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EX POST VERSUS EX ANTE MEASURES OF THE USER COST OF CAPITAL

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Should we use ex post or ex ante measures of user costs to calculate the contribution of capital in a growth accounting exercise? The answer, based on a simple model of temporary equilibrium, is that ex post is better in theory. In practice researchers usually calculate ex post user costs by assuming that the rate of return is equalized across assets. But this is only true if expectations are correct. In general, the ex post rate of return differs between assets, even though ex ante it is the same. I propose a hybrid method. The index of capital services is estimated using ex ante weights; the contribution of capital is the growth of this index multiplied by the ex post income share of capital. I show that this method is theoretically correct if the production function is CES. I compare the ex post, ex ante and hybrid methods using data for 31 U.K. industries from 1970 to 2000.

1. INTRODUCTION

Measuring the cost of capital services is important for many purposes, for example, for growth accounts as well as for the construction of capital measures in the national accounts. But there has been a long-standing debate over whether an ex post or an ex ante measure of the user cost is better. The present paper proposes a hybrid method, incorporating elements of both approaches. I argue that this new method is both consistent with economic theory and can readily be implemented in practice by national statistical agencies.

In order to do growth accounting we need to estimate the contribution of capital to the growth of output. This contribution equals the elasticity of output with respect to capital services multiplied by the growth of capital services. In the real world there are many types of capital so we need to estimate an index of the growth of capital services. For the latter we need estimates of the user cost (rental price) of each asset to employ as weights, on the assumption that user costs measure marginal products. Some of the elements of the user cost, e.g. asset prices, are known ex post but not with certainty ex ante. Another element, the rate of return, is still more problematic. The standard approach (e.g. Jorgenson and Griliches, 1967; Christensen and Jorgenson, 1969; Jorgenson *et al.*, 1987) has been to use an ex post measure; this is sometimes also called the endogenous approach. In the ex post approach it is assumed that the rate of return is equalized

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across assets. Then this unknown rate can be found by using the condition that the sum of the returns across assets (where the return on an asset is the product of its user cost and the flow of capital services that it yields) equals observed, total profit (gross operating surplus in national accounts language). The alternative, ex ante approach, sometimes also called the exogenous approach, employs a rate of return derived from external information, e.g. from financial market data, together often with estimates of expected, rather than actual, asset price inflation (though the latter could also be employed).

Many have felt uncomfortable with the ex post approach (e.g. Schreyer *et al.*, 2003; Schreyer, 2004). After all, investment decisions have to be made in advance of knowing all the relevant facts. Surely agents employ some notion of the required rate of return in deciding how much to invest, and this required rate may differ from the actual, realized rate? Equally, they must base their decisions on expected, not actual, capital gains and losses. Using the ex post measure would seem to imply either that all expectations are realized (a world of perfect certainty) or that the quantities of capital can be instantaneously adjusted to the desired levels, after all uncertainties have been resolved. Neither assumption seems attractive a priori.[1] This suggests using an ex ante approach. On the other hand, when doing growth accounting we are interested in what the contribution of capital actually was, not in what it was expected to be, and for this the ex post approach seems preferable (Berndt and Fuss, 1986).

Resolving the ex post versus ex ante issue is also quite topical in the light of the current revision of the 1993 System of National Accounts (SNA). The 1993 SNA requires that gross operating surplus be included as a category of income, on all fours with compensation of employees. But though it is widely recognized that gross operating surplus (and probably a part of mixed income too) equals the return to non-financial assets, just as compensation of employees equals the return to labor, the 1993 SNA does not spell this out. Following the publication of the OECD manual on capital measurement (OECD, 2001b), there has been an increasing desire on the part of several national statistical agencies to produce statistics of capital consumption, capital stocks and capital services that are internally consistent. Now the "Advisory Expert Group for the Update of the System of National Accounts, 1993" has recommended that countries which wish to do so may include a breakdown of gross operating surplus into the returns accruing to different assets; such a breakdown will not be included in the core accounts but may be included in supplementary accounts (Intersecretariat Working Group on National Accounts, 2007). The arguments in favor of this approach are spelled out in Schreyer *et al.* (2005), which however leaves unresolved the issue of whether an ex ante or ex post measure of the cost of capital should be employed.

