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WELFARE MEASUREMENT, INVOLUNTARY UNEMPLOYMENT, AND HETEROGENEITY

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This paper concerns welfare measurement in an economy with union wage setting, where the equilibrium is characterized by unemployment. Contrary to results derived in the first best, the current value Hamiltonian is not an exact welfare measure in an economy with unemployment. Instead, the welfare measure also depends on "employment effect," which are caused by the discrepancy between supply and demand in the labor market. In addition, since unemployment gives rise to heterogeneity, distributional effects will also characterize the welfare measure.

1. INTRODUCTION

The welfare economic foundation for social accounting originates, to a large extent, from Weitzman (1976), who was able to show that the comprehensive net national product (NNP) constitutes an exact welfare measure in a dynamic representative-agent economy. The concept of comprehensive NNP is meant to imply an extension of the conventional NNP, where the extension is designed to reflect all relevant aspects of consumption and capital formation for society. To be more specific, Weitzman's result means that, if the resource allocation is first best and the technology stationary, then the current value Hamiltonian underlying the optimal resource allocation will be proportional to the present value of future utility facing the representative consumer.[1] The current value Hamiltonian is, in turn, interpretable as the comprehensive NNP in utility terms in the sense of representing the instantaneous utility of all current consumption plus the utility value of all current net investments. Furthermore, Weitzman's result has clear practical relevance, as it seems to imply that we can measure welfare in a dynamic economy solely by using information that refers to the time when the measurement is conducted (and in some special cases solely by observables).

The purpose of this paper is to analyze the consequences of involuntary unemployment in the context of social accounting. Such a study is clearly moti-

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[1]In the context of environmental economics, the comprehensive NNP is sometimes referred to as the "green NNP." See Hartwick (1990) for an early application of the theory of social accounting to an economy, where environmental and/or natural resources are important parts of the economic system. There is also an empirical literature that tries to quantify the green NNP or parts thereof; see, e.g., the literature review by Vincent and Hartwick (1997) and the more recent application to Sweden by Ahlroth (2003). See also Oulton (2004) for a comparison between output measures and a (potential) welfare measure of NNP-type.

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vated because of the high unemployment rates in many countries, while the consequences of unemployment for social accounting are not yet fully understood. It is also motivated because earlier comparable literature on welfare measurement in imperfect market economies[2] often focuses on issues other than the labor market, which makes it interesting to extend the theory in order to address the welfare measurement problem under unemployment.

To my knowledge, only two previous theoretical studies on social accounting have considered the possibility of imperfect competition in the labor market. Aronsson (1998) adds the assumption that the wage rate exceeds the market clearing wage rate at each instant to an otherwise standard Ramsey growth model. The results show that the present value of future utility facing the representative consumer becomes proportional to the sum of the current value Hamiltonian and the present value of future changes in employment. The intuition is that, if the equilibrium is characterized by excess supply of labor at each instant, future increases in the employment will contribute to higher welfare and vice versa. Aronsson and Löfgren (1998) take the analysis a step further by briefly addressing welfare measurement under monopoly-union wage formation. The assumption that trade-union wage formation causes unemployment makes it possible to relate the present value of future changes in employment to the wage elasticity of the labor demand. However, both these earlier studies are based on representative-agent models, where the unemployment is measured in terms of work hours.

In this paper, I will extend the study of social accounting under imperfect competition in the labor market to a multi-consumer economy, in which agents may be either employed or unemployed. Such a framework also captures that unemployment gives rise to heterogeneity among agents; an aspect that is not addressed by a representative-agent model. The analysis to be carried out in the paper is based on a dynamic general equilibrium model, which is a variant of the Ramsey growth model. Following earlier research on welfare and fiscal policy under imperfect competition in the labor market (e.g. Fuest and Huber, 1997; Koskela and Schöb, 2002; Aronsson and Sjögren, 2004), I assume that trade-union wage setting is the mechanism behind the existence of involuntary unemployment. However, note that this assumption is not necessarily of great importance per se; the main results will also apply if the unemployment is caused by mechanisms other than trade-union wage formation. Arguably, this strengthens the analysis to be carried out below.

