverty measure among the family Π_1 and the value of the poverty line z if and only if:*

$$g_1(p; \boldsymbol{y}_t, \boldsymbol{y}_{t+1}) - g_1(p; \boldsymbol{y}_t, \gamma(\boldsymbol{y}_t)) \geq 0 \quad \forall p \in [0, F(z^+, \boldsymbol{y}_t)], \tag{7}$$

with at least one value $p^* \in [0, F(z^+; \boldsymbol{y}_t)]$ such that:

$$g_1(p^*; \boldsymbol{y}_t, \boldsymbol{y}_{t+1}) - g_1(p^*; \boldsymbol{y}_t, \gamma(\boldsymbol{y}_t)) > 0. \tag{8}$$

Proposition 1 and Corollary 1 are very interesting since they mean that the condition (2) is fulfilled for all poverty measures from the class defined by equation (1) and all poverty lines in the range $[0, z^+]$ (Duclos, 2009). As a consequence, it is very robust from an ethical point of view since it requires minimal agreement for the assessment of "pro-poor" growth for a given benchmark scenario γ.

At this point, it is worth noting that most researchers prefer using GIC than *cdf* when checking for growth "pro-poorness." Indeed GICs are appealing because it seems quite intuitive to compare growth rates, and the distribution of economic gains can easily be compared to the overall performance of the economy. Moreover, they provide an elegant picture of the evolution of the relative distribution of income for a given growth spell. However, GICs suffer from limitations that deserve to be noted. In particular, the use of population quantiles conceals the depth of poverty for those at the lowest quantile of the income distribution since the first centile of the distribution may be either close or very far from the poverty line with very different consequences in terms of poverty evaluation. On the contrary, as the *cdf* is plotted for each value of income, it is easier to infer the extent of poverty changes. In a sense, GICs are more informative about poverty changes for rank-based poverty measures like Sen's index, while the use of *cdf* comparisons is more consistent with the analysis of poverty for subgroup additive poverty measures like Foster, Greer, and Thorbecke's family of poverty measures.

[7]It is worth noting that the idea of performing welfare comparisons on the basis of the quantile functions is not new and can be traced back at least to Mahalanobis (1960).

Despite the simplicity and usefulness of Proposition 1 and Corollary 1, it is well known that first-order dominance tests are likely to be inconclusive in a significant number of cases. It is then necessary to add further restrictions to the type of poverty measures used for "pro-poor" growth assessments and to turn to higher-order stochastic dominance conditions. For instance, if the class of poverty measures defined in equation (1) is restricted to indices that respect $\partial^2\theta/\partial y^2 \geq 0$, we obtain the class of poverty measures Π_2 that is contained within Π_1 and complies with the weak transfer axiom (Sen, 1976). According to the weak transfer axiom, an income loss for a poor individual does not raise poverty if it is at least compensated by an increase of the same amount for a poorer person. Robustness tests based on this axiom are then more powerful than first order stochastic conditions as they do not require income improvement at each quantile of the population during the corresponding period. More specifically, second order dominance tests require the use of the poverty gap function G such that:

$$G(z;\boldsymbol{y}) := \int_0^z (z-y)\,dF(y;\boldsymbol{y}). \tag{9}$$

This function simply returns the average shortfall with respect to the poverty line z given the income distribution $\boldsymbol{y}$. The relationship between growth "pro-poorness" and the class of poverty measures Π_2 is then summarized by the following proposition:

Proposition 2. *For a given counterfactual scenario* γ *and a given maximum value* z^+ *for the poverty line, the statement that the growth pattern observed between years* t *and* $t+1$ *is "pro-poor" is weakly robust with respect to the choice of the poverty measure among the family* Π_2 *and the value of the poverty line* z *if and only if:*

$$G(z;\boldsymbol{y}_{t+1}) - G(z;\gamma(\boldsymbol{y}_t)) \leq 0 \quad \forall z \leq z^+, \tag{10}$$

with at least one value $z^* \in [0, z^+]$ *such that:*

$$G(z^*;\boldsymbol{y}_{t+1}) - G(z^*;\gamma(\boldsymbol{y}_t)) < 0. \tag{11}$$

A test for growth "pro-poorness" proposed by Son (2004) and related to the class of poverty measures Π_2 is based on the poverty growth curve (PGC) that plots the growth rate of the mean income of the bottom $100p$ percent of the population with individuals ranked by increasing order of income. More formally, the PGC is defined by:

$$g_2(p;\boldsymbol{y}_t,\boldsymbol{y}_{t+1}) := \int_0^p \frac{F^{-1}(u;\boldsymbol{y}_{t+1})}{F^{-1}(u;\boldsymbol{y}_t)} - 1\,du. \tag{12}$$

It can easily be checked that the comparison of the observed PGC with the one corresponding to the counterfactual distribution yields a criterion that is equivalent to the one presented in Proposition 2.[8]

[8]On the power of the PGC approach for "pro-poorness" tests, see Davis (2007).

Corollary 2. *For a given criterion* γ *and a given maximum value* z^+ *for the poverty line, the statement that the growth pattern observed between years* t *and* $t+1$ *is "pro-poor" is weakly robust with respect to the choices of the poverty measure among the family* Π_2 *and the value of the poverty line* z *if and only if:*

$$g_2(p;\boldsymbol{y}_t,\boldsymbol{y}_{t+1})-g_2(p;\boldsymbol{y}_t,\gamma(\boldsymbol{y}_t))\geq 0 \quad \forall p\in[0,F(z^+;\boldsymbol{y}_t)], \tag{13}$$

with at least one value $p^* \in [0, F(z^+; \boldsymbol{y}_t)]$ *such that:*

$$g_2(p;\boldsymbol{y}_t,\boldsymbol{y}_{t+1})-g_2(p;\boldsymbol{y}_t,\gamma(\boldsymbol{y}_t))>0. \tag{14}$$

3. "Pro-Poorness" with Multidimensional Poverty

The previous section reviewed the conditions that have to be met in order to obtain a judgment that is not likely to be contingent on choices for the functional form of the poverty measure or for the value of the poverty line. However, the results depend on the crucial assumption made in the previous section that poverty should be considered only in monetary terms. Yet, most researchers agree that other dimensions of poverty like education, health, access to public services, or real freedoms should be taken into account for the analysis of poverty. The inclusion of such elements logically modifies the definition of poverty indices and induces a change in the scope of the anonymity axiom.[9] Moreover, in most cases, it may entail that the marginal contribution of income to poverty is determinated by the level of the other dimensions chosen to assess poverty.