The purpose of this paper is to assess the case for and against ex post and ex ante measures. The correct measure of the user cost depends on the underlying model. So I set out in Section 2 a simple model that is very similar to that of Berndt and Fuss (1986). Here firms have to choose the levels of their asset stocks before knowing for sure the price of output, the level of TFP, and the wage. Once these

[1]The OECD capital and productivity manuals (OECD, 2001a, 2001b) mention the ex post and ex ante alternatives but without substantive discussion as to which is preferable.

data are revealed, firms can choose their labor input but cannot adjust their capital inputs until the next period. This model, if true, tells us how we ought to do growth accounting. According to this model, the ex post user costs are the correct measures to use. But Section 3, which considers how both methods are applied in practice, shows that the common method of calculating ex post user costs is not in general correct. The reason is that unless all expectations are realized, ex post rates of return differ between assets even though ex ante they are expected to be the same. So the growth of capital services may be better measured using the ex ante method, though this is not guaranteed to be the case in practice, since firms' expectations cannot be measured exactly. However, within the usual assumptions of growth accounting, the overall weight to be applied to the index of capital services growth when calculating TFP growth should be the ex post, not the ex ante, profit share, which is easily observed. I then go on to suggest a hybrid method of estimating the capital services index, which uses elements of both the ex ante and the ex post approaches. An advantage of this hybrid approach is that it uses exactly the same readily observable data as does the ex post approach. Section 4 checks how much difference method makes to the results using data for 31 industries covering the whole U.K. market sector (i.e. excluding only health, education, and government), 1970–2000. Section 5 concludes.

2. The Model

I consider an economy with many industries where capital assets are quasi-fixed. Investment decisions have to be made at the beginning of the period, before the price of output, the prices of assets, the prices of other inputs, or the level of technology are known with certainty.[2] At the end of the period prices and technology are revealed and firms get the chance to choose their variable inputs, but they cannot at that point change the level of their capital inputs. In the next period, the problem is repeated. I assume that firms are risk-neutral. The problem of the representative firm in each industry is then to maximize the expected present value of the future stream of receipts, net of payments including tax.

The representative firm has to make its decisions in two stages. In the first stage it chooses the levels of the asset stocks in order to obtain the desired flows of capital services in the second stage. It must make this decision without knowing for certain what prices, wages or the level of technology (TFP) will be in the second stage. In the second stage, prices, wages and TFP are revealed and then the firm gets the chance to choose its variable inputs. At the end of the second period the firm gets to choose again the level of its capital stocks. I assume that second hand markets for capital goods are available, so any investment decisions that the firm turns out to regret can then be undone. I also assume constant returns to scale and that firms are price-takers in input markets and in the output market. There are m capital goods, which are quasi-fixed; for ease of exposition I assume that there is only one variable input ("labor").

I assume a neo-classical production function given by

[2]This model is essentially the same as the one considered by Berndt and Fuss (1986). In their model short period equilibrium is where capital stocks are pre-determined and firms adjust variable factors to maximize profits.

(1) $$Y_t = B_t f(K_{1t}, K_{2t}, \ldots, K_{mt}, L_t)$$

Here Y_t is the firm's output during period t, B_t is the level of TFP, assumed stochastic, L_t is labor input, and K_{it} is the flow of capital services from the i-th asset during period t.[3] The evolution of the capital stocks is given by:

(2) $$A_{is} = I_{is} + (1-\delta_i)A_{i,s-1}$$
$$K_{is} = A_{i,s-1}, \quad i = 1, \ldots, m; \quad s = t-1, t, \ldots$$

where A_{is} is the physical stock of the i-th type of capital at the end of period s, δ_i is its depreciation rate, and I_{is} is gross investment in the i-th type. The flow of capital services during s is assumed to be proportional to the stock at the end of $s-1$ (with the constant of proportionality normalized to equal one).

