

AN AXIOMATIC APPROACH TO NATIONAL ACCOUNTING:
AN OUTLINE

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This article deals in an axiomatic manner with problems of definition, classification, and measurement in the national accounts. It argues that the elementary units which must be classified in national accounting are economic objects (real and financial), rather than transactions. The article defines briefly a set of postulates, and shows that the structure of a simple system of national accounting can be derived from them. There are twenty postulates—certain of them establishing basic categories such as sector, time, economic object, value (price); others establishing relations between categories (for example the notion of ownership); and others describing operations in which economic objects can be involved, such as production, final consumption, change of ownership, and change of debtor and creditor (in the case of financial objects). It is shown that the system of postulates makes it possible to consider a large number of accounting concepts (flows or stocks) as classes (baskets) of real objects (e.g., exports, real capital) or financial objects (e.g., payments, total debt of a sector). These concepts can be defined without reference to prices, although prices are necessary to measure them. Other concepts cannot be defined in this way in this system of postulates, for example value added, foreign balance, saving, net worth. However, it is possible to define magnitudes of the latter type and measure them in terms of value: for example, value added can be defined as the difference between the value of receipts and the value of outlays of a sector. In this way it is possible to establish algebraic relations among the national accounting concepts. (This article is a summary of certain parts of the doctoral thesis of the author, published in Norwegian in 1955.)

A. INTRODUCTION

This brief note deals with fundamental problems of definition, classification, and measurement in the national accounts. The approach is, however, somewhat unusual: the purpose is to suggest a set of postulates from which the structure of a national accounting system can be deduced. The exercise, if successful, should help to establish, in a less loose and imprecise way than is usually the case, the concepts and relationships used in national accounting work, and to uncover the categories which are basic to the design of a descriptive system for the economy. Furthermore, it should help to make clear to us the thought processes which are presupposed by the derivation of such a system.

The presentation that follows aims merely at sketching the main line of thought, rather than at giving a rigorous exposition of the argument. In fact, this note summarizes parts of my doctoral thesis, published in Norwegian 10 years ago, where such a rigorous presentation was attempted.¹ There is no need to repeat, on this occasion, the apparatus of symbols originally used.

¹Odd Aukrust: *Nasjonalregnskap. Teoretiske prinsipper* (National Accounts. Theoretical Principles), *Samfunnsøkonomiske Studier* No. 4, Statistisk Sentralbyrå, Oslo 1955.

B. THE MAIN IDEAS

A summary presentation of the main ideas set out below may be helpful at this stage. In an axiomatic approach to national accounting the aim will be to find and introduce by way of postulates a set of basic categories (e.g., “sector”, “object”, “time”) and relationships (e.g. “owned by”), whose nature is such that, from them, we can derive the most important concepts and relationships of which we make use.

One basic question which must be answered is the following: What are the elementary units which we try to observe classify and measure in national accounting work? The traditional answer is that they are *economic transactions*; economic transactions can be grouped into flows, and the entries in the national accounts are said to represent the money values of these flows. This approach may not be a happy one.²

We want to explore in this note another approach. We shall consider the elementary units to be classified in national accounting work to be *economic objects* (real and financial) rather than economic transactions. The universe studied comprises—if we like—all economic objects ever existing, whether in the past or in the future. We want to demonstrate that, when these elements (the economic objects) are postulated as being well defined, some of the most important aggregates in the national accounts can be defined as classes of such elements.

For a classification to be possible, we have to postulate that certain distinguishing characteristics (properties) attaching to the individual objects are given, which can form the basis for their classification. In selecting these characteristics we note that, typically, the aggregates to be defined have reference to particular transactors (sectors), and that they have a time dimension. This suggests that the categories of “sector” and “time” will have to be introduced into the system. Since we want to define stocks as well as flows, time may be conceived of as consisting of time points and of intermediate intervals in such a way that a set of consecutive intervals defines a period. Finally, we shall have to assume that certain types of events (transactions or transformations) are given, to which objects may be subject and which are of interest to us in the national accounts, e.g., sales.

