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UPPER BOUNDEDNESS FOR THE MEASUREMENT OF RELATIVE DEPRIVATION

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A new index of relative deprivation is derived axiomatically. Thanks to an asymptotically concave individual contribution function, the new measure provides a sounder quantification to the concept of relative deprivation as conceptualized in the seminal work of Runciman (1966) and better reflects the sociological connotations of the phenomenon.

1. INTRODUCTION

Stouffer *et al*. (1949) coined the locution *relative deprivation* to refer to the feeling of frustration arising in the comparisons with more successful individuals. Runciman (1966, p. 10) characterizes more precisely the phenomenon by stating that an individual "is relatively deprived of X when (i) he does not have X; (ii) he sees some other person or persons, which may include himself at some previous or expected time, as having X; (iii) he wants X; (iv) he sees it as feasible that he should have X." Interpersonal comparisons typically take place within the so-called *reference group*, defined as the subset of society composed by the persons an individual compares to. The criteria adopted for the demarcation of reference groups are demographic lines and "similarities" such as race, gender, and education (Eibner and Evans, 2004) or ethnicity, age, class, religion and political values, and geographical proximity (Bylsma and Major, 1994). The choice of a whole country as reference group is often made for the sake of simplicity.

In the first contribution addressing the quantification of relative deprivation, Yitzhaki (1979) proposes a measure which is equivalent to the absolute Gini index—the product of mean income in society and the Gini index of inequality. Building upon Runciman's remark that "The magnitude of a relative deprivation is the extent of the difference between the desired situation and that of the person desiring it" (1966, p. 10), Hey and Lambert (1980) provide an alternative motivation for Yitzhaki's result by extending his approach to the utility space and considering interpersonal comparisons explicitly (see also Yitzhaki, 1980, 2010).

The focus on one-to-one comparisons proposed by Hey and Lambert is at the base of the common approach to the measurement of relative deprivation in society. A desirable individual deprivation function should first quantify the rela-

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tive deprivation arising in the comparison between individual *i* and individual *j*. A two-stage summation is then implemented. The sum of these values across the members of *i*'s reference group represents her *total* relative deprivation; all individual magnitudes are then added together to obtain the *aggregate* figure of relative deprivation in society.

In this contribution we will abstract from the multidimensional nature of the phenomenon of relative deprivation. For the sake of simplicity we will assume that there exists a unique relevant domain referred to as "income." A new index of relative deprivation is derived axiomatically and is proved to be the only Dalton-type measure as defined by Hagenaars (1987) and Foster and Jin (1998) possessing a certain set of desirable properties. The distinctive feature of the new measure consists of an asymptotically concave individual contribution function which makes the index bounded above yet able to accommodate monotonicity in reference incomes. As argued in the paper, this functional form better reflects the sociological connotations of relative deprivation and enables one to account for Runciman's (1966) so far neglected hypothesis of irrelevance of "fantasy wishes."

The paper develops as follows. In Section 2 we provide the motivation for an index of relative deprivation to be not only less-than-linear but also bounded above. The notation used throughout the paper and the axiomatic framework appear in Section 3, while Section 4 introduces our new index of relative deprivation. Section 5 concludes.

2. Measuring Relative Deprivation: The Desirability of Bounded Concavity

The existing income-based indices of relative deprivation can be usefully categorized according to whether they are sensitive to transfers among better off individuals or not. The latter are based upon linear individual contribution functions where the relative deprivation between the *i*-th and the *j*-th individual is quantitatively equivalent to the income gap between them.[1]

Measures that are sensitive to mean-preserving changes in the income distribution of better-off individuals have been proposed by Paul (1991), Chakravarty and Chattopadhyay (1994), and Podder (1996). For all these measures the individual contribution function is less-than-linear in the reference income.[2] In those works, the justification for concavity has been grounded on the economic principle of diminishing marginal utility of income as well as on the well-established belief in sociologic theory that people's deprivation is more sensitive to advancements achieved by members of the reference group who are closer to their condition (see, *inter alia*, Festinger, 1954). Suppose that a person richer than individual *i* wins the lottery. It seems plausible that the increment in individual *i*'s relative deprivation

[1]See the contributions of Yitzhaki (1979, 1980), Hey and Lambert (1980), Chakravarty and Chakraborty (1984), Berrebi and Silber (1985), Chakravarty (1997), and Chakravarty and Mukherjee (1999). For two alternative characterizations of the Yitzhaki index, see Ebert and Moyes (2000) and Bossert and D'Ambrosio (2006).

