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INTANGIBLE ASSETS AND NATIONAL INCOME ACCOUNTING

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In this paper I relate the measurement of intangibles to the project of measuring the sources of growth. I focus on three related and difficult areas of the measurement of national income: the measurement of new goods, the deflation of intangible investment, and the divergence between the social and private valuations of intangible assets. I argue that the economic theory and practice underlying measurement of these items is currently controversial and incomplete, and I point toward how concretely to move forward.

Introduction

How do we measure intangible investment—investment to create new products and processes—in the national income accounts so that it can help us explain measured total factor productivity growth? And how do we measure the economic growth due to new products that arise from intangible investment?

In this paper I argue that this dual measurement task for the national income accounts has become central to economics today. Improved measurement is likely to be necessary if we are to understand how best to encourage economic growth via public funds and intellectual property rights and to understand the sources of wealth. However, constructing these measures is complicated by the fact that it requires a shift in the fundamental paradigm of economics, from the "invisible hand" to "creative destruction." The paradigm shift requires some adjustments as well in how we measure consumption, investment, and capital.

For example, when we measure new consumer products' contribution to growth, we must consider the consumer surplus that arises from them and the fact that consumers must learn about these products. It is not sufficient to measure the change in prices of existing goods; in addition, the assumption that the consumer's perception of the utility of goods is unchanging is untenable.

Second, when new products are created, the real value of the creation,[1] that is, the real intangible investment in their creation, which includes research, develop-

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[1]These are the costs which must be incurred to think up and develop a new product, and inform retailers and consumers about it. It does not include the direct production costs of the new products that are sold to the consumer.

ment, marketing, and institutional innovation, is each time somewhat sui generis. The units of measurement for these investments are not intrinsic, in the sense that the units of measure for a given model of car or laptop are intrinsic. While we might use proxy units of measurement (such as patents or citations), empirical work (e.g. Alexopoulos, 2006; Bessen, 2008) has shown that the link between such units and the economic value of the creation or invention is quite weak. Thus, the units of measurement for deflating intangible investment either must be extrinsic (related to inputs or to alternative uses) or must depend on a research project's future success.

Third, in measuring intangible investments we need to recognize that the assets so created are non-rival. As a consequence, the private value of these assets may diverge from their social value. The social valuation is needed in order to explain growth, while the private valuation is needed in order to explain wealth creation. In addition, the creation and introduction of new products typically requires expenditures on marketing and organizational change that need to be recognized as intangible investments too.

I begin this article by pointing out how the U.S. economy has changed over the past half century, extending work by Griliches (1994). I argue that mass production and tangible investment have become less important, while new products (including goods and services increasingly tailored to the individual such as medical care, education, and business services) and intangible investment have become more important. This switch highlights the importance of measuring the contribution to welfare from new products.

I then present two simple stylized models—one macro and the other micro—to clarify the measurement issues that must be confronted. The macro model is a stylization of growth theory models as set forth, for example, in Aghion and Howitt (1998). The micro model sets forth and expands ideas on new goods, taking off from Diewert (1998, 2007). I conclude with a brief discussion based on the ideas of Hicks (1956).

The Changing U.S. Economy and Growth Measurement

Cell phones and the internet have had a dramatic impact on our lives. Romer (1994) has pointed out that the gains from trade that arise from new products such as these may substantially outweigh the gains from Hecksher–Ohlin–Samuelson factor equalization. A country such as Somalia or even China would be unlikely to produce these products entirely on their own; the ability to trade for them acquires a value that is qualitatively different from a shift in relative prices.

Mass production lies at the heart of the "invisible hand" paradigm; the example of the pin factory and the benefits of division of labor in Adam Smith's *Wealth of Nations* makes this clear. Perfect competition and the law of one price assume that more than one firm can make many copies of the same product. In his American Economic Association presidential address, Zvi Griliches (1994) named "agriculture, mining, manufacturing, transportation and utilities" as being the most readily measurable industries. These are mass production industries. I report Griliches's data from that paper in the first column in Table 1; these industries are shown as declining from 49 percent of U.S. nominal gross national product in 1947 to 31 percent in 1990—from nearly one-half to less than one-third of the economy,

TABLE 1

Zvi Griliches's (1994) "Measurable" Sectors* of GNP (GDP) (nominal, percent)

	(1) Griliches' Data (SIC) % of GNP	(2) Current SNA Data % of GDP
1947	48.7	43.4
1959	44.3	38.4
1969	40.3	34.2
1977	38.2	32.3
1990	30.9	24.8
2007		19.9

Note: *Agriculture, mining, manufacturing, transportation, and utilities.

TABLE 2

The Rising Importance of Measurement Difficulties in Consumption

	% of Total PCE in Current Dollars						Inflation Rate, %
	1947	1959	1969	1977	1990	2006	1977–06
1. PCE	100.0	100.0	100.0	100.0	100.0	100.0	3.6
2. Medical care	4.6	6.5	8.9	11.1	16.5	20.6	5.3
3. Personal business services	3.2	4.1	4.8	5.3	6.5	7.5	4.3
4. Education and research	0.9	1.3	1.9	1.9	2.2	2.6	5.3
5. Religious and welfare	1.3	1.6	1.7	1.8	2.3	2.6	4.2
6. Total hardest-to-measure (sum of items 2–5)	10.0	13.4	17.2	20.1	27.6	33.3	4.9
7. Other	90.0	86.6	82.8	79.9	72.4	66.7	3.2

based on the SIC classification in use at the time. In the second column I show currently available statistics, using the SNA industrial classification now in use, which show these industries declining from 43 percent of U.S. nominal gross domestic product in 1947 to 20 percent in 2007. The products of easily measurable mass production industries have shrunken to just one-fifth of value added.

An alternative perspective on the changing U.S. economy can be seen by looking at the hardest-to-measure consumption expenditures in Table 2. These include medical care, personal business services, education and research, and religious and welfare services (charitable contributions). (These data and categories are taken from the U.S. national income accounts, personal consumption expenditures by type of expenditure, table 2.5.5, as published in the Haver Analytics database in August 2008.) As a group, these have risen from 10 percent of nominal personal consumption expenditures in 1947 to 33 percent in 2006. In this group, precisely what is consumed is in many cases controversial, and there are no market prices in some cases.

The inflation rate in this group of products is measured to be faster than the average for all consumption expenditures; for the period from 1977 to 2006, the average inflation rate is more than 50 percent faster than for other consumption. In part, this faster inflation rate may reflect our inability to measure inflation for these products; in many cases, we measure something closer to inputs than true outputs.

