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STATUS QUO IN THE WELFARE ANALYSIS OF TAX REFORMS

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The welfare analysis of tax reforms most often consists of comparing the post-reform distribution of individual welfare with the pre-reform distribution or possibly that obtained from another reform as if they were completely independent. Such an "anonymous" approach does not take into account "changes" in individual situations, generally the main source of contention in any tax reform debate. This paper proposes a welfare criterion that allows comparison of reforms while taking into account individual status quo—i.e. pre-reform—situations. This is done by extending standard utilitarian social welfare criteria to the case where individual utilities depend on initial income and income change.

1. Introduction

The welfare analysis of tax-benefit reforms most often consists of comparing pre- and post-reform income distributions, or two post-reform distributions associated with two distinct reforms, without real consideration for how individual tax units fare in the reform. An acceptable tax reform using that approach is a reform that improves the distribution of income according to some social welfare criterion. Out of two tax reforms, the best one is the one that yields the highest level of social welfare. Such comparison is made using standard welfare economics. Either a specific social welfare function is chosen to make the comparison as in the optimal taxation literature in the line of Mirrlees (1971), or the social choice criterion is based on a family of welfare functions, as with welfare dominance analysis (see, for instance, Lambert, 1989). In the latter case, the social welfare ranking is only partial so that a tax reform cannot always be said to be socially better than another.

The social welfare comparison made in standard tax reform analysis is "anonymous," in the sense that the actual income trajectory of individual agents does not matter. Comparing two tax reforms B and C is generally done without taking into account the distribution of individual welfare in the initial situation A. As the initial situation A imposes economic constraints such as budget neutrality, possible reforms B and C might depend on A. In this approach this dependence is not because the welfare of individuals in post-reform situations B or C might

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somehow depend on their initial situation. In reality, however, some reforms appear socially undesirable, in view of the initial tax system or "status quo," because they would entail too large an income loss for particular individuals or groups of people.

Marginal tax reform analysis, pioneered by Guesnerie (1977) or Ahmad and Stern (1984), is not subject to the same criticism. It consists of exploring the distribution of individual welfare that can be achieved through specific marginal reforms of the existing tax system in the neighborhood of that system. It is not anonymous since the change in total welfare depends on how various individuals in society are affected by the reform and the social valuation of the marginal change in their welfare. Yet, marginal tax reform analysis is infrequently used, possibly because few reforms actually considered in the real world appear to be truly "marginal." Political economy models of tax reforms also account explicitly for the pre-reform situation of voters or, more generally, decision-makers. Individual agents are in favor of a reform vs. the status quo only if it improves their personal situation. Political economy models of tax reforms can often be reformulated as social welfare maximization problems but, logically, social welfare should then depend on the status quo situations of economic actors. For instance, in models *à la* Romer (1975), the social welfare function is the utility of the median voter as defined by the status quo distribution.

In welfare economics, the concept that should permit addressing explicitly the issue of the status quo in evaluating any policy that modifies individual incomes is that of horizontal equity. According to that principle, a reform is equitable only if it provides "equal treatment to equals" (Musgrave, 1959). In other words, people who have the same welfare level in the status quo situation must have the same welfare level after the reform. When this principle applies, reforms from the same base situation can be compared in terms of post-reform welfare. Only vertical inequality matters and no reference to the status quo is needed for analyzing this in standard welfare economics approaches. However, this concept of horizontal equity raises some difficulty when it does not strictly apply to the reforms being compared. When two reforms do not treat equals in the same way, or when people are all unequal in the status quo, how should they be compared? For instance, should it make a difference when transferring a euro from X to Y that this transfer changes their relative ranks in the distribution, and how much of a difference should it make?

Considerable attention has been devoted to these issues in the recent literature (see Aronson and Lambert, 1994; Kaplow, 1995; Jenkins and Lambert, 1999; Duclos and Lambert, 2000; Duclos, 2008). Most of it is devoted to the design of inequality measures that allow for the identification of these various types of inequality when comparing two reforms. For instance, Duclos *et al*. (2003) show how a specific family of inequality measures permits one to distinguish the role of vertical inequality, horizontal inequality, and reranking when analyzing the distributional impact of a reform. Yet, the underlying framework is traditional in the sense that social welfare is defined only on individual post-reform utilities. The status quo enters that decomposition because of the desire to isolate in the overall effect of a reform what may be due to reranking or a breach in horizontal equity, not because the status quo is given any individual or social value *per se*.

The main contribution of this paper is precisely to explore the implications of explicitly recognizing the importance of the status quo by incorporating it within a standard utilitarian framework. Put another way, the approach taken is to allow the status quo to affect the individual evaluation of post-reform incomes. Individuals are more or less sensitive to a given income change due to a reform depending on their initial income. The social welfare of a tax reform is thus defined on individual utilities that depend on both pre-reform and post-reform incomes. Such utility functions may be justified in different ways. The simplest one is that individual utility actually encompasses past and future incomes, social welfare being thus defined over a transition period where the marginal utility of post-reform income depends on the pre-reform income. Although this may be found too short-run a social welfare criterion, it will be seen that, by considering families of such utility functions, it is actually possible to cover a rather wide range of time horizons, including long-run horizons with an increasing relative weight given to the post-reform situation. By doing so, this paper seeks to provide the tax reform welfare analysis literature with an analytical instrument that explicitly takes status quo situations into account while incorporating at the same time elements of the standard utilitarian welfare approach. In this framework, reforms that would satisfy some welfare optimality criterion in the long-run but would be difficult to implement in the short- and medium-run because they would entail a drastic reranking of disposable incomes, might appear much less attractive. At the same time, the possibility of some inconsistency between such long-run and medium-run evaluations of tax reforms has to be considered.

Another contribution of this paper is to conduct the analysis not for specific social welfare and personal utility functions but for a wide family of them, thus generalizing welfare dominance analysis initially developed for income distribution comparisons (Atkinson, 1970) in this specific context. This is done by adapting the "sequential dominance criterion" developed in Atkinson and Bourguignon (1987) to this particular context where individual income utility depends on both pre-reform and post-reform incomes.

