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THE MONETARY METHOD AND THE SIZE OF THE SHADOW ECONOMY: A CRITICAL ASSESSMENT

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A widely applied approach to measure the size of the shadow economy, known as the "monetary method" or the "currency approach," is based on econometric estimates of the demand for money. These estimates are used to get the currency held by economic agents in excess of the amount they need to finance registered transactions. This excess of currency multiplied by the income-velocity of circulation (assumed to be equal in the registered and shadow economies) gives a measure of the hidden GDP. This paper shows that the monetary method only produces coherent estimates if the income-elasticity of the demand for currency is one and suggests a way to correct the estimated size of the shadow economy when such elasticity is not one. The correction is applied to existent measures for different countries.

> . . . cash leaves no tracks, and makes no demands on anybody else's integrity.
> (Benjamin Friedman, *The Economist*, July 22, 2000, p. 76)

1. Introduction

Undeclared, under-declared, non-measured and under-registered transactions made to avoid the burden of taxes or to circumvent regulations, illegal transactions connected with crime and corruption and legal but non-market activities are included in the concept of shadow economy. For the last 30 years economists have been interested in the study of these transactions under the names of hidden, unrecorded, underground, parallel, black or shadow economy. A special volume of *The Economic Journal* (1999) and a survey by Schneider and Enste (2000) thoroughly document such interest. The average estimated size of the shadow economy varies from 12 percent of registered GDP for OECD members to 23 percent for transition economies and to 39 percent for developing countries (see, for example, Schneider, 2000).

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The very nature of the shadow economy makes its measurement a difficult task. Furthermore, different estimation methods target different concepts.[1] Therefore estimation methods have become an important theoretical and empirical issue.

A widely applied way to measure the size of the shadow economy, known as the "monetary method" or the "currency approach", assumes that cash is used to make transactions that agents want to keep hidden from official records and it is essentially based on the computation of discrepancies between declared income and the income implied by the observed currency demand.

In this paper we focus on an internal inconsistency of the technique that has repeatedly appeared in applied work and has not been addressed so far in the literature. We point out that in almost every empirical application the steps followed to "measure" the size of the shadow economy are inconsistent with the method itself. That is, rather than an extrinsic critique, we stress an intrinsic flaw. In particular the method assumes that the velocity of circulation of cash is the same in both reported and hidden sectors and we show that this equality is only true when the income elasticity of the demand for cash is one.[2] However, given that the hypothesis of equal velocities is the usual assumption, the literature has made the calculations "as if" the income elasticity were one even when it was not the case.[3]

The next section presents a brief outline of the evolution of the "currency approach". Section 3 discusses the method based on the econometric estimation of the demand for currency within an aggregation framework to present some requirements that the income-elasticity of the demand for cash should meet to get coherent results. In Section 4 a correction for incoherent results is suggested and applied to existent estimates. Some closing remarks are made in the last part of the paper.

2. An Outline of the "Currency Approach" Evolution

The "currency approach" to measure the size of the shadow economy is based on the assumption that cash is used to make transactions that agents want to keep hidden from official records. Transactions made using cash are difficult to trace because they leave no tracks. Other assets are registered in financial institutions and their uses are recorded in such a way that transactions made with them can be easily inspected. If the amount of currency used to make hidden transactions can be estimated, then this amount could be multiplied by the income-velocity of money to get a measure of the size of the shadow economy.

The "monetary method" was first presented by Gutmann (1977) and Feige (1979) and it has evolved to use econometric tools in estimates made by Tanzi

[1]Authors do not seem to be always aware of this observation.

[2]This is a general statement and it applies to any proxy of income used by different researchers. We refer to "almost every empirical application" because some find unitary income elasticities.

[3]The method has been strongly criticized on different grounds. Caridi and Passerini (2001) argued that it wrongly considers the concepts of unreported and unrecorded activities to be equivalent and Thomas (1999) and Breusch (2005a, 2005b), among others, doubt its quantitative accuracy based on econometric grounds concerning time series properties, structural breaks and sensitivity to units of measurement.

(1982, 1983), which are based on Cagan (1958). The technique was then applied to measure the size of the shadow economy in the U.S., Italy, Norway, Canada, South Africa, Tanzania, Mexico, India, Australia, Austria, Belgium, Denmark, France, Germany, Great Britain, Ireland, Netherlands, New Zealand, Spain, Sweden, Switzerland Argentina, etc.[4]

Neither Gutmann (1977) nor Feige (1979) used econometric estimates of the demand for currency. They reckoned the amount of money held to finance hidden transactions by reference to a past point in time in which, it was postulated, no shadow economy existed. The size of the hidden sector is then obtained multiplying the income-velocity of circulation—assumed to be equal for the registered and the shadow economies—by that amount of money.