The paper contributes to the literature in primarily two ways. The first is by showing what the exact welfare measure looks like in an economy, where some individuals are unemployed. In addition to the current value Hamiltonian (or a Hamiltonian-like component), such a measure is likely to contain one or several aspects of future changes in the number of employed persons along the general equilibrium path as well as distributional effects following because unemployment gives rise to heterogeneity; entities that would vanish from the welfare measure in a first best setting. The second is by more thoroughly discussing the distributional aspects of unemployment in the context of social accounting. As I will show below,

[2]For an overview of the literature, see Aronsson *et al.* (2004). See also Usher (1994) for a discussion about the interpretation of the Hamiltonian.

the presence of unemployment does not only imply additional terms in the welfare measure (terms in addition to the current value Hamiltonian); it also affects the way in which we should calculate an analogue to real comprehensive NNP. The latter stands in sharp contrast to the earlier literature on social accounting and unemployment referred to above, which focuses on representative-agent models.

The outline of the paper is as follows. In Section 2, I present the model and also briefly discuss welfare measurement in a first best equilibrium. Section 3 deals with welfare measurement under equilibrium unemployment and presents the main results along with their empirical implications. Finally, Section 4 provides a summary and discussion of the results.

2. The Model

The production side of the economy is competitive with labor and capital being the variable production factors. To simplify the notations as much as possible, I describe the production side as if a single competitive firm produces a homogenous good. The objective function is written

$$\Pi(t) = F(N(t), K(t)) - w(t)N(t) - r(t)K(t) \tag{1}$$

where $F(\cdot)$ is the production function, N employment (measured as the number of employed persons, see below), K the aggregate capital stock, w the wage rate (or labor income) per employee, and r the interest rate. I assume that the production function is increasing in each argument and strictly concave, and that labor and capital are technical complements in production in the sense that the marginal product of labor (capital) increases with the size of the capital stock (employment). The firm treats w and r as exogenous, and it obeys the first order conditions $F_N(N, K) - w = 0$ and $F_K(N, K) - r = 0$ at each instant, where the subindices denote partial derivatives. Solving this equation system for N and K gives the demands for labor and capital, respectively. For further use, note that the labor demand can be written as

$$N = N(w, r) \tag{2}$$

in which $N_w, N_r < 0$. I assume that w and r (and, consequently, N) are continuously differentiable with respect to time.

Turning to the consumption side, there are M consumers in the economy. Let me follow the convention in the literature on social accounting by neglecting population growth, as well as by assuming that the consumers are endowed with perfect foresight. The consumers share a common instantaneous utility function, and each consumer supplies one unit of labor inelastically[3] at each instant. The instantaneous utility facing consumer i at time t is given by

[3]This assumption simplifies the analysis. Although the neglect of leisure in the utility function will affect the exact form of the welfare measure to be derived below, it is not of major importance for how changes in the number of employed persons, or distributional effects of changes in the factor prices, affect the welfare measure, which are the issues of main concern here.

(3) $$u_i(t) = u(c_i(t))$$

where c is consumption. The function $u(\cdot)$ is increasing in its argument and strictly concave.

In this paper, I am not primarily concerned with a detailed description of the labor market opportunities at an individual level; only that part of the workforce may be unemployed at each instant. With this idea in mind, and to make the optimal control problem manageable, it is convenient to find a simple mechanism that assigns individuals either to employment or unemployment in future periods. To achieve this basic objective, I assume that: (i) there is a unique seniority ranking among individuals, which is known and given exogenously at time zero; (ii) available jobs are distributed on the basis of seniority;[4] and (iii) seniority increases with the time spent in employment. Given the size of total employment, which is determined in the general equilibrium (see below), these additional assumptions make it possible to assign individuals to employment or unemployment in future periods. They also imply that the seniority ranking among individuals does not change over time.[5] Finally, since individuals endowed with perfect foresight know the employment level at each instant in the future as well as their own seniority ranking, it follows that each individual, at time zero, knows his/her employment status along the whole general equilibrium path.