In order to take into account this aspect, we propose to make use of the tools of sequential stochastic dominance. Originally, this technique was developed in order to assess robust comparisons of income distributions when households differ in needs. Although differences in households' needs are most of the time based on differences in their size or composition in the studies using the framework of sequential stochastic dominance, other characteristics that are of interest for social welfare analysis or multidimensional approaches to poverty can also be taken into account as proxies of needs.

For this purpose, we will consider that the additional information to be included in the poverty measure can be summed up by the variable x, though our framework can easily be extended so as to use more additional variables. In many situations, the satisfaction of non-monetary needs cannot be assessed by continuous variables. For instance, the health status is generally assessed using some ordered categorical variable. In the same spirit, the education level is often measured using the number of years of schooling. As a consequence, we assume that the variable x is discrete and takes $K \in \mathbb{N}^* \backslash 1$ values that can be ordered in the following manner $x^1 \leq x^2 \leq \ldots x^K$. This index may have a cardinal content, but,

[9]However, it is worth stressing, as noted by Kolm (1977), that the inclusion of additional information to assess individuals' well-being may ease the agreement on the principle of equal treatment of equals.

for our present purpose, we are only interested in its ordinal properties.[10] We also assume that the well-being of the *i*-th person increases with the value of the index x. The distribution of this variable in the population is then described by the n-vector $\boldsymbol{x} := \{x_1, \ldots, x_n\}$ whose elements x_i are ordered in the same manner as in the income vector $\boldsymbol{y}$. Let $\boldsymbol{X}$ be the $n \times 2$-matrix obtained by placing the vectors $\boldsymbol{y}$ and $\boldsymbol{x}$ side by side. Then the *i*-th line of the matrix summarizes all the relevant characteristics of person *i*, that is his or her income and the level of his or her non-monetary attribute.

Considering many attributes for the analysis of poverty also implies a change in the definition of the poverty domain. In the multidimensional poverty measurement literature, many rival definitions have been suggested (Bourguignon and Chakravarty, 2002; Duclos *et al*., 2006; Alkire and Foster, 2007). In this section, we will consider a very general definition of the poverty domain that is close to the one used in Duclos *et al*. (2006) by assuming that that the income poverty line is a non-increasing function of the value of the index x_i.

For instance, let us consider income y and health x as the relevant dimensions for poverty analysis, and two poor individuals A and B with the same income $y_A = y_B < \tilde{z}$, but different levels of health. More specifically, suppose that individual B does not suffer from any deprivation with respect to health while individual A has a disability, i.e. $x_A < x_B$. Indeed, this health deficiency implies that A is poorer than B since he or she suffers from deprivation in this dimension, but we may go a step further and consider that person A's disability also has consequences in the income dimension. The disability generates specific outlays (long-term medical treatment, prostheses, etc.) and increases the cost of other outlays like transport. As a consequence, we may conclude that a general income poverty line $\tilde{z}$, defined with respect to the needs of a healthy person, is inappropriate for person A as its income level cannot yield the same consumption level as it can for individual B. While A and B share the same income level, we may then consider that A suffers from a larger degree of deprivation than B in the income dimension. Thus, we should observe $z(x_A = x^k) > z(x_B = x^{k+s})$, $k \in \{1, \ldots, K-1\}$, $s \in \{1, \ldots, K-k\}$. In order to save space, let z_k denote the value of income poverty line for those individuals whose value of x_i is equal to x^k. Income deprivations are then assessed using the K-vector $\boldsymbol{z} := \{z_1, \ldots, z_K\}$ such that $z_1 \geq z_2 \ldots \geq z_K \geq 0$.

It is important to stress that for our analysis we do not need to explicitly specify a poverty line for the non-income dimension since all the relevant information concerning the shape of the poverty domain is included in the vector $\boldsymbol{z}$. For instance, the traditional "intersection" approach of poverty identification consists of tagging a person as poor if she is deprived with respect to both the income and the non-income attribute. It then is obtained for $z_k = c \in \mathbb{R}_{++}$, $k \in \{1, \ldots, j\}$, and $z_k = 0$, $k \in \{j+1, \ldots, K\}$, $1 \leq j \leq K$. On the other hand, a "union" approach—an

[10]In empirical applications of the propositions suggested in the present section, we may face some difficulties in ranking some categories of individuals. A first solution is to gather together categories whose ranking is not straightforward. A more robust solution proposed by Atkinson (1992) is to use the sequential criteria for all relevant orderings of the categories used for x. For instance, let the members of the population be of types x^a, x^b, and x^c. Assuming that the individuals of types x^a are the neediest but that x^b and x^c cannot easily be compared, it would then be necessary to perform our tests for $x^a < x^b < x^c$ and $x^a < x^c < x^b$.

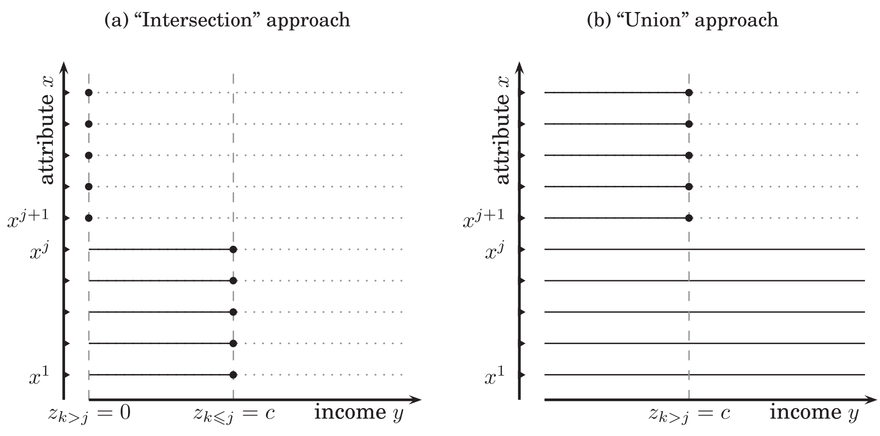

Figure 1. The Definition of the Poverty Domain under Different Rival Approaches

individual is then regarded as poor if she is deprived with respect to at least one attribute—can be used by imposing $z_k = +\infty$, $k \in \{1, \ldots, j\}$, and $z_k = c$, $k \in \{j + 1, \ldots, K\}$, $1 \leq j \leq K$. These two cases are illustrated in Figure 1 where the poverty domain is depicted each time by the set of horizontal thick lines. Of course, it is still possible to restrict the analysis to a sole monetary view of poverty identification if $z_k = c$, $k \in \{1, \ldots, K\}$. The procedure described above will then still prove to be useful since it gives the focus on the more fragile part of the population (considering the non-monetary attribute) when analyzing income poverty.