This remains true despite very vigorous efforts on the part of the U.S. Bureau of Labor Statistics and the Bureau of Economic Analysis to improve these measures.

The intrinsic difficulties are compounded by the rapid introduction of new products. Rapid innovation in medical care and in personal business services makes it even harder to develop reasonable inflation measures. The difficulties of measuring output for non-market sectors have been emphasized by Diewert (forthcoming).

When we cannot properly measure inflation, we cannot properly measure growth. A theory of endogenous growth seeks to link intangible investments to growth, which means we need ways of measuring the real growth due to new products. A further difficulty is that the connection between intangible investments and output is bedeviled by uncertainty—the probability distributions of returns to investments in intangibles are highly skewed (Scherer, 1984; Scherer and Harhoff, 2000). The skewed uncertainty in returns implies that learning about rates of return takes place relatively slowly. It also suggests that aggregate data (by firm or industry, say) may be more informative than information at the project level.

New Measures of Investment

New measures of intangible investment expenditures by Nakamura (2003), Corrado *et al*. (2005), and Corrado and Hulten (2008) suggest that these investment expenditures have risen from roughly 4 percent of U.S. GDP in 1977 to 9–10 percent in 2006. The U.S. economy is making an ever larger investment in new product development (Figure 1). These measures of intangible investment include investment in software, research and development, marketing, and organizational change.

The U.S. Bureau of Economic Analysis has, since 1998, included software in its measures of private business investment in the national income accounts. In addition, research and development is now included in a satellite account. Figure 2 shows four concepts of U.S. private business fixed investment, as a nominal

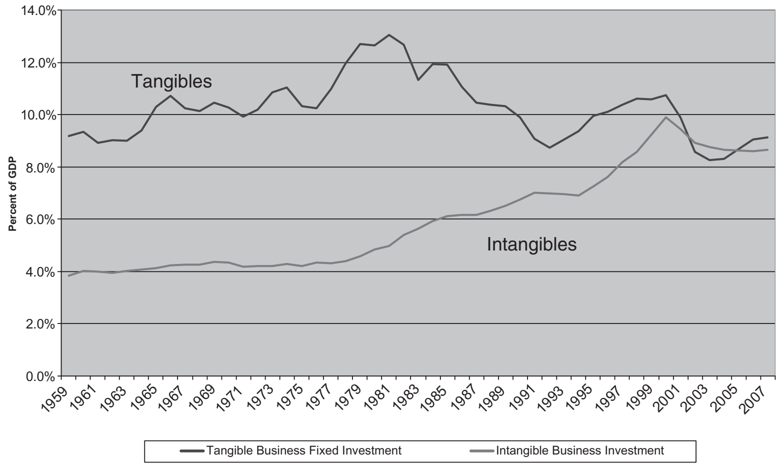

Figure 1. Investment: In the U.S., Intangibles are as Important as Tangibles

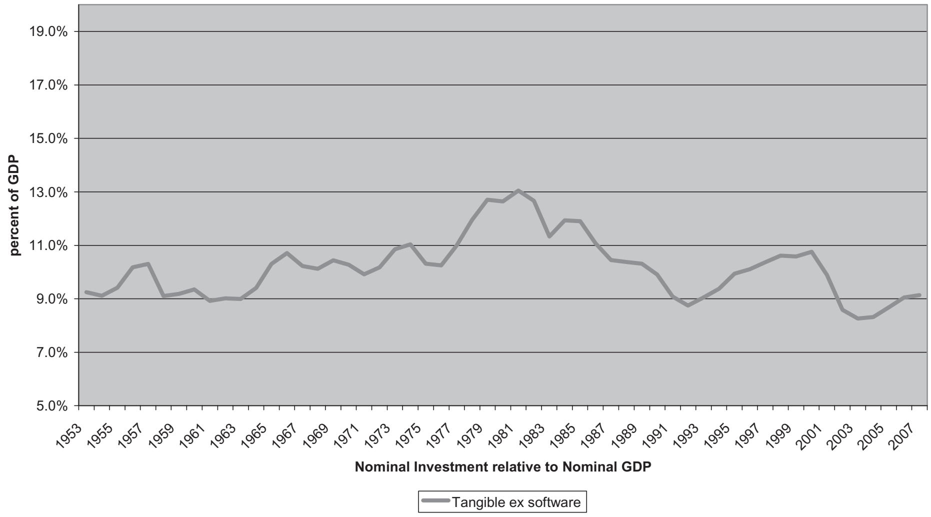

Figure 2a. Four views of U.S. Private Business Investment: View 1: Old Definition—Excludes Software Investment Near Post-War Low

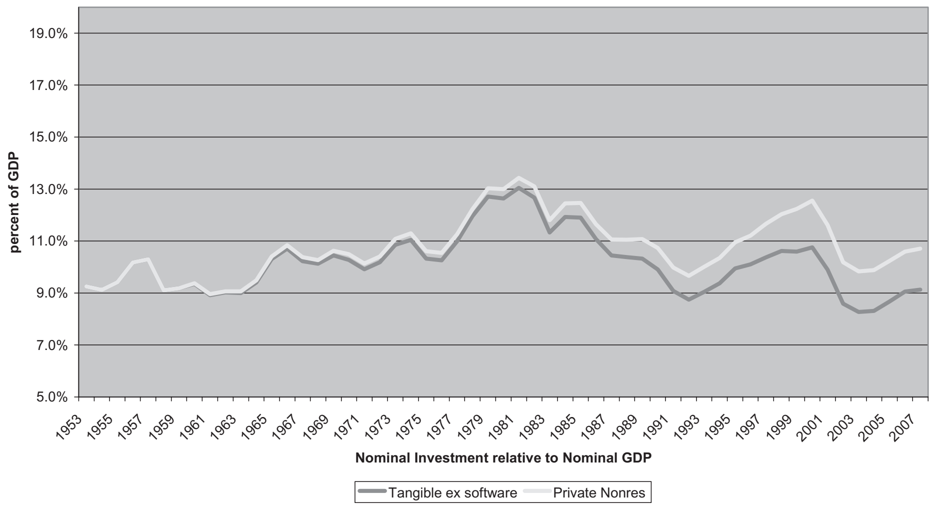

Figure 2b. Four views of U.S. Private Business Investment: View 2: Current Official Definition Investment Near Post-War Average

percent of U.S. nominal gross domestic product: the first is based on tangible investment only, using the pre-1998 definition of business fixed investment. If we were to use this concept, we would think that U.S. investment was at roughly its low for the postwar period. This would depict an economy where investment was faltering. The second concept is the current one in the accounts, which includes software. With this change, we see investment neither rising nor falling; investment in 2006 is about at its postwar average.

