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MEASURING RICHNESS AND POVERTY: A MICRO DATA APPLICATION TO EUROPE AND GERMANY

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In this paper, we define a new class of richness measures. In contrast to the often used headcount, these new measures are sensitive to changes in rich individuals' incomes and, therefore, allow for a more sophisticated analysis of richness. We demonstrate the application of these new measures in analyzing the development of poverty and richness over time in Germany. Moreover, we compare Germany to many other European countries and investigate the impact of tax reforms on poverty and richness. Using these examples, we show the importance of taking the intensity of changes into account and not only the number of people beyond a given richness line (headcount). We propose to use the new measures in addition to the headcount index for a more comprehensive analysis of richness.

1. INTRODUCTION

The problems of financing European welfare states and the increasing pressure of global economic competition have given rise to a debate whether the gap between rich and poor is generally widening, in particular as a consequence of recently implemented reforms of the tax and transfer system. Poverty at the bottom of the income distribution has long been in the spotlight of both academic research and political discussion. Quantitative studies of income poverty have been conducted by, for example, Krause and Wagner (1997) or Hanesch *et al*. (2000) for Germany, and Atkinson (1997) and de Vos and Zaidi (1997) have compared European countries.[1] While it is indisputable that society should ensure a certain minimum subsistence level, the top of the income distribution has just recently become a particular focus of attention, especially in the context of income tax reform. Studies on income richness[2] have been carried out by Krause and Wagner (1997) and Merz (2004). Since 2000, the German parliament has been demanding

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[1]A microsimulation study of the effects of a minimum pension policy to reduce poverty in several European countries can be found in Atkinson *et al*. (2002).

[2]We use richness as a synonym for affluence in this paper.

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regular governmental reports on poverty and richness (see Bundesregierung, 2001, 2005, 2008). Many recent tax reforms proposals with a tendency to lower (marginal) tax rates have been criticized for redistributing from the poor to the rich (see, e.g. OECD, 2006). It is widely believed that the rich are getting richer and the poor are getting poorer.

Given this debate, appropriate summary measures, which provide additional information beyond analyzing the inequality of the whole income distribution, are of key importance for an empirical assessment of the development of poverty and richness. Several poverty indices have been developed in the long tradition of the literature on measuring income poverty. Measuring income richness is a less examined field. As far as we know, empirical studies mainly use the population share of rich individuals (headcount ratio) to measure income richness.[3] However, the headcount is not a satisfying measure for either poverty or richness. It is only concerned with the number of people below (above) a cutoff line. Therefore, if nobody changes his or her status, this index is unaffected by a change in income.

This paper aims to find a more sophisticated measure of richness and contribute to the literature by defining a new class of richness indices analogous to well-known measures of poverty. Our approach is more sophisticated because it takes into account the intensity of the changes and not only the number of people beyond a given richness line. To demonstrate the usefulness of these new measures, we analyze three empirical problems: first, we look at the development of poverty and richness indices over time in Germany (ex post longitudinal analysis). Second, we compare the values of these indices for Germany with different European countries (cross-country analysis). Third, we compute the values of these indices for different reform proposals of the German tax and transfer system (ex ante analysis). Our analysis is based on household micro data provided by GSOEP, EU-SILC, and the microsimulation model FiFoSiM.

The empirical application reveals that our new measures expand the results beyond a pure headcount analysis. We find distinctive differences in our longitudinal analysis of the development of richness in Germany. It depends on the measure whether richness is increasing (headcount) or decreasing (some of the new measures) regarding various time periods. The new measures also clarify differing effects of German reunification. When comparing affluence in countries across Europe, our new measures reveal additional information beyond the proportion of rich people. The composition of "the rich" is also accounted for by the newly defined measures. The cross-country analysis yields different groups of countries according to their values of poverty and richness indices. In general, Eastern and Southern European countries as well as Anglo-Saxon countries are characterized by rather high poverty and richness; whereas Continental and Northern European countries can be distinguished by rather small values of poverty and richness. Finally, our analysis of flat tax reform proposals for Germany shows the difference empirically between concave and convex measurement. These empirical examples

[3]There is a series of recent papers using income shares to analyze the top income distribution; see, e.g. Atkinson (2005), Dell (2005), Piketty (2005), Saez (2005), Saez and Veall (2005), Piketty and Saez (2006), Atkinson and Piketty (2007), Aaberge and Atkinson (2008), and Roine and Waldenström (2008).

demonstrate the usefulness of our new measures; and thus, we suggest using them in addition to the headcount index for a more comprehensive analysis of richness.

The paper is organized as follows. Section 2 describes well-known poverty indices. In Section 3, we define analogue indices of richness and report the main differences. In Section 4, we describe the micro data used for the analysis. Section 5 reports the results of our empirical analysis for Germany and the European cross-country analysis, and Section 6 concludes.

2. Poverty Indices

Many poverty indices have been proposed in the literature.[4] We focus on a class of indices that contains the two most common measures, the headcount and the Foster, Greer, and Thorbecke (1984) (FGT) indices.

Consider a net income distribution $x = (x_1, x_2, \dots, x_n) \in R_+^n$, where n is the number of individuals or households. Let π be the poverty line, e.g. 60 percent of the median income, and $p = \#\{i | x_i < \pi,\ i = 1, 2, \dots, n\}$ the number of poor individuals.

We consider poverty indices φ of the form

$$\varphi(x) = \frac{1}{n}\sum_{i=1}^{n} u\left(\frac{x_i}{\pi}\right), \tag{1}$$

where $u : R_+ \to R_+$ is decreasing on [0, 1) and vanishes on [1, ∞). Examples are:

- The proportion of poor persons (headcount) is defined as

$$\varphi^{HC}(\mathbf{x}) = \frac{1}{n}\sum_{i=1}^{n} \mathbf{1}_{x_i<\pi} = \frac{p}{n}, \tag{2}$$

  with $\mathbf{1}_{x_i<\pi} = 1$, for $x_i < \pi$ and $\mathbf{1}_{x_i<\pi} = 0$ elsewhere.
- The FGT indices are defined by

$$\varphi_\alpha^{FGT}(x) = \frac{1}{n}\sum_{i=1}^{n}\left(\left(1-\frac{x_i}{\pi}\right)_+\right)^\alpha = \frac{1}{n}\sum_{i=1}^{n}\left(\max\left\{1-\frac{x_i}{\pi}, 0\right\}\right)^\alpha, \tag{3}$$

  with $\alpha > 0$ and $(y)_+ := \max\{y, 0\}$. The coefficient α may be interpreted as a parameter of poverty aversion, since greater values of α attach increasingly greater weight to large poverty gaps.
- Other examples of this form (1) are the indices by Watts (1968) and Chakravarty (1983).

3. New Measures of Richness

Before we define new measures of affluence or richness, we give a short review of the sparse literature on affluence measurement. Then we discuss desirable properties of affluence measures.

[4]See Zheng (1997) and Chakravarty and Muliere (2004) for recent surveys of the vast literature.

3.1. *Review of the Literature*

While all poverty indices of the previous section are well-known, little research has been done on the measurement of richness at the top of the income distribution. For an overview of the sparse literature, see Medeiros (2006). The first challenge is to define an affluence or richness line. We define it analogously to the poverty line: a cutoff income point above (below) which a person or household is considered to be rich (non-rich). Similar to the poverty line, it is possible to define the richness line in absolute terms (e.g. 1 million euros) or relative terms (e.g. 200 percent of the median or the mean income).