We shall postulate, therefore, that for each individual object the following characteristics are given:

- i. Information as to whether the object is a real or a financial object.
- ii. Information as to the points of time at which the object is in existence.
- iii. Information as to which transactors the objects are related to at any particular point of time during their existence.
- iv. Information as to which transactions (events of the given types) the object is subject to during its existence; and, for every such transaction, further information as to the time interval when it takes place.

²There are at least three reasons why the traditional approach appears unsatisfactory. (i) Economic transactions do not seem to be a well-defined category; hence ambiguity arises when flows are defined as groups of such transactions. (ii) Economic transactions cannot be used to define stocks, which are needed in addition to flows in the national accounts. (iii) We are forced to interpret the entries in the national accounts as payment flows (“flows of payables”) which, to my mind, is unfortunate. For instance, the statement that a country’s commodity exports are X mill.kr. must be interpreted as saying that “payables of X mill.kr. in respect of commodities exported became due to that country” whereas a simpler understanding is that “a basket of commodities worth X mill.kr. was exported”.

These types of information constitute the distinguishing characteristics which will make a classification possible. We may, if we like, conceive of each object as carrying a label containing this information.

We can now define stock items (e.g., a sector's real capital at a given point of time) by selecting a class of objects which have in common, among other things, the characteristic of being in existence at the point of time we consider. In a similar way, a flow (e.g., a country's exports over a definite period) can be defined through the selection of those objects which have in common, among other things, the characteristic of having been involved in specific types of transactions during the time intervals constituting that period.

Next a set of evaluation coefficients (prices) is postulated for all objects. (Each object carries, as it were, a price tag in addition to the information label already referred to.) With such a set of evaluation coefficients given, the way is open for establishing, in the form of a scalar number, what we may call the value of a class; furthermore, it can be shown that those national accounting entries that can *not* be defined as classes in the sense suggested in the paragraph above, such as value added or saving, can instead be defined as value concepts in terms of such scalar numbers. Finally, if we further postulate that "exchange of objects always occurs according to the prices that are postulated", it can be shown that simple relationships (referred to in the following as the eco-circ relationships) will exist between the value concepts established.

Simple as these ideas are, we shall nevertheless find that we need as many as twenty postulates in order to establish what is no more than a crude outline of a national accounting system. These postulates are numbered in what follows in roman numerals.

C. THE REAL CIRCULATION

Nine postulates serve to define stocks and flows of real objects. The first three postulates establish the categories:

- I. *Sector* or transactor; a sub-set of the sectors define the domestic economy,
- II. *Time*, conceived of as consecutive "time intervals" separated by "points of time",
- III. *Economic* objects, of which there are two kinds (real or financial). Each economic object has a known existence over time, and the universe of real objects has no element in common with the universe of financial objects.

The fourth postulate establishes a relationship between sectors, time, and real objects, corresponding to the idea of "ownership":

- IV. For every real object, at any one point of time during its existence, one-and only one-sector can be denoted as the "owner" of the object.

The next group of postulates describes the transactions which real objects may be subject to, and which are of interest in national accounting. In the present note only three types of transactions in real objects are considered. They are:

- V. *Production*, conceived of as transformation processes whereby real objects are "created" (start to exist) at the same time as other real objects, used as inputs, are cancelled out (cease to exist).

VI. *Final Consumption*, conceived of as processes whereby real objects are cancelled out (cease to exist), other than by being used as inputs.

VII. *Change of ownership*.

Postulates V, VI, and VII all are to the effect that the time interval when the transactions take place can be precisely established.

It is finally postulated:

VIII. No real object comes into existence by any other way than by being produced.

IX. No real object can be involved in more than one transaction of the types described by postulates V through VII in the shortest time interval we have under consideration.

Through these nine postulates a number of flow and stock items of interest in the national accounts can be defined. For instance, a sector's real capital can be defined as the class of real objects which exist and are owned by that particular sector at a given point of time (follows from I, II, III, and IV). Other classes of real objects can be selected which will define for any given sector and for any given period, the flow items:

output

input

final consumption

sales made to any sector or group of sectors whatsoever, including sales abroad

total sales

total purchases

Corresponding national aggregates can now be defined through a simple (logical) summation. For instance, if there are v national sectors with real capital $K_1 \dots K_v$, respectively, we can define a class K as the logical sum (union) of the $K_1 \dots K_v$, which is the real capital of the nation. In a similar manner total domestic output, total domestic input, domestic consumption, total exports, and total imports may be defined.