The paper is organized as follows. It starts (Section 2) by recalling some basic criteria to compare two income distributions, focusing on the dominance criteria applied to the incomplete mean income curve (or Bonferroni curve[1]). It discusses in particular the importance of the underlying ranking of the populations for defining these curves. Section 3 establishes a general social welfare dominance criterion when the status quo enters individual utility functions on top of post-reform income and derives dominance properties that generalize incomplete mean income curve dominance. Section 4 then derives some simple necessary conditions for dominance and discusses the relationship they may have with the measurement or evaluation of vertical and horizontal inequity. An application that compares two reforms of the way the French income tax system deals with family size is given in Section 5. Some implications of the whole analysis in this paper and future research directions are discussed in the concluding section.

[1]See Silber and Son (2010).

2. The Utilitarian Approach to Tax Reform Comparisons: Dominance Criteria

Consider a population of n people with initial (status quo) incomes y_i^0 for $i = 1, 2, \ldots n$ and then two tax reforms leading to disposable incomes y_i^1 and y_i^2 respectively. For further reference, define the income change of individual in reform j as:

$$x_i^j = y_i^j - y_i^0$$

for $j = 1,2$. We are interested in comparing the two reforms 1 and 2, or the two distributions $Y^1 = \{y_1^1, y_2^1, \ldots y_n^1\}$ and $Y^2 = \{y_1^2, y_2^2, \ldots y_n^2\}$ according to the standard utilitarian social welfare criterion:

$$W_u(Y^j) = \sum_{i=1}^{n} u(y_i^j) \quad j = 1, 2.$$

As it is well known from the income inequality literature, Y^2 is said to 2nd order welfare dominate Y^1 iff[2]:

$$W_u(Y^2) \geq W_u(Y^1) \quad \forall u: \ u'(\ ) \geq 0 \quad and \quad u''(\ ) \leq 0. \tag{1}$$

In other words, reform 2 dominates reform 1 if utilitarian social welfare is higher with reform 2 for all individual utility functions that are increasing and concave. It is equally well-known that dominance (1) is equivalent to Generalized Lorenz curve dominance:

$$W_u(Y^1) \geq W_u(Y^2) \quad \forall u: \ u'(\ ) \geq 0 \quad and \quad u''(\ ) \leq 0$$
$$\Leftrightarrow G^1(k) \geq G^2(k) \quad k = 1, 2, \ldots n$$

where the Generalized Lorenz curve $G^j(k)$ is defined as:

$$G^j(k) = \sum_{i \in I^j(k)} y_i^j; I^j(k) = \{i : y_i^j \leq y_m^j \quad \forall m \notin I^j(k)\}; \quad card\,[I^j(k)] = k. \tag{2}$$

In other words, the Generalized Lorenz curve for distribution Y^j is simply the total income of the k poorest individuals in the population with tax reform j as a function of k.

The "incomplete mean income" (or Bonferroni) curve is simply the Generalized Lorenz curve divided by k, i.e. the mean income of the k poorest.

$$IMI^j(k) = G^j(k)/k. \tag{3}$$

It is obvious from (3) that Generalized Lorenz curve dominance is strictly equivalent to the dominance of incomplete mean income (IMI) curves. Much use will be made of these incomplete mean income curves in what follows.

[2]See, for instance, Cowell (2000).

Note that the definition of both the Lorenz and the incomplete mean income curves would be much simpler if it were assumed that individuals in the population were initially ranked according to income. When comparing distribution Y^1 and Y^2, however, this would mean that the ranking of individual incomes is the same with the two tax reforms, which is most unlikely in general. This paper is precisely about the case where rankings are different.

The preceding welfare dominance or IMI dominance criteria applied to tax reforms 1 and 2 clearly ignore the status quo, or initial distribution of disposable income. The income ranking $I^j(k)$ in the definition of the IMI curve is post-reform, not pre-reform. It is "anonymous" in the sense that the pre-reform identity of a person ranked k in the post-reform distribution is not known. Yet, it is possible to redefine the IMI curve in a " non-anonymous" way on the basis of any arbitrary ranking of people, and in particular the status quo ranking. For instance, one can define "status quo based incomplete mean income curves" (sq IMI) as:

(4) $$sq\ IMI^j(k) = \sum_{i \in I^0(k)} y_i^j / k;\ I^0(k) = \{i : y_i^0 \le y_m^0 \quad \forall m \notin I^0(k)\};$$
$$and\ card\left[I^0(k)\right] = k.$$

Some use will be made of these curves below. To simplify and without loss of generality, we shall also assume that individuals are ranked by increasing income in the status quo distribution, so that:

$$I^0(k) = \{1, 2, :\ldots k\}.$$

Figure 1 shows examples of sq IMI curves for two tax reforms as well as the IMI curve for the status quo, the horizontal axis being rescaled through $p = k/n$ so that all curves are defined on the [0,1] interval. The main difference between sq IMI and IMI curves is that the former are not necessarily increasing, precisely because

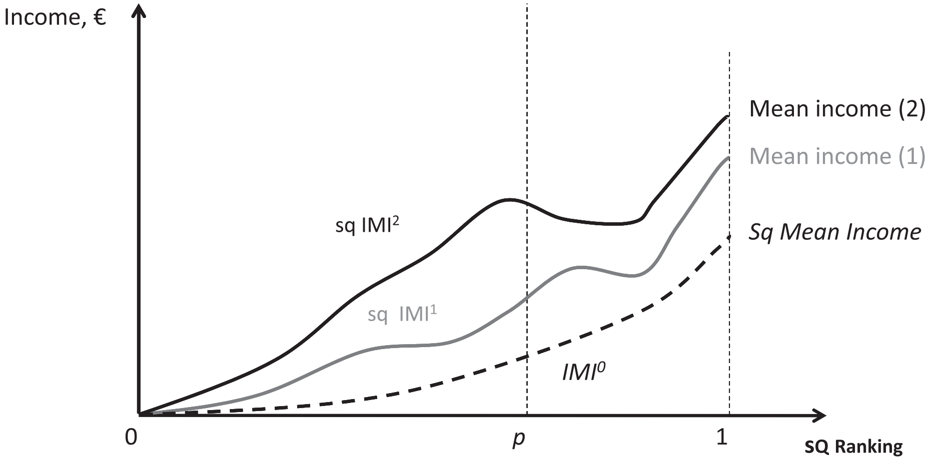

Figure 1. Status Quo (sq) Incomplete Mean Income Gain Curves

individuals are not ranked by increasing income any more. If the individual ranked $k + 1$ in the status quo distribution has a disposable income in tax reform 1 lower than the mean income of the k previous individuals, the sq IMI curve is locally decreasing at $k + 1$. Figure 1 also shows the sq IMI curve for the status quo distribution Y^0 which happens to be identical to the IMI curve since individual incomes are then ranked in ascending order.