Let ρ be the richness line and $r = \#\{i|x_i > \rho, i = 1, 2, \ldots, n\}$ the number of rich individuals. In many studies on income richness, only the proportion of rich people is used as a measure of richness:

$$(4) \qquad R^{HC}(\mathbf{x}) = \frac{1}{n}\sum_{i=1}^{n}\mathbf{1}_{x_i>\rho} = \frac{r}{n}$$

with $\mathbf{1}_{x_i>\rho} = 1$, for $x_i > \rho$ and $\mathbf{1}_{x_i>\rho} = 0$ elsewhere. Its definition resembles that of the poverty headcount ratio. However, if we want to compare different tax and transfer reform scenarios, this is not a satisfying definition of richness: if nobody changes his or her status (rich or non-rich), neither a change in a rich person's income nor a transfer between rich individuals will change this index.

Medeiros (2006) defined an affluence gap by

$$(5) \qquad R^{Med}(\mathbf{x}) = \frac{1}{n}\sum_{i=1}^{n}(x_i - \rho)_+ = \frac{1}{n}\sum_{i=1}^{n}\max\{x_i - \rho, 0\}.$$

The advantage of this definition compared to the headcount is that this affluence gap is increasing in income. However, Medeiros' index of richness is not standardized and is an absolute measure of richness. R^{Med} is implicitly proportional in income, i.e. a transfer between two rich people will not change the index. Furthermore, this absolute index is not scale invariant, i.e. multiplying all incomes with a scalar increases R^{Med} by this factor. To overcome these drawbacks, we propose a different approach in the following subsections.

3.2. *Desirable Properties of Richness Indices*

The general idea of measuring richness analogously to poverty is to take into account the number of rich people as well as the intensity of richness. Thereby, an index of affluence is constructed as the weighted sum of the individual contributions to affluence. The weighting function of the index shall have some desirable properties which are derived following the literature on axioms for poverty indices.

Multiple axioms have been suggested in the literature on poverty measurement (see, e.g. Sen, 1976; Foster *et al.*, 1984; Chakravarty and Muliere, 2004). We translate these axioms to the measurement of richness to form the desired properties that an index of affluence should satisfy:[5]

- *Focus axiom:* a richness index should be independent of the incomes of the non-rich.
- *Continuity axiom:* the index should be a continuous function of incomes, i.e. small changes in the income structure should not lead to discontinuously large changes in the richness index.
- *Monotonicity axiom:* a richness index should increase if, *ceteris paribus*, the income of a rich person increases.
- *Subgroup decomposability axiom:* the overall degree of richness may be decomposed into the (population) weighted sum of subgroup richness indices.

The transfer axiom of poverty measurement cannot be translated one-to-one to richness measurement and has to be discussed in more detail. A poverty index satisfies the transfer axiom if the index decreases when a rank-preserving progressive transfer from a poor person to someone who is poorer takes place. This property can be translated to the richness measurement in two different ways:[6]

- *Transfer axiom T1 (concave):* a richness index should increase when a rank-preserving progressive transfer between two rich people takes place.
- *Transfer axiom T2 (convex):* a richness index should decrease when a rank-preserving progressive transfer between two rich people takes place.

The question behind the definition of these two opposite axioms is: "Should an index of richness increase if (i) a billionaire gives an amount x to a millionaire, or (ii) if the millionaire gives the same amount x to the billionaire?" This question cannot be answered without moral judgment. In the following subsection, we define a general class of richness measures which allows the application of both transfer axioms.

3.3. *Defining a New Class of Richness Measures*

3.3.1. General Class

In general, a richness index satisfying the four axioms and either T1 or T2 can be defined analogously to the general class of poverty indices in equation (1) as

$$R(\mathbf{x}, \rho) = \frac{1}{n}\sum_{i=1}^{n} f\left(\frac{x_i}{\rho}\right), \tag{6}$$

where f is a continuous (except for the headcount), strictly increasing function that is either concave (for T1) or convex (for T2). We use strictly increasing transformations because the indices of affluence should be sensitive to higher incomes, i.e. satisfy the monotonicity axiom. To fulfill the focus axiom, a person with an income not greater than ρ should not influence the measure of richness, i.e. $f\left(\frac{x_i}{\rho}\right) = 0$, for

[5]We do not give a formal notation of these axioms but rather state them informally, although they can be easily noted mathematically precise.

[6]Unless otherwise stated, concave and convex are meant in the strict sense.

$x_i \leq \rho$. To fulfill the subgroup decomposability axiom, the index of richness has to be additively decomposable, i.e. the affluence index is a weighted sum of several household subgroups:

$$R(\mathbf{x}, \rho) = \sum_{m=1}^{M} \frac{n_m}{n} R_m(\mathbf{x}, \rho) \tag{7}$$

for any given richness line ρ, M population subgroups indexed $m = 1, \ldots, M$, n_m is the number of people and $R_m(\mathbf{x}, \rho)$ the richness index of subgroup m with the same overall richness line ρ. As mentioned above, an important difference between the measurement of poverty and richness concerns the transfer axiom. In poverty measurement decreasing the income of a very poor person has a larger effect than increasing the income of a less poor person (minimal transfer axiom). Choosing between a concave or convex individual affluence function f is a normative choice. In the following subsections, we define a concave and a convex version of this general class and discuss assets and drawbacks of each approach in Section 3.4.

3.3.2. Concave Class (T1)[7]

If one believes that an affluence index should be less sensitive to changes of very high incomes, i.e. a progressive transfer between rich people increases affluence (concave transfer axiom (T1)), then, formally spoken, f has to be concave. In this case the relative incomes $\frac{x_i}{\rho}$ have to be transformed by a function that is concave on $(1, \infty)$.

Many poverty indices are standardized, so that the individual extent of poverty has the value one if a person has no income at all, and zero if a person has income at or above the poverty line. If we translate this into measurement of affluence, a person with income at or below the affluence line would contribute zero to the affluence index and contribute nearly one if he were "very, very rich." However, there is an obvious difference between the income classes of the poor and the rich: the incomes of the poor are bounded by 0 and π, but the incomes of the rich only have a lower bound ρ. The problem here is that it is not obvious how to transform the incomes of the rich to the unit interval.[8] We transform the incomes of the rich relative to the affluence line, $\frac{x_i}{\rho}$, to the unit interval with a strictly increasing transformation function f, with $\lim_{y \to \infty} f(y) = 1$. Alternative functions that are non-concave, i.e. either linear or convex, do not allow for a standardization and, therefore, individual affluence will be unbounded in these cases. However, "standardisation of the measure in a given range (such as 0 to 1) has only a superficial attractiveness to recommend it" (Cowell, 1995, p. 67).

We define standardized measures of affluence R by

[7]We call the richness index concave if the individual affluence function f is concave, similarly for convex.

[8]If the individual contribution to the affluence index, e.g. for the affluence gap $x_i - \rho$, is divided by $x^{max} - \rho$ (or x^{max}), i.e. the affluence of the richest person with income x^{max}, the index will not fulfill the monotonicity axiom, since increasing the income of the richest person will decrease the index.