All concepts established in the two preceding paragraphs are in terms of classes of real objects, i.e. they are defined as "baskets of goods and services". Note that there are certain very important national accounting concepts which cannot be defined in this way. A prominent example is the concept of "value added" (for a sector or a nation). The explanation is that, in logic, while classes may always be added, subtraction is not always meaningful.³ For this reason, to talk of value added as a class—the class of output minus the class of input—just does not make sense. We shall see later, however, that value added can be defined under certain conditions in terms of a *numerical* (scalar) value.

D. THE FINANCIAL CIRCULATION

In order to describe the financial circulation, six more postulates are needed in addition to the nine already introduced. The first of these, which establishes a

³For instance, we may conceive of a basket containing three oranges plus two apples but not of a basket containing three oranges minus two apples.

relationship between the categories of sectors, time, and financial objects that correspond to the *idea of creditor and debtor*, says in effect:

- X. For every financial object, at any point of time during its existence, one creditor sector and one debtor sector can be identified.

The next group of postulates serves to describe the transactions in which financial objects can be involved and which are of interest in national accounting. The present system considers four types of such transactions:

- XI. Financial objects can be *created*, or
 XII. *Cancelled out* (cease to exist), or
 XIII. Undergo a *change of creditor*, or
 XIV. Undergo a *change of debtor*.

This group of four postulates are all to the effect that the time interval when the transaction takes place can be precisely established. We further postulate:

- XV. No financial object can be involved in more than one transaction of the types described by postulates XI through XIV in the shortest time interval we have under consideration.

By means of postulates I through III and X through XV we can define as classes of financial objects a sector's positive, negative, and net financial assets, financial contributions to any other sector or group of sectors whatsoever including financial contributions abroad; total financial contributions received; and total financial contributions to others. For instance, a sector's positive financial assets (a stock item) can be defined as the class of financial objects which exist and of which that particular sector is the creditor at a given point of time. The definition of a financial flow is more complicated. For example, total financial contributions (this term is used here about a "payment" in the widest sense of the word) made during a period to a sector A from a sector B can be defined as the sum (the logical sum) of the following four sub-classes:

- financial objects created (as defined by XI) during the period which, when created, had A as creditor and B as debtor,
- financial objects cancelled out (as defined by XII) during the period which, when cancelled, had B as creditor and A as debtor,
- financial objects undergoing, during the period, an operation (as defined by XIII), whereby A was made creditor to the object instead of B,
- financial objects undergoing, during the period, an operation (as defined by XIV), whereby A was made debtor to the object instead of B.

This corresponds to the four ways in which a "payment" from B to A may be effectuated: (i) A obtains a fresh claim on B, (ii) B cancels a claim held against A, (iii) B hands over to A a claim (e.g. a bank-note) held against some third sector, (iv) B accepts responsibility for A's debt to a third sector.

Likewise, we may define as classes of financial objects a number of national financial aggregates. Some of them (flow aggregates) can be derived simply as the (logical) sum of aggregates already defined above; for instance, national financial contributions abroad is the sum of financial contributions abroad by individual national sectors. Others (stock aggregates) must be defined in much the same way as the corresponding sector aggregates were defined, e.g., a nation's positive financial assets at a given point of time must be defined as the class of financial objects having a national sector as creditor and a non-national sector as debtor at that

particular point of time rather than as the (logical) sum of the positive financial assets of all national sectors.⁴

E. THE INTERPLAY BETWEEN THE REAL AND THE FINANCIAL CIRCULATION

Having studied in the two foregoing sections the real circulation and the financial circulation each taken in isolation, it is now time to study the connections between the two. For this purpose three additional postulates are established which introduce the distinction between *requited and unrequited transactions*. They say in essence:

- XVI. A real flow from one sector to another is always associated with a financial contribution in the opposite direction. (Taking “associated with” to mean something like “paid for” this postulate rules out the existence of real transfers, i.e. the system as here set out has no room for unrequited real flows.)
- XVII. A financial contribution from one sector to another *can* (but need not) be associated with financial contributions in the opposite direction.
- XVIII. No financial contribution from one sector to another can be associated with both a real flow and a financial contribution in the opposite direction.