Differences between IMI and sq IMI curves disappear when no reranking takes place with the tax reforms. Standard dominance results then apply:

$$sq\ IMI^1(p) \geq sq\ IMI^2(p)\ \forall p \Rightarrow IMI^1(p) \geq IMI^2(p)\ \forall p \Rightarrow$$
$$W_u(Y^1) \geq W_u(Y^2) \quad \forall u:\ u'(\ ) \geq 0 \quad and \quad u''(0) \leq 0.$$

3. General Dominance Criterion with Status Quo

Introducing the status quo into the preceding familiar type of dominance analysis requires making individual utility functions depend on both post-reform and pre-reform incomes. Utilitarian social welfare will thus now be based on individual utilities of the type $u(y_i^0, y_i^j)$. There are various justifications in adopting such a specification. First, the recent economic literature on happiness suggests that both income change and initial (or terminal) income matter, although the effect of the income change tends to dissipate after a few years.[3] If it is so, well-being effects of a tax reform should depend in the medium-run on both the income change that it causes and the initial income of a person.

Second, one may use a genuine permanent income argument. Measuring well-being on a period that includes both the pre-reform and the post-reform periods, permanent income may be defined as: $\tilde{y}_i^j = ay_i^0 + by_i^j$ where a and b are weights given to the two periods. However, with imperfect foresight of the reform, costly replanning, and possibly imperfect capital markets, there is no reason to aggregate pre- and post-reform incomes linearly.

Third, introducing both pre- and post-reform incomes in individual utility functions is a way to take into account the political cost of a reform for a social planner, without getting into political economy modeling. Finally, note that y_i^0 need not be the pre-reform income. It may be some normative reference that a social planner would like to use when evaluating actual individual incomes. More precisely, the planner may want to ground his/her comparison between the two distributions Y^1 and Y^2 on their distance from a reference distribution, Y^0, for instance market incomes.

It will be convenient to rewrite individual utilities as a function of initial income and income change:

$$u(y_i^0, y_i^j) = \upsilon(y_i^0, x_i^j). \tag{5}$$

With this specification it is reasonable to assume that the marginal utility of the income gain is positive and decreasing, that it decreases with initial income, and

[3]See, for instance, Easterlin (2003) and Di Tella *et al.* (2010) on the time profile of the effect of an income change on happiness.

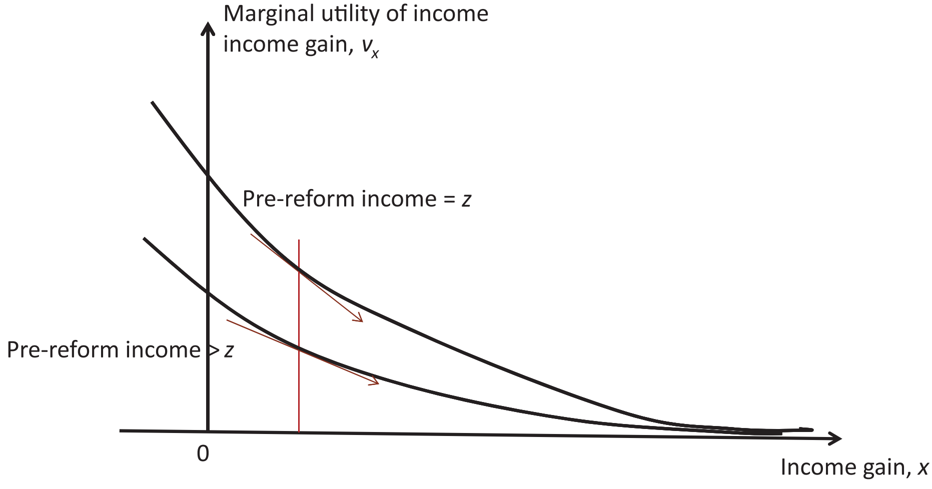

Figure 2. Shape of the Marginal Utility of Income Gain Curves

also that people with high initial incomes are less sensitive to marginal changes in income gains. Formally, this means:

$$\upsilon_x \geq 0, \quad \upsilon_{xx} \leq 0, \quad \upsilon_{xy} \leq 0, \quad \upsilon_{xxy} \leq 0. \tag{6}$$

The first two assumptions are common. The third term in (6) means that income change and initial income are supposed to be substitutes, as in the permanent income interpretation above. The fourth term in (6) extends the idea that richer people are less sensitive to income changes one step further. Figure 2 shows the shape of the marginal utility of income gain curves consistent with these assumptions.[4]

Based on the preceding assumptions, it is possible to generalize the standard welfare dominance criterion to a "status quo based social welfare dominance criterion." It is now grounded on utility functions that depend on both pre-reform income and income changes. Denoting V the family of functions that satisfy (6), (1) becomes:

$$W_\upsilon(Y^0, X^1) \geq W_\upsilon(Y^0, X^2) \quad \forall \upsilon \in V \tag{7}$$

$$with \quad W_\upsilon(Y^0, X^j) = \sum_{i=1}^{n} \upsilon(y_i^0, x_i^j) \quad j = 1, 2.$$

This is the kind of multi-dimensional dominance studied by Atkinson and Bourguignon (1982) with the peculiarity that the marginal distribution of one of the two arguments of the individual utility function has the same marginal distribution in the two two-dimensional distributions being compared. Atkinson and

[4]Note that it would have been possible also to define $u()$ in log incomes, in which case $\upsilon()$ would be defined on income and relative, rather than absolute income change. This kind of transformation is not neutral in terms of the signs of second and third derivatives, though.