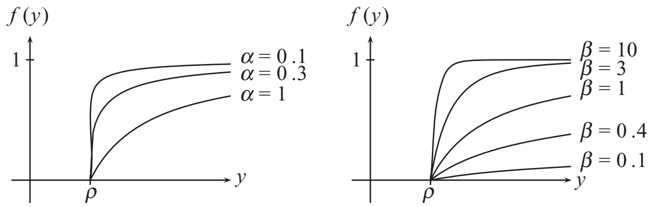

Figure 1. Graphs of $f(y)$ for $R_{\alpha}^{FGT,T1}$ (Left) and R_{β}^{Cha} (Right), for Different α and β

$$(8)\qquad R(\mathbf{x},\rho)=\frac{1}{n}\sum_{i=1}^{n} f\left(\frac{x_i}{\rho}\right),$$

where $f: R_+ \to [0, 1]$ is strictly increasing and concave on $(1, \infty)$.[9]

If we use $f(y):=\left(1-\frac{1}{y}\right)^{\alpha}\cdot\mathbf{1}_{y>1}$, with $\alpha \in (0, 1)$, we obtain an affluence index $R_{\alpha}^{FGT,T1}$, that resembles the FGT index of poverty satisfying T1:

$$(9)\qquad R_{\alpha}^{FGT,T1}(\mathbf{x},\rho)=\frac{1}{n}\sum_{i=1}^{n}\left(1-\frac{1}{\left(\frac{x_i}{\rho}\right)}\mathbf{1}_{x_i>\rho}\right)^{\alpha}=\frac{1}{n}\sum_{i=1}^{n}\left(\left(\frac{x_i-\rho}{x_i}\right)_+\right)^{\alpha},\quad \alpha\in(0,1).$$

The new affluence index increases with a progressive transfer between a rich and a very rich person (T1), since $\left(\frac{x-\rho}{x}\right)^{\alpha}$ is concave on (ρ, ∞), for $0 < \alpha \leq 1$.

We may also employ $f(y)=\left(1-\frac{1}{y^{\beta}}\right)\cdot\mathbf{1}_{y>1}$, $\beta > 0$ and obtain an index analogous to the poverty index of Chakravarty (1983):

$$(10)\qquad R_{\beta}^{Cha}(\mathbf{x},\rho)=\frac{1}{n}\sum_{i=1}^{n}\left(1-\left(\frac{\rho}{x_i}\right)^{\beta}\right)_+,\ \beta>0.$$

Obviously, $f(y)=\left(1-\left(\frac{\rho}{y}\right)^{\beta}\right)$ is concave for $y > \rho$ and $\beta > 0$ (T1). One can easily see that $R_1^{FGT,T1}(\mathbf{x},\rho)=R_1^{Cha}(\mathbf{x},\rho)$ for $\alpha=\beta=1$. Furthermore, for $\alpha \to 0$ and $\beta \to \infty$, $R_{\alpha\to 0}^{FGT,T1}$ and $R_{\beta\to\infty}^{Cha}$ respectively resemble the headcount index R^{HC}.

The advantage of R_{β}^{Cha} over $R_{\alpha}^{FGT,T1}$ is the possibility of constructing indices with "slowly" increasing functions f (see Figure 1) whereas $R_{\alpha}^{FGT,T1}$ is not concave for $\alpha > 1$. Therefore, we use $R_{\beta}^{Cha}(\mathbf{x},\rho)$ in the remainder of this paper as our concave measure of affluence.

[9]A case without standardization is the Watts (1968) measure of affluence, where $\pi = \rho$, $f(y) = \ln(y)$ for $y > 1$.

3.3.3. Convex Class (T2)

If one believes that a convex weighting function f is more appropriate, it is possible to define measures of affluence $R(\mathbf{x}, \rho)$ satisfying the convex transfer axiom T2 by

$$R(\mathbf{x}, \rho) = \frac{1}{n}\sum_{i=1}^{n} f\left(\frac{x_i}{\rho}\right), \tag{11}$$

where f is strictly increasing and convex on $(1, \infty)$.

If we use $f(y) := (y - 1)^{\alpha}$ for $y > 1$, with $\alpha > 1$, we obtain an affluence index $R_{\alpha}^{FGT,T2}$ that resembles the FGT index of poverty satisfying T2:[10]

$$R_{\alpha}^{FGT,T2}(\mathbf{x}, \rho) = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i}{\rho} - 1\right)^{\alpha} \cdot \mathbf{1}_{x_i > \rho} = \frac{1}{n}\sum_{i=1}^{n}\left(\left(\frac{x_i - \rho}{\rho}\right)_{+}\right)^{\alpha}, \quad \alpha > 1. \tag{12}$$

This affluence index decreases with a progressive transfer between a rich and a very rich person (T2), since $\left(\frac{x-\rho}{\rho}\right)^{\alpha}$ is convex on (ρ, ∞) for $\alpha > 1$, We use this index in the remainder of this paper as our convex measure of affluence, which we compare with the concave R_{β}^{Cha}.

3.4. *Concave vs. Convex Measures*

Choosing a concave or convex weighting function is a normative judgment. In this subsection, we discuss arguments in favor of each approach.

3.4.1. Discussion

From a policy-oriented point of view, the interpretation of the different approaches has to be clearly kept in mind when analyzing different scenarios. A concave measure that increases with a transfer from a billionaire to a millionaire, can lead to completely different conclusions compared to a convex measure that increases with a transfer from the millionaire to the billionaire. A more equal distribution of the rich will lead to a more homogenous group with probably more equal interests and, therefore, more influence on decisions affecting society; hence, the concerns that the rich are more visible and important in that population. If one is interested in this internal homogeneity of the rich, the concave approach is more suitable. This view can be somehow seen as the "polarization view," i.e. richness increases when the homogeneity of the top of the distribution increases and, therefore, also the polarization, *ceteris paribus*. This is in contrast to the "inequality view," which would be satisfied if richness increased when inequality (among the rich) increased. This view is satisfied with the convex transfer axiom (T2), which is more appropriate if one is worried about the concentration of richness in the hands of a few. This view is also more consistent with the views of Atkinson (2007) and Leigh (2009), who consider richness as a source of power.

[10]Note that for $\alpha \in (0, 1)$, f is concave and we have a non-standardized version of the concave FGT.

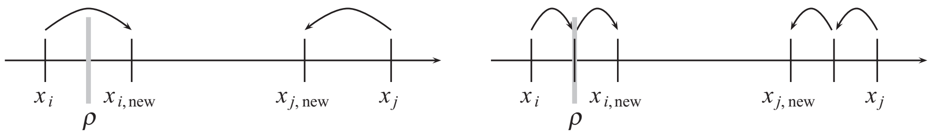

Figure 2. Transfers Between a Rich and a Non-Rich

An argument in favor of the concave view is that people are rather envious of a rich dentist living next door gaining several hundred thousand euros; but they admire superstars far away gaining several millions or marvel at self-made billionaires, like Bill Gates or Steve Jobs.[11] This assertion supports the concave view, i.e. Bill Gates' individual affluence and its contribution to a measure of richness does not increase very much if he receives another million (diminishing marginal utility). Whereas the individual affluence of the dentist would increase tremendously if he received a million.

However, there is a serious drawback to the concave approach: it is not compatible with the weak transfer axiom, i.e. how a progressive transfer from a rich individual to another person (rich or non-rich) will change the affluence index. It is only compatible with the minimal transfer axiom (concerning progressive transfers between two rich people). For poverty analysis, both axioms are almost equivalent because minimal transfer and weak monotonicity imply weak transfer (see, e.g. Donaldson and Weymark, 1986; Zheng, 1997). However, this is not the case for a concave affluence index. It is only true for the convex case. As the weak transfer axiom is considered to be one of the core axioms in poverty analysis (Zheng, 1997), this is a tremendous drawback of the concave approach. The following example shows this problem:

If we split a progressive transfer between a rich person j and a non-rich person i that becomes rich in two transfers (see Figure 2, right hand side), the first transfer decreases the (concave) affluence of person j because of the monotonicity axiom, whereas we do not care about person i because of the focus axiom. The second transfer increases affluence. Altogether, for a progressive transfer with a change from non-rich to rich, it is not clear whether affluence increases or decreases. This is different to the poverty measurement, in which we can split such a transfer in two transfers and poverty decreases in both steps.