The assumptions above enable me to formalize the optimization problem facing each consumer as if he/she solves a standard optimal control problem with the additional restriction that the wage rate is zero during times of unemployment.[6] The optimization problem facing individual i is to choose $c_i(t)$ at each instant such as to maximize

(4) $$U_i(0) = \int_0^\infty u(c_i(t))e^{-\theta t}dt$$

subject to the asset accumulation equation and an initial capital stock

(5) $$\dot{k}_i(t) = \pi(t) + r(t)k_i(t) + w_i(t) - c_i(t)$$

(6) $$k_i(0) = k_0,$$

where $w_i(t) = w(t)$ if individual i is employed at time t and zero otherwise. For later use, note that $K = \int_0^M k_i di$.

[4]The idea that employment opportunities are allocated according to seniority rules, and not by random draw, has been formalized in the context of trade-union wage formation by Oswald (1993). In practice, rules governing lay-offs and/or reemployment differ considerably across countries. For instance, Sweden applies seniority rules (based on years of employment) in case of lay-offs, and workers who were recently laid off will become reemployed if the employer needs to increase employment again.

[5]This aspect simplifies the analysis considerably by comparison with a more general framework, where the seniority ranking may change over time. Recall that the idea of the paper is to capture fundamental aspects of unemployment for welfare measurement at the aggretate level: not to provide considerable detail as to who is employed and who is not (which are of no concern here).

[6]This means that I disregard unemployment benefits. As long as unemployment implies a loss of income relative to being employed, this is not important for the qualitative results derived below.

The aggregate profits are assumed to be shared equally among the consumers, i.e. $\pi = \Pi/M$. In addition to equations (5) and (6), I impose a No Ponzi Game (NPG) condition. Note also that I am disregarding other possible assets than physical capital. Each individual treats the wage rate, the interest rate and the profit income as exogenous. The necessary conditions involve (the time indicator has been suppressed for notational convenience)

$$u_c(c_i)e^{-\theta t} - \lambda_i = 0 \tag{7}$$

$$\dot{\lambda}_i = -\lambda_i r \tag{8}$$

$$\lim_{t\to\infty} \lambda_i = 0 \tag{9}$$

where λ_i represents the costate variable associated with equation (5) and is interpretable as the marginal utility value of capital (or the marginal utility of consumption) in present value terms. Equation (7) is the standard first order condition for consumption, whereas equations (8) and (9) are, respectively, the equation of motion for the marginal utility value of capital and the transversality condition.[7]

Let me turn to the labor market part of the model. As mentioned in the introduction, trade-unions are assumed to have an influence on wage formation. In addition, I add three characteristics, which are common in the literature on trade-unionized labor markets:

(i) All individuals are trade-union members.
(ii) Trade-unions only decide (or bargain over) the wage rate.
(iii) The firm unilaterally chooses the number of persons to employ.

These characteristics typically imply that trade-unions have some degree of market power, and that the equilibrium wage rate is likely to exceed the market clearing wage rate. The employment is assumed to be determined by the labor demand. One specific wage formation process discussed in earlier literature that obeys these characteristics is a monopoly trade-union maximizing the wage sum of its members subject to the only constraint that the labor demand decreases with the wage rate. However, for the analysis to be carried out below, it suffices to assume that the wage rate is such that

$$N = N(w, r) < M \tag{10}$$

at the equilibrium, implying that the labor demand, measured in terms of the number of employed persons, falls short of the labor force.