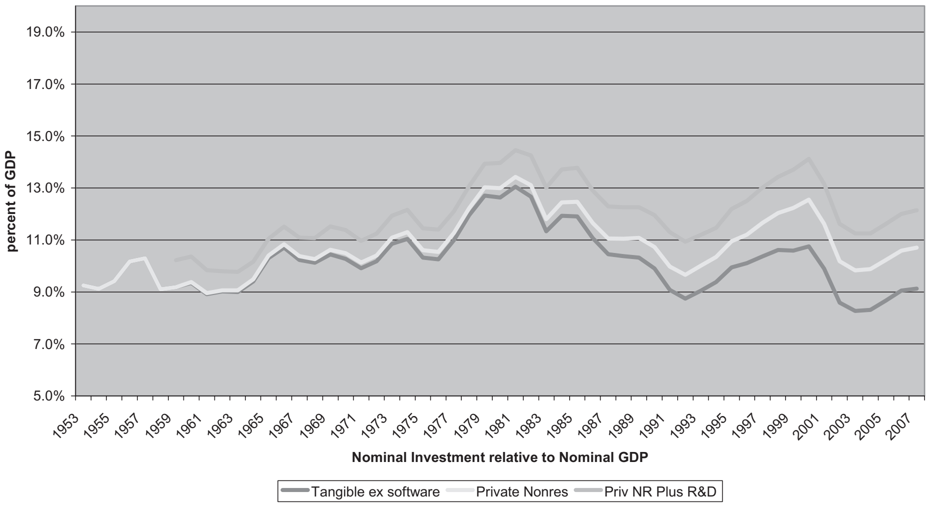

Figure 2c. Four views of U.S. Private Business Investment: View 3: With Private R&D from Satellite Account Investment Above Pre-1977

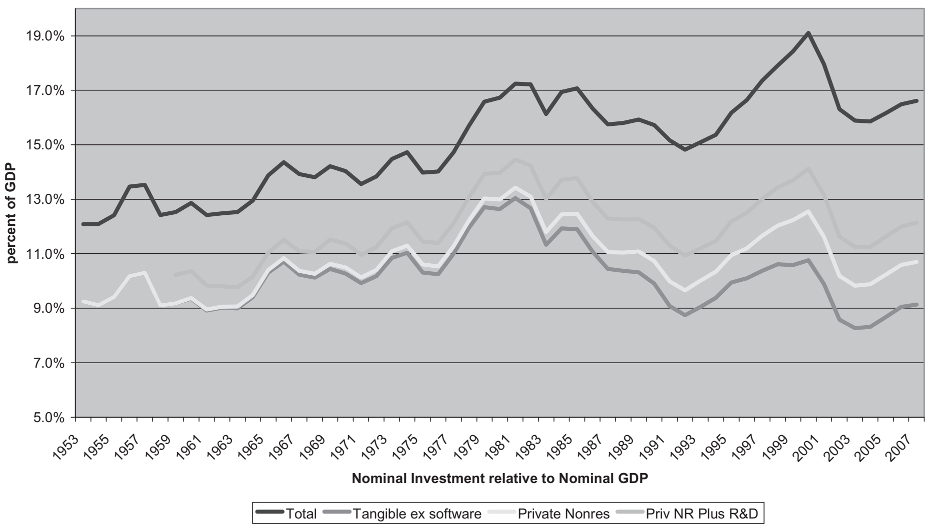

Figure 2d. Four views of U.S. Private Business Investment: View 4: All Intangibles Included Investment Near Post-War High (Excluding Internet Bubble)

The third concept adds in research and development from the BEA satellite account for R&D (Fraumeni and Okubo, 2002); I have updated it with data from the U.S. National Science Foundation on U.S. R&D paid for by firms. This shows investment as a proportion of U.S. GDP to be above the rates before 1977 but appearing roughly trendless since the late 1970s.

The fourth concept adds in marketing and organizational change. This includes data on advertising and other administrative and training expenses associated with new products (Nakamura, 2003; Corrado *et al.*, 2005). While the public may not need to be informed about existing products such as butter and milk, new products such as new video game platforms, organic soy milk, or new mortgage, insurance, or medical products may require learning on the part of consumers. In addition, success at developing and producing new products may require scarce managerial talents and new management methods (see, e.g. Andersson *et al.*, 2009).

When we include marketing and organizational change, total business investment continues to trend upward throughout the period, with 2006 investment at the highest it has been with the exception of the internet bubble (Nakamura, 2003; McCann–Erickson, 2007; Corrado and Hulten, 2008). This shows an economy that is very dynamic and becoming more so, with one-sixth of all expenditures devoted to investment.

These four different views of the economy illustrate the proposition that how we measure the economy deeply affects our views of its dynamics. How we define investment is a crucial element in our understanding of economic growth.

These data show U.S. industries, consumption expenditures, and investment expenditures, all changing very substantially away from mass production and toward new product development, with only some of the new products being ones that can be mass produced. Properly analyzing this economy requires new measures and new theory.

Changing Growth Theory

We want to consider how to measure investment in intangibles, with a view to completing the Solow–Jorgenson program of measuring inputs so as to provide an economic answer to the question: What causes total factor productivity growth? This question was first raised concretely in Solow (1957), and Jorgenson has proposed a series of frameworks, beginning with Jorgenson (1966) and continuing through Jorgenson and Landefeld (2006), for analyzing this question through the national accounts.

The production function that Solow (1956) introduced to study growth takes the following well-known form: $Q_t = A_tF(K_t,N_t)$, where Q is output, A is a measure of the state of technology, F is a constant-returns-to-scale production function, K is capital, and N is labor.

Tangible capital accumulation takes the general functional form g, with both partial derivatives positive, which Solow assumed to take the following linear form, where δ is the rate of depreciation of capital, and I is new gross investment in tangibles:

$$K_t = g(K_{t-1}, I_t) = (1-\delta)K_{t-1} + I_t.$$

Omitting government and net exports for simplicity, Solow's economy has a GDP composed of consumption and investment: $C_t + I_t = Q_t$. Solow (1957) considered technological progress A to be exogenous.