The works by Tanzi (1982, 1983) and all the papers based on them use econometric estimates of the demand for currency. This approach recognizes that the income-velocity depends not only on variables that induce economic agents to make hidden transactions but also on income and the opportunity cost of holding cash. The estimated equation of the demand for currency is useful to get the extra cash held by economic agents to finance hidden transactions without postulating that there was some past time period in which no shadow economy existed. It is again assumed that the income-velocity of circulation for registered and hidden transactions is equal, so the size of the shadow economy is measured by multiplying the extra cash by that income-velocity of circulation.

3. The Standard "Monetary Method" Within an Aggregation Framework

We devote this section to build a formal aggregation framework useful to present the "monetary method" based on the econometric estimation of the demand for currency defined as real cash holdings and to derive the condition that the income-elasticity should meet to obtain equal velocities in both the registered and the hidden sectors.

A currency demand function in Cagan's (1958) tradition can be expressed as:

$$C_O = A(1+\Theta)^{\alpha} Y_O^{\beta} \exp(-\gamma i) \tag{1}$$

where C_O denotes observed cash balances, Θ is a vector of variables which induce agents to make hidden transactions (for example the ratio of taxes or government expenditure to GDP), Y_O is a scale variable (for example, registered GDP), i measures the opportunity cost of holding cash (the interest rate or the rate of inflation); A, α, β and γ are positive parameters. Observed currency, C_O, is equal to total currency, C_T, which includes cash used for recorded transactions, C_R, plus cash used for hidden transactions, C_H,

$$C_O = C_T = C_R + C_H \tag{2}$$

[4]Tanzi (1999) gives a skeptical view given the wide diversity of the results obtained.

Observed GDP, Y_O, is the real registered GDP, Y_R, which does not include hidden GDP, Y_H,

$$Y_T = Y_O + Y_H = Y_R + Y_H \tag{3}$$

Since observed currency includes C_H but observed GDP excludes Y_H, the usual econometric regression of C_O on Y_O would result in biased and inconsistent estimates.

The empirical applications based on this technique follow four steps. First, a demand for currency is estimated as in (1).[5] Second, under the implicit assumption that the demands for C_R and C_H have the same functional form with equal parameters, Θ is set equal to zero to get an estimate of the amount of cash demanded under no incentives to hide transactions, $\hat{C}_R$,

$$\hat{C}_R = \hat{A} Y_O^{\hat{\beta}} \exp(-\hat{\gamma} i) \tag{4}$$

Third, since $\hat{C}_R$ is known from (4) and C_T is observed currency, C_O, $\hat{C}_H$ can be obtained by difference,[6]

$$\hat{C}_H = C_T - \hat{C}_R \tag{5}$$

Fourth, to get the size of the shadow economy it is assumed that the velocity of circulation for both, registered and hidden transactions, is the same, so

$$v_R = \frac{Y_R}{C_R} = \frac{Y_H}{C_H} = \hat{v}_R \tag{6}$$

and then,

$$\hat{Y}_H = \hat{v}_R \hat{C}_H \tag{7}$$

$\hat{Y}_H$ is "the" estimation of the size of the shadow economy and it is obtained using $\hat{C}_H$ from (5) and $\hat{v}_R$ from (6).

So far we have described the procedures followed in the literature. The key assumption made explicit in (6) requires $\beta = 1$, which is evident if we recall that the income-velocity for registered money is

$$v_R = \frac{Y_R}{C_R} = \frac{Y_R}{A Y_R^{\beta} \exp(-\gamma i)} = \frac{Y_R^{1-\beta}}{A \exp(-\gamma i)} \tag{8}$$

while the velocity for the hidden economy is

[5]To take into account that the time series are integrated, some works consider equation (1) as a long run relation. Other papers estimate first difference equations, partial adjustment models or hybrids.