We now present the main results of the paper and show how "pro-poor" growth can be robustly assessed using sequential dominance procedures that mirror the first- and second-order stochastic dominance conditions expressed in the previous section.[11]

3.1. *First-Order Stochastic Sequential Dominance and Sequential Growth Incidence Curves*

The theoretical developments in this section are very close to the one developed in Gravel and Moyes (2008) and Duclos and Échevin (2009), the main difference being that the final distribution is compared to a counterfactual distribution that is not necessarily the joint distribution observed at the beginning of the growth spell. We define $F_k(y; \boldsymbol{y})$ as the income *cdf* of those individuals whose level of x_i is equal to x^k. Poverty is then assessed using the following class of additive poverty measures:

$$\Theta(\boldsymbol{X}_t, \boldsymbol{z}) := \sum_{k=1}^{K} q_k(\boldsymbol{x}_t) \int_0^{z_k} \theta(y, x, z_k)\, dF_k(y; \boldsymbol{y}_t), \tag{15}$$

[11]In the present paper, we do not explore higher-order sequential dominance conditions. However, our results can easily be extended to third-order stochastic dominance tests using Lambert and Ramos (2002) results.

with $q_k(\boldsymbol{x}_t)$, $\sum_{k=1}^{K} q_k(\boldsymbol{x}_t) = 1$, being the share of the population belonging to the group of individuals whose level of the non-monetary attribute is equal to x^k. Here, we consider the class of poverty measures $\bar{\Pi}_1$ such that θ in equation (15) satisfies the following properties: $\theta(z_k, x^k, z_k) = 0$, $\partial\theta/\partial z_k \geq 0$, $\partial\theta/\partial y \leq 0$ if $y < z_k$, $\partial\theta/\partial y = 0$ if $y \geq z_k$. Thus, as in the case of monetary poverty, the indices are supposed to comply with the multidimensional counterparts of the focus, weak monotonicity, continuity, anonymity, population, non-decreasingness with respect to the poverty line, and subgroup additivity axioms.[12] Moreover, it is also assumed that:

$$(16) \qquad \frac{\partial}{\partial y_i}\theta(y_i, x^k, z_k) \leq \frac{\partial}{\partial y_i}\theta(y_i, x^{k+1}, z_{k+1}), \quad \forall k \in \{1, \ldots, K-1\}.$$

It can easily be seen that $\Pi_1 \subset \bar{\Pi}_1$ since this class of unidimensional poverty measures is obtained for $z_k = z \ \forall k \in \{1, \ldots, K\}$ and when the value of $\partial\theta/\partial y$ does not vary with the level of x_i. Finally, the condition expressed in equation (16) is standard in the literature on multidimensional inequalities and poverty (Atkinson and Bourguignon, 1982; Tsui, 2002) and is related to the axiom known as the non-decreasingness under correlation switches. This axiom stipulates that, given two individuals with endowments (y_A, x_A) and (y_B, x_B), a permutation of the values of these two vectors so that A can be said to be unambiguously poorer than B, should not lower the poverty level, all other things being equal. In other words, the social evaluator is supposed to be averse to the accumulation of severe deprivations for a limited number of individuals.

We now turn to the issue of "pro-poorness" evaluation. In the previous section, the assessment of the "pro-poorness" of growth was performed on the basis of a counterfactual income distribution $\gamma(\boldsymbol{y}_t)$. This definition of "pro-poorness" is consistent with the income-based approach of poverty but may not be appropriate when other attributes are taken into account. Indeed, in the context of our setting, we are concerned with the evolution of the whole matrix $\boldsymbol{X}$, so that growth between the years t and $t + 1$ will be deemed "pro-poor" for a given counterfactual benchmark Γ, a given poverty measure Θ, and a given set of poverty lines $\boldsymbol{z}$ if and only if:

$$(17) \qquad \Theta(\boldsymbol{X}_{t+1}, \boldsymbol{z}) - \Theta(\Gamma(\boldsymbol{X}_t), \boldsymbol{z}) \leq 0.$$

The main difference with the definition corresponding to equation (2) consists of the definition of the counterfactual scenario that gives more latitude for the social evaluator. Indeed, as we may observe simultaneous variations of the vectors $\boldsymbol{y}$ and $\boldsymbol{x}$, it is then necessary to ask whether the evaluation should be performed on the basis of a counterfactual distribution for the distribution of the index x. We then have to distinguish the situations in which the counterfactual matrix $\Gamma(\boldsymbol{X}_t)$ is obtained from $\boldsymbol{X}_t$ by simply changing its income vector $\boldsymbol{y}_t$, and cases in which the non-income vector $\boldsymbol{x}_t$ is not necessarily left unchanged. Let the first situation be called the "income pro-poorness" of growth and the second one "well-being

[12]For a comprehensive review of the axioms used for multidimensional poverty measurement, see Bresson (2009).

pro-poorness" of growth. To avoid confusion, let γ_y and γ_x respectively denote the functions used to define the counterfactual distributions of the income and non-income variables.

The case of "well-being pro-poorness" warrants some interest because the counterfactual distribution $\gamma_x(\boldsymbol{x}_t)$ of the non-income item may be slightly more complex than the one corresponding to individual incomes. The most important question is whether $\gamma_x(\boldsymbol{x}_t)$ should be exogeneously or endogeneously defined with respect to the observed growth pattern.

In the former case, we may choose to define $\gamma_x(\boldsymbol{x}_t)$ using the initial and final distributions $\boldsymbol{x}_t$ and $\boldsymbol{x}_{t+1}$, and dissociate it from observed changes in the income dimension. It is worth emphasizing that, due to the particular nature of the variable x, the choice of γ_x is obviously more complicated than with income. Hence the relative and absolute counterfactual functions γ^r and γ^a cannot be used with our general setting as the variable x is ordinal—it would not make sense for instance to apply a given growth rate to qualitative data. In fact, this criticism prevails for all conceptions of the counterfactual scenario that rely on a distributional neutral approach to growth and uses mean-based definitions of inequality. With respect to this issue, a promising solution is the use of median-based approaches with inequality being considered in terms of "distance" from the median value (Allison and Foster, 2004).[13] Whatever the chosen procedure, it is important to stress that the nature of the variable used for the index x has also to be taken into account as it may be bounded (Klasen, 2008).