Furthermore and especially in the concave case, the choice of the richness line is much more important than in the convex case, since a transfer from a rich to a less rich household has a different impact on the richness measure depending on whether the poorer household is above or below the richness line. The sensitivity of the individual affluence function is very high just above the richness line (the largest slope of f at ρ, see Figure 1). Hence, the choice of the affluence line is sensitive with respect to the specification of the cutoff and should be carefully checked.

[11]Cf. Lockwood and Kunda (1997) and the psychological literature cited within.

3.4.2. Examples

We now illustrate some properties of our new measures with the help of some small examples:

Example 1: A change in a rich person's income should change the measure of richness (monotonicity axiom). Consider two populations with income distribution

$$\mathbf{x} = (5, 5, 5, 11, 11) \text{ and } \mathbf{y} = (5, 5, 5, 100, 100).$$

Let $\rho_{\mathbf{x}}$, $\rho_{\mathbf{y}}$ be 200 percent the median income. Then $\rho_{\mathbf{x}} = \rho_{\mathbf{y}} = 10$ and we obtain

$$R^{HC}(\mathbf{x}, \rho = 10) = R^{HC}(\mathbf{y}, \rho = 10) = 0.400,$$

and

$$R^{Cha}_{\beta=1}(\mathbf{x}) = 0.036 \quad \text{and} \quad R^{Cha}_{\beta=1}(\mathbf{y}) = 0.360,$$

$$R^{Med}(\mathbf{x}) = 0.400 \quad \text{and} \quad R^{Med}(\mathbf{y}) = 36,$$

$$R^{FGT,T2}_{\alpha=2}(\mathbf{x}) = 0.004 \quad \text{and} \quad R^{FGT,T2}_{\alpha=2}(\mathbf{y}) = 32.4.$$

The results for the measures $R^{Cha}_{\beta=1}$, R^{Med} and $R^{FGT,T2}_{\alpha=1}$ all indicate that (the intensity of) richness is lower in population **x**, i.e. $R(\mathbf{x}) < R(\mathbf{y})$.[12]

Example 2: Transfer axiom T1 (T2): A richness index shall be less (more) sensitive to changes of very high incomes. Let

$$\mathbf{x} = (5, 5, 5, 11, 9989) \text{ and } \mathbf{y} = (5, 5, 5, 1000, 9000),$$

where **y** is obtained from **x** by a progressive transfer of 989 monetary units between the two rich individuals. Again we obtain

$$R^{HC}(\mathbf{x}) = R^{HC}(\mathbf{y}) = 0.400.$$

However, the results for the intensity measures are quite different:

$$R^{Cha}_{\beta=1}(\mathbf{x}) = 0.218 \quad \text{and} \quad R^{Cha}_{\beta=1}(\mathbf{y}) = 0.398,$$

$$R^{Med}(\mathbf{x}) = 1996 \quad \text{and} \quad R^{Med}(\mathbf{y}) = 1996,$$

$$R^{FGT,T2}_{\alpha=2}(\mathbf{x}) = 19{,}916{,}088 \quad \text{and} \quad R^{FGT,T2}_{\alpha=2}(\mathbf{y}) = 16{,}360{,}039.$$

According to $R^{Cha}_{\beta=1}$, richness is lower in population **x** which is in line with the concave transfer axiom T1, i.e. a richness index should increase with a progressive

[12]Note that multiplying all incomes by a scalar, e.g. 2, leaves all relative richness measures unchanged (scale invariance), whilst the value of the absolute richness measure R^{Med} doubles.

transfer. The value of the absolute R^{Med} index is the same in both populations, indicating that redistribution takes place only among the rich subpopulation. As expected, $R^{FGT,T2}_{\alpha=2}(\mathbf{x}) > R^{FGT,T2}_{\alpha=2}(\mathbf{y})$, i.e. this richness index decreases with a progressive transfer between two rich individuals.

4. Data and Methodology

Our analyses are based on three different data sources. For the analysis of the development of the indices in Germany, we use panel data from the GSOEP. Data from the EU-SILC are used for the cross-country comparison, whereas data provided by the microsimulation model FiFoSiM are used for the analysis of tax reforms. All three sources are described in the following subsections.

4.1. *GSOEP*

The German Socio-Economic Panel (GSOEP) is a representative panel study of private households in Germany and has existed since 1984. Each wave includes the incomes of the previous year. The 2007 GSOEP consisted of around 12,000 households, a total of more than 30,000 individuals. The data include information on earnings, employment, occupational and family biographies, health, personal satisfaction, household composition, and living situation.[13]

4.2. *EU-SILC*

EU-SILC (European Union Statistics on Income and Living Conditions) is the successor of ECHP data. The EU-SILC collects comparable cross-sectional and longitudinal multidimensional micro data on income and social exclusion in European countries. Since 2005, the dataset has covered 25 EU member states, plus Norway and Iceland, and is the largest comparative survey of European income and living conditions.

4.3. *FiFoSiM*

FiFoSiM is a behavioral microsimulation model for the German tax and transfer system using income tax and household survey micro data. The approach of FiFoSiM is innovative insofar as it creates a dual database using two micro datasets for Germany: FAST01 and GSOEP.[14] FAST01 is a micro dataset from the 2001 German federal income tax statistics containing the relevant income tax data of nearly 3 million households in Germany. For our second data source, GSOEP, see Section 4.1.

The layout of the tax benefit module follows several steps. First, the database is updated using the static ageing technique, which allows controlling for changes

[13]See SOEP Group (2001) and Haisken De-New and Frick (2003) for a more detailed introduction to GSOEP.

[14]Several tax benefit microsimulation models for Germany have been developed in recent years (see, e.g.Wagenhals, 2004). Most of these models use either GSOEP or FAST data. FiFoSiM is so far the first model to combine these two databases.

in global structural variables and a differentiated adjustment for different income components of the households. Second, we simulate the current tax and benefit system of 2006, using the uprated data. This allows us to compute the disposable income for each person, taking into account the detailed rules of the complex tax benefit system. The modeling of the tax and transfer system uses the technique of microsimulation.[15] FiFoSiM computes individual tax payments for each case in the sample, considering gross incomes and deductions in detail. The individual results are multiplied by the individual sample weights to extrapolate the fiscal effects of the reform with respect to the whole population. After simulating the tax payments and the received benefits, we can compute the disposable income for each household. The result of this simulation is the benchmark for different reform scenarios which are also modeled by using the modified database and applying the different tax benefit rules using the technique of microsimulation. A detailed description of the FiFoSiM simulation model can be found in Peichl and Schaefer (2008).

4.4. *Income Concept and Methodology*

Disposable income is defined as market income minus direct taxes and social contributions plus cash benefits (including pensions) for our analyses. The unit of analysis is the individual. To compensate for different household structures and possible economies of scale in households, we use equivalent incomes throughout the analysis. For each person, the equivalent (per-capita) total net income is the household's total disposable income divided by the equivalent household size according to the modified OECD scale.[16]

The poverty (richness) line is chosen as 60 percent (200 percent) of the median equivalent income. Choosing the richness line as twice the median is an arbitrary but common practice (see Medeiros, 2006). It can be argued that choosing the richness line is not as problematic as choosing the poverty line (usually 60 percent of median income) because the upper parts of the income distribution are not as dense as the lower parts. However, in the case of the concave measure, the choice of the richness line can become crucial, depending on the respective application.[17]

To account for regional differences in Germany after reunification, we adjust incomes with the consumer price index of the respective region provided in the GSOEP data. We therefore express all incomes in prices of 2000. To account for regional differences across Europe, we use PPP-adjusted incomes.