Solow (1957) used this model to analyze the sources of growth by measuring the contributions of added capital, added labor, and improved technology over the period of 1909 to 1949.[2] During that time, real output in the non-farm business sector (a convenient grouping that avoids the measurement problems of the agricultural and government sectors) grew at an annual rate of 2.9 percent.[3] Of this rate, 1.1 percentage points were due to an increase in the total number of hours worked (a product, in turn, of more people working a shorter number of hours each year, with the increase in workers outweighing the shortening of the work year). Of the remaining 1.8 percentage points, Solow reckoned that one-fifth (0.4 percent annually) was due to an increase in capital per worker, that is, people having more equipment with which to do their work. The remaining four-fifths (1.4 percent annually) was due to an increase in technological progress, that is, having superior procedures and equipment. Subsequent work by Edward Denison (1974) for the period 1929 to 1969 pushed up the annual contribution of technology to 1.7 percent annually.[4] The clear-cut evidence from these and other studies is that for the greater part of the 20th century, most of American economic growth per worker was due to improvements in our technology—i.e., to intangible capital growth—rather than to increases in the amount of tangible capital per worker. Exactly how technological advance of this type occurs and to what extent the improvements in technology reside in organization of the workforce (working smarter) or equipment (smarter tools) is unclear. Indeed, technological advance came to be known as the "black box" of economic growth.[5]

The statistical basis that Solow established for our understanding of the economy was, and remains, founded on the national income accounts. National income statistics form our basis for tests of macroeconomic propositions. Hill (1999) has documented how the material basis for investment and assets came to be a foundational assumption and notes that this leaves open the issue of how intangible investments should be dealt with.

In the intervening half century since Solow's paper, progress on addressing the question of how to measure intangible investment has been modest, despite the efforts of many of the best minds of the profession. Beginning with Romer (1986) and summarized in Aghion and Howitt (1998), a new endogenous growth theory has been set forth that emphasizes sunk costs associated with new product development. New theory implies some changes in the national accounts.

Intangible capital accumulation drives growth in technology. Technological progress is created by intangible investment, denoted here by H. Parallel to the general form for tangible capital g, we can set $A_t = h(A_{t-1}, H_t)$. However, the general functional relationship between H and A embodied by h (like g, with positive partial derivatives) has not yet been clarified. See, for example, Madsen (2008). It is generally assumed that in the absence of additional investment in intangibles, technological progress stops; that is, $A = h(A, 0)$. Linearized, this would imply:

[2]The data on labor hours are from Solow's source, later published as Kendrick (1961).

[3]In agriculture, the difficulty is counting the hours of farm owners and their families. In the government sector the outputs—compulsory schooling, criminal justice—are hard to count because they are not priced in the marketplace.

[4]The 1.7 percent figure represents Denison's semiresidual, which includes both pure technological advance and economies of scale—productivity gains due to the increased scale of production. Here, I am lumping the two together. It is now generally recognized that technological advance and economies of scale are, in the long run, inseparable. Output *per person* grew 2.1 percent during this period.

[5]See, for example, the preface to Rosenberg (1982). See also Diewert and Nakamura (2007).

$$A_t = h(A_{t-1}, H_t) = A_{t-1} + H_t.$$

That is, technological progress is not lost and does not depreciate from the social perspective.

Both tangible and intangible investments are valued not in themselves but for their consequences. We value a flour mill and the work of novelists because they contribute to the production of bread and books. These are investments that can result in benefit flows over extended periods of time, and thus, from an accounting perspective, we will typically want to amortize costs over an extended period of time.

However, the lifetime of a tangible product is limited by physical wear and tear as well as by obsolescence. A tangible product is usually rival too—when put to the use of one set of workers, it cannot be used by others. An intangible is non-rival in the sense that it can be used by additional agents without preventing use by those originally using it. Piracy—the unlawful duplication of copyrighted works—is a demonstration of this non-rival character.

Moreover, the intellectual property associated with the private value of an intangible is typically of limited duration. For example, the basic integrated circuit patents—issued to Robert Noyce of Fairchild Semiconductor and Jack Kilby of Texas Instruments—expired in the 1990s. The copyright to Jane Austen's novels—which remain as popular as ever—has similarly expired. In both cases, the social value of these ideas and expressions is greater than ever. But their value as private wealth to their inventors or creators (or their heirs or assigns) has vanished. The social and private values of a tangible tend to coincide, while the social and private values of an intangible almost invariably differ.[6]

Thus, we need a separate equation for intangible private capital accumulation, B. This measure of wealth does depreciate, as the private value of intangible investment is subject to either the expiration of intellectual property rights or to the development of a superior alternative that makes it obsolete. That is, we need an equation of the form:

$$B_t = (1-\delta) B_{t-1} + H_t.$$

The linear functional form here is more likely to be correct, since under profit maximization, the private investment in intangibles is made on the expectation that a dollar of investment will create (at least) a dollar's worth of assets. On the other hand, H is not mass produced. Thus, the real units in which to denominate A and B are not as clear as they are for K. With mass production, capital investment is composed of multiple copies of tools and structures that provide units, such as square or cubic feet, or computers, or tractors, or even copies of software programs. There is a clear sense in which a price can be applied to a given unit of such an investment, whether produced in the same period or in different periods.

Work by Hulten and Hao (2008) shows that intangible investment can account for a large proportion of private wealth creation in the pharmaceutical

[6]A complicating factor is that some intellectual property takes the form of brands and trademarks that are rival and fully appropriable. A customer's knowledge of a new product is, typically, a mixture of knowledge about the product itself (which could be made by any firm), which is non-rival, and about the firm making the product (which may include reputational capital), which is rival.

industry. Work in the accounting literature such as Lev and Sougiannis (1996) and Lev (2001) presents evidence that intangible investment results in private wealth creation across industries.

However, the intellectual properties that form this wealth—ideas and expressions that are created with intangible investment—are by their nature unique. The second invention of a given idea (in cases of multiple discovery) has nothing like the value of the first such invention.