[2]In particular, the functional forms chosen by Paul (1991), Chakravarty and Chattopadhyay (1994), and Podder (1996) are, respectively, a rank-order-adjusted radical function, a simpler radical function, and a logarithmic function.

would be larger in the case that this person had an income slightly larger than hers rather than in the case that this person was a millionaire. Indeed, the extent to which people perceive a variation of given amount in others' incomes decreases the larger the reference income. This consideration explains the increase in the relative deprivation experienced by individual i in the comparison with two richer individuals as a result of a rank-preserving progressive transfer[3] between them. Because of that "proximity effect," in the conflict between the two opposite directional changes in individual i's deprivation (an increase when comparing with the recipient and a decrease when comparing with the donor) the impact of the nearer individual getting farther outweighs that of the more distant individual getting correspondingly closer. Further, consider a thought experiment with more sociological overtones: suppose that in period one all N girls in a village have a necklace except for girl G_a, while, in period two, $N - 1$ girls do not have any but girl G_b has $N - 1$ necklaces. Again, girl G_a's relative deprivation is most likely to decrease from period one to period two.

For these reasons, we follow Paul (1991), Chakravarty and Chattopadhyay (1994), and Podder (1996) in the proposal of an index that is monotonically increasing but concave in the reference income. However, we introduce an innovation with respect to the behavior of the index for very large reference incomes. All the indices of relative deprivation proposed thus far in the literature increase indefinitely when the reference income does.[4] We find such behavior unsatisfactory on two counts. Firstly, it fails to reflect a relevant aspect in Runciman's (1966) conceptualization of relative deprivation, neglected in fact by all existing deprivation literature. Despite allowing that "a man may say with perfect truth that he wants to be as rich as the Aga Khan," Runciman considers these as "fantasy wishes" (1966, p. 10) which are concretely irrelevant in evaluating deprivation. Indeed, the unlimited-wants assumption may well lend support to the monotonically increasing behavior of a deprivation function defined in an unbounded income space. Yet, it does not at all imply that the failure in fulfilling this kind of aspiration causes her boundless frustration. Secondly, following the Adam Smith–Amartya Sen linen shirt argument, we can view relative deprivation "in the form of exclusion from social interaction . . . or—more generally—[from] taking part in the life of the community" (Sen, 2000, p. 7); for a conceptual link between relative deprivation and the multidimensional phenomenon of social exclusion, see also Bourguignon (1999), Atkinson and Bourguignon (2001), and Bossert *et al*. (2007). If the individual deprivation function is not upper-bounded then, *whatever* the income distribution and the size of the reference group, the addition of a sufficiently rich individual may increase i's total relative deprivation by *any* magnitude. However, it seems unlikely that the relative deprivation experienced by the poorest

[3]A transfer is called "progressive" if the donor is richer than the recipient, and "regressive" if the recipient is richer than the donor. We ask for the transfer to be rank-preserving merely for expositional simplicity.

[4]Note that the individual deprivation function in Chakravarty (1997) is "indirectly" upper bounded since the interpersonal income gaps are normalized by the mean income in society. However, such normalization implies that the relative deprivation between individual i and individual j is affected, via the change in the mean income, by the variation in the income of a third person; also, the index is insensitive to mean-preserving distributional changes.

member of a very large reference group where income is unequally distributed would increase, say, a thousand times as a consequence of the arrival of *one* particularly well-off individual.