[15]Cf. Gupta and Kapur (2000) and Harding (1996) for an introduction to the field of microsimulation.

[16]The modified OECD scale assigns a weight of 1.0 to the head of household, 0.5 to every household member aged 14 or older, and 0.3 to each child under 14. Summing up the individual weights gives the household specific equivalence factor.

[17]As our application is more an illustration of the different concepts rather than a thorough analysis of an economic question, we do not report the results for different richness lines here. The results do change, especially with respect to the magnitude; however, the main findings concerning the differences between the different measures remain robust.

TABLE 1
Development of Poverty and Richness in Germany

Year	$p50$	I_G	φ^{HC}	$\varphi^{FGT}_{\alpha=1}$	$\varphi^{FGT}_{\alpha=2}$	R^{HC}	$R^{Cha}_{\beta=0.3}$	$R^{Cha}_{\beta=3}$	R^{Med}	$R^{FGT,T2}_{\alpha=2}$
1983	13.0	0.264	12.8	3.1	1.4	5.8	0.48	2.7	650	3.2
1984	13.0	0.271	13.0	3.3	1.5	5.7	0.51	2.8	753	5.6
1985	13.1	0.263	12.7	3.2	1.5	5.8	0.43	2.5	601	3.7
1986	13.8	0.253	12.0	3.0	1.4	5.4	0.39	2.4	519	2.0
1987	14.2	0.254	12.1	3.3	1.6	6.1	0.39	2.4	521	1.8
1988	14.6	0.259	12.5	3.3	1.5	5.5	0.41	2.4	631	3.4
1989	14.7	0.262	12.3	3.4	1.8	5.5	0.44	2.6	688	3.7
1990	15.1	0.259	13.1	3.4	1.6	5.6	0.42	2.5	625	2.4
1991	14.7	0.262	12.9	3.5	1.7	5.9	0.39	2.4	564	2.4
1992	14.9	0.268	13.4	3.7	1.9	6.3	0.44	2.7	630	2.2
1993	14.6	0.273	13.3	3.8	1.9	6.9	0.52	3.2	740	2.7
1994	14.3	0.281	14.7	4.5	2.4	7.0	0.53	3.1	773	3.6
1995	14.6	0.275	14.0	4.3	2.3	6.8	0.50	3.0	715	3.0
1996	14.6	0.271	13.3	4.1	2.2	6.8	0.51	3.1	724	2.6
1997	14.5	0.268	13.8	4.2	2.1	7.0	0.42	2.7	596	2.4
1998	14.7	0.266	12.3	3.7	1.9	7.0	0.46	2.9	637	2.1
1999	15.2	0.275	14.0	4.2	2.2	7.3	0.49	3.1	718	2.8
2000	15.5	0.270	13.7	4.2	2.1	6.7	0.46	2.9	684	2.3
2001	15.4	0.288	15.3	4.5	2.2	7.8	0.58	3.5	901	3.8
2002	15.8	0.283	15.6	4.6	2.2	7.6	0.52	3.3	775	2.2
2003	15.6	0.287	16.4	4.9	2.3	7.4	0.52	3.2	777	2.7
2004	15.3	0.291	17.3	5.0	2.4	8.1	0.54	3.4	778	2.7
2005	15.1	0.311	18.8	5.5	2.5	8.4	0.72	4.1	1157	5.6
2006	15.1	0.307	17.3	5.1	2.3	8.7	0.72	4.1	1150	7.0

Note: Values (in % except $p50$ (in 1000 euros), I_G, R^{Med}) of the poverty and richness indices using GSOEP data (equivalent disposable income), modified OECD-Scale, until 1990 only West Germany, incomes in prices of 2000.

5. Empirical Applications

In this section, we show that the new measures are not only theoretically interesting, as explained by the hypothetical examples in Section 3.4.2, but also provide extra explanatory value when analyzing empirical data.[18] We present three empirical applications to illustrate the difference of our affluence measures to the common headcount index.

5.1. *Development of Poverty and Richness in Germany*

The first empirical application is the longitudinal analysis of the development of poverty and richness in Germany since the early 1980s. Table 1 presents the values for the median equivalent income ($p50$), which is used to define the poverty (60 percent) and richness (200 percent) lines, the Gini index of inequality (I_G), the poverty indices (headcount and φ^{FGT}), and several richness measures (headcount, the concave R^{Cha}_{β} and the convex R^{Med}, and $R^{FGT,T2}_{\alpha}$).

[18]Notice, that we show that in fact from year to year, country to country, or scenario to scenario, the different indices develop differently. However, we do not claim that the data are accurate enough so that every change of a measure reflects an actual change in the population. Nevertheless, we show that under usual conditions the choice of indices and the underlying judgments does matter.

The values of various indices for both poverty and richness increased overall during the 23 years of our analysis. Therefore, one could make the case for increasing poverty and affluence in Germany.

When taking a closer look at the development of the indices over time, one has to divide the data into the periods of 1983–90 (only West Germany) and 1991–2006 (East and West Germany). Between 1990 and 1991, there was an increase in the number of rich people but a small decrease in richness measured by $R^{Cha}_{\beta=0.3}$ and $R^{Cha}_{\beta=3}$. Inequality increased according to the Gini index, as well as poverty according to FGT, although the number of poor people decreased. How can this different behavior of the headcount and the concave intensity measures be explained? The increasing number of people above the richness line is due to the overall decreased median income since East Germany is covered by the data as of 1991. The non-standardized absolute Medeiros measure R^{Med} indicates that richness decreased in absolute terms. Both effects, the absolute decline in richness and the higher number of rich people, contribute to the change in the intensity measures. The converging income differences between East and West Germany explain the overall increase in the measures of richness and poverty after reunification.

The new measures of richness can yield distinctively different results than the ordinary headcount index. As for the comparison of 1990 and 1991, from 1996 to 1997 the headcount index indicates an increase in richness, whereas R^{Cha}_{β} indicates a decrease. These effects can be explained by changes in the income structure. If R_{HC} increases while R^{Cha}_{β} decreases, the number of people above the richness line grows (headcount), whereas the intensity of richness and inequality is decreasing. For other years, such as 1995–96, the headcount remains constant, but the concave and convex measures move in different directions. This clearly indicates changes among the distribution of the rich. The increase in the concave measure with the simultaneous decrease of the convex measure reveals that the distribution of the rich is more equal in 1996. This example illustrates the relevance of the choice of the transfer axiom T1 or T2.

All the results show that a more sophisticated analysis of the development of richness yields different results compared to just counting the number of people above a certain affluence line. Therefore, we propose using the new measures, in addition to the headcount index, for a more comprehensive analysis of richness.

5.2. *Poverty and Affluence in Europe*

Our second application is the cross-country comparison of 26 European countries in 2005. These countries include the EU-25 countries, excluding Malta but including Iceland and Norway.[19] Below we use the term "Europe" for this

[19]The EU-SILC countries include: Austria (AT), Belgium (BE), Cyprus (CY), Czech Republic (CZ), Denmark (DK), Estonia (EE), Finland (FI), France (FR), Germany (DE), Greece (GR), Hungary (HU), Iceland (IS), Ireland (IE), Italy (IT), Latvia (LV), Lithuania (LT), Luxembourg (LU), Netherlands (NL), Norway (NO), Poland (PL), Portugal (PT), Slovak Republic (SK), Slovenia (SI), Spain (ES), Sweden (SE), and United Kingdom (UK).