A Brief Digression to the First Best

To simplify the analysis in later parts of the paper, it is convenient to begin by briefly considering welfare measuremcnt in the first best. This provides a reference

[7]The transversality condition is necessary, provided that certain growth conditions are fulfilled. These growth conditions serve as upper bounds for the influence of the state variable on the functions involved. For further details, the reader is referred to Seierstad and Sydsaeter (1987, Theorem 16 of Chapter 3).

case by which to compare the welfare measure to be derived under imperfect competition in the labor market. In the special case where the resource allocation is first best, we have $N = M$, and the first order conditions look as if they are derived from the following present value Hamiltonian for the economy as a whole;

$$H^*(t) = \int_0^M [u(c_i^*(t))e^{-\theta t} + \lambda_i^*(t)\dot{k}_i^*(t)]\,di = Mu(c^*(t))e^{-\theta t} + \lambda^*(t)\dot{K}^*(t) \tag{11}$$

where the superindex "*" is used to the denote the first best equilibrium. The second line of equation (11) is explained by the fact that individuals have identical preferences and initial capital stocks; they are, therefore, identical in the first best equilibrium associated with the model set out above. The equation of motion for the aggregate capital stock (which is derived by using the individual asset accumulation equations and the objective function of the firm) is given by

$$\dot{K}^*(t) = F(N^*(t), K^*(t)) - Mc^*(t) \tag{12}$$

where $N^* = M$. Differentiating equation (11) totally with respect to time, and using the first order conditions facing the firm together with equations (7) and (8), gives

$$\frac{dH^*(t)}{dt} = -\theta Mu(c^*(t))e^{-\theta t} \tag{13}$$

which means that the only non-autonomous time dependence of the economic system originates from the utility discount factor. The indirect effects of time via control, state and costate variables vanish as a consequence of optimization. Since the present value Hamiltonian approaches zero when time goes to infinity,[8] one can solve equation (13) and transform the solution to current value to obtain

$$\theta V^*(t) = H^{c*}(t) \tag{14}$$

where

$$V^*(t) = M\int_t^\infty u(c^*(s))e^{-\theta(s-t)}\,ds$$

is the optimal value function and $H^{c*}(t) = H^*(t)e^{\theta t}$ the current value Hamiltonian. Defining the consumer surplus,[9] $s^* = u(c^*) - u_c(c^*)c^*$ and $S^* = Ms^*$, the current value Hamiltonian can be rewritten as

$$H^{c*}(t) = Mu(c^*(t)) + \lambda^{c*}(t)\dot{K}^*(t) = \lambda^{c*}(t)\left[C^*(t) + \dot{K}^*(t)\right] + S^*(t) \tag{15}$$

[8]This property of well behaved optimal control problems is due to Michel (1982).

[9]For a more detailed analysis of the role of the consumer surplus in social accounting, the reader is referred to Li and Löfgren (2002).

in which $C^* = Mc^*$, while $\lambda^{c*}(t) = \lambda^*(t)e^{\theta t}$ is the marginal utility value of capital measured in current value terms at time t.

Equation (14) is Weitzman's welfare measure applied to the economy described above. It means that the current value Hamiltonian constitutes an exact welfare measure in the first best equilibrium. Note that the current value Hamiltonian is interpretable as a measure of comprehensive NNP in utility terms; it measures the utility of the current consumption plus the utility value of the current net investments. For the economy discussed here, which has a relatively simple structure, the consumption concept only contains private goods and services, and the net investment concept only contains physical capital; this explains why the comprehensive NNP coincides with the conventional NNP. The expression after the second equality in equation (15) shows that the welfare measure can also be written as the sum of the linearized current value Hamiltonian and the consumer surplus. The linearized current value Hamiltonian is, in turn, equal to the marginal utility of wealth times an expression that will be interpreted as the real comprehensive NNP.[10]

3. The Decentralized Equilibrium with Unemployment

Let me now return to the decentralized equilibrium under imperfect competition in the labor market. What does the relationship between the optimal value function and the sum of individual Hamiltonians look like in this case? The sum of individual present value Hamiltonians can be written as

$$H^0(t) = \int_0^M \left[u\left(c_i^0(t)\right)e^{-\theta t} + \lambda_i^0(t)\dot{k}_i^0(t)\right]di \tag{16}$$