The firm's real, net receipts after tax in period t are:

$$(1-u_t)(p_{Yt}Y_t - w_tL_t - D_t) - \sum_{i=1}^{m} h_{it}p_{it}I_{it}$$

where u_t is the corporate tax rate; p_{Yt} is the price of output in this industry; w_t is the wage; p_{it} is the price of the i-th asset; D_s is the real value in period s of depreciation allowances on capital acquired prior to $t-1$; and h_{it} is the proportion of the price of the i-th type of capital that the firm has to pay, i.e. it is one minus the effective subsidy rate to capital provided by the tax system (for example, through depreciation allowances in excess of true economic depreciation). I measure all prices and wages (p_{Yt}, p_{it}, w_t) relative to some index of the aggregate price level, e.g. the GDP deflator or the price of consumption goods. This is the sense in which "real value" is to be understood.

The firm's problem at the first stage, maximize the expected present value of net receipts, can be written as:

(3) $$\max_{K_{1t},\ldots K_{mt}, L_t} H_{t-1} = -\sum_{i=1}^{m} h_{i,t-1}p_{i,t-1}I_{i,t-1} +$$
$$(1+r_t^*)^{-1}E_{t-1}\left[(1-u_t)(p_{Yt}Y_t - w_tL_t - D_t) - \sum_{i=1}^{m} h_{it}p_{it}I_{it}\right] +$$
$$(1+r_t^*)^{-1}(1+r_{t+1}^*)^{-1}E_{t-1}[(1-u_{t+1})(Y_{t+1} - w_{t+1}L_{t+1} - D_{t+1}) -$$
$$\left.\sum_{i=1}^{m} h_{i,t+1}p_{i,t+1}I_{i,t+1}\right] + \ldots$$

Here r_t^* is the real, required rate of return, assumed to be known to the firm, and E_{t-1} denotes the expectations operator, which is conditional on information available at the end of period $t-1$. This objective function is to be maximized subject to the constraints, the production function and the capital accumulation equations. The maximization is in principle to be done for all the firm's decision variables, i.e. all inputs over all time from now to infinity. But from the structure of the problem, there is no need to do the maximization over all future time, just the current period ($t-1$) and the next period (t), because the existence of second

[3]To simplify the notation, I omit the subscript indicating the industry.

hand markets for capital goods allows the firm to re-optimize at the end of periods t, $t + 1$, etc. Mathematically, we shall see that the first order conditions for the K_{it} depend only on variables dated $t - 1$ and t.

After substituting the constraints (1) and (2) into the objective function (3), the first order conditions for capital services in period t (i.e. for asset stocks at the end of $t - 1$) are:

$$\frac{\partial H_{t-1}}{\partial K_{it}} = -h_{i,t-1}p_{i,t-1} + (1+r_t^*)^{-1}E_{t-1}\left[(1-u_t)\,p_{Yt}\frac{\partial Y_t}{\partial K_{it}} + h_{it}(1-\delta_i)(1+\pi_{it})\,p_{i,t-1}\right] = 0,$$
$$i = 1, \ldots, m$$

where I set $\pi_{it} = (p_{it} - p_{i,t-1})/p_{i,t-1}$, the real rate of capital gain on the i-th asset. Solving for the expected marginal product, these conditions reduce to:

(4)
$$\begin{aligned} E_{t-1}\left[p_{Yt}\frac{\partial Y_t}{\partial K_{it}}\right] &= \frac{h_{it}}{1-u_t}\Big[\big(1+r_t^*\big)-(1-\delta_i)(1+E_{t-1}[\pi_{it}])\big)\Big]p_{it-1} \\ &= T_{it}\left[r_t^* + \delta_i\left(1+\pi_{it}^e\right) - \pi_{it}^e\right]p_{it-1}, \quad i = 1, \ldots, m \end{aligned}$$

putting $\pi_{it}^e = E_{t-1}[\pi_{it}]$ and $T_{it} = h_{it}/(1 - u_t)$, the tax factor, and assuming for simplicity that $h_{it} = h_{i,t-1}$. In other words, to maximize profits the expected real value of the marginal product of each type of capital must be set equal to the expected rental price, where the expected rental price is defined as:

(5)
$$q_{it}^e = T_{it}\left[r_t^* + \delta_i\left(1+\pi_{it}^e\right) - \pi_{it}^e\right]p_{it-1}, \quad i = 1, \ldots, m$$

This is of course the familiar Hall–Jorgenson formula in discrete time, extended to allow for risk (Hall and Jorgenson, 1967; Jorgenson, 1989).[4] Note that the real rate of return is the same for all assets. Due to constant returns to scale, equations (4) pin down only the labor–capital ratios (or equivalently, the output–capital ratios). To pin down the desired *level* of the capital stocks, firms also need to form expectations about the output price, the level of demand and the level of TFP in the next period.[5] This closes the model and fixes the asset stocks.

At the second stage, firms must choose labor input so as to maximize post-tax profit, where the firm takes as given the output price, the wage, the level of TFP, and the pre-determined levels of the capital stocks. So the problem is now:

$$\max_{L_t}[(1-u_t)(p_{Yt}Y_t - w_tL_t - D_t)]$$

subject to

[4]There is also a first order condition for labor which says that the expected real value of the marginal product of labor should equal the expected real wage. But this condition is not required for the solution since the firm gets the chance to revise its plan for labor in the second stage.

[5]For the industry as a whole, market demand helps to determine output and price. Then some process, including entry and exit, is needed to allocate market demand amongst surviving firms. But for present purposes, we do not need to be specific about this.

$$Y_t = B_t f(A_{1,t-1}, A_{2,t-1}, \ldots, A_{m,t-1}, L_t)$$

and the first order condition is:

$$(6) \qquad p_{Yt}\left[\frac{\partial Y_t}{\partial L_t}\right]_{K_{jt}=A_{j,t-1},\forall j} = w_t$$

This last equation determines the optimal labor input. It also defines implicitly the short run supply curve of output. To the extent that expectations are not realized, firms will move up or down the short run supply curve, hence output and labor input may differ from what was expected when the levels of the capital stocks were fixed, as too will the marginal products of capital. We can define the real, ex post user cost of capital (q_{it}) as equal to the ex post real value of the marginal product:

$$(7) \qquad q_{it} = T_{it}[r_{it} + \delta_i(1+\pi_{it}) - \pi_{it}]\, p_{i,t-1} = p_{Yt}\left[\frac{\partial Y_t}{\partial K_{it}}\right]_{K_{jt}=A_{j,t-1},\forall j, L_t=L_t^*}, \quad i = 1, \ldots, m$$

where r_{it} is the ex post rate of return. This last equation could be thought of as an implicit definition of the ex post rate of return: if we knew the parameters of the production function, we could solve for the rates of return (the r_{it}'s), since all the other elements of the user cost can be taken as known. Note that here I do *not* assume that the ex post rate of return is equalized across assets, so I write r_{it}, not r_t. In fact I shall show in a moment that in general ex post rates of return will differ as between assets, even though ex ante they are the same.