TABLE 2
Poverty and Affluence in Europe

	$p50$	I_G	φ^{HC}	$\varphi^{FGT}_{\alpha=1}$	$\varphi^{FGT}_{\alpha=2}$	R^{HC}	$R^{Cha}_{\beta=0.3}$	$R^{Cha}_{\beta=3}$	R^{Med}	$R^{FGT,T2}_{\alpha=2}$
Europe	13.2	0.339	23.1	8.4	4.5	9.6	0.76	4.4	1073	9.2
AT	17.3	0.242	11.1	2.5	1.1	4.8	0.36	2.2	605	1.5
BE	16.7	0.264	14.9	3.7	1.5	5.0	0.38	2.2	695	17.5
CY	9.9	0.270	12.6	2.5	0.8	7.0	0.54	3.1	593	6.8
CZ	8.6	0.249	11.1	2.3	0.8	4.9	0.42	2.4	402	4.6
DE	14.8	0.255	11.3	3.2	1.6	5.3	0.43	2.4	749	5.9
DK	17.3	0.222	10.5	2.8	1.5	2.7	0.26	1.3	529	2.8
EE	6.4	0.321	18.2	5.7	2.9	10.5	0.88	5.2	549	4.3
ES	12.2	0.323	20.8	7.6	4.4	9.5	0.68	4.2	797	3.6
FI	16.0	0.251	10.4	1.8	0.6	4.6	0.42	2.2	893	10.9
FR	14.9	0.267	12.9	3.0	1.2	6.6	0.51	3.0	788	3.7
GR	11.9	0.343	21.5	6.6	3.3	11.0	1.01	5.5	1267	6.8
HU	6.3	0.330	17.4	5.0	2.4	8.4	0.90	4.3	758	16.2
IE	16.3	0.311	17.4	3.5	1.1	8.8	0.77	4.1	1595	16.7
IS	19.2	0.249	9.2	2.5	1.3	5.2	0.50	2.6	1099	5.4
IT	14.1	0.318	19.7	6.3	3.3	9.0	0.72	4.2	1033	5.4
LT	5.2	0.341	20.6	7.0	3.6	11.3	1.05	6.0	532	4.7
LU	27.2	0.275	13.7	3.1	1.1	7.9	0.60	3.7	1525	2.3
LV	5.0	0.380	22.3	7.9	4.2	13.7	1.38	7.1	809	16.7
NL	16.1	0.257	9.7	3.0	1.8	5.7	0.47	2.7	827	4.6
NO	21.7	0.268	9.6	2.5	1.2	4.0	0.49	2.2	2087	75.1
PL	5.3	0.332	19.3	5.7	2.7	11.4	1.00	5.8	530	5.2
PT	8.9	0.368	18.2	5.3	2.5	14.1	1.62	8.3	1641	16.4
SE	15.2	0.227	11.5	3.9	2.6	2.6	0.17	1.0	271	1.4
SI	13.1	0.227	10.0	2.2	0.8	4.4	0.26	1.7	291	0.6
SK	6.3	0.282	13.2	3.6	1.5	5.3	0.52	2.6	516	35.9
UK	17.2	0.318	19.3	5.8	2.9	9.3	0.72	4.2	1347	7.3

Note: Values (in % except *p50* (in 1000 PPP-adjusted euros), I_G, R^{Med}) of the poverty and richness indices using EU-SILC data (household equivalent disposable income), modified OECD-Scale, 2005, variable poverty and richness lines.

group of 26 European countries. Table 2 presents the values of the various indices where the poverty (60 percent) and richness (200 percent) lines are computed for each country respectively.[20]

The values of these indices vary significantly across countries. In general, poverty and richness are correlated with the level of inequality. The highest (lowest) values of richness in terms of the headcount measure can be found in Portugal (Sweden), whereas poverty is the highest (lowest) in Latvia (Iceland). When looking at more sophisticated measures of richness, these extremes remain. There are, however, differences when comparing particular countries. We see, for instance, that there is a higher percentage of rich people in Luxembourg than in Norway (see R_{HC}). We also find higher values for the concave R^{Cha}_{β} in Luxembourg than in Norway; however, it is the other way round for the convex $R^{FGT,T2}_{\alpha=2}$. One

[20]One should note that due to these variable poverty and richness lines, the values of the European poverty and richness indices cannot be decomposed into the population weighted sum of individual country contributions. For this exercise, fixed European poverty and richness lines are necessary. However, this analysis (which can be obtained from the authors upon request) does not lead to new insights, besides the trivial fact that richer (poorer) countries contribute, *ceteris paribus*, more (less) to European richness and below (above) average to European poverty.

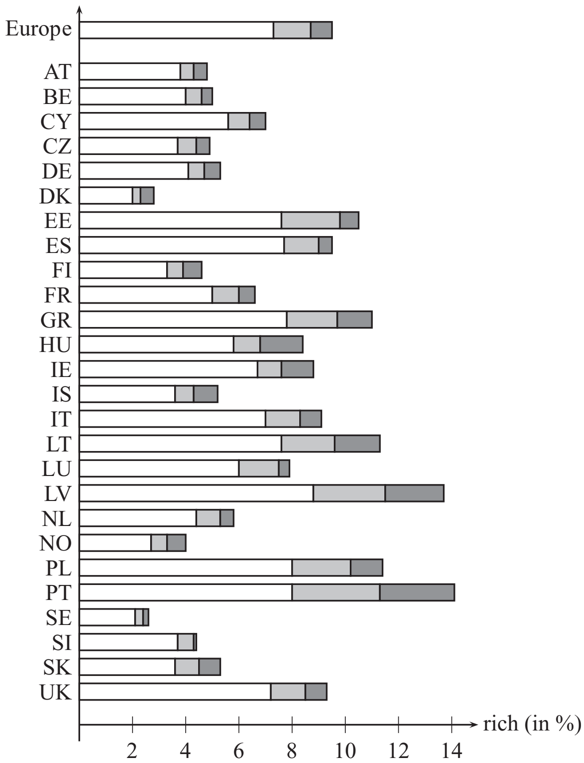

Figure 3. Percent of Population with Income Between 200% and 30% (White), Between 300% and 400% (Light Gray), and Above 400% (Dark Gray)

reason for this finding is the higher portion of people with income above 400 percent of the median income in Norway than in Luxembourg (see Figure 3).

When comparing Germany to Slovakia we find equal shares of rich individuals (R^{HC}) in the respective countries; whilst the concave intensity of richness (R_{β}^{Cha}) appears to be higher in Slovakia. Figure 3 shows that this can be explained by a larger number of "less rich" people with income between 200 and 300 percent median in Germany. Furthermore, because of some people with extremely high incomes in Slovakia (the income share of the top 1 percent is 7.1 percent in Slovakia compared to 5 percent Germany), the convex $R_{\alpha=2}^{FGT,T2}$ index is much higher than in Germany.

The cross-country analysis yields two groups of countries in comparison to the EU average. In most cases, we find neighboring countries in the same group. This classification is in line with the literature on welfare state typologies (see Arts and Gelissen (2002) for an overview). In general, Continental and Nordic welfare states are among the countries with low poverty and richness, whereas Eastern and Southern European countries as well as the Anglo-Saxon countries have high poverty and richness, indicating less redistribution in these welfare state regimes:

(A) High poverty and high richness: Estonia, Greece, Lithuania, Latvia, Hungary, Ireland, Italy, Poland, Portugal, Spain, United Kingdom.
(B) Low poverty and low richness: Austria, Belgium, Cyprus, Czech Republic, Denmark, Finland, France, Germany, Iceland, Luxembourg, Netherlands, Norway, Slovak Republic, Slovenia, Sweden.