For technological progress, A, the units are determined ultimately by $Q/F(K,N)$. That is, how to properly deflate A may be not be determined until after we have solved the riddle of economic growth. For example, Copeland and Fixler (2008) have proposed a mixture of input prices and patents to construct a deflator for R&D. Yet it is well known that patents alone are not a good quantity measure of R&D output, precisely because patents are not closely related to either economic or social value. Patents weighted by citations may be more closely related to economic value but the linkage is still loose; see Alexopoulos (2006), however, for a new sort of measure based on published works. Neither patents nor patents weighted by citations are available in real time, since patents may not be granted until years after the intangible expenditures that give rise to them, whereas the Alexopoulos measure, relying on published technical books about an important new product or process, is potentially available in real time to the extent that the inventions are important enough to inspire authors to write about them as they are being discovered and brought to market. Discovering how best to deflate intangibles as output—and thus the productivity of the sectors that produce intangibles—is an important project that is unlikely to be complete until we have a relatively settled understanding of endogenous growth.

We may wish to deflate private wealth, B, with an input deflator (reflecting the accumulation of real inputs, from the income side, according to the Jorgenson–Landefeld proposal) or with an output deflator (reflecting the opportunity costs and consumption potential represented by the wealth). But, again, there are no intrinsic units for this private wealth.

We draw two tentative conclusions from this brief look at a stylized model of endogenous growth. First, we no longer have a single measure of capital accumulation; we have two: social capital, to explain growth, and private capital, to explain wealth. Second, there are no intrinsic units by which to deflate either private or social investment as an output.

The private amortization of the private wealth—the temporary monopoly rents granted to the holder of intellectual property—is in principle comparatively straightforward, even though we have not yet solved this problem. Hall (2007) has discussed the considerable empirical problems with measuring depreciation of research and development. The different rates of amortization in turn imply that social and private accounting for national income will differ for both nominal and real values.

Additional challenges for macroeconomic measurement and for international measurement have been pointed out by McGrattan and Prescott (2001, 2008). However, Diewert and Huang (2008) have pointed out that the assignment of amortization costs over the possibly infinite lifetime of an intangible is essentially arbitrary.

Now we consider the microeconomic counterparts of this growth theory. In the next section, we make the point that we need to measure new products' contribution to consumer utility if we are to quantitatively link intangible investments and growth.

Microeconomics and Relative Price and Output Measurement

If economic growth takes the form of a greater quantity of existing goods, then measuring output is not too difficult. If one year there are 10 million cars produced and the next year there are 11 million of the same kinds of cars, then output growth is likely to be in the vicinity of 10 percent (providing composition does not change too much). That kind of output growth is straightforward and easily understandable. However, when economic growth is linked to new consumer products, output measurement becomes much harder.

Work by John Hicks (1956) and Erwin Diewert (1976, 1978) has created a remarkably rigorous foundation for the measures of price and output in the national income accounts when the goods are the same in two periods. A true cost-of-living index gives the cost of maintaining a given level of utility; it is the ratio of expenditure functions that corresponds to a given utility level and two given sets of prices. A money metric utility function can then be constructed that relates the increase in actual expenditures to the true cost-of-living index to tell us by what proportion actual expenditures exceed or fall short of the level necessary to maintain constant utility; these in turn give us the compensating and equivalent variations of Hicks.

Diewert's work (1976) built on that of Hicks by defining a superlative price index as one that approximates a true cost-of-living index for a flexible functional form utility function to the second order. He went on to show that certain families of price indexes are superlative for different utility functions. In a remarkable paper, he then showed that standard superlative price indexes tend to closely approximate one another (Diewert, 1978, but see Hill, 2006). A Fisher ideal index is one such superlative index, so we know that the Fisher ideal price index is likely a very good approximation to the true cost-of-living index. Put another way, the Fisher ideal quantity index likely provides us with a reasonable mapping to consumer surplus.

A true cost-of-living index answers the question: How much must I spend to maintain my living standard? For example, if I must spend 5 percent more money this year to give me the same enjoyment as last year, a true cost-of-living index would rise by 5 percent. Such an index ought to take into consideration the changing relative prices of pre-existing products and new goods and services that become available, because to take advantage of these changes as they occur, I will likely buy a different bundle of goods this year, even if I can still afford to buy what I bought last year.

Relative Price Changes

Suppose I buy only clothing and computer supplies. In 2007, I had $200 to spend on articles of clothing priced at $10 each or on computer supplies also at $10 each, and I bought 10 clothing items and 10 computer items. In 2008, I earn 5 percent more, or $210, and clothing increases in price to $12, while computer supplies decrease in price to $9. I could again buy 10 clothing items and 10 computer items, but I choose to buy 7 clothing items and 14 computer items.

It can be inferred that my standard of living has improved: I could have bought exactly what I did last year, but I did not. I prefer what I am buying, which I could not buy last year, so I am better off. A true cost-of-living index should help us measure, as precisely as possible, this improvement in living standards.

In general, a Laspeyres price index (which compares today's cost of the *base year consumption bundle* to what it cost back then) will tend to understate improvements in welfare and overstate price increases, since its comparison is based on a bundle that was optimized with respect to its prices in the base year, but not in the current year. Similarly, a Paasche price index (which compares today's cost of *today's consumption bundle* to what it would have cost back then) will tend to exaggerate improvements in welfare and implicitly understate price increases. These effects increase as prices diverge further from those in the base year. So when there are divergent price trends, these effects accumulate over time until the base year is updated.

A remarkably good fix for the problem, proposed early in the last century by economist Irving Fisher, is to average the quantity-based index and the price-based index and chain them. Fisher's so-called ideal index multiplies the two indexes and then takes their square root. In our example, this would result in an inflation rate of 2.4 percent, about midway between the Laspeyres price index and the Paasche price index. The UN System of National Accounts has begun emphasizing chain-weighted price and output indexes in its reporting of gross domestic product and components.[7] What Diewert's remarkable result tells us is that this price index will in all likelihood closely approximate a true cost-of-living index and that the real output growth rate will approximate a money metric utility index. For a world without endogenous growth, this fulfills Pigou's goal of finding a measure of national income that represents "a measure of national welfare brought into relation with the measuring rod of money" (see Nakamura, 2007, for additional discussion and references).

Suppose there are new goods. Then we have a more difficult problem. Fisher and Shell (1972) have shown that to approximate the utility index for a new product, what is needed is to price the new product in the period *before its introduction* at the price at which the product would just have zero consumption. (This idea can be traced back to Hicks; see Diewert, 1998.) Determining this Fisher–Shell shadow price is a daunting empirical challenge but not an impossible one. Unfortunately, new goods are an important part of progress in many areas of consumption, such as the health and finance fields.