Bourguignon (1987) showed that, in that case, the preceding welfare dominance criterion is equivalent to a "sequential dominance" of Generalized Lorenz curves. In the present case, this sequential dominance is equivalent to:[5]

$$(8) \qquad Z^1(p,q) \geq Z^2(p,q) \quad \forall p, q \in [0,1]$$

where $Z^j(p, q)$ is the incomplete mean income gain among the p poorest individuals in the status quo distribution and the lowest q income gainers in reform j. Formally:

$$Z^j(p,q) = \sum_{i \leq k, i \in I_l^k} x_i^j / l \quad with \quad I_l^k = \{i : x_i^j \leq x_m^j \quad \forall m \notin I_l^k\};$$

$$card\ I_l^k = l;\ p = k/n; \quad q = l/k.$$

Practically, the Z^j curves may be evaluated as follows. Select the k poorest in the status quo distribution Y^0 ($i = 1, 2, \ldots\ k$), For reform j, rank these k individuals by increasing income gains x_i^j and consider the l lowest. Normalizing k and l into [0, 1] through the transformation $p = k/n$; $q = l/k$, $Z^j(p, q)$ is simply the mean income gain of all these individuals. Dominance of reform 1 over reform 2 requires the surface Z^1 (p, q) to be above the surface Z^2 (p, q) everywhere in the space $[0,1] \times [0,1]$.

One way of interpreting the preceding dominance condition in the line of Atkinson and Bourguignon (1987) is to consider the rank of an individual tax unit in the status quo distribution as an indicator of the "need" for a positive income change. The Z-dominance criterion (8) above then compares the distribution of income changes among people who are the neediest, starting with the neediest group and then adding less and less needy groups. Under the assumption that the marginal utility of income gains is positive and decreasing, the same dominance criterion as for incomes applies, that is criterion (1). The only difference is that the criterion applies to incomplete mean *income gain* curves rather than incomplete mean *income* curves for each group of neediest people. Note, however, that this comparison relies on a ranking that is specific to each tax reform and does not depend any more on the status quo ranking. The latter is used only to define the needs of tax units for more income.

Figure 3 illustrates the application of the Z-dominance criterion when tax reform 2 dominates reform 1. Instead of representing the whole surface $Z(p, q)$, a few projections on the (Z, q) space are shown: the 10 and 50 percent poorest individuals in the status quo distribution and the whole population ($p = 100$ percent). Within each of these three groups, people are ranked by increasing income gain for each reform and the comparison relies on the q smallest income gains. Of course the q smallest income gains are not the same for tax reforms 1 and 2. The graph shows the mean income gain for these subgroups when q goes from 0 to 1. As they are drawn, the Z curves start being negative because people ranked first in terms of increasing income gains may have negative income gains. In this

[5]See Bourguignon (2011).

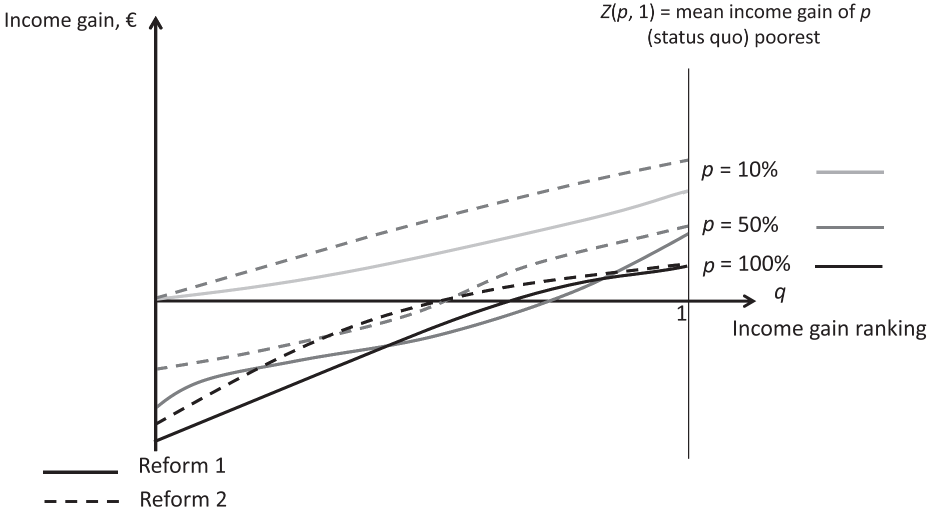

Figure 3. Illustration of Dominance with Status Quo Social Welfare Criterion (comparison of incomplete mean income gain curves $Z^j(p, q)$ for selected values of p)

regard, note that if Z(p, q) is initially negative for some p_0, it must be so for all $p \geq p_0$. Since more people are added, the minimum income gain cannot go up. The incomplete mean income gain curves $Z(p, q)$ are increasing with respect to q but they are not necessarily concave or convex.[6] They have no particular shape with respect to p. Note, however that two projections on the (Z, q) plane may cross each other. This is the case for the curves corresponding to p = 100 percent and p = 50 percent in Figure 3. Some people in the upper half of the status quo distribution have a smaller (or more negative) income change than people in the bottom half. However, people in the upper half have larger income gains at the middle range of income gain whereas their overall mean gain is smaller.

4. Some Simple Necessary Conditions for Dominance and Relationship with Other Dominance Criteria

Examining whether a surface is above another is not a simple matter. Comparing a few sections of two surfaces as done in Figure 3 may be restrictive because it may miss the regions where they actually cross each other. It is thus useful to identify simpler criteria that might allow an easier and quicker identification of situations where dominance is clearly violated. At the same time, this will permit exploring the relationship between the status quo dominance criterion in this paper and the standard utilitarian dominance approach of tax reform evaluation.

[6]Similar curves are shown in a growth incidence curve context in Bourguignon (2011). Note, however, that their shape is erroneous because they have been inadvertently constrained to go through the origin.