[10]Nate that the welfare measure is expressed in a utility-metric, i.e. both the linearized current value Hamiltonian, $\lambda^c[C+\dot{K}]$, and the consumer surplus are measured in units of utility. Although the choice of metric is not important for the characterization carried out here, practical applications necessitate that the utility-based welfare measure is transformed into a money-metrics welfare measure. The practical problem is that, even if one were to neglect the consumer surplus, one cannot in a meaningful way compare $C+\dot{K}$ (referred to as "real comprehensive NNP" above) at two different points in time, since the marginal utility of wealth changes over time. The key to a money-metrics transformation of utility-based welfare measures was provided by Weitzman (2001) and further elaborated on by Li and Löfgren (2002). Applied to the present model, their approach is to introduce a "price index," $p(t)$ such that

$$p(t) = \lambda^c(t_0)/\lambda^c(t) = \exp\left(-\theta(t-t_0) + \int_{t_0}^t r(s)\,ds\right).$$

This makes it possible to re-scale the welfare measure such that

$$H^c(t) = \lambda^c(t_0)\left[\frac{C(t)}{p(t)} + \frac{\dot{K}(t)}{p(t)}\right] + S(t)$$

where $\lambda^c(t_0)$ is a constant, and the expression within the square bracket is interpretable as a money-metricized version of real comprehensive NNP.

where the superindex "0" is used to denote the equilibrium in the imperfect market economy. By analogy to equation (11), equation (16) can also be interpreted as measuring the comprehensive NNP in utility terms for the economy set out above discounted to present value, which is why it is interesting to consider as a welfare measure. Let $\bar{\lambda}(t) = (1/M)\int_0^M \lambda_i(t)\,di$ denote the average marginal utility value of capital (i.e. the average marginal utility of consumption) at time t measured in present value terms. For further use, note that (where the time indicator has been suppressed)

$$\int_0^M \lambda_i \dot{k}_i di = \bar{\lambda}\Pi + r\int_0^M \lambda_i k_i di + w\int_0^N \lambda_i di - \int_0^M \lambda_i c_i di$$

since those $M - N$ individuals who are unemployed receive no labor income.[11]

We have now come to the position of being able to study social accounting. By analogy to the welfare concept used in the first best, define the optimal value function,[12]

$$V^0(t) = \int_t^\infty \int_0^M u\left(c_i^0(s)\right)e^{-\theta(s-t)} di\, ds,$$

and consider Proposition 1 below:

Proposition 1. *Within the given framework, and if the equilibrium is characterized by unemployment, the welfare measure can be written as*

$$\begin{aligned}\text{(17)}\qquad \theta V^0(t) = H^{c^0}(t) &+ \int_t^\infty \left[\bar{\lambda}^0(s)\frac{\partial \Pi^0(s)}{\partial w(s)} + \int_0^{N^0(s)} \lambda_i^0(s)\,di\right] e^{\theta t}\dot{w}^0(s)\,ds \\ &+ \int_t^\infty \left[\bar{\lambda}^0(s)\frac{\partial \Pi^0(s)}{\partial r(s)} + \int_0^{M} \lambda_i^0(s)k_i^0(s)\,di\right] e^{\theta t}\dot{r}^0(s)\,ds \\ &+ \int_t^\infty \lambda_N^0(s)w^0(s)\dot{N}^0(s)e^{\theta t}\,ds\end{aligned}$$

where $H^{c^0}(t) = H^0(t)e^{\theta t}$ *is the sum of the current value Hamiltonians.*

Proof: See the Appendix.

[11]Although c_i, and λ_i, as well as the aggregate variables w and r, arc continuously differentiable with respect to time by the assumptions made earlier, k_i is not smooth for individuals switching between employment and unemployment (as the wage facing the individual will jump from w to 0 or vice versa at a switch point). I will assume that this problem is of minor practical importance (e.g., that the number of switchers at each instant is small relative to the number of nonswitchers), and proceed as if the *sum* of Hamiltonians is continuously differentiable with respect to time. An alternative would have been to approximate the capital stock for the switchers by a smooth function. However, as this is of no concern for the basic mechanisms via which unemployment affects the welfare measure, I abstain from such extensions here.