A simple way to measure the gain from new goods may be to measure the decline in price of an existing, and now obsolete, good of the same type as the new good. If the new good's introductory price relative to the existing good reflects the difference in quality between the new good and existing good, then the decline in price of the existing good will tend to reflect the gain in consumer utility. Here we must be careful to distinguish seasonal goods whose utility changes over time (a

[7]For further discussion, see Triplett (1992).

spring frock, fall apples) from obsolescence. Obsolescence properly reflects the increase in quality of incoming new products.

One way in which economists have modeled new goods is as new varieties. International trade theory, in particular, has seized upon the notion of new varieties to explain the fact that trade occurs mainly among developed countries rather than between developed (capital-rich), high-wage countries and (labor-rich) low-wage countries as the Heckscher–Ohlin–Samuelson theory would suggest. This viewpoint is evident in the undergraduate textbook of Krugman and Obstfeld (2008) and the graduate textbook of Feenstra (2004). However, the issue of how to measure variety remains controversial (see, e.g., Bils and Klenow, 2001; Diewert, 2007). Moreover, Feenstra (1995) makes clear that the measured consumption value of variety is typically biased downward.

A further challenge is posed by the fact that with new goods, consumer learning may be occurring. Indeed, in one of the small but growing number of studies performed on new products, Fisher and Griliches (1995) take as their preferred measure of inflation for a pharmaceutical product the view that learning on the part of consumers is taking place. When consumers learn about the characteristics of a new product, the product on average is likely to rise in value for those who use it. Consider a drug whose efficacy is 1 for N consumers and 0 for another N consumers. If initially it is not known how to tell the first type of consumer from the second, then the drug's expected efficacy will be one-half compared with when it is known how to differentiate the two types. As knowledge increases, the sales base may shrink, but the monopoly price may rise. The Fisher–Shell shadow price must now be calculated using the utility function *after the learning has taken place*.

By contrast, under a standard neoclassical model of perfect competition, learning is ruled out by assumption. Moreover, if goods have existed for some time, learning dynamics are unlikely to be important in the aggregate, even if individual consumers must learn about each product over time, provided that the distribution of knowledge remains stable. Thus, it is most likely that learning will be important in the aggregate when there are new goods. Of course, consumers can learn negative as well as positive things about products. But the greater weight will be on the products that consumers learn positive things about, so the average impact will be biased.

A similar issue applies to network externalities (see, for example, Economides, 1996). With network externalities, products become more valuable as the number of users increases. A classic example is the telephone: one telephone is useless, and as more telephone users sign up, each telephone becomes more useful. In this case as well, the utility of a consumer good rises over time; as a consequence, its price history is not a good guide to its utility: the demand curve shifts outward as the number of users increases. Also, if two products compete to create a network, at first consumers may be reluctant to buy either. This will bias the initial shadow price of the winning product downward. Again, for products that have existed for a long time, network externalities are likely to be unimportant if the network has become stabilized.

Solving these problems is a formidable but not insurmountable economic task. To gain confidence in our solutions, we must use a variety of methods to estimate the value to households of new products. In addition to attempting to measure direct gains from particular innovations as in Fisher and Griliches, there are aggregative methods that can apply to whole industries or consumption categories. Some broad methods have been suggested by Nakamura (1997, 1999).

For example, to understand medical care we have the preliminary work of Aizcorbe and Nestoriak (2008) and Song *et al.* (forthcoming), which measures output by diagnoses and allows us to recognize the rising number of diagnoses that improved medical technology has made susceptible to treatment. To move beyond that, we have to follow the controversial direction of measuring and quantifying medical outcomes, as Cutler and co-authors (e.g., Cutler and Richardson, 1998; Cutler *et al.*, 1998, 2007) have done in a series of studies. A number of different approaches are valuable: studies relating progress in diagnoses to changes in measures of health outcomes; studies relating expenditures on new product development (new drugs and instruments) to intellectual property and private corporate and non-corporate wealth; and studies relating changes in longevity (both unadjusted and quality-adjusted) to measures of consumer welfare.

A Formal Microeconomics Model

The following model formalizes a simple linear demand curve for a new product. Despite its simplicity, the model can incorporate learning and a quality ladder. We investigate the impact of monopoly pricing, competitive pricing, and intangible investment.

There is a representative household with a unit mass of labor, supplied inelastically. The labor can be used to produce two goods, a numeraire good q_0 produced at unit cost, and a second good, q_1, produced at a labor cost of c.

For the representative household (also mass unity) the numeraire good has linear utility and the second good has quadratic utility. By giving the first good linear utility, we keep the marginal utility of income constant, effectively removing income effects. This may be reasonable for modeling cases where the second good is small relative to income. Quadratic utility implies a first order approximation to a general utility function; to this approximation, our model is actually quite general.

At time t the consumer's utility is: $U_t = q_0 + \lambda q_1 - \frac{1}{2} q_1^2$, with $\lambda << 1$. The consumer maximizes this subject to the second good's relative price p, and income y. Household income at time t is $y = q_0 + pq_1$. The demand for the second good will be $q_1 = \lambda - p$.

Invisible Hand Case

To understand the usefulness of this model for our purposes, first consider the "invisible hand" case in which the second good is priced competitively. Cost of production in period t is $c_t < \lambda$; under competition, $p_t = c_t$, and consumption is $\lambda - c_t$. Producing this quantity of the second good requires $c_t(\lambda - c_t)$ units of labor, so production of the numeraire good is $1 - c_t(\lambda - c_t)$. Utility is $1 + {}^1/_2(\lambda - c_t)^2$. The expenditure function to attain utility u is $u - {}^1/_2(\lambda - c_t)^2$. Nominal income, equal to nominal consumption, is 1. Now suppose that cost falls from c_t to $c_t - a$. Nominal income remains 1; a true cost-of-living index to maintain the old utility is $1 + {}^1/_2(\lambda - c_t)^2 - {}^1/_2(\lambda - c_t + a)^2$. Real income under our money metric utility function is $1/(1 - a(\lambda - c_t + 1/2\ a))$.

Diewert's papers tell us that real income under money metric utility will be closely approximated by a Fisher ideal index of real output. What that result tells us—and our model affirms—is that what we measure as real output growth in the national income accounts for existing consumer goods, as cost falls, is approximately the gain to consumers, in the form of consumer surplus, of using a money metric.