Conversely, we may feel that $\gamma_x(\boldsymbol{x}_t)$ should be computed on the basis of some statistical or theoretical relationship between x and y, and the counterfactual distribution of income $\gamma_y(\boldsymbol{y}_t)$. For instance, it is well known that the income level of an individual is one of the main determinants of his or her health level. If this dimension of well-being is measured with a discrete variable, one may think of using a multinomial choice model to estimate the role of income using the observations at time t. Then the estimated model could be used to construct $\gamma_x(\boldsymbol{x}_t)$ by predicting the health status that should be observed for each individual using the corresponding income level in $\gamma_y(\boldsymbol{y}_t)$. Of course, more complex designs can also be chosen, using for example CGE models with micro-simulation exercises so as to fully take the effects of economic growth into account.

In order to save space, we will now introduce the following notation:

$$\text{(18)} \qquad \Delta^{\gamma}_{t,t+1} F_k(z;\boldsymbol{y}) := q_k(\boldsymbol{x}_{t+1}) F_k(z;\boldsymbol{y}_{t+1}) - q_k(\gamma_x(\boldsymbol{x}_t)) F_k\big(z;\gamma_y(\boldsymbol{y}_t)\big),$$

with $q_k(\boldsymbol{x})$ being the share of observations from $\boldsymbol{x}$ whose values are equal to x^k. The properties of the class of poverty measures $\bar{\Pi}_1$ then lead to the following result:

Proposition 3. *For a given counterfactual scenario* Γ *and a given vector* z^+ *of maximum values for the specific poverty lines, the statement that the growth pattern observed between years* t *and* $t+1$ *is "pro-poor" is weakly robust with respect to the*

[13]Here, the word "distance" does not refer to the traditional Euclidean distance but to the number of categories separating two values of the index x.

choice of a poverty measure from the class $\bar{\Pi}_1$ *and the value of the poverty lines z if and only if:*

$$(19) \qquad \sum_{k=1}^{j} \Delta^{\gamma}_{t,t+1} F_k(z; \boldsymbol{y}) \leq 0 \quad \forall z \leq z_j^+, j \in \{1, \ldots, K\},$$

with at least one integer $j^* \in \{1, \ldots, K\}$ *and one value* $z^* \in [0, z_{j^*}^+]$ *such that:*

$$(20) \qquad \sum_{k=1}^{j^*} \Delta^{\gamma}_{t,t+1} F_k(z^*; \boldsymbol{y}) < 0.$$

In less formal terms, growth then can be said to be "pro-poor" in a multidimensional sense for the observed period if the multidimensional headcount index is not greater for the final distribution than for the counterfactual distribution at each point of the poverty domain. The criterion suggested in Proposition 3 refers to the one first suggested by Bourguignon (1989) and developed by Atkinson (1992), Jenkins and Lambert (1993), and Chambaz and Maurin (1998), but applied to the question of the assessment of "pro-poor" growth. So we are not restricted to comparisons of observed distributions as the initial distribution is replaced by a counterfactual distribution based on this initial distribution. The second difference with respect to these studies is that the heterogeneity of the population is not understood according to the household size, but according to any set of individual characteristics that can be considered as relevant dimensions of poverty.

In the previous section, we pointed out that the conditions that have to be met to conclude in a robust manner whether growth is "pro-poor" could also be expressed with the help of the GIC (cf. Corollary 1). In most cases, such an equivalence cannot be observed with our bidimensional definition of poverty, except when the counterfactual vector $\gamma_x(\boldsymbol{x}_t)$ exhibits the same distribution as the marginal distribution of x observed in year $t + 1$, since the income growth rates cannot yield information about the changes in the marginal distribution of the non-monetary attribute. However, an extension of the GIC that uses the additional information provided by the variable x to discriminate different population subgroups may still prove to be useful for the analysis of "pro-poor" growth so that this particular case may warrant some interest.

For this, we first define partial quantile functions as:

$$(21) \qquad F_k^{-1}(p; \boldsymbol{X}) := \min\{y_{it} \in \boldsymbol{y}_t^k \,|\, F(y_{it}; \boldsymbol{y}_t^k) \geq p\},$$

with $\boldsymbol{y}_t^k$ being the subset of values from $\boldsymbol{y}_t$ corresponding to individuals whose value of the index x is not greater than x^k. The function $F_k^{-1}(p; \boldsymbol{X})$ thus returns the value of income y corresponding to the p-th centile of the subpopulation of types 1 to k ranked by increasing value of income. For $k = K$, this function simply becomes the traditional quantile function presented in equation (5). Using this instrument, we can then propose the use of the following set of sequential growth incidence curves (SGICs) function:

$$g_{1,k}(p; \boldsymbol{X}_t, \boldsymbol{X}_{t+1}) := \frac{F_k^{-1}(p; \boldsymbol{X}_{t+1})}{F_k^{-1}(p; \boldsymbol{X}_t)} - 1, \tag{22}$$

that corresponds to the income growth rate of the p-th percentile of the subpopulation of type 1 to k taking into consideration the non-income attribute. Dominance can then assessed by comparing the values of this function with the corresponding SGICs for the counterfactual distribution $\Gamma(\boldsymbol{X}_t)$ for the bottom part of the population. Our results are then summarized by the following corollary:

Corollary 3. *For a given counterfactual scenario γ and a given vector z^+ of maximum values for the specific poverty lines, the statement that the growth pattern observed between years t and t + 1 is "pro-poor" is weakly robust with respect to the choice of poverty measure among the family $\overline{\Pi}_1$ if and only if:*

$$g_{1,k}(p; \boldsymbol{X}_t, \boldsymbol{X}_{t+1}) - g_{1,k}(p; \boldsymbol{X}_t, \Gamma(\boldsymbol{X}_t)) \geq 0 \quad \forall p \leq F\left(z_k^+, \boldsymbol{y}_t^k\right), \quad k \in \{1, \ldots, K\}, \tag{23}$$

with at least one integer $j^* \in \{1, \ldots, K\}$ and one value $p^* \in \left[0, F\left(z_{j^*}^+, \boldsymbol{y}_t^{j^*}\right)\right]$ such that:

$$g_{1,j^*}(p^*; \boldsymbol{X}_t, \boldsymbol{X}_{t+1}) - g_{1,j^*}(p^*; \boldsymbol{X}_t, \Gamma(\boldsymbol{X}_t)) > 0. \tag{24}$$