4.1. *Mean Income Gain Dominance*

Figure 3 illustrates the dominance criterion (8) by showing intersections of the $Z^j(p, q)$ surfaces with $p = cst$ planes. It is possible to do the same with $q = cst$ planes, a special case being $q = 1$. The interpretation of the $Z(p, 1)$ curves is simply the mean income gain in tax reform j of the p poorest people in the status quo distribution. Hence the proposition:

Proposition 1. *A necessary condition for tax reform 2 to status quo welfare dominate tax reform 1 in the sense of (6)–(7) is that the mean income gain of the p poorest people in the status quo distribution be higher in reform 2 than in reform 1 for all values of p in* [0, 1]. *This condition implies also that the "status quo based incomplete mean income" curve defined in (4) to be everywhere higher for reform 2.*

This proposition is rather intuitive. It simply says that, on average, the poorest people in the status quo distribution are better off with reform 1 than with reform 2 because their income gain is higher. Note, however, this is only *on average*. In effect, it is quite possible that some people among the p poorest are worse off with reform 2 because their income gain is lower than in reform 1.

Proposition 1 may be interpreted in terms of "growth incidence curve." Dividing $Z^j(p, 1)$ by the mean income of the p poorest, the resulting curve would simply show the mean growth rate of income of the p poorest people when moving from the status quo to reform j. This curve has been called the "non-anonymous cumulative growth incidence curve" in Bourguignon (2011) and was previously used by Grimm (2007). It is related to the "income mobility profiles" in Jenkins and van Kerm (2006) and van Kerm (2009). It differs from the standard (anonymous cumulative growth incidence curves that would compare the income gain of the p poorest people ranked according to post-reform income rather than pre-reform (status quo) incomes (see, for instance, Ravallion and Chen, 2003).

4.2. *A Necessary Condition in Terms of Anonymous Incomplete Mean Post-Reform Income*

Interestingly enough, the general dominance criterion (7)–(8) also implies some dominance property in terms of the incomplete mean income curves of post-reform incomes, as they were recalled earlier in this paper—see (3).

Proposition 2. *A necessary condition for tax reform 2 to welfare dominate tax reform 1 in the sense of (6)–(7) is that the (anonymous) incomplete mean income (IMI) curve of the post-reform distribution defined in (3) be higher with reform 2 than with reform 1 or for the IMI curve for reform 2 to cross the IMI curve for reform 1 the first time from above.*

The proof of that proposition is easy and shows at the same time the generality of the status quo based welfare dominance criterion (7). A particular family of individual utility functions that satisfy (5)–(6) indeed consists of functions that depend only on the post-reform income:

$$\upsilon(y_i^j, x_i^j) = h(y_i^j + x_i^j).$$

In other words, the general dominance criterion (5) that explicitly takes into account both status quo and post-reform incomes includes the simpler and more familiar utilitarian welfare criterion based exclusively on post-reform incomes. As seen above, this criterion leads to comparing the incomplete (with respect to *p*) mean income curves of the reforms being evaluated.[7] There is a slight restriction in that equivalence, however. It can be seen that the assumptions (6) made on utility functions imply in this particular case:

$$h'(\ ) \geq 0; \quad h''(\ ) \leq 0 \quad \textit{and} \quad h'''(\ ) \geq 0.$$

As in (1), the post-reform utility function, *h*(), must thus be increasing and concave, but, unlike in (1), its third derivative must also be positive. The latter property, known as "transfer sensitivity," requires a progressive transfer of *k* euros made between two individuals with an income difference *m* to yield more social welfare gain when performed at the bottom of the income distribution rather than at the top. It is known from Shorrocks and Foster (1987) that under such conditions, social welfare dominance is equivalent to third degree dominance, which in turn requires the generalized Lorenz curve of the dominating reform to be everywhere above that of the other reform or to cross it, for the first time, from above.

4.3. *Necessary Condition on Minimum Income Gains*

Put in another way, the preceding proposition implies that a sufficient condition for reform 2 not to dominate reform 1 is for the poorest post-reform incomes to be lower with reform 2. A closely related, but different necessary condition for dominance is the following:

Proposition 3. *For reform 2 to dominate reform 1 in the sense of (6)–(7), the minimum income gain among all individuals in reform 2 must not be smaller than the minimum income gain in reform 1.*

The proof can be seen in Figure 3. If the minimum income gain with reform 2 is smaller than with reform 1, then $Z^2(1, 0+) < Z^1(1, 0+)$ and there cannot be dominance in the sense of (8).

4.4. *Inequality Corrected Mean Income Gain Curves*

The last necessary condition for dominance is obtained by integrating below the (Z, q) projection of all the $Z^j(p, q)$ curves from a horizontal line at $-a$, as in Figure 4. Call $\Sigma^j(p)$ the corresponding area. Clearly if $Z^2(p, q) \geq Z^1(p, q)$ for all (p, q), then it is the case that $\Sigma^2(p) \geq \Sigma^1(p)$ for all *p*. Now, $\Sigma^j(p)$ can be defined as the

[7]It is important to stress the possible ambiguity of the "incomplete mean income" in the present framework where incompleteness could be with respect to *p*, the status quo ranking, or with respect to *q* given *p*, the income gain ranking specific to a tax reform. In Proposition 1 the "mean income gain for the *p* poorest" could be reworded as the "incomplete (with respect to *p*) mean income gain" as used in Proposition 2. When there is no risk of ambiguity, the former expression will be used.

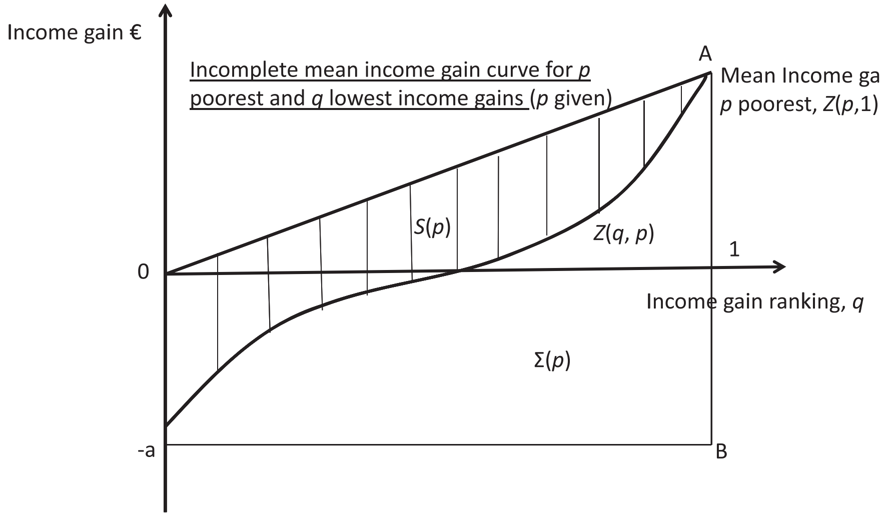

Figure 4. Measuring the Inequality of Income Gains

difference between the trapezoidal area ($-a$, O, A, B) which is equal to $a + Z(p, 1)/2$ and the area between the ray OA and the $Z^j(p, q)$ curve, which is labeled $S^j(p)$ in Figure 4. Transforming $S^j(p)$ into:

$$\Gamma^j(p) = 2 \cdot S^j(p) / Z^j(p, 1),$$

the index $\Gamma^j(p)$ can be interpreted as an inequality measure of income gains similar to the Gini coefficient for incomes. If all income gains are the same, then $\Gamma^j(p) = 0$. If they are all equal to 0 except one, then $\Gamma^j(p) = 1$. The difference with a Gini coefficient is that $\Gamma^j(p)$ may be larger than unity when the minimum income gain is negative.

With the above definition, it follows that

$$\Sigma^j(p) = a + (1/2) Z^j(p, 1) \cdot [1 - \Gamma^j(p)] \tag{9}$$

where $\bar{Z}^j(p) = Z^j(p,1).[1 - \Gamma^j(p)]$ can be interpreted as the "inequality corrected mean income gain" among the p poorest individuals in the status quo distribution. Indeed, it can be seen that the terms in a in (9) cancel, when comparing the areas $\Sigma^j(p)$. This implies the following:

Proposition 4. *A necessary condition for reform 2 to dominate reform 1 in the sense of (6)–(7) is for the inequality corrected mean income gain curve $\bar{Z}^2(p)$ for the p poorest individuals in the status quo distribution to be above the inequality corrected mean income gain curve $\bar{Z}^1(p)$ for all p in* [0, 1].

The practical interest of that proposition is to summarize the dominance property of the $Z^j(p, q)$ surface through the dominance of single curves that go much beyond the simple mean income gain criterion in Proposition 1 by taking into account the distribution of income gains, and not only their mean.

From a more conceptual point of view, what is interesting in this necessary condition for dominance is that it combines horizontal and vertical inequality concepts. The inequality coefficients $\Gamma^j(p)$ are related to horizontal inequality in the sense that they measure the diversity of income gains of people who are similar in the status quo, i.e. they are among the p poorest. Strictly speaking, a true measure of horizontal inequality would refer to those people *at rank p* in the status quo distribution, instead of people *at rank p and below*. Thus, $\Gamma^j(p)$ can be interpreted as a kind of integral of such horizontal inequality measures. The mean income gain $Z^j(p, 1)$ terms, on the contrary, convey information on changes in vertical inequality by showing how the income of the poorest people in the status quo is modified depending on the degree of poverty that is considered. Combining the two necessary conditions in Propositions 1 and 4, it can be seen that the dominance of reform 1 over reform 2 requires the mean income gain to be higher for the poorest people in the status quo, whatever the poverty threshold, but also the inequality of income gains among the poorest not to be too high.

5. An Example of Application: Family Size in the French Income Tax

The French income tax system is based on a standard piecewise linear tax schedule. However, this tax schedule is applied to taxable income adjusted by family size. Formally, the income tax T is given by:

$$T = Y \cdot t\,[Y/h(N)] \tag{10}$$

where Y is taxable income, N is the number of persons in the tax unit, $t()$ is the tax rate schedule for single persons, and $h(N)$ is the "quotient familial," or the number of adult equivalents, or consumption units in the tax unit. Actually, $h()$ depends on both the size and the composition of the tax unit. For instance, $h(N) = 2$ for a couple, each of the first two children adding 0.5 and each subsequent one adding 1. For a single, $h(N) = 1$, the first child adds 1, the second one 0.5 and the third one 1 again, etc. Beyond this somewhat arbitrary equivalence scale, there has long been some discussion about whether such a tax system was "fair." Indeed, as the tax rate schedule is increasing and convex, the tax deduction for one child increases with the level of taxable income of a tax unit, which may be seen as inequitable.

Two alternative reforms are explored in what follows. The first one consists of a constant income tax deduction, A, for each consumption unit, the tax function becoming then:

$$T_1 = Y \cdot t\,[Y - Ah(N)]. \tag{11}$$

The second reform consists of replacing the income-dependent tax deduction for the number of persons in the tax unit by an allowance, B, for each consumption unit independently of income. The tax schedule is then:

$$T_2 = Y \cdot t\,(Y) - B \cdot h(N). \tag{12}$$

In all cases $t()$ remains the tax rate schedule for single persons, whereas the constants A and B are calibrated so as to keep the tax receipt constant. On the other hand, no change is introduced in the benefit part of the redistribution system, including child benefits.

These two reforms have been designed essentially for the sake of illustrating the comparison criterion developed in this paper. In practise, one could think of many other, less extreme, reforms that would combine these two functional forms or alternative definitions of the "quotient familial," $h(N)$.

As $t()$ is a convex function—the marginal tax rate increases with income—it is pretty obvious that reform 1 is preferable to reform 2 for high income people, for a given N, whereas the opposite holds for low incomes. On the other hand, it is also easy to see that both reforms entail some reranking of disposable income per consumption unit. To see this, consider two tax units with taxable income, Y_1 and Y_2, and quotient familial, $h(N_1)$ and $h(N_2)$, such that their taxable income per consumption unit is the same. It follows from (10) that they have the same disposable income per consumption unit, y^0, and pay the same tax per consumption unit in the status quo situation. Their disposable income with reform 1 may be expressed as:

$$y_i^1 = y^0 \cdot \frac{1 - t[Y_i - Ah(N_i)]}{1 - t[Y_i/h(N_i)]} \quad i = 1, 2.$$

As taxable income per consumption unit is the same for the two tax units, the ratio of the disposable incomes per consumption unit with reform 1 is as follows:

$$\frac{y_1^1}{y_2^1} = \frac{1 - t[Y_1 - Ah(N_1)]}{1 - t[Y_2 - Ah(N_2)]}.$$

If tax rates are the same with the status quo taxation system, this is not the case with reform 1. Disposable incomes per consumption unit are not equal any more. The largest tax unit now has lost relatively to the smallest one. In other words, reranking takes place across tax units with different demographic compositions. The same can be shown for reform 2 and when comparing reforms 1 and 2.