Now let us consider how growth accounting should be done in this model. We should estimate the contribution of capital services using the actual (ex post) marginal products (Berndt and Fuss, 1986; Berndt, 1990). This is because, under our assumptions of perfect competition on both the output and input sides and constant returns to scale, the basic growth accounting equation holds in this model. From (1):

$$(8) \qquad \frac{d\ln B_t}{dt} = \frac{d\ln Y_t}{dt} - \frac{L_t}{Y_t}\frac{\partial Y_t}{\partial L_t}\frac{d\ln L_t}{dt} - \sum_{i=1}^{m}\frac{K_{it}}{Y_t}\frac{\partial Y}{\partial K_{it}}\frac{d\ln K_{it}}{dt}$$

Now by Euler's Theorem

$$(9) \qquad \begin{aligned} Y_t &= \frac{\partial Y_t}{\partial L_t}L_t + \sum_{i=1}^{m}\frac{\partial Y_t}{\partial K_{it}}K_{it} \\ &= \frac{w_t}{p_{Yt}}L_t + \sum_{i=1}^{m}\frac{q_{it}}{p_{Yt}}K_{it} \end{aligned}$$

from (6) and (7). Now define S_t as aggregate real profit (gross operating surplus), i.e. aggregate nominal profit deflated by the overall price level: $S_t = p_{Yt}Y_t - w_tL_t$. Hence from (9)

(10) $$S_t = \sum_{i=1}^{m} q_{it} K_{it}$$

i.e. the sum of the ex post returns equals ex post profit. Finally, substituting (6), (7) and (10) into (8), we obtain

(11) $$\frac{d \ln B_t}{dt} = \frac{d \ln Y_t}{dt} - \left(1 - \frac{S_t}{p_{Yt} Y_t}\right) \frac{d \ln L_t}{dt} - \left(\frac{S_t}{p_{Yt} Y_t}\right) \sum_{i=1}^{m} \frac{q_{it} K_{it}}{S_t} \frac{d \ln K_{it}}{dt}$$

This proves that ex post rental prices and ex post profits are the correct weights to use for growth accounting. So if there were only one type of capital, there would be no difficulty in estimating TFP growth from the right hand side of (11), employing a discrete approximation such as the Törnqvist. We would just use the share of profit in the value of output to measure the elasticity of output with respect to capital. But if there is more than type of capital, how are we to measure the elasticity of output with respect to each type separately?

To make further progress, we need to be more explicit about the production function. From now on, I assume that the production function is CES:

(12) $$Y_t = B_t \left[\sum_{i=1}^{m} a_i K_{it}^{(\sigma-1)/\sigma} + a_{m+1} L_t^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)}, \quad B_t > 0, \quad a_i > 0$$

where $\sigma > 0$ is the elasticity of substitution. Later, I consider the effect on the results of using a more general production function.

In the CES case, the ex post real value of the marginal product of the *i*-th type of capital is

(13) $$q_{it} = p_{Yt}(\partial Y_t / \partial K_{it}) = a_i \left[p_{Yt} (B_t^{\sigma-1} Y_t)^{1/\sigma} / K_{it}^{1/\sigma} \right].$$

and the expected (ex ante) real value of the marginal product is

(14) $$q_{it}^e = E_{t-1}[p_{Yt}(\partial Y_t / \partial K_{it})] = a_i E_{t-1} \left[p_{Yt} (B_t^{\sigma-1} Y_t)^{1/\sigma} \right] / K_{it}^{1/\sigma}$$

The first order condition for labor at the second stage is:

(15) $$w_t / p_{Yt} = a_{m+1} (B_t^{\sigma-1} Y_t)^{1/\sigma} / L_t^{1/\sigma}$$

By solving (15) for L_t and substituting the result into (12) we can find the short run supply curve of output as a function of the output price (for a given wage), at least implicitly (no closed form solution for Y_t is possible, except if $\sigma = 1$, the Cobb–Douglas case). This completes the account for the representative firm. To complete the account for the economy as a whole we would need to add the demand side, but for present purposes this is not necessary.