[12]It is straightforward to extend the model by assigning different distributional weights to the employed and the unemployed.

Equation (17) means that welfare, measured by the present value of future utility, is proportional to the sum of four terms. The first is the sum of individual current value Hamiltonians, which is interpretable as the comprehensive NNP in utility terms (this will be further discussed below). One important aspect of Proposition 1 is to invalidate the welfare interpretation of the sum of current value Hamiltonians in case of unemployment; information referring to time t is no longer sufficient for measuring welfare at that time. This is understood from the appearance of the remaining terms on the right hand side of equation (17), which are forward looking and, therefore, not measurable by using entities referring to time t. These additional terms were absent in the first best welfare measure given by equation (14).

The second and third terms on the right hand side of the welfare measure (which are proportional to the changes in w and r) represent distributional effects of the wage rate and the interest rate, respectively. These distributional effects arise because unemployment gives rise to heterogeneity among the consumers. Consider first the distributional effect of a change in the wage rate. It consists of two parts; the first is the decrease in profit income from an increase in the wage rate which influences all consumers, and the second is the increase in labor income among the employed if the wage rate increases. Since $\partial\Pi/\partial w = -N$, the distributional effect at a given point in time (the terms within the square bracket in the first row) can be written as

$$\bar{\lambda}^0 \frac{\partial \Pi^0}{\partial w} + \int_0^{N^0} \lambda_i^0 di = \left(\bar{\lambda}_n^0 - \bar{\lambda}^0\right) N^0 \tag{18}$$

where $\bar{\lambda}_n^0 = (1/N)\int_0^N \lambda_i di$ is the average marginal utility value of capital in present value terms among the employed. On average, the employed have more life-time resources than the unemployed and will, therefore, be able to consume more at each instant. This suggests that $\bar{\lambda}_n^0 < \bar{\lambda}^0$, in which case the right hand side of equation (18) is negative. The intuition in terms of equation (17) is that future increases (decreases) in the wage rate lead to lower (higher) welfare. The instantaneous distributional effect of a change in the interest rate (the terms within the square bracket in the second row) is equal to the number of individuals in the economy times the covariance between λ and k;

$$\bar{\lambda}^0 \frac{\partial \Pi^0}{\partial r} + \int_0^{M} \lambda_i^0 k_i^0 di = M\, cov\left(\lambda^0, k^0\right). \tag{19}$$

As the marginal utility value of capital is likely to decline with the size of the capital stock, one would clearly expect $cov(\lambda, k) < 0$. In this case, therefore, future increases (decreases) in the interest rate contribute negatively (positively) to the welfare level. The intuition behind equations (18) and (19) is that a higher wage rate or interest rate tends to increase the income (earnings and capital income, respectively) more among individuals with a low marginal utility value of capital than among individuals with a high marginal utility value of capital. In other words, the consumption possibilities are redistributed toward those with a low marginal utility of consumption.

The fourth term on the right hand side of equation (17), which is proportional to $\dot{N}$, measures the present value of future changes in the number of employed person.[13] The intuition is that a marginal increase in the number of employed persons leads to a welfare gain for the individual who becomes employed at the ruling wage rate.[14] By using the labor demand, the time derivative of N can be decomposed into two effects:

$$\dot{N} = N_w \dot{w} + N_r \dot{r}. \tag{20}$$

Therefore, since $N_w < 0$ and $N_r < 0$, the employment effect depends on how the factor prices adjust along the general equilibrium path. If $\dot{w}$ and $\dot{r}$ are positive (negative) along the general equilibrium path, then the employment effect contributes to lower (higher) welfare at time t.