This classification of countries and their ranking, in general, remains robust when looking at different measures. However, there are some distinct differences across indices. For instance, Norway, Slovakia, and Belgium are among the countries with relative low richness according to all measures apart from the convex $R_{\alpha}^{FGT,T2}$, which ranks them first to third. On the other hand, Estonia, Spain, Lithuania, and Poland are ranked much lower according to this measure.

In comparison to the other countries, the $R_{\alpha=2}^{FGT,T2}$ index for Norway is extremely high. This high value is due to a few very, very rich individuals in the Norwegian dataset, which drive the results in the case of a convex measure. A concave measure is less sensitive to whether very high incomes are well represented in the data or not.

5.3. *Poverty and Richness Effects of Flat Tax Reform Proposals in Germany*

The introduction of flat rate tax systems is widely seen as a reform which may boost efficiency, employment, and growth through simplification and higher incentives (see Keen *et al*., 2008). However, these efficiency effects do not come for free. Inequality is likely to increase as a consequence of a flat tax reform, implying redistribution from the poor to the rich. We try to shed some light on the question whether the gap between rich and poor is widening as a consequence of a flat tax reform by analyzing the effects of two flat tax reform proposals on poverty and richness in Germany as a third application of our measures of richness.[21]

5.3.1. Current System and Scenarios

The basic steps for the calculation of the personal income tax under German tax law are as follows.[22] The first step is to determine a taxpayer's income from different sources and to allocate it to the seven forms of income. For each type of income, the tax law allows for certain income related deductions. The second step is to sum up these incomes and to take into account losses carried forward to obtain the adjusted gross income. Third, deductions, such as contributions to pension plans or charitable donations, are taken into account, which finally results in the taxable income. The income tax is calculated by applying the tax rate schedule to the taxable income. The tax liability T is calculated on the basis of a mathematical formula which, as of 2006, is structured as follows:

[21]In this paper, we focus on questions of poverty and richness. We have analyzed the effects of these tax reforms on equity and efficiency elsewhere (see Fuest *et al*., 2008).

[22]A more detailed description of the (modeling of) German tax rules can be found in Peichl and Schaefer (2008).

TABLE 3

Poverty and Richness Effects of Flat Tax Reform (variable poverty and richness lines)

	$p50$	I_G	φ^{HC}	$\varphi^{FGT}_{\alpha=1}$	$\varphi^{FGT}_{\alpha=2}$	R^{HC}	$R^{Cha}_{\beta=0.3}$	$R^{Cha}_{\beta=3}$	R^{Med}	$R^{FGT,T2}_{\alpha=2}$
Status quo	17945	0.289	18.1	4.1	1.4	8.0	0.55	3.4	917	2.0
Flat tax 1	17657	0.298	17.8	4.0	1.4	8.7	0.66	4.0	1136	3.2
Flat tax 2	17968	0.287	17.9	4.1	1.4	7.6	0.54	3.3	920	2.3

Note: Values (in % except *p50*, I_G, R^{Med}) of the poverty and richness indices using FiFoSiM (variable poverty and richness lines).

$$(13)\quad T(x)=\begin{cases}0, & \text{if } x\le 7{,}644,\\ \left(883.74\cdot\dfrac{x-7664}{10000}+1500\right)\cdot\dfrac{x-7664}{10000}, & \text{if } 7{,}664<x\le 12{,}739,\\ \left(228.74\cdot\dfrac{x-12739}{10000}+2397\right)\cdot\dfrac{x-12739}{10000}+989, & \text{if } 12{,}739<x\le 52{,}151,\\ 0.42\cdot x-7914, & \text{if } 52{,}151<x,\end{cases}$$

where x is the taxable income. For married taxpayers filing jointly, the tax is twice the amount of applying the formula to half of the married couple's joint taxable income.

The modeled flat tax reform scenarios are revenue-neutral combinations of tax base simplification with single tax rates as described in Fuest *et al.* (2008). Tax base simplification is modeled as the abolition of a set of specific deductions from the tax base included in the German income tax system.[23] The first one has a low marginal tax rate of 25.1 percent and a basic tax allowance of 7664 euros (which corresponds to the current tax system). The second flat tax scenario has a higher marginal tax rate of 32 percent and a higher allowance of 12,100 euros.

5.3.2. Results

The effects of the flat tax reform scenarios are calculated in the microsimulation model FiFoSiM. In this paper, we abstract from behavioral adjustments, i.e. we assume that the economic agents do not change their labor supply or savings in response to these tax reform scenarios. Table 3 presents the values of the measures for the different tax reform scenarios in the manner of the governmental reports on poverty and richness. In this methodology, the median and, therefore, the poverty and the richness line vary in each case.[24]

The values for the poverty indices do not change significantly for the revenue-neutral reform scenarios in comparison to the status quo, although the median

[23]Our choice of simplification measures is influenced by the German policy debate about existing tax breaks and deductions. Naturally, this analysis is restricted by the availability of data. The complete tax base adjustment bundle consists of the abolition of deductibility of commuting costs, the abolition of the saver's allowance, the restriction of labor income related expenses to €1000, as well as the abolition of several tax allowances for age, single parents, and children, and deductions for tax accountancy costs, church tax, and donations (charitable and for political parties).

[24]Our results when using the same methodology are in line with these reports (see Bundesregierung, 2001, 2005).

TABLE 4

Poverty and Richness Effects of Flat Tax Reform (fixed poverty and richness lines)

	φ^{HC}	$\varphi^{FGT}_{\alpha=1}$	$\varphi^{FGT}_{\alpha=2}$	R^{HC}	$R^{Cha}_{\beta=0.3}$	$R^{Cha}_{\beta=3}$	R^{Med}	$R^{FGT,T2}_{\alpha=2}$
Status quo	18.1	4.1	1.4	8.0	0.55	3.4	917	2.0
Flat tax 1	19.5	4.3	1.5	8.4	0.63	3.7	1086	3.0
Flat tax 2	17.9	4.1	1.4	7.6	0.55	3.3	924	2.3

Note: Values (in % except R^{Med}) of the poverty and richness indices using FiFoSiM (fixed poverty and richness lines).

income and thus the poverty line vary.[25] The richness indices, however, change due to the fact that the tax base simplification measures affect higher income groups the most. For instance, the effective marginal tax rates are reduced for the highest income decile in scenario 2, whereas in the first scenario they are reduced in the three highest income deciles. The two reform scenarios change the R^{Cha}_{β} indices in different directions. The flat tax with a high marginal rate and basic allowance (flat tax 2) very slightly decreases these indices, whereas the flat tax with a low marginal rate and basic allowance (flat tax 1) increases the R^{Cha}_{β} measures. The increase in richness is unanimously confirmed by R^{HC}, R^{Med}, and $R^{FGT,T2}_{\alpha}$, but only for flat tax 1. For flat tax 2, R^{HC} decreases, R^{Med} remains almost unchanged, whereas $R^{FGT,T2}_{\alpha}$ increases. The latter effect for flat tax 2 can be explained by its design. The rather high marginal rate of 32 percent implies that only the "super rich" are getting richer when compared to the status quo; whereas some people who were rich before change their status to non-rich. As $R^{FGT,T2}_{\alpha}$ puts greater emphasis on the very top of the distribution, its value is increasing; whereas R^{Cha}_{β}, which gives more importance to people just above the richness line, decreases.