These three postulates serve mainly to define aggregates which divide the various classes of financial contributions—between two sectors, between one sector and all other sectors, or between all national sectors and countries abroad—each into three sub-classes: (i) contributions (“payments”) that relate to the exchange of real objects, (ii) contributions (“payments”) that relate to the exchange of financial objects, (iii) contributions (“payments”) that are transfers. Note that the postulates introduce an important asymmetry between the real and the financial circulation: Real inter-sectoral flows always have a financial counterpart, but the opposite does not hold true; real objects are never bartered, while financial objects sometimes are; the existence of real transfers is ruled out, while the existence of financial transfers is not.

Finally, as part of the interplay between the real and the financial circulation we can define the total assets or wealth of a sector in the following way. The wealth (“net worth”) of a sector is the sum (the logical sum) of three sub-classes, namely (i) the class of real objects owned by the sector, (ii) the class of financial objects of which the sector is creditor, (iii) the class of financial objects of which the sector is debtor. The wealth of a nation can be similarly defined.

F. THE PROBLEM OF MEASUREMENT. DEFINITION OF VALUE CONCEPTS

So far, the exposition has dealt exclusively with classes (“baskets”) of real and financial objects and with logical relationships between classes. No reference has been made to prices or values. In this section we turn to the problem of measure-

⁴The latter is of course a much broader class than the former since it includes all claims held by national sectors, whether against the rest of the world or against other national sectors.

ment and *numerical* (as opposed to logical) relationships; we want to show, *inter alia*, how content can be given to those entries in the national accounts which cannot be defined as classes (e.g. value added, saving). Since “value” is an important property which objects have in common, and a property which will render measurement possible, valuation becomes the central issue.

Value is not a clearly defined property of the objects that can be determined by experiment, in the same sense as “weight” can be determined. As a result, a number of questions arise which we shall here side-step by simply postulating:

XIX. There exists a “*national accounts price list*” in the sense that for every object, real or financial, one non-negative, rational number is given which can be taken to express the value of the object.

XX. Two required flows (real/financial or financial/financial) always have the same value (the postulate of “the preservation of values in exchange”).

The condition imposed by XX is fulfilled in a system where purchases and sales always take place according to the prices given in the national accounts price list.

Postulate XIX allows us to define the values of any class of real and/or financial objects whatsoever as specific functions of the prices stated. Example: The class consists, say, of n elements, and $w_1 \dots w_n$, are the prices given for these elements according to XIX. We can then define a number $w = w_1 + \dots + w_n$, which we will call-by convention-the value of the class. In other words, we define the value of a class as the algebraic sum of the figures which express the value of the objects contained in the class. However, for classes defined as stocks of financial objects we shall choose to include negative financial objects-objects in respect of which the sector we have under consideration is debtor-with a negative sign. With this convention the value of the negative financial objects, or debts, of a sector, will always be a negative number. We shall refer to these conventions as “rules for defining the value of classes”.

When the values of the various classes are known, algebraic operations on the numbers expressing these values are permitted. For instance, we can define new magnitudes as algebraic relationships—for example, differences—between these values. In this way we can define—as value concepts—a number of national accounting entries which cannot be defined as classes.⁵ Thus we define, for any sector and any period whatsoever:

value added as the difference between the value of the output and the value of the input of the sector,

net investment as the difference between the value of the real objects owned by the sector at the end of the period and the value of those owned by it at the beginning of the period,

net financial investment as the difference between the value of the financial objects (claims or debts) held by the sector at the end of the period and the value of those held by it at the beginning of the period,

⁵Though we cannot conceive of a basket containing three oranges minus two apples, it obviously makes sense to talk about the value of three oranges minus the value of two apples.

saving as the difference between the value of all objects (real and financial) held by the sector at the end of the period and the value of those owned by it at the beginning of the period,

disposable income as the value added of the sector plus the value of transfers received by the sector minus the value of transfers paid by the sector.