Reranking justifies using the comparison criterion developed in this paper. The two reforms have been simulated using the tax-benefit model, Sysiff.[8] The model is based on the 2006 household budget survey, updated to 2008, and the 2008 tax-benefit system. Individual utility is assumed to depend on disposable income per consumption unit, as measured by $h(N)$, and individuals are ranked according to that criterion in the pre-reform situation.[9] Figure 5 shows the standard utilitarian welfare dominance criterion (2)–(3). Individuals are ranked according to post-reform disposable income per consumption unit in their tax unit.

[8]See http://microsimula.parisschoolofeconomics.eu/sysiff.htm.

[9]The income per adult equivalent $Y/h(N)$ is assumed to measure the level of welfare of the N persons in the tax unit. Note also that it would be possible to use another equivalence scale than $h(N)$, including the number of persons in the tax unit, N.

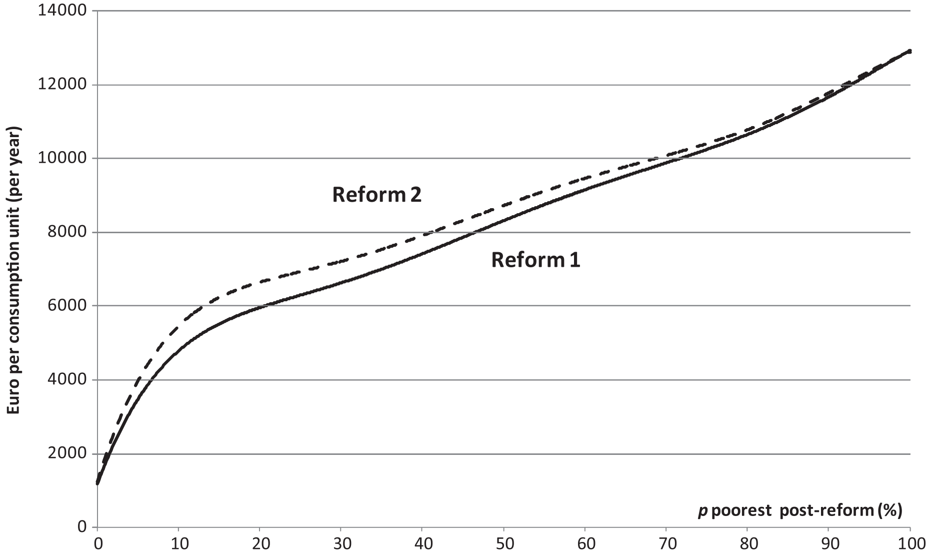

Figure 5. Comparing Reforms: Incomplete Mean Income Curves With Own (Post-Reform) Ranking

The incomplete mean income curves show that reform 2 unambiguously dominates reform 1. This means that the necessary condition in Proposition 2 is satisfied.

We now switch to criteria based on status quo dependent individual utility functions, analyzing other necessary conditions for dominance identified in the preceding section. Figure 6 shows the mean income gain (MIG) with respect to the status quo for the p poorest individuals in the status quo ranking, $Z^j(p, 1)$. The curve for reform 1 is equal to zero for the poorest individuals in the status quo, essentially because only those tax units that pay a strictly positive income tax can benefit from the reform. This is not true for reform 2 that dissociates demographic size and income in the determination of the income tax. Poorest people thus gain with reform 2, which thus appears more favorable than reform 1. When considering the 60 percent poorest people, it can be seen that they gain on average 270 euros (annually) per consumption unit with reform 2 and only 50 euros with reform 1. At the other extreme, the two curves converge toward zero when the whole population is included. This is simply because the reforms are budget neutral and the mean income gain for the whole population is zero for the two reforms.

A second necessary condition is concerned with the inequality corrected mean income gain curves, $\bar{Z}^j(p) = Z^j(p,1)\cdot[1-\Gamma^j(p)]$. These curves are also shown in Figure 6. Although this is not necessarily the case *a priori*, it turns out that the coefficients $\Gamma^j(p)$ that describe the inequality of the income gains among the p poorest in the status quo distribution are all less than unity. It follows that the inequality corrected mean income gain curves are positive and below the mean income gain curves. Although there are differences in the inequality of income

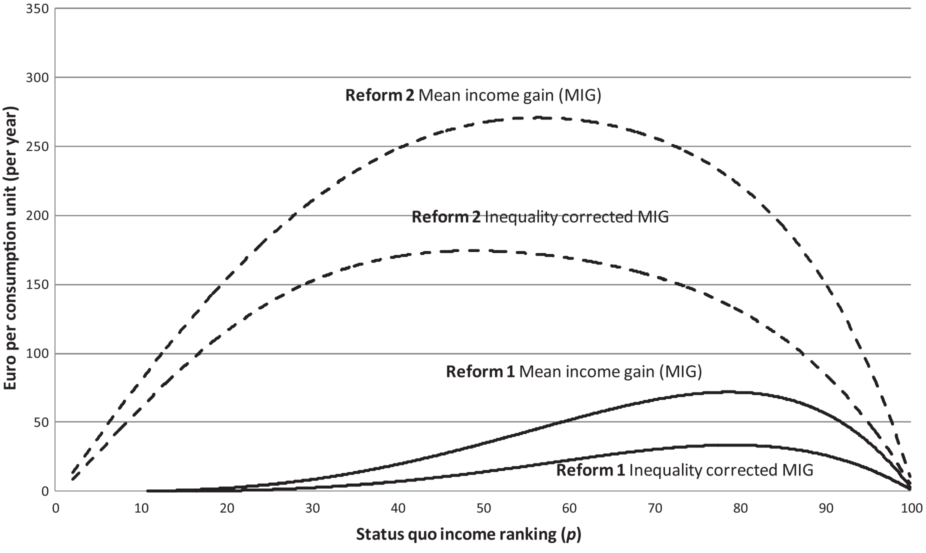

Figure 6. Mean Income Gain and Inequality Corrected Mean Income for p Poorest in Status Quo

gains across the two reforms, reform 2 still dominates reform 1. The necessary conditions in Propositions 1 and 4 for the dominance of reform 2 are thus satisfied.