[6]Tanzi (1982) uses $\hat{C}_T$ instead of C_T in (5).

$$v_H = \frac{Y_H}{C_H} = \frac{Y_H}{A Y_H^{\beta} \exp(-\gamma i)} = \frac{Y_H^{1-\beta}}{A \exp(-\gamma i)} \tag{9}$$

The velocity is the same in both sectors if $\beta = 1$.[7] Those studies that find $\beta \neq 1$ but follow the steps described above are therefore incorrect.[8]

The assumption that the demands for C_R and C_H follow Cagan's type forms with equal parameters allows an explicit aggregation of (2) as:

$$C_T = A Y_R^{\beta} \exp(-\gamma i) + A Y_H^{\beta} \exp(-\gamma i) = A Y_R^{\beta} \exp(-\gamma i) \left(1 + \left(\frac{Y_H}{Y_R}\right)^{\beta}\right) \tag{10}$$

This formulation does not need to be restricted to currency. It is also valid for any wider aggregate (e.g. M_1) as long as the interest rate or the inflation rate is the opportunity cost of holding it. However β obtained from (1) only matches β in (10) if the ratio $\frac{Y_H}{Y_R}$ is independent of Y_R.[9] It must be stressed that all papers using this approach implicitly make this assumption.

4. A Correction of the Method

In this section we show that there is no need to impose ad hoc restrictions on the velocity of circulation when the "currency approach" is followed to compute the size of the hidden economy. The problem addressed in Section 3 can be solved by an explicit recognition that v depends on the value of β.[10] We also suggest a way to "correct" wrong estimates made by imposing $\beta = 1$ when it is not the case.

Equation (10) can also be written as:

$$C_T = A Y_R^{\beta} \exp(-\gamma i)(1 + \Theta)^{\alpha} \tag{11}$$

which is equal to (1). In fact equation (10) is always behind expression (11). Recalling that $Y_O = Y_R$, $C_T = C_O$ and that C_T and Y_R are observed variables, (11) can be econometrically estimated as in (1).[11] Next, as we already described in the previous section, setting $\Theta = 0$ provides an estimate for C_H. The ratio between C_R and C_H is

[7]The velocity is also the same for the improbable case in which $Y_R = Y_H$ for any β.

[8]It should be stressed that currency is the money aggregate whose demand should have an income elasticity equal to one. While this value may appear reasonable and theory-based in the case of the demand for the aggregate used to finance all transactions (e.g. the demand for M_1), it may not necessarily be correct for narrower definitions of money. For instance, in Baumol-Tobin's model, the value of transactions elasticity is 1/2.

[9]For this reason it is convenient to measure Θ normalized by registered GDP.

[10]It should be noticed that the income elasticity of currency demand is assumed to have the same value for the two sectors but this assumption cannot be tested.

[11]The variable $(1 + \Theta)^{\alpha}$ is sensitive to changes in the units in which Θ is measured, as pointed out in Breusch (2005a, 2005b).

$$\frac{C_R}{C_H} = \frac{AY_R^{\beta}\exp(-\gamma i)}{AY_H^{\beta}\exp(-\gamma i)} = \left(\frac{Y_R}{Y_H}\right)^{\beta} \tag{12}$$

Equation (12) provides an expression for Y_H given Y_R, C_R, C_H and β. Consequently there is no need to make the ad-hoc assumption on the equality of income velocity in both sectors.

The assumption about the equality of v for hidden and registered transactions is accurate if $\beta = 1$. This can also be seen from (12) since,

$$\frac{Y_R}{C_R} = \left(\frac{Y_R}{Y_H}\right)^{1-\beta}\frac{Y_H}{C_H} \tag{13}$$

or

$$v_R = \left(\frac{Y_R}{Y_H}\right)^{\beta-1} v_H \tag{14}$$

Equation (14) shows in another way that it is inaccurate to assume that v is equal for registered and hidden transactions when the hypothesis $\beta = 1$ is rejected. Equation (12) can be used to "correct" wrong estimates made by imposing $\beta = 1$ when it is not the case. From (12) it follows that

$$\frac{C_R}{C_H} = \frac{\overline{Y}_R}{\overline{Y}_H} \tag{15}$$

where $\frac{\overline{Y}_R}{\overline{Y}_H}$ is the (faulty) ratio obtained under the restriction $\beta = 1$. From (12) and (15)

$$\frac{Y_H}{Y_R} = \left(\frac{C_H}{C_R}\right)^{\frac{1}{\beta}} = \left(\frac{\overline{Y}_H}{\overline{Y}_R}\right)^{\frac{1}{\beta}} \tag{16}$$

which shows how to correct results obtained assuming $\beta = 1$ when $\beta \neq 1$.