Empirical Implications

Let us now turn to the empirical implications of equation (17) and, in particular, how the additional components associated with equilibrium unemployment affect the welfare measure. To make the interpretations economically meaningful, I will assume throughout this section that $\bar{\lambda}_n < \bar{\lambda}$ and $cov(\lambda, k) < 0$ for all t. The following result is an immediate consequence of Proposition 1:

Corollary 1. *Rising (falling) factor prices along the general equilibrium path imply that the sum of current value Hamiltonians at time t overestimates (underestimates) welfare at time t.*

The intuition behind Corollary 1 is that higher factor prices lead to a redistribution from the unemployed to the employed—which reduces the welfare given the social welfare function assumed above—as well as lead to lower employment. In other words, future increases (decreases) in the factor prices imply redistribution away from (toward) those with a high marginal utility of consumption. Note also that discounting means that the near future is given a higher weight in the welfare measure than the distant future. The latter is practically relevant for the interpretation of the corollary; if $\dot{w}$ and $\dot{r}$ are both predominantly positive (negative) in the near future, the current value Hamiltonian is likely to overestimate (underestimate) the welfare at time t.

Distributional effects do not only arise because future changes in the factor prices affect the employed and the unemployed in different ways (the effects

[13]A similar result, yet with the interpretation in terms of future changes in the hours of work, is derived in the representative-agent models referred to in the introduction.

[14]A similar qualitative result applies if the unionized labor market is replaced by dual labor market, where those who do not become employed in the unionized sector will find work in another (competitive) sector, provided that the wage rate is higher in the unionized sector than in the other sector. If the wage rate were higher in the other sector, no individual would work in the unionized sector, in which case the welfare measure takes the same general form as equation (14).

discussed above). Such effects also appear because the distribution is suboptimal at present; an aspect which is captured by the analogue to comprehensive NNP. In the first best resource allocation, briefly addressed above, we saw that the sum of linearized current value Hamiltonians is interpretable as the marginal utility of consumption times the real comprehensive NNP. We shall here derive the corresponding expression for the economy with equilibrium unemployment. Let $\bar{c}(t) = (1/M)\int_0^M c_i(t)\,di$ and $\bar{\kappa}(t) = (1/M)\int_0^M \dot{k}_i(t)\,di$ denote the average private consumption and average capital accumulation, respectively, in the economy as a whole at time t. The following result is a direct consequence of Proposition 1:[15]

Corollary 2. *The sum of current value Hamiltonians can be written as (neglecting the time indicator)*

$$H^{c^0} = \bar{\lambda}^{c^0}\left[C^0\left(1+\rho_c^0\right) + \dot{K}^0\left(1+\rho_k^0\right)\right] + S^0 \tag{21}$$

where $\rho_c = cov(\lambda, c)/\left(\bar{\lambda}\bar{c}\right)$ *and* $\rho_k = cov\left(\lambda, \dot{k}\right)/\left(\bar{\lambda}\bar{\kappa}\right)$.

Equation (21) is an analogue to equation (15). The first part is interpretable as the linearized current value Hamiltonian for the economy as a whole (i.e. sum of linearized current value Hamiltonians over all individuals) and is measured by the average marginal utility of consumption in current value terms, $\bar{\lambda}^c$, times an analogue to real comprehensive NNP, whereas the second part, S, is the consumer surplus (measured in the same way as above). By comparison with the first best, where only the sum of aggregate consumption and aggregate net investments constitutes the comprehensive NNP, one can see that the appearance of equilibrium unemployment implies that the distribution of consumption and savings (i.e. the income distribution) among individuals at present is also relevant, which is the reason why the distributional characteristics, ρ_c, and ρ_k, are part of the linearized current value Hamiltonian. This is so because the marginal utility of consumption differs across agents, which it did not in the first best reference case discussed above.

Note that $\rho_c < 0$ according to equation (7). Therefore, the weight attached to the aggregate consumption in the context of real comprehensive NNP is lower here than in the first best. The intuition is that the marginal welfare contribution of the aggregate private consumption is smaller than it would have been, had the marginal utility of consumption been equalized across agents. It is not (as far as I can see) possible to sign the distributional characteristic for the net investments, ρ_k.