3. Notation and Axiomatic Framework

Let $\aleph$, $\Re_+$, and $\Re_{++}$ be the sets of positive integers, and non-negative and positive real numbers, respectively. For $n \in \aleph$, $\Re^n_{++}$ denotes the positive orthant of the Euclidean n-space $\Re^n$. The finite set of individuals $A = \{1, \ldots, n\}$ with $n \geq 2$ represents society and $y^A = (y_1, \ldots, y_n) \in \Re^n_{++}$ is the corresponding vector of incomes arranged in increasing order, with y_i denoting person i's income. For simplicity, let us apply Yitzhaki's (1982) assumption of closed reference groups[5] to the whole society so that A is the reference group for all individuals. By considering the case of unequal incomes ($y_i \neq y_j \; \forall \; i, j \in \aleph$) and the income ranking of individuals in A as fixed, the exposition of our axioms is significantly streamlined without loss of generality.[6] The deprivation of the i-th individual when comparing to the j-th is evaluated by a function D_i mapping j's income to the non-negative part of the real line, i.e. $D_i : \Re^n_{++} \rightarrow \Re_+$. We apply a partition of A into two subsets according to whether the reference person j is richer than individual i or not: respectively, $A^r_i(y^A) = \{j \in A : y_j > y_i\}$ of size $n_r \in \aleph$ and $A^p_i(y^A) = \{j \in A : y_j \leq y_i\}$ of size $(n - n_r) \in \aleph$. The following are desirable properties for an index of relative deprivation D_i.

Focus (F): For all $y^A, \tilde{y}^A \in \Re^n_{++}$, $D_i(y^A) = D_i(\tilde{y}^A)$ whenever $\tilde{y}^A$ is obtained from y^A by a variation in individual j's income, for some $j \in A^p_i(y^A)$.

Following Runciman (1966) and Sen (1976), individual i suffers from relative deprivation only from the comparison with better-off individuals. Similarly to the measurement of absolute poverty, this axiom does not require relative deprivation to be independent of the *number* of individuals poorer than individual i but merely of their *income distribution*.

Anonymity (A): For all $y^A, \tilde{y}^A \in \Re^n_{++}$, $D_i(y^A) = D_i(\tilde{y}^A)$ whenever $\tilde{y}^A$ is derived from y^A by a permutation.

This property, sometimes referred to as *Symmetry*, postulates the irrelevance of individual identities.

Separability (S): For all $y^A \in \Re^n_{++}$, let $A = A_1 \cup A_2 \cup \ldots A_l$ where $A_h \cap A_k = \varnothing$ for all $h, k = 1, 2, \ldots, l$. Then, denoting by y^{A_m} the income vector of the m-th population subset $A_m \subseteq A$, we have that $D_i(y^A) = D_i(y^{A_1}) + D_i(y^{A_2}) + \ldots + D_i(y^{A_l})$.

[5]Yitzhaki (1982, p. 106) writes that by *closed reference groups* "is meant that if person A is in person B's reference group, then B is in A's reference group." That device allows the notation to be kept simple in the aggregation process.

[6]For example, in this way it will possible to require i's relative deprivation to be insensitive to income changes affecting poorer individuals without distinguishing the case in which an increase in their incomes makes them richer than i; even more valuable simplifications are gained in the exposition of axioms related to income transfers among individuals.

S enables us to split individual i's reference group into mutually exclusive subgroups and derive her relative deprivation as the sum of the deprivations experienced towards each of those subgroups.

Scale Invariance (SI): For all $y^A, \tilde{y}^A \in \Re^n_{++}$, $D_i(y^A) = D_i(\tilde{y}^A)$ whenever $\tilde{y}^A = \lambda y^A$ for some $\lambda \in \Re_{++}$.

This axiom requires relative deprivation to remain unchanged if all incomes are multiplied by a positive scalar.

Monotonicity (M): For all $y^A, \tilde{y}^A \in \Re^n_{++}$, $D_i(y^A) < D_i(\tilde{y}^A)[D_i(y^A) > D_i(\tilde{y}^A)]$ whenever $\tilde{y}^A$ is derived from y^A by augmenting [diminishing] individual j's income, for some $j \in A_i^r(y^A)$.

According to M, the larger the reference income the larger the deprivation experienced by individual i. This assumption allows us to rule out all measures of relative deprivation merely based on the headcount of persons richer than individual i.

Normalization (N): For all $y^A \in \mathbb{R}^n_{++}$, $D_i : \Re^n_+ \to \Re^{UB}_+$, where $\Re^{UB}_+ = [0, n_r/n)$.