The condition suggested by Corollary 3 best suits situations where the distribution of the non-monetary variable x is time-invariant. However, this result can easily be extended to the case of variable distributions of the index x. Indeed it can be shown that a sufficient, but not necessary, condition for growth to be deemed "pro-poor" between t and $t + 1$ given the courterfactual scenario Γ and the set of poverty lines $\boldsymbol{z}^+$ is to comply simultaneously with the conditions expressed in Corollary 3 and:

$$F\left(x^j; \boldsymbol{x}_{t+1}\right) \leq F\left(x^j; \gamma_x(\boldsymbol{x}_t)\right) \quad \forall j \in \{1, \ldots, K\} \text{ such that } z_j > 0. \tag{25}$$

If this condition is not met, the use of SGICs may still be considered as relevant for the analysis of the "pro-poorness" of growth with our multidimensional framework as a complementary tool to the conditions presented in Proposition 3.[14] Since the SGIC dominance criterion assumes perfect equality for the distribution of the non-income attribute between the two distributions to be

[14]When presenting the paper at a conference, it was suggested that we might be able to rely on SGICs in practice when the marginal distributions of x differ only marginally in the counterfactual and the final distributions of x. Though the suggestion may sound reasonable, our experience is that even non-significant differences in the distribution of the non-income attribute may yield conflicting results when comparing those obtained with the SGICs to those obtained with the sequential first order dominance procedure.

Of course, this remark is only from a purely theoretical point of view since empirical applications are generally performed using samples and not observations for the whole population. Taking statistical inference into account, we can admit that we should be confident with results obtained with the SGICs whenever we fail to reject the assumption that the counterfactual distribution $\gamma_x(\boldsymbol{x}_t)$ is equal to the one observed at time $t + 1$ using, for instance, the Kolmogorov–Smirnov test.

compared, i.e. $F(x^j; \boldsymbol{x}_{t+1}) \leq F(x^j; \gamma_x(\boldsymbol{x}_t))\ \forall_j$, it may still be used to decompose the effects in the assessment of the "pro-poor" nature of growth. Indeed, the set of SGICs essentially captures the effects of changes in the conditional distribution of income so that it helps estimate what could be called a pure income growth effect. Consequently, comparing the differences between the sequential first order dominance curves and the SGICs is a simple way of obtaining the effects of changes in the marginal distribution on the non-monetary attribute in our "pro-poor" assessment.

3.2. *Second-Order Stochastic Sequential Dominance and Sequential Property Growth Curves*

As in the unidimensional case, the tests suggested in the previous section may be inconclusive. In order to increase the power of the test, it is then necessary to turn to a reduced set of poverty measures $\bar{\Pi}_2$. Starting with the conditions used to define the class $\bar{\Pi}_1$, we impose the following additional restriction:

$$\text{(26)} \qquad \frac{\partial^2}{\partial y^2}\theta\left(y_i, x^k, z_k\right) \geq \frac{\partial}{\partial y^2}\theta\left(y_i, x^{k+1}, z_{k+1}\right) \geq 0, \quad \forall k \in \{1, \ldots, K-1\}.$$

The condition (26) can be decomposed into two parts. The first part relates to the non-negativity of the second-order derivative of the function θ with respect to income. This non-concavity assumption is well-known in the poverty and inequality literature, and signifies that progressive transfers of income—a transfer is said to be progressive if it reduces inequalities—within the set of individuals with the same value of the attribute x do not raise the poverty level. The second part of condition (26) is the non-increasingness of $\partial^2\theta/\partial y^2$ with respect to the value of x. This assumption indicates that there are diminishing returns from progressive transfers as we move to less needy individuals for given levels of income.[15,16]

Let $G_k(z; \boldsymbol{y})$ denote the value of $G(z; \boldsymbol{y})$ when $F(z; \boldsymbol{y})$ is replaced by $F_k(z; \boldsymbol{y})$ in equation (9). This function indicates the value of the average income gap among individuals of the k-th type for a given income poverty line z. Using the following notation:

$$\text{(27)} \qquad \Delta^{\gamma}_{t,t+1}G_k(z; \boldsymbol{y}) := q_k(\boldsymbol{x}_{t+1})G_k(z; \boldsymbol{y}_{t+1}) - q_k(\gamma_x(\boldsymbol{x}_t))G_k\left(z; \gamma_y(\boldsymbol{y}_t)\right),$$

we obtain the "pro-poorness" condition expressed in Proposition 4 when poverty measures of the class $\bar{\Pi}_2$ are considered.

Proposition 4. *For a given counterfactual scenario* Γ *and a given vector* z^+ *of maximum values for the specific poverty lines, the statement that the growth pattern*

[15]As emphasized in Lambert and Ramos (2002), it is worth mentioning that a class of poverty measures that belongs to $\bar{\Pi}_1$ and includes $\bar{\Pi}_2$ can also be used if only the non-concavity of θ is assumed. It is then necessary to turn to the sequential dominance criterion proposed by Bourguignon (1989) to obtain a robust evaluation of growth "pro-poorness" using this intermediate class of poverty measures.

[16]For a discussion on the generalization of the Pigou–Dalton transfer principle, see in particular Ebert (2000).

observed between years t and t + 1 is "pro-poor" is weakly robust with respect to the choice of a poverty measure among the family $\bar{\Pi}_2$ *if and only if:*

$$\sum_{k=1}^{j} \Delta_{t,t+1}^{\gamma} G_k(z; \boldsymbol{y}) \leq 0 \quad \forall z \leq z_j^+, \, j \in \{1, \ldots, K\}, \tag{28}$$

with at least one integer $j^* \in \{1, \ldots, K\}$ *and one value* $z^* \in [0, z_{j^*}^+]$ *such that:*

$$\sum_{k=1}^{j^*} \Delta_{t,t+1}^{\gamma} G_k(z^*; \boldsymbol{y}) < 0. \tag{29}$$