Inventor Case

Hypothetically at least, we can model new goods easily. Figure 3 shows the following:

Consider the case of an entrepreneur working alone. Since new product inventions bestow temporary monopoly powers even without patents, and since inventors do also sometimes take out patents, one way of modeling the activity of such an entrepreneur is as follows. We view the inventor as a one-person monopolistic business owned by a household. At time t, the monopolist, owned by the representative household, sells the new good to maximize profit. Facing the household's linear demand curve, the monopolist sets the new good's price to $\frac{\lambda + c}{2}$. At this price, the household buys $q_1 = \frac{\lambda - c}{2}$. Monopoly profit is then $\pi = \frac{(\lambda - c)^2}{4}$.

The monopoly firm at time t produces ${}^1/_2(\lambda - c)$ units of the new good, and the household contributes ${}^1/_2c(\lambda - c)$ units of labor. The household contributes $1 - {}^1/_2c(\lambda - c)$ units of labor to producing the existing good. Nominal income is $1 + {}^1/_4(\lambda - c)^2$; nominal income increases because of monopoly profit.

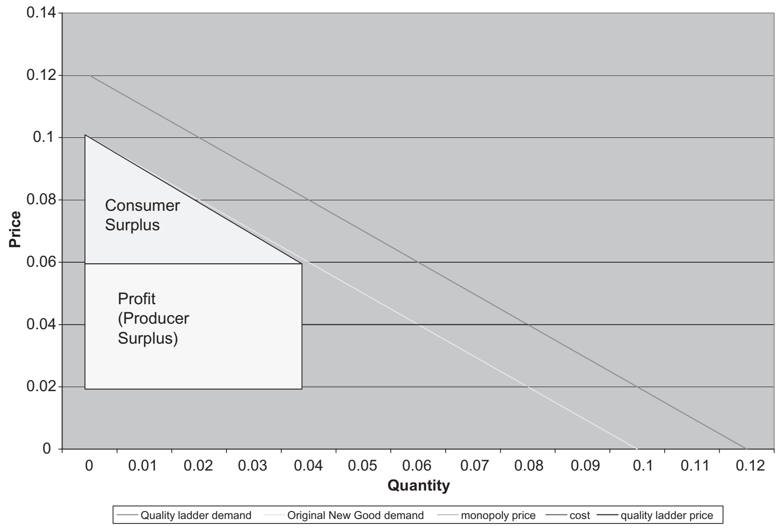

Figure 3. Monopoly Case

TABLE 3

When Does the Fisher Ideal Index Work? Comparing Modeled Money Metric Utility with Fisher Ideal Index Measured Growth in Nine Scenarios

Scenario	New Good Utility: lambda	New Good Cost	Real Growth: Money Metric	Real Growth: Fisher Ideal
1. Competitive: new good introduction	0.1	0.02	0.3210%	0.3195%* (0.0000%)**
2. Competitive: fall in cost	0.1	0.02 to 0.01	0.0851%	0.0850%
3. Monopoly to competitive	0.1	0.02	0.0801%	0.0798%
4. Competitive to monopoly	0.1	0.02	–0.0798%	–0.0798%
5. Monopoly intro new good	0.1	0.02	0.2402%	0.2400%* (0.1600%)**
6. Monopoly quality ladder I	0.1 to 0.12	0.02	0.2598%	0.1502%
7. Monopoly quality ladder II	0.12 to 0.144	0.02	0.2694%	0.0508%
8. Competitive learning	0.1 to 0.12	0.02	0.1803%	0.0000%
9. Monopoly learning	0.1 to 0.12	0.02	0.1348%	0.0449%

Notes:
*Using Fisher–Shell shadow price.
**Ignoring new good pricing.

Consumer utility under the price p is $q_0 + {}^1/_2(\lambda - p)^2$; the second term is the consumer surplus from the purchase of the new good. Under the monopoly price, consumer utility is $q_0 + 1/8(\lambda - c)^2$. The expenditure function $e(p,u)$ of the representative household is the cost of reaching utility level u given price p. This is $u - {}^1/_2(\lambda - p)^2 = u - 1/8(\lambda - c)^2$. The compensating variation $C = e(p_t, u_{t-1}) - e(p_{t-1}, u_{t-1})$, and the equivalent variation $E = e(p_t, u_t) - e(p_{t-1}, u_t)$ will be the same, since the marginal utility of income is the same in both time periods.

Table 3 shows the results when we parameterize λ and c, under perfect competition and monopoly. Scenario 1 shows the case when a new good is introduced without intellectual property rights, so that pricing is competitive, when $\lambda = 0.1$ and $c = 0.02$. In this case, money metric utility rises by 0.3201 percent, and real output, as measured by a Fisher ideal quantity index, rises 0.3195 percent, provided that the Fisher–Shell price is used in the period before the new good is introduced. On the other hand, if the new good price is ignored, then there is no increase in real output.

In the competitive case of a fall in cost, and in simple movements of the new good from monopoly to competitive pricing and vice versa, the Fisher ideal index performs just as one would expect from Diewert's work, very closely approximating the money metric utility.

The same close approximation holds good in the monopoly case, where a firm has intellectual property rights over the new good, if the Fisher–Shell price is used. Figure 3 illustrates this case. However, when the quality of the good rises as a result of learning, whether pricing is competitive or monopoly, the Fisher ideal index does not closely approximate a true money metric index.

Quality Ladder Case

Finally, suppose there is a quality ladder. In case *I*, the original innovator's good is made obsolete by a new, improved version of the same good, produced by