N asks for individual relative deprivation to be bounded above by n_r/n for all conceivable income distributions. Alternative normalizations could be devised, but the one we suggest appears valuable since the value of the lowest upper bound for D_i equals the headcount ratio of the persons richer than individual i, i.e. the size of the subset $A_i^r(y^A)$ relative to that of the whole population. It can be also noted that the range specified by axiom N allows relative deprivation to be expressed in average terms. As illustrated in Section 2, this axiom originates from the desirability for an index of relative deprivation to be upper-bounded.

Proximity (PR): For all $y^A, \tilde{y}^A \in \Re^n_{++}$, $D_i(y^A) < D_i(\tilde{y}^A)[D_i(y^A) > D_i(\tilde{y}^A)]$ whenever $\tilde{y}^A$ is the result of a progressive [regressive] transfer between any two individuals in the subset $A_i^r(y^A)$.

The motivation for this requirement has been exhaustively provided in Section 2. Here we observe that M and N together yield concavity for very large values of the reference income but do not allow exclusion of the presence of inflection points at lower income values. PR ensures less-than-linearity for all reference incomes larger than y_i.

4. A New Index of Relative Deprivation

The class of Dalton-type poverty measures in Hagenaars (1987) and Foster and Jin (1998), as well as the relative poverty index in Vaughan (1987) are based upon individual deprivations measured as the (normalized) welfare gap between the desired and the existing situation: DS and ES, respectively. For individual i such index can be written as $D_i^W(DS_i; ES_i) = \dfrac{W(DS_i) - W(ES_i)}{W(DS_i)}$, where $W(\cdot)$ can be thought of as a welfare or utility function. Consider the following function for individual i's relative deprivation, where the existing situation is individual i's own income and the desired situations are the incomes of the individuals in $j \in A_i^r(y^A)$—i.e. $ES_i = y_i$ and $DS_i = y_j$ for some $j \in A_i^r(y^A)$:

$$D_{i,\beta} = \frac{1}{n} \sum_{j \in A_i^r(y^A)} \frac{(y_j)^\beta - (y_i)^\beta}{(y_j)^\beta}. \tag{1}$$

Proposition 1. An index of relative deprivation of the form D_i^W satisfies axioms F, A, S, SI, M, N, and PR if and only if it is identical to $D_{i,\beta}$ with $\beta \in \Re_{++}$.

Proof. See the Appendix.

The interpretation of parameter β is intimately connected with the conceptualization of relative deprivation of Runciman (1966) described above. As β increases, the spread in the deprivation values associated with reference incomes of different magnitudes levels off. In the limit, as β increases indefinitely, individual *i*'s deprivation in the comparison with any one richer individual approaches $1/n$ and her total relative deprivation approaches the headcount of people richer than her. Hence, larger β's strengthen the relative importance of unfulfilled "closer" aspirations and lower the imaginary threshold for Runciman's fantasy wishes. Beyond its direct interpretation as normalized welfare gap between the desired and the existing situation, the simplicity of the functional form of $D_{i,\beta}$ is valuable in terms of empirical applicability. The aggregate figure of relative deprivation in society is obtained by summing up across individuals and normalizing by the population size:

$$D_\beta = \frac{1}{n} \sum_{i=1}^{n-1} D_{i,\beta}. \tag{2}$$

As noted by Hey and Lambert (1980) a normalization by population size provides, strictly speaking, an *average* rather than an *aggregate* figure of relative deprivation in society. The desirability of average figures typically rests in the possibility to compare the (per capita) degree of deprivation in societies with different population sizes. This approach is found unsatisfactory, however, if the focus is the *overall* amount of deprivation suffered in a country. For a discussion of this issue in the realm of poverty measurement, see Chakravarty *et al.* (2006).

5. Conclusions

A deeper reading of Runciman (1966) and the sociological essence of the phenomenon motivate our proposal for a new index of relative deprivation. The new measure is based on an asymptotically concave individual deprivation function allowing the joint accommodation of monotonicity in reference incomes and upper boundedness. The index is axiomatically characterized and presents a simple analytical formulation which fosters empirical applicability.