As in the previous section, it may be interesting to look for an alternative way of expressing Proposition 4 when the marginal distributions $\boldsymbol{x}_{t+1}$ and $\gamma_x(\boldsymbol{x}_t)$ do not differ. A natural extension of the concept of PGCs can be obtained using the set of functions $g_{2,k}$ that returns the growth rate of the mean income of the bottom $100p$ percent of the subpopulation of types 1 to k taking into consideration the non-income attribute, i.e.:

$$g_{2,k}(p; \boldsymbol{X}_t, \boldsymbol{X}_{t+1}) := \int_0^p \frac{F_k^{-1}(u; \boldsymbol{X}_{t+1})}{F_k^{-1}(u; \boldsymbol{X}_t)} - 1 \, du. \tag{30}$$

Corollary 4. *For a given counterfactual scenario* γ *and a given vector* z^+ *of maximum values for the specific poverty lines, the statement that the growth pattern observed between years t and t + 1 is "pro-poor" is weakly robust with respect to the choice of a poverty measure among the family* $\bar{\Pi}_2$ *if and only if:*

$$g_{2,k}(p; \boldsymbol{X}_t, \boldsymbol{X}_{t+1}) - g_{2,k}(p; \boldsymbol{X}_t, \Gamma(\boldsymbol{X}_t)) \geq 0 \quad \forall p \leq F(z_k^+, \boldsymbol{y}_t^k), \, k \in \{1, \ldots, K\}, \tag{31}$$

with at least one integer $j^* \in \{1, \ldots, K\}$ *and one value* $p^* \in [0, F(z_{j^*}^+, \boldsymbol{y}_t^{j^*})]$ *such that:*

$$g_{2,j^*}(p^*; \boldsymbol{X}_t, \boldsymbol{X}_{t+1}) - g_{2,j^*}(p^*; \boldsymbol{X}_t, \Gamma(\boldsymbol{X}_t)) > 0. \tag{32}$$

Let us call sequential poverty growth curves (SPGCs) the graph of the set of functions $g_{2,k}$ against the population quantiles p. Of course the limitations noticed for the use of SGICs apply to SPGCs.

4. Illustration: Testing the Pro-Poorness of Growth in Turkey 2003–05

The proposed method will now be applied using data from the 2003, 2004, and 2005 Turkish household consumption and expenditure surveys provided by the Turkish Statistics Institute (Turkstat). The time span is slightly short for an assessment of the "pro-poorness" of growth, but the purpose of the present exercise is merely illustrative. However, Turkey is an interesting case for an in-depth examination of the "pro-poor nature" of growth. After the 2001 crisis, Turkey entered a period of high growth and structural transformation. Following a rebound in 2001, annual growth rates averaged nearly 7 percent over the years 2003–07. According to international standards, income poverty is low in comparison with other Middle-Eastern and North African countries, but inequalities remain high and are to a large extent driven by high differentials across regions. Moreover, despite improvements in social indicators, education records quite weak levels in comparison with countries with equivalent levels of GDP per capita (Akkoyunlu-Wigley and Wigley, 2008). The country also faces wide income and education gaps between urban and rural areas (World Bank, 2005, 2008), and education seems to hold an important role in understanding discrepancies of development within the country (Duman, 2008).

In order to illustrate the usefulness of our method, poverty is defined here using education alongside the more traditional income component. The income component corresponds to the disposable equivalent individual income adjusted with the OECD equivalence scale. In order to take inflation into account, all incomes are expressed with reference to the 2003 consumer price index provided by Turkstat. Education deprivations are measured on the basis of educational level attainments. Our datasets allow the distinction between the following six categories: (1) illiterate; (2) literate but without schooling; (3) some primary education; (4) primary school; (5) secondary education and occupational education equivalent to secondary school; and (6) higher education. Since children have not achieved their final educational level, the analysis focuses solely on the adult population (more than 20 years old). In line with our framework, we hold the reasonable assumptions that well-being is an increasing function of educational attainments and that any income gain improves well-being the most at low educational levels. Consequently, each sample has been split into six groups of educational levels ranked by decreasing needs with respect to income: for a given income level, illiterate people are thus associated with the highest level of need and highly educated people with the lowest level. Regarding the pro-poorness of growth, the illustrations rely on a very traditional counterfactual scenario, that is a relative approach to "pro-poor" growth for the income dimension $\left(\gamma_y(\boldsymbol{y}_t) = \gamma^r(\boldsymbol{y}_t) = \boldsymbol{y}_t \frac{\mu_{t+1}}{\mu_t}\right)$ while using just the observed changes for education $(\gamma_x(\boldsymbol{x}_t) = \boldsymbol{x}_t)$.

In Section 3, we mentioned that the classes of bidimensional poverty measures used for "pro-poorness" checks imply the definition of different monetary poverty lines for each value of the non-monetary attribute. For the sake of simplicity, we define a general income poverty line expressed as some percentage of the median income as usually carried out in poverty analyses in OECD countries, and consider that this poverty line is appropriate for the least deprived group regarding education. More specifically, as stochastic dominance tests are notably designed to assess the robustness of poverty comparisons to the level of the poverty line, we have opted for a very conservative maximum value of this income poverty line z_6^+, that is 90 percent of the median income for the whole population. With the choice of a strictly positive value of the poverty line for people with high educational endowments, we have assumed that improving the education level of any individual should raise his or her level of well-being in a significant manner but never results in a move out of poverty if she is still deprived with respect to income after an improvement in her educational level.

For the remaining education groups, instead of choosing some particular values, we have opted to leave that issue unresolved *a priori* since one can hardly conclude how high the income level of a poorly educated individual should be for him or her to escape poverty compared to the level for a well-educated person. Nevertheless, we still required these specific income poverty lines to never be inferior to the ones corresponding to better educated groups ($z_k \geq z_{k+1}\ \forall k = 1, \ldots, K-1$ referring to the education level). After sequentially aggregating the population by their educational attainment, we then estimated the income level $\hat{z}_k^+$ each time such that the sign of the dominance curve changed and considered this value as a maximum for the definition of the poverty frontier such that the "pro-poor" judgment still holds. As a consequence, we let the data show the limits for the poverty domain (the "critical set" in Duclos *et al.*, 2006) compatible with an ethically robust "pro-poor" conclusion. If the poverty line chosen for the better educated subpopulation was included in this set ($z_6^+ \leq \hat{z}_k^+\ \forall k = 1, \ldots, 6$), we did not reject the possibility of accepting a "pro-poor" judgment, but added that the result was valid only for sets of poverty lines below the critical poverty frontier ($z_k \leq \hat{z}_k^+\ \forall k = 1, \ldots, 6$).

From the several comparisons carried out using the data from the three household surveys mentioned above, we have extracted three cases that highlight the relevancy of our approach.