A drawback of the approach of recomputing the (variable) poverty and richness lines is that an increasing measure of poverty (or a decreasing index of richness) does not necessarily indicate a worse situation for people with low (high) incomes as a result of the changing poverty (richness) line. To account for this weakness of relative measurement, we fix the poverty and richness lines at the value of the status quo taxation and recalculate the measures (see Table 4).[26]

Not surprisingly, there is once again no large variation in the values of the poverty measures for the flat tax with a high basic allowance (flat tax 2). However, the flat tax with the smaller basic allowance increases the poverty indices due to the abolishment of certain deductions such as commuting costs and tax free bonuses for irregular working hours.

The flat tax alternative with a low marginal rate and basic allowance (flat tax 1) increases the richness indices. The scenario with a high marginal rate and basic allowance (flat tax 2) decreases the headcount measure as well as R^{Cha}_{β}; whereas

[25]When analyzing poverty, one has to take into account that the lowest deciles of the income distribution seldom pay income taxes, since there are always high basic tax allowances. Therefore, a reduction of income poverty through tax reforms is naturally restricted. A reform of the benefit system, like an increase in the social assistance for instance, would be a more effective measure. Furthermore, the minor distributional effects can be explained to some extent by the revenue neutrality of the flat tax reforms.

[26]Median income and Gini coefficient remain unchanged compared to Table 3.

TABLE 5
Subgroup Decomposition

	φ^{HC}	$\varphi^{FGT}_{\alpha=1}$	$\varphi^{FGT}_{\alpha=2}$	R^{HC}	$R^{Cha}_{\beta=0.3}$	$R^{Cha}_{\beta=3}$	R^{Med}	$R^{FGT,T2}_{\alpha=2}$
Single, no children (9%)								
Status quo	26.3	7.0	3.1	6.2	0.49	3.0	836	2.2
Flat tax 1	26.6	7.1	3.1	6.9	0.59	3.4	1073	3.5
Flat tax 2	26.1	7.1	3.1	6.2	0.51	3.0	894	2.7
Single Parent (2%)								
Status quo	51.8	8.8	2.3	0.0	0.0	0.0	0	0.0
Flat tax 1	53.1	8.9	2.3	0.2	0.0	0.0	2	0.0
Flat tax 2	51.2	8.8	2.3	0.0	0.0	0.0	0	0.0
Couple, no children (52%)								
Status quo	14.4	3.7	1.3	11.4	0.82	5.1	1354	3.1
Flat tax 1	14.8	3.7	1.4	11.8	0.89	5.3	1546	4.3
Flat tax 2	14.3	3.6	1.3	11.1	0.79	4.9	1343	3.3
Couple with children (37%)								
Status quo	19.5	3.9	1.1	4.0	0.23	1.5	371	0.7
Flat tax 1	22.7	4.1	1.2	4.3	0.29	1.8	502	1.2
Flat tax 2	19.1	3.8	1.1	3.3	0.23	1.4	391	0.8

Note: Values (in % except R^{Med}) of the poverty and richness indices using FiFoSiM (fixed poverty line) for subgroups.

$R^{FGT,T2}_{\alpha}$ increases, i.e. the difference between the concave and the convex measure persists when comparing flat tax 2 to the status quo.

This latter example clearly shows that it depends on normative judgments whether flat tax scenario 2 increases or decreases richness. Therefore, if one has to judge which tax scenario is preferable, for example in a sense that affluence should be limited, the decision depends on the chosen measure and its underlying assumptions.

To further analyze the effects of the flat tax reforms on population subgroups, we decompose the measures into four groups according to the family status of the household: single, single parents, and couples with and without children. The results are presented in Table 5. The population shares are given in parentheses.

As expected, poverty is highest among single parents, of whom more than half are considered poor. Poverty is lowest among couples without children. Apart from couples with children, the indices change little after the introduction of a flat tax and the changes are in line with the results for the overall population.

Richness is highest among couples without children; whilst single parents are never among the rich. We once again see that R^{Cha}_{β} is in line with the headcount variation. As before, the convex index $R^{FGT,T2}_{\alpha=2}$ moves in the opposite direction when flat tax 2 is compared to the status quo, as it puts more emphasis on the "super rich."

6. Conclusions

In this paper, we propose a new general class of affluence measures. In contrast to the headcount, the values of these new indices increase with a rich person's income. We apply several indices to longitudinal data of Germany and cross-country data of Europe, and we simulate different flat tax reform scenarios for Germany. We find that our measures of richness are a useful addition to pure headcount calculations.

We discover distinctive differences between headcount ratios and the new affluence indices in our longitudinal analysis of the development of richness in Germany. The new measures also clarify differing effects of the German reunification. While the number of rich people increased (from 1990 to 1991) as a result of the decreasing median due to the inclusion of the East German population, the absolute measure of richness decreased as well as the concave measures, whereas the convex measure remained unchanged.

Our new measures reveal additional information beyond the proportion of rich people, as the composition of "the rich" is also accounted for. This becomes evident when affluence in countries is compared across Europe. We show, for instance, structural differences between the rich populations of Germany and Slovakia, although the headcount index shows equal values in both countries. We also find that poverty and richness (compared to the local median) are rather high in Eastern and Southern European as well as Anglo-Saxon countries, but low in Continental and Nordic countries.

Not surprisingly, we find that the (revenue-neutral) flat tax reform scenarios have only small effects on poverty but some influence on richness. The results show the difference between concave and convex measurement. A flat tax with a low marginal tax rate and basic allowance decreases the concave measures, whereas a flat tax with higher tax parameters increases them. The convex measure increases in both scenarios. This clearly shows the normative implication of the question whether a progressive transfer between rich individuals should increase or decrease an affluence index.

Our general class of measures allows for concave and convex weighting functions for individual affluence. The empirical applications, however, show that the different conclusions drawn from concave and convex measures are, in general, not that large. Nonetheless, striking differences become evident in particular cases. However, a qualification has to be made. When comparing concave and convex indices, one has to take into account that we base our analysis on survey data. If we use a convex function instead of a concave one, the estimates of the affluence indices depend greatly on the very high incomes. In many datasets, however, high incomes can be excluded (due to non-response), top-coded, anonymized, or less representative than other income ranges. One solution could be to construct a dataset of the highest incomes based on tax return data. However, it is generally not advisable to compare tax return data across countries, as income tax systems differ considerably (see, e.g. Atkinson and Piketty, 2007). Constructing a homogenous cross-country top income dataset is subject to further research and could lead to important insights for future cross-country comparisons. To still be able to apply the $R(\mathbf{x},\rho)$ measures, information about the whole income distribution, or at least the median income (for the richness line), is indispensable. Therefore, it would be useful to merge such a top income dataset with information on the bottom of the income distribution. As a result, a reliable picture of the entire income distribution can be obtained (see, e.g. Bach *et al.*, 2009).

To sum up, our analysis shows that the measurement of richness is a complex field. The empirical applications reveal that the new richness measures presented here lead to different results compared to the headcount index for some of the time periods, countries, and reform scenarios. Our approach accounts for changes in the intensity of high incomes and, therefore, allows for a distinct analysis of structural changes at the top of the income distribution. We propose using the new measures in addition to the headcount index for a more sophisticated analysis of richness. We leave the choice of the weighting function up to the researcher, depending on the research question and the available data: this is ultimately a normative decision.

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