However, by using $\bar{\lambda}_n < \bar{\lambda}$ and $cov(\lambda, k) < 0$, one may actually show that $\bar{\lambda}^c\left[C\rho_c + \dot{K}\rho_k\right] < 0$ (if evaluated along the general equilibrium path), which is the expression by which the linearized current value Hamiltonian in equation (21) differs from its counterpart in equation (15). To see this, note that

[15]Note that the principles behind the money-metrics transformation discussed in footnote 10 apply here as well. See Löfgren (in press) for a treatment of externalities in the context of money-metrics welfare measures.

$$\int_0^M \lambda_i^c \left(c_i + \dot{k}_i\right) di = \bar{\lambda}^c \left[C(1+\rho_c) + \dot{K}(1+\rho_k)\right]$$
$$= \bar{\lambda}^c [\Pi + rK + wN] + rM\,cov\left(\lambda^c, k\right) + wN\left[\bar{\lambda}_n^c - \bar{\lambda}^c\right]$$

and $\dot{K} + C = \Pi + rK + wN = F(N, K)$. Therefore,

$$\bar{\lambda}^c \left[C\rho_c + \dot{K}\rho_k\right] = rM\,cov\left(\lambda^c, k\right) + wN\left[\bar{\lambda}_n^c - \bar{\lambda}^c\right] < 0$$

in the general equilibrium. The following result is also a consequence of Proposition 1:

Corollary 3. *If the equilibrium is characterized by unemployment, then* $\bar{\lambda}^{c^0}\left[C^0 + \dot{K}^0\right]$ *overestimates the linearized current value Hamiltonian.*

Therefore, since the linearized current value Hamiltonian is interpretable as the average marginal utility of consumption times an analogue to real comprehensive NNP, Corollary 3 is interpretable to mean that $C + \dot{K}$ overestimates the real comprehensive NNP.

4. Summary and Discussion

The general conclusion from this paper is that unemployment is important to consider explicitly in social accounting. This is so for several reasons. First, if the equilibrium is characterized by unemployment, the current value Hamiltonian has no obvious welfare interpretation. Although this conclusion may seem obvious at a general level, the results show why this discrepancy occurs as well as what the exact welfare measure looks like. Second, unemployment gives rise to heterogeneity which, in turn, means that distributional effects of changes in the factor prices are present in the welfare measure, and that distributional characteristics are part of the linearized current value Hamiltonian. Third, the exact welfare measure becomes dependent on future changes in the number of employed persons along the general equilibrium path.

Note that the basic idea behind the paper is to characterize and interpret the welfare measure; not to determine the signs or magnitudes of the extra terms. One direction of future research is to apply the welfare analysis in practice in order to assess the relative welfare contribution that is due to the appearance of imperfect competition in the labor market. A possible approach here would be to use a numerical general equilibrium model calibrated against real world data. This topic is both interesting and complex enough to motivate a study of its own. I leave these and other extensions for future research.

5. Appendix

Differentiating equation (16) totally with respect to time and using equation (1), (7), (8), and (9) together with the first order conditions of the firm gives

$$\text{(A1)} \qquad \begin{aligned} \frac{dH^0(t)}{dt} = &-\theta \int_0^M u\left(c_i^0(t)\right) e^{-\theta t} di \\ &+ \left[\bar{\lambda}^0(t) \frac{\partial \Pi^0(t)}{\partial w(t)} + \int_0^{N^0(t)} \lambda_i^0(t) di \right] \dot{w}^0(t) \\ &+ \left[\bar{\lambda}^0(t) \frac{\partial \Pi^0(t)}{\partial r(t)} + \int_0^M \lambda_i^0(t) k_i^0(t) di \right] \dot{r}^0(t) \\ &+ \lambda_N^0(t) w^0(t) \dot{N}^0(t). \end{aligned}$$

By assuming that the present value Hamiltonian approaches zero when time goes to infinity, one can derive equation (17) by solving equation (A1).

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