Our first illustration is related to the contrast between the urban and rural areas in Turkey considering the 2003–04 growth spell. For both populations, we used the first order sequential dominance procedure so that our results refer to the very inclusive class of multidimensional poverty measures $\bar{\Pi}_1$. As shown in Section 3.1, the sequential first order stochastic dominance procedure consists of comparing the value of the multidimensional headcount index, i.e. the share of the population whose incomes and educational level are less than the chosen values, for each pair of income and educational levels included in the poverty domain. As the non-income attribute is described by a discrete variable, the appropriate approach consists of first estimating the difference in the share of illiterate people whose incomes are less than a given threshold between the final distribution and the counterfactual distribution up to the value such that the sign of that difference changes. Then, if this value is consistent with the income poverty line chosen for that group, that is any value above the income poverty line z_6^+ referring to the highly educated people in our setting, we can then add the set of individuals belonging to the second education group and proceed with the comparison of multidimensional headcount indices along the range of income values and so on.

For the present paper, we preferred to contrast the results yielded by the sequential procedure with the more traditional first-order stochastic dominance procedure for the whole population and thus began with this traditional approach. In Figure 2, the difference in the headcount index between the final and the counterfactual distributions is depicted by the thick continuous black curve for the whole population. We can observe that the standard first-order stochastic dominance was satisfied for both the urban (Figure 2a) and the rural (Figure 2b) areas between 2003 and 2004 since the corresponding curves are in both cases below the horizontal axis up to the income poverty line (represented by the dashed vertical line). Consequently, with a traditional monetary approach to poverty, we would

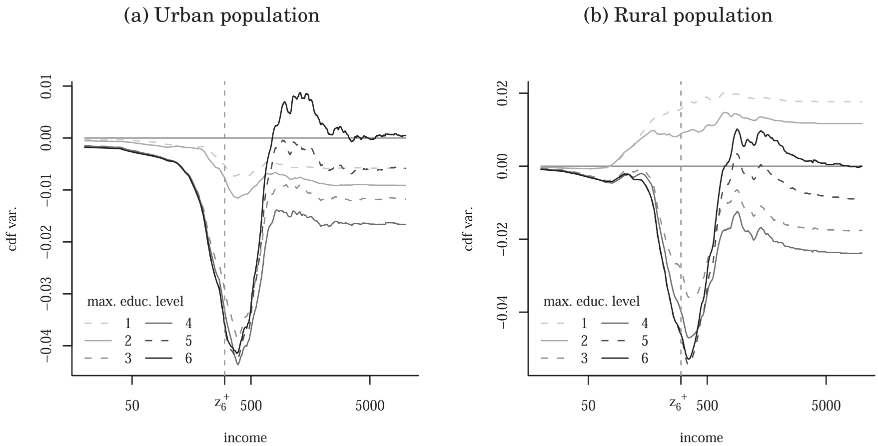

Figure 2. “Pro-Poor” Growth Check: Education and Income in Turkey, 2003–04, Urban and Rural Populations

conclude that growth has been “pro-poor,” in the relative sense, in both the urban and the rural areas.

However, the picture becomes slightly different once we turn to multidimensional poverty with the inclusion of the education dimension. While the classical “pro-poor” result holds with the multidimensional analysis in urban areas, it would be misleading to rely solely on the standard monetary analysis for the rural areas. Indeed, bringing together all individuals whatever their education level, that is without putting a particular emphasis on the poorly educated households, would lead to the wrong conclusion that the growth pattern was biased in favor of the neediest between 2003 and 2004. Nevertheless, focusing on the first two groups of educational attainment (the continuous and dashed light grey curves in Figure 2b), we see that the share of low income and low education individuals has not decreased as much as would have been the case with a “neutral” growth pattern during the period. It is worth noting that we could even conclude that growth was rather “anti-poor” in Turkey between 2003 and 2004 if the poor are defined as those people whose income is below z_6^+ and who did not complete primary school.

The second illustration is related to the usefulness of a second-order sequential stochastic dominance check. Looking at Figure 3, it can be seen that we cannot conclude whether or not growth was “pro-poor” in a robust manner taking into consideration the whole Turkish population for the 2004–05 growth spell since dominance curves are sometimes above and sometimes below zero for income levels below the income poverty line z_6^+. However, since the curves are above this level for the very bottom part of the income range, it may be interesting to focus on the more limited set $\bar{\Pi}_2$ of distribution-sensitive multidimensional poverty measures and consequently to turn to the second-order sequential stochastic dominance procedure. Contrary to the first-order procedure, the second-order procedure relies on the use of income gaps, that is the average value of income shortfalls with respect to the poverty line times the value of the corresponding multidimensional headcount index.

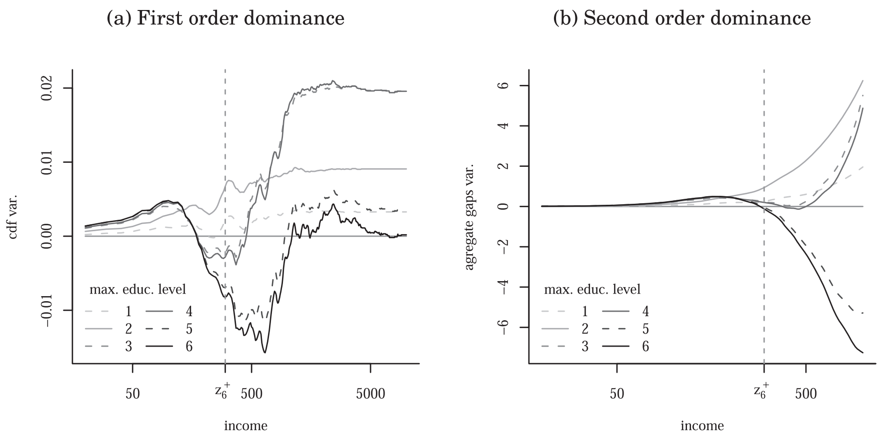

Figure 3. "Pro-Poor" Growth Check: Education and Income in Turkey, 2004–05, Whole Population

The results, plotted in Figure 3b, show that the joint distribution of education and income in 2005 is dominated by the corresponding counterfactual based on the 2004 distribution up to some admissible poverty frontier. In other words, we can conclude in a robust manner that growth can be deemed "anti-poor," in the relative sense, in Turkey during the period 2004–05 taking into consideration distribution-sensitive multidimensional poverty measures. It is worth noting that, in this specific case, our conclusions are similar to the ones obtained with the traditional dominance checks (cf. the thick black curves in Figures 3a and 3b), but our approach yields more information on the distribution of the economic growth "cake" and may even provide some insights into the forces that shape that distribution when the non-income attributes are known determinants of income (like education in our case). Indeed, it can be seen that welfare gains were less important for the four lowest education groups, whatever their initial income level, as would have been the case with an inequality preserving growth pattern, suggesting that Turkish growth could not be deemed "pro-poor" for the corresponding period as it was fostered by sectors requiring high-skilled workers.