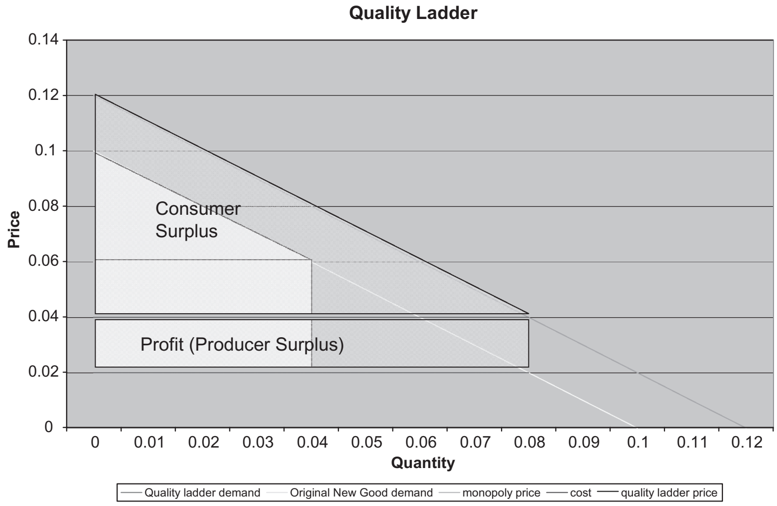

Figure 4. Quality Ladder

a rival. The quality of the new good is 0.12, up 20 percent from 0.1. This is not a drastic innovation, and the rival must price the new good exactly so as to make the original monopolist's good obsolete. The price of this new good is then 0.024. The original monopolist at best can set a break-even price of 0.02, but at this price no one will buy. (Technically, the price of the new good is 0.024 minus epsilon, and we let epsilon shrink toward zero.) If we measure the decline in price due to the new good by the decline in the obsolete good's price from 0.06 to 0.02 in our Fisher ideal price index, we still understate the gain in consumer utility due to the new good (scenario 6). To understand the understatement of consumer utility, in our model as λ rises, the shadow Fisher–Shell price rises, but this is not included in the measured gain. The measured price falls from 0.06 to 0.02; the measured quantity rises from 0.04 to 0.11502. The measured quantity is exactly right, in the sense that the quantity consumed of the new good is 0.096 and its quality has risen by 20 percent, so the quality-adjusted consumption has gone up by 0.11502. Why is there a problem? Because the average price as measured by the Fisher ideal index is the average of 0.06 and 0.02, or 0.04. But the average valuation along the demand curve is from 0.12 (the Fisher–Shell shadow price) to 0.024, or 0.072. Thus the Fisher ideal index is undervaluing the gain in consumer utility. This is shown in Figure 4. Whereas the Fisher–Shell shadow price in Figure 3 enabled us to capture the entire triangle of consumer surplus, the average Fisher ideal index omits the gain in consumer surplus above the old monopoly price (here, 0.06).

In case II, the second monopolist's good is made obsolete by a third innovator, whose good's quality is up another 20 percent, for a quality of 0.144. The second innovator's price falls from 0.024 to 0.020. Even less of the gain in consumer utility is captured by nominal income growth divided by the Fisher ideal price index (scenario 7).

It can readily be shown that the real growth rate of output (with the money metric utility function) is closely approximated by a Fisher ideal quantity index provided that the price of the new good is set so that consumption exactly equals zero, that is, so that the price equals λ.

Learning Case

Learning or network externalities in this model are represented as a change in λ. If λ rises from one period to the next, the impact on real quantity under the money metric is not captured accurately by a Fisher ideal quantity index. These are illustrated for the competitive case and the monopoly case in scenarios 6 and 7.

In brief, mechanical measures of output growth are unlikely to capture the full gains from outward shifts in the demand curve, whether these shifts arise from quality ladders, learning, or network externalities. We need to rely upon econometric methods and articulated economic theory to estimate Fisher–Shell shadow prices and to measure the utility gains represented by movements in demand curves. A key question will be whether it is possible to do so in a "conservative" way so that we are reasonably sure our answer brings us closer to the true underlying consumer surplus gain.

Once we have measured the social value of new products via these or other methods, we need to systematically relate them to intangible investment in their own and related industries. By building up a series of case studies that join private and social measures of intangible capital to private wealth measures and national growth measures, we can develop a sturdy theory of measurement and powerful theories of endogenous growth.

We can extend this model to include intangible investments and intangible capital. As modeled above, under the assumption of free entry into intangible investment, at the margin, expected investment must equal the discounted present value of future returns.

A Scientific Revolution in Economics?

Ever since Adam Smith's *The Wealth of Nations* was published in 1776 (Smith, 1963), most economists have espoused the view that a specific aspect of competition called *perfect competition* is the main spur to economic efficiency. A key aspect of perfect competition is that many firms have access to the set of production possibilities and that economies of scale are sufficiently small that no single firm can dominate any market through low costs. The paradigm of perfect competition carries with it the first welfare theorem: that an economy of perfect competition uses economic resources efficiently. Combinations of firms to monopolize markets are inefficient, and antitrust legislation is an instrument of good government in maintaining an efficient economy. This confident welfare judgment is a hallmark of perfect competition.

A key factor omitted from perfect competition is the potential for new products. Missing from the model of perfect competition is the possibility that firms can use economic resources to develop new products and that firms may simply gain, or might be legally granted, temporary monopoly rights—intellectual property—to the new products they have developed. Similarly missing is learning and technological diffusion—that consumers and producers might need to learn about new products before being able to use them efficiently. The paradigm of perfect competition underlies our national income accounts and is reflected in our theory of price measurement as well as our measures of nominal economic activity. In particular, stocks of private intangible assets have historically not been included in our measures of private wealth, and temporary monopoly, based on intellectual property, is not considered a relevant factor of production.

In *Capitalism, Socialism, and Democracy*, Schumpeter (1942) argued for an alternative paradigm for economic theory in which creativity was the prime mover in a modern economy, and profits were the fuel. He argued that what is most important about a capitalist market system is precisely that it rewards change by allowing those who create new products and processes to capture some of the benefits of their creations in the form of short-term monopoly profits. Competition, if too vigorous, would largely deny these rewards to creators and instead pass them on to consumers, in which case firms would have scant reason to create new products. These monopoly profits provide individual entrepreneurs with the means to fund innovation in response to perceived opportunities and to make them known to the potential purchasers of the new products, or firms with the expected returns to justify supporting in-house research departments.

The alternative paradigm has not been easy to make into a theory that can stand on the same footing as the new one. Beginning with Romer (1986) and Aghion and Howitt (1992), and summarized in Aghion and Howitt (1998), a new endogenous growth theory has been set forth that emphasizes sunk costs associated with new product development. But the new theory implies that the national accounts must be set up somewhat differently.

How should economists and non-economists think about the possibility of a paradigm shift in economics? British economist and Nobel laureate John Hicks (1983) argues that economic science must adapt to the nature of the economy. The growing importance of creative endeavors appears to be what is new in the economy. If so, this represents a significant change in the nature of the U.S. economy, one that is difficult to align with the paradigm of perfect competition. This new economy is highly competitive, but creative destruction is the center of the competition. This implies, in line with Hicks's views, that for understanding economics now, Joseph Schumpeter's "creative destruction" paradigm may be superior to Adam Smith's "invisible hand."

Economic theory and economic measurement will need to develop together if economists are to understand the economic sources of economic growth. The national income accounts framework was initially constructed on the basis of more than a half-century of industry and plant-level studies and a detailed analysis of consumer welfare that incorporated insights from Pigou to Hicks. To develop new economic measures that are able to quantify the economic sources of aggregate economic growth, we will need a similar foundation of industry and plant-level studies.

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