We can now answer the question posed above (How are we to measure the elasticities of output with respect to each of the many types of capital?) by noting three implications of our model:

1. By definition, the (true) ex post user cost equals the marginal product. In other words, asset-specific, ex post user costs are the correct weights for constructing capital services when capital is quasi-fixed, leaving aside for the moment how to measure user costs (the q_{it}) in practice. A Törnqvist index of the growth of aggregate capital services (K) can be written as:

$$\ln[K_t/K_{t-1}] = \sum_{i=1}^{m}(1/2)(v_{it}+v_{i,t-1})\ln[K_{it}/K_{i,t-1}] \tag{16}$$

where

$$v_{it} = q_{it}K_{it}\Big/\sum_{i=1}^{m} q_{it}K_{it}.$$

We have already noted (see (10)) that, under the assumption of constant returns to scale, the sum across assets of the ex post returns equals real profit: $S_t = \Sigma_{i=1}^{m} q_{it}K_{it}$.

2. The ex ante and ex post user costs, and the ex ante and ex post marginal products, are proportional to each other *and the factor of proportionality is the same for all assets*:

$$\frac{q_{it}^e}{q_{it}} = \frac{E_{t-1}[p_{Yt}B_t^{(\sigma-1)/\sigma}Y_t^{1/\sigma}]}{p_{Yt}B_t^{(\sigma-1)/\sigma}Y_t^{1/\sigma}}, \quad i = 1, \ldots, m, \tag{17}$$

from (13) and (14). This means that we could equally well use the ex ante user costs in place of the ex post ones for the weights in the index of capital services since

$$v_{it} = q_{it}K_{it}\Big/\sum_{i=1}^{m} q_{it}K_{it} = q_{it}^e K_{it}\Big/\sum_{i=1}^{m} q_{it}^e K_{it} = v_{it}^e \text{ (say)}, \quad i = 1, \ldots, m. \tag{18}$$

However, even if we use ex ante weights in constructing the index of capital services, we should still use the ex post aggregate profit share ($S_t/p_{Yt}Y_t$) as the weight to apply to that index when estimating TFP growth: see (8) above. So the contribution of capital to output growth, using for example a Törnqvist index, is

$$(1/2)[(S_t/p_{Yt}Y_t) + (S_{t-1}/p_{Y,t-1}Y_{t-1})]\ln[K_t/K_{t-1}] \tag{19}$$

with the weights in the capital services index measured either by (16) or (18). The interest of this result is that, as the next section will show, there is a practical method of estimating the ex ante user costs, but the usual method of estimating ex post user costs produces the wrong answer, unless all expectations turn out to be correct.

3. Ex ante profits are $S_t^e = \Sigma_{i=1}^{m} q_{it}^e K_{it}$ and these stand in the same proportion to ex post profits as do ex ante to ex post user costs:

$$\frac{S_t^e}{S_t} = \frac{\sum_{i=1}^{m} q_{it}^e K_{it}}{\sum_{i=1}^{m} q_{it}K_{it}} = \frac{E_{t-1}[p_{Yt}B_t^{(\sigma-1)/\sigma}Y_t^{1/\sigma}]}{p_{Yt}B_t^{(\sigma-1)/\sigma}Y_t^{1/\sigma}}. \tag{20}$$

These results have been established for a CES production function with only one variable input. But still sticking with CES, equation (17) would continue to hold whatever the number of variable inputs. However, dropping the CES assumption and assuming a more general neo-classical production function would change the results (even with constant returns to scale). For example, with a translog production function, the ratio of expected to actual marginal products would no longer be the same for all assets, i.e. (17) would not hold. But the difference between the CES and translog cases depends on the share elasticity parameters which may in practice be small (see Appendix). If so, the CES assumption could still be defended as a good approximation in this context. Note that we only need the CES to be a good approximation for two consecutive points; we do *not* need to assume that the same CES production function holds at all points.

3. Measuring User Costs in Practice

Now I turn to how ex ante and ex post user costs can be estimated in practice. Tax factors, asset prices and the output price are directly observable, at least in principle. Depreciation rates present more of a problem but for the present purpose I ignore this and assume that data on these are available too.

Ex Ante User Costs

To estimate ex ante user costs we require one-period-ahead forecasts of asset prices: see equation (5). These could be generated by univariate time series models (Harper *et al.*, 1989). In addition, we need the required rate