The significance of our contribution goes beyond the measurement of relative deprivation. Thanks to the asymptotic behavior of our index, new possibilities emerge for the combined evaluation of the absolute and relative aspects of a person's deprivation. Consider, for example, the simple product between our individual relative deprivation function and individual absolute deprivation functions such as those suggested by Chakravarty (1983) or Foster *et al.* (1984). For the absolute poor the resulting index would be a measure of relative deprivation where the asymptote (hence the ceiling to its largest possible value) is set at the value of the person's absolute deprivation. Such an index would maintain the desirable features of $D_{i,\beta}$ as well as incorporate the insight of Atkinson and Bourguignon (2001), who conceptualize a "hierarchy of capabilities" between absolute and relative deprivation with priority granted to the former—an idea which echoes the earlier contribution of Maslow (1942). Research is underway to fully expound and empirically test the system of measurement hinted at.

Appendix

Proof of Proposition 1. By F all incomes of individuals belonging to the subset $A_i^p(y^A)$ are deleted, and by repeated application of S we have that $D_i(y^A) = \sum_{j \in A_i^r(y^A)} f_i(y_i, y_j)$. A allows us to write $f_i = f_e \; \forall e \in \aleph$, $e \neq i$, so that $f_i \equiv f$ $\forall i \in \aleph$. By N, $f(y_i, y_j) = \frac{g(y_i, y_j)}{n}$ with the codomain of $g(\cdot)$ being [0,1), hence $D_i(y^A) = \frac{1}{n} \sum_{j \in A_i^r(y^A)} g(y_i, y_j)$. M and N together imply that for reference individuals belonging to $A_i^r(y^A)$ the function assumes strictly positive values. The reason is that by M $g(y_i, y_j) > g[y_i, (y_j - \varepsilon)]) > g(y_i, (y_j - \varepsilon - \mu)]) > \; \ldots$, with $\varepsilon, \mu, \ldots \in \Re_{++}$ being "small," and by N the minimum image of the function is zero. It follows that for reference incomes larger than y_i the greatest lower bound of the function is zero, which is only approached as $(y_j - y_i) \to 0$. The need for $g(y_i, y_j)$ to be of the form $\frac{w(y_j) - w(y_i)}{w(y_j)}$ and the fact that $w(y_j) > w(y_i)$ ensure that the least upper bound of $g(y_i, y_j)$ is 1; consequently, that the one of $D_i(y^A)$ is $\frac{n_r}{n}$. By writing D_i^W as $1 - \frac{W(y_i)}{W(y_j)}$, SI can be conceived to concern only the ratio $\frac{W(y_i)}{W(y_j)}$, which is therefore required to be of the form $\varphi\left(\frac{y_i}{y_j}\right)$ where $\varphi(\cdot)$ is continuous. Following Aczél (1966, p. 144), the solutions to the above Pexider functional equation are $W(x) = \vartheta(x)^\beta$ for some constants ϑ and β. In order to assume the required form, $g(y_i, y_j)$ must hence be identical to $\frac{(y_j)^\beta - (y_i)^\beta}{(y_j)^\beta}$. M and PR require $\beta > 0$. Substituting in (1) we derive the proposed index $D_{i,\beta}$. This proves the necessity side of the proposition. The sufficiency side can be verified by investigating the functional form of $D_{i,\beta}$. Simple algebraic manipulations make the asymptotic behavior evident, with either y_j or the ratio $(y_j/y_i) > 1$ as independent variable. *QED*.

References

Aczél, J., *Lectures on Functional Equations and their Application*, Academic Press, New York, 1966.

Atkinson, A. B. and F. Bourguignon, "Poverty and Inclusion from a World Perspective," in J. E. Stiglitz and P. A. Muet (eds), *Governance, Equity and Global Markets*, Oxford University Press, Oxford, 151–64, 2001.

Berrebi, Z. M. and J. Silber, "Income Inequality Indices and Deprivation: A Generalization," *Quarterly Journal of Economics*, 100, 807–10, 1985.

Bossert, W. and C. D'Ambrosio, "Reference Groups and Individual Deprivation," *Economics Letters*, 90, 421–6, 2006.

Bossert, W., C. D'Ambrosio, and V. Peragine, "Deprivation and Social Exclusion," *Economica*, 74, 777–803, 2007.