Corresponding national aggregates can be defined in an analogous way.

G. THE ECO-CIRC RELATIONSHIPS

It follows from our “rules for defining the value of classes,” as set out above, that all logical relationships between classes established in sections B–D have simple counterparts in valid algebraic relationships between the numbers expressing the values of these classes. In particular, if one class is the (logical) sum of two or more others, the value of the first class is simply the (algebraic) sum of the values of the latter. For instance, if class *A* is the logical sum of classes *B* and *C*—i.e. *A* contains those elements which are contained either in *B* or in *C*—and if *a*, *b*, and *c* are the numbers expressing the values of these classes, then, algebraically, $a = b + c$. This fact, together with the postulate of “the preservation of values in exchange” laid down by XX and the definitions introduced above, makes it possible to establish a set of algebraic relationships—essentially the simple Keynesian definitional equations—which must hold in a national accounting system derived from the twenty postulates set out above.

For any sector the following relationships (understood as algebraic relationships between values), *inter alia*, can be shown to hold:

net value added (or net product) = output – input

net real investment = increase in real capital

net financial investment = increase in net financial assets (claims less debts)

saving = increase in net wealth

disposable income = net product (as generated by the sector) + transfers received – transfers paid

saving = net real investment + net financial investment

net value added (or net product) = investment + consumption + (real objects sold – real objects bought)

Similar relationships can be shown to hold for the nation, though it may be found convenient in this case to change the terminology slightly. These relationships are not reproduced here.

Finally, it can be proved that a large number of national accounting entries relating to the nation are, in this system, simple sums of corresponding sector entries over all sectors, *inter alia*:

national capital (real, financial or total) = the sum of the capital of all sectors (real, financial or total)

domestic consumption = the sum of the consumption of all sectors

net domestic product = the sum of the net products of all sectors

net domestic consumption = the sum of the consumption of all sectors

net domestic real investment = the sum of the net real investment of all sectors

net domestic financial investment = the sum of the net financial investment of all sectors

domestic saving = the sum of the saving of all sectors

disposable national income = the sum of the disposable income of all sectors

H. CONCLUSIONS AND IMPLICATIONS

The set of twenty postulates used above to derive a national accounting system is, of course, not the only one which could be conceived of. Others are equally feasible. Some would lead to national accounting systems different from the one described here, in much the same sense as non-Euclidian geometries are different from Euclidian geometry. Little more can be claimed for the system outlined above than that it represents one possible attempt among many to add precision to the formulation of national accounting concepts. Still it may be of interest to point out some of the conclusions suggested by our analysis.

An outstanding feature of the present system is that it defines the national accounting entries—as far as possible—as baskets of real and/or financial objects. One important demonstration is that the national accounting entries fall into two broad groups: those that can be defined as classes in the sense suggested above, and those that cannot—at least not within the present system of postulates—be so defined. The latter group includes such key entries as net product, income, net real investment, net financial investment and saving (for a sector or a nation). The distinction throws light on the difference, long recognized by national accountants, between entries which are directly measurable and those which must be measured as “balancing items”.

The distinction also throws light on the problem of deflation. For entries defined as baskets of real and/or financial objects all we have to do in order to obtain figures “at constant prices” is to assume the existence of a list of base year prices similar to the list of current prices postulated by XIX. For entries existing merely as numbers, on the other hand, estimates “at constant prices” must again (as were the corresponding current price estimates) be defined as numbers. It is well known that starting from a set of national accounts which “balances” at current prices and revaluing the entries in this system at constant prices will result in a system which generally will not balance. (“Balance”, in this context, means that the eco-circ relationships are satisfied.) It can now be seen why this must be so: two required flows, though they may have identical values when valued at current prices, will generally not have identical values when re-valued at constant prices; hence the requirements of postulate XX (“the preservation of values in exchange”) are not met; therefore, in a system at constant prices, the eco-circ relationships—which depend on the validity of XX—cannot be expected to hold either.