Three out of the four simple necessary conditions for dominance derived above are satisfied. What about the third one, and in effect the simplest one? It turns out that it is not. This can be seen at the same time as we examine the whole surfaces $Z^j(p, q)$ in Figure 7. As in Figure 3, this figure shows the projections on the (Z, q) plane of $Z^j(p, q)$ curves for selected values of p.

The first thing to notice in Figure 7 is that for both $p = 80$ percent and $p = 100$ percent, the minimum income gain is smaller in reform 2. In both cases the incomplete mean income gain curve for reform 2 starts from below that of reform 1. Under these conditions, Proposition 3 is not satisfied and reform 2 cannot dominate reform 1 when the status quo situation is taken into account in evaluating social welfare. This is essentially because the restrictions (5) put on individual utility do not rule out a very large disutility from income losses even for people with high pre-reform incomes. It turns out that reform 2 is more progressive than reform 1 in the sense that it redistributes more from rich to poor. This appears quite clearly in Figures 5 and 6. But, precisely because of this progressivity, income losses at the top of the distribution are higher than with reform 1 so that reform 2 cannot dominate.[10] As shown by the top curves in Figure 7, this does not prevent reform 2 from doing much better than reform 1 when considering the 40 percent poorest people in the status quo distribution. This is because there is no income loss for these people with either reform and reform 2 redistributes more toward poorer people.

[10]As mentioned earlier, things would be different here if individual utility functions had been specified in terms of initial income and relative, rather than absolute income gain. This is a direction for further research.

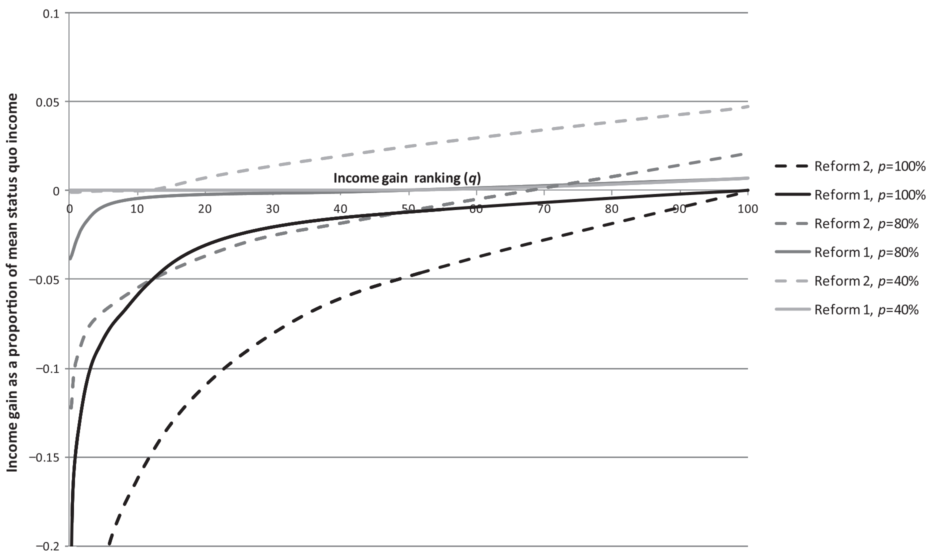

Figure 7. Comparing Reforms' Incomplete Mean Income Gain Curves $Z^j(p, q)$ for Selected Values of p

Note: Mean income gain expressed as a proportion of mean status quo income of p poorest.

This analysis of the full map of $Z^j(p, q)$ curves seems to suggest that it is difficult to design budget neutral reforms that would status quo welfare dominate each other. In the present case, reform 2 must be smoothed so that income losses are lower at the top, which requires modifying the tax rate schedule t (). But then, reform 2 will become much less progressive and it is not clear that it will be able to do better than reform 1.

Yet, it is clearly possible to find a tax reform that would fully dominate reform 1 in the sense of the status quo based welfare criterion (7). This requires that income gains in reform 1 are not perfectly and negatively correlated with status quo ranks. If this is the case, then there exist two individuals k and m such that $k \leq m$ and $x_m^1 \geq x_k^1$. Transferring a small amount from m to k so as not to change the income gain ranking then generates a new tax reform that dominates reform 1 in the sense of (7) and (8). To see this, note that all the $Z(p, q)$ curves such that k and m are among the p poorest shift upward between the two ranks in the income gain distribution occupied by k and m. The same is true for all ranks above that of k when k is among the p poorest and m is not. Finally, the $Z(p, q)$ curves remain the same when neither k nor m are among the p poorest.

6. Conclusion

This paper has presented a social welfare criterion to compare tax reforms that takes into consideration the pre-reform, or status quo situation of tax units as a way of making tax reform analysis more realistic in terms of what is possible and what would probably be rejected on the basis of too many people losing or a few people losing too much in the reform. Based on a utilitarian framework where individual utilities depend on both pre-reform income and income gain, some simple criteria have been derived that amount to comparing the distribution of individual income gains in two reforms sequentially for the p ($\in [0, 1]$) poorest people in the status quo distribution of incomes.

If the reforms being compared cause some reranking of people, or, equivalently, the income gains to be imperfectly correlated with initial ranks in the status quo distribution, then it is likely that this criterion will yield different conclusions than the standard utilitarian approach based on individual utilities that depend on post-reform income only. An example of such an occurrence has been given in this paper. Yet, more analysis of plausible tax reforms is needed to evaluate how partial the ordering based on this criterion is in practice. If it is too partial, then restricting further the underlying set of welfare functions might be considered. With the present specification, for instance, it is sufficient that the maximum income loss be higher in a reform to prevent it from dominating another. This may be found unduly restrictive.

Another direction for further research is the link between status quo based welfare dominance criteria as the one analyzed in this paper and the standard utilitarian post-reform approach to tax reform. Clearly the former approach is most adequate to study the transition from a tax system to another in the short- and medium-run, whereas the latter is more appropriate for the long-run. It would be interesting to develop criteria that could explicitly deal with such a dynamic setting.

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