5. Corrected Values for Some Empirical Estimations

In this section we present some results only as an exercise to illustrate the main point of the paper. We do not intend to provide accurate measures of the size of the underground economy. We only show estimates from other papers to apply the correction described above.[12] In fact it should be pointed out that the level of econometric analysis in these studies is rather basic and casts doubts on their

[12]Unfortunately, many studies applying the monetary method do not show the results of the estimation of the demand for currency, and they focus on the "one" number: the size of the shadow economy as a percentage of the GDP.

results, especially when the key role played by the estimated income elasticity is hardly recognized.[13] Authors generally present their results as percentages of GDP, implying that they computed the portion of GDP not registered by statistics.[14]

Argentina 1930–83

Guissarri (1986) measures the size of the shadow economy in Argentina for 1930–83 using annual data. He estimates a demand for currency which could be understood as a long run equation. The share of government expenditure in GDP and the ratio between the official and black exchange rates of the U.S. dollar were the chosen variables to quantify the incentives to hide transactions in Θ. The econometric estimate for $\hat{\beta}$ is 0.508. He follows the standard technique described in Section 3, that is, the assumption of equal velocities or $\beta = 1$. According to his calculations, the size of the shadow economy in 1983 (his last observation) represented 56 percent of registered GDP. Nevertheless, our correction implies that the magnitude of the hidden sector was 32 percent.[15]

Bolivia 1990–2003

Humérez Quiroz (2005) tries to estimate the size of the informal economy in Bolivia between 1990 and 2003. For 2000 the author finds that the underground sector was 51 percent of the registered GDP under the hypothesis of $\beta = 1$. The econometric estimations presented in the paper are such that $\hat{\beta} = 2.19$, so our correction rises the percentage to 73 percent.

Australia 1967–2000

Bajada and Schneider (2003) produce a time series estimate of the size of the "cash economy" in Australia between 1967 and 2000. The results are based on an error correction model for money demand in which $\hat{\beta}$ is 0.852. This is the long run estimate obtained by assuming the static equilibrium value of all variables. They calculate that the size of the shadow economy between 1990 and 2000 averaged 14.6 percent of GDP. They also compare this figure with the one resulting from a MIMIC approach: 14.82 percent. They stress the similarity of estimates. However, if the method is correctly followed, the fraction of the shadow income was 10.4 percent of the registered economy.

Norway 1952–78

Isachsen *et al.* (1982) get an income elasticity of the demand for currency of 0.85 between 1952 and 1978. The correction changes the share of the shadow economy from 8 to 5.1 percent of the registered economy.

[13]A pair of examples illustrates the point. Thomas (1986) re-estimated Tanzi's model for 1930–80 and found evidence of a structural break in 1945 while the tax variable was statistically not significant after 1946. Tanzi (1982) had found an income elasticity very close to one. Smith (1986) showed that the model of Matthews and Rastogi (1985) was mispecified.

[14]Another recent paper that applies the monetary method to estimate the size of the shadow economy and also displays the consistency flaw discussed here is Gadea and Serrano-Sanz (2002).

[15]See Ahumada *et al.* (2003) for a detailed analysis of this paper.

Isachsen and Strom (1985) use the "currency approach" to get an estimate of the size of the hidden economy of 6.3 percent for 1978. This estimate is based on a partial adjustment currency demand equation with a long run income elasticity equal to 0.663. The corrected estimate gives 1.51 percent.

Tanzania 1968–90

Bagachwa and Naho (1995) present two different time series estimates of the size of the shadow economy, based on different real currency demand functions which differ only in the way the tax and government intervention variables enter the equation. The ratio of hidden to registered economy for 1990 is 33.24 percent from the first equation and 20.96 percent from the second. However, the income elasticities they find are 2.323 and 2.569 respectively. For 1990 the corrections give estimates of 62.2 percent and 54.4 percent.

6. Final Remarks

The "monetary method" to measure the size of the shadow economy is based on econometric estimates of the demand for money. These estimates are used to get the currency held by economic agents in excess of the amount they need to finance registered transactions. The standard monetary approach uses the excess of currency multiplied by the velocity of circulation (assumed to be equal in the registered and the shadow economies) to measure hidden GDP.

This paper builds a formal aggregation framework to show that this procedure is accurate only when the income elasticity of the demand for money is one. The paper also suggests a way to correct faulty estimates and applies it to some published results to show that the assumption of equal velocities together with income elasticity estimates lower (higher) than one result in figures biased upwards (downwards) for the shadow economy.

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