Our analysis has brought out clearly that, in national income statistics, the numbers representing the various national accounting aggregates, or relationships derived from such numbers (for instance, percentage distributions and growth rates), are functions equally of quantities and of prices. This applies to estimates at current prices as much as to estimates at constant prices. This is a disagreeable conclusion, which seems to deprive the quantitative relationships in national accounts of much of their deeper significance. The figures will always have to be interpreted in the light of the prices used in their estimation. If market prices are used, they will readily be accepted as “plausible” and “natural”, since

market prices are a set of valuation coefficients which people are accustomed to use in everyday life. However, in this general acceptance lies also the risk that the quantitative relationships in the national accounts will be given a more absolute interpretation than is warranted.⁶ (When, for example, we are informed that the net value added of one industry is twice as high as that of another, we are easily led to accept this as in some sense “absolutely true”; however, we should not ignore the fact that the truth of the statement depends upon the prices used in the calculation.)

From the above, certain terminological requirements seem to follow. In formulating verbal definitions we should ensure that the terminology chosen reflects the quantity/price dimensions of the aggregates to be defined. For instance, we should take care to present our estimates of consumption, investment, exports, etc. (concepts defined as classes of real and/or financial objects), as estimates of “the *value*—at current prices or at the prices of a base year as the case may be—of goods and services consumed, invested or exported”. When it comes to concepts defined as numbers rather than as classes, phrases such as “the export surplus is the excess of exports over imports” (which is a false statement) should be avoided and replaced by “the export surplus is the *value* of exports minus the *value* of imports”. (If this amount of caution was shown our readers would perhaps accept more easily the fact that an export surplus at current prices may turn into an import surplus when measured at constant prices, i.e., at the prices of some other year.) Similarly, we should not say “GDP is a measure of the net output of the economy” but rather something like “GDP is a number expressing (i) the value of goods and services used for home consumption and investment plus (ii) the value of goods and services exported *minus* (iii) the value of goods and services imported (values measured at current or constant prices as the case may be)”. (By conveying the idea that our measure of GDP depends as much on prices as on quantities we would prepare the reader for the discovery—which he is bound to make some day—that the growth-rate of the economy at constant prices may change whenever the national accountant chooses to change his base-year.)

The set of postulates used in this note is seen to lead, quite naturally, to a set of definitions which satisfies the definitional equations given by the eco-circ relationships of section G. It is a delicate question whether this should lead to the conclusion that, in the revised SNA, the main concepts should be made to conform to the same relationships. If the general philosophy of this paper is accepted, the answer probably is yes. The implications are that “domestic” concepts should be given predominance over “national” concepts, that “market price” concepts

⁶If a statement on the quantitative relationship between two national accounting magnitudes is to have a more than conventional significance, it must be possible to demonstrate that the valuation coefficients (prices) chosen represent, in some sense or another, a transformatory relationship between the objects. Market prices represent in a sense a transformatory relationship of this kind; they indicate that the objects—under the prevailing market conditions, whatever this may imply—can be exchanged for one another at the prevailing market prices, so long as we have in mind the exchange of marginal quantities. The crucial question is whether these exchange-value relationships can be taken to represent more fundamental transformatory relationships between the objects. Such interpretations may be (i) that the exchange-value relationships are proportional to the utility afforded by the objects to any one individual, or (ii) that the exchange-value relationships express a technical transformatory relationship, in the sense that the community will be able to procure n units more of an object by relinquishing one unit of another object, whose market price is n times higher than the price of the first object.

should be given predominance over “factor cost” concepts, and that the main aggregates be defined in a way which does not make them dependent on a sub-classification (e.g. into current and capital) of the category transfers.

There are, obviously, a great number of problems of definition which have been left completely open by the present note. For example, we have not raised such questions as: Where precisely is the borderline of production to be drawn? When do transactions take place, e.g., are commodities exported when sold or when actually moved across the border? Is gold in gold-producing countries—or small coins anywhere—to be considered a real or a financial object? The answers to these and similar questions depend on the correspondence which we choose to establish—by convention—between the logical structure laid down by our set of postulates and observable (“real world”) phenomena. Here the range of choice is very wide. However, it is outside the scope of this note to enter into a discussion of such problems of conventions.