Bourguignon, F., "Absolute Poverty, Relative Deprivation, and Social Exclusion," in G. Kochendorfer-Lucius and B. Pleskovic (eds), *Inclusion, Justice and Poverty Reduction*, Villa Borsig Workshop Series, Berlin, 1999.

Bylsma, W. H. and B. Major, "Social Comparisons and Contentment: Exploring the Psychological Costs of the Gender Wage Gap," *Psychology of Women Quarterly*, 18, 241–9, 1994.

Chakravarty, S. R., "A New Index of Poverty," *Mathematical Social Sciences*, 6, 307–13, 1983.

———, "Relative Deprivation and Satisfaction Orderings," *Keio Economic Studies*, 34, 17–31, 1997.

Chakravarty, S. R. and A. B. Chakraborty, "On Indices of Relative Deprivation," *Economics Letters*, 14, 283–7, 1984.

Chakravarty, S. R. and N. Chattopadhyay, "An Ethical Index of Relative Deprivation," *Research on Economic Inequality*, 5, 231–40, 1994.

Chakravarty, S. R. and D. Mukherjee, "Measures of Deprivation and their Meaning in Terms of Satisfaction," *Theory and Decision*, 47, 89–100, 1999.

Chakravarty, S. R., R. Kanbur, and D. Mukherjee, "Population Growth and Poverty Measurement," *Social Choice and Welfare*, 26, 471–83, 2006.

Ebert, U. and P. Moyes, "An Axiomatic Characterization of Yitzhaki's Index of Individual Deprivation," *Economics Letters*, 68, 263–70, 2000.

Eibner, C. E. and W. N. Evans, "The Income–Health Relationship and the Role of Relative Deprivation," in K. N. Neckerman (ed.), *Social Inequality*, Russell Sage Foundation, New York, 2004.

Festinger, L., "A Theory of Social Comparison Processes," *Human Relations*, 7, 117–40, 1954.

Foster, J. E. and Y. Jin, "Poverty Orderings for the Dalton Utility-Gap," in S. P. Jenkins, A. Kapteyn, and B. M. S. van Praag (eds), *The Distribution of Welfare and Household Production*, Cambridge University Press, Cambridge, 1998.

Foster, J. E., J. Greer, and E. Thorbecke, "A Class of Decomposable Poverty Measures," *Econometrica*, 3, 761–6, 1984.

Hagenaars, A., "A Class of Poverty Indices," *International Economic Review*, 3, 583–607, 1987.

Hey, J. D. and P. J. Lambert, "Relative Deprivation and the Gini Coefficient: Comment," *Quarterly Journal of Economics*, 3, 567–73, 1980.

Maslow, A. H., "A Theory of Human Motivation," *Psychological Review*, 50, 370–96, 1942.

Paul, S., "An Index of Relative Deprivation," *Economics Letters*, 36, 337–41, 1991.

Podder, N., "Relative Deprivation, Envy and Economic Inequality," *Kyklos*, 3, 353–76, 1996.

Runciman, W. G., *Relative Deprivation and Social Justice*, Routledge, London, 1966.

Sen, A. K., "Poverty: An Ordinal Approach to Measurement," *Econometrica*, 44, 219–31, 1976.

———, "Social Exclusion: Concept, Application and Scrutiny," *Social Development Papers No. 1*, Office of Environment and Social Development, Asian Development Bank, Manila, 2000.

Stouffer, S. A., E. A. Suchman, L. C. De Vinney, S. A. Star, and R. M. Williams, *The American Soldier: Adjustments During Army Life 1*, Princeton University Press, Princeton, 1949.

Vaughan, R., "Welfare Approaches to the Measurement of Poverty," *Economic Journal*, 97, 160–70, 1987.

Yitzhaki, S., "Relative Deprivation and the Gini Coefficient," *Quarterly Journal of Economics*, 93, 321–4, 1979.

———, "Relative Deprivation and the Gini Coefficient: Reply," *Quarterly Journal of Economics*, 3, 575–6, 1980.

———, "Relative Deprivation and Economic Welfare," *European Economic Review*, 17, 99–113, 1982.

———, "Is there Room for Polarization?" *Review of Income and Wealth*, 56, 7–22, 2010.