Finally, Figures 4a and 4b present the results obtained using the set of SGICs and SPGCs, respectively, for the same 2004–05 growth spell. The value of the headcount index associated with z_6^+ and corresponding to each curve is depicted using vertical lines. As explained in Section 3, it is worth stressing that these tools should theoretically be used for a growth "pro-poorness" check only if the counterfactual distribution of education, that is the observed distribution in 2004 in our example, was the same as the one observed in 2005. However, applying the Kolmogorov–Smirnov test to our samples leads to the rejection of the null hypothesis that the samples are drawn from the same distribution at the 5 percent level. This is quite a surprising result since it is hardly believable that the distribution of education in 2005 was really significantly different from the one observed in 2004. Indeed, differences between the two samples are sometime non-marginal with respect to education and thus may have driven the results observed in Figures 3a

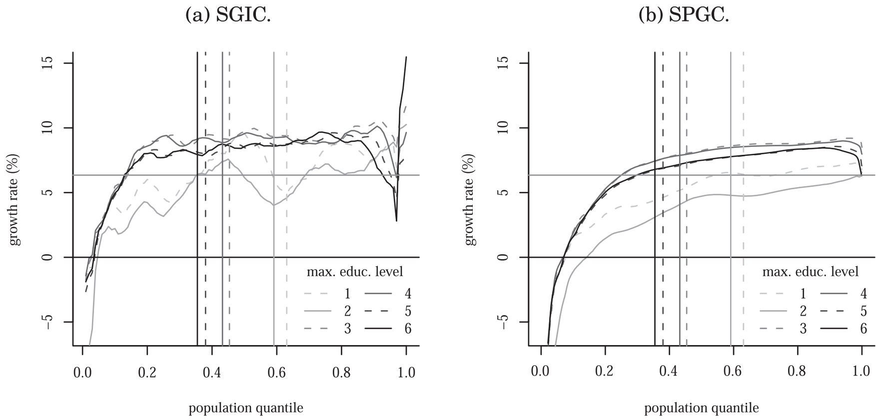

Figure 4. "Pro-Poor" Growth Check: Education and Expenditure in Turkey, 2004–05, Whole Population

and 3b. These slight differences in the marginal distribution of education explain why the "anti-poor" statement observed with the sequential second-order dominance test (Figure 3b) does not hold when looking at the set of SPGCs (Figure 4b). More specifically, it can be seen that the mean growth rate of the poor is sometimes above the average growth rate once individuals with some formal education are introduced into the analysis. With a longer time span, we could have relied on the results provided by Figure 3b, but with our present illustration, we contend that it may be preferable to ignore the changes in education observed for our samples and thus to focus on the sole variations in conditional distributions of income using the sets of SGICs and SPGCs.

5. Concluding Remarks

In this paper, we have proposed extending the use of sequential stochastic dominance techniques in order to assess robust judgments of the "pro-poorness" of growth within the framework of a multidimensional approach to poverty measurement. Indeed the traditional tools used to check for "pro-poor" growth focus on the sole monetary aspect of poverty. As is well-known, the inclusion of other dimensions of poverty induces a change in the definition of poverty. In particular, the poverty domain is generally slightly different when poverty is considered in a multidimensional sense so that the analysis may require considering individuals that were previously not deemed poor or rejecting individuals that were regarded as poor with a unidimensional analysis. Here, we propose the use of sequential dominance procedures suggested by Bourguignon (1989) and developed by many authors like Atkinson (1992) and Jenkins and Lambert (1993) in order to define first-order and second-order dominance criteria that make it possible to assess the robustness of a statement of "pro-poor" growth in income as well as in other well-being attributes for a class of poverty measures and a wide range of poverty lines.

Unlike the traditional studies that use sequential stochastic dominance, the heterogeneity of the population is not defined on the basis of household size and composition. On the contrary, individuals' needs differ according to non-income attributes as in the study of Duclos and Échevin (2009) for poverty measurement. Unlike the attempt made by Grosse *et al.* (2008), who extend Ravallion and Chen's (2003) growth incidence curve (GIC) to non-monetary dimensions of poverty, our method takes into account the changes in the joint distribution of the well-being attributes. For this purpose, our method only adds two weak conditions to the traditional mathematical conditions used for unidimensional poverty measurement. The first one is that the income poverty line does not increase with the level of the non-monetary indicator. The second one requires the marginal contribution of income to well-being to decrease with the level of non-income attributes. As a special case for our approach, it is possible to define the equivalence of GICs and PGCs, named SGICs and SPGCs, that are based on partial quantile functions and may be used to obtain robust conclusions when the marginal distribution of the non-monetary attribute is left unchanged. It is worth noting that the use of these curves can be extended so as to take changes in the distribution of the non-income attributes into account. Moreover, though the social evaluator has more latitude in defining the counterfactual situation in order to make judgments on the "pro-poorness" of growth, the definition of this counterfactual is challenging from an empirical point of view as it entails considering the relationships between income and non-income attributes. Our feeling is that this issue should be a matter of scrutiny for further empirical studies.

Finally, the usefulness of the proposed method has been illustrated using Turkish household surveys for the period 2003–05. Thus, three illustrative cases have been considered. Using first-order sequential dominance, the first case contrasts the results obtained from the traditional monetary approach to poverty and shows that both perspectives can yield diverging conclusions regarding the "pro-poor" nature of growth in rural areas for the period 2003–04. The second case provides an application of second-order dominance while highlighting the additional information provided when considering the non-income attribute. Finally, this latter case has served to underline the issues raised by the design of the traditional GICs and PGCs within the multidimensional framework of poverty and to stress the complementarity between SGICs (SPGCs) and sequential dominance curves.

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Supporting Information

Additional Supporting Information may be found in the online version of this article:

Appendix. Proof of Propositions and Corollaries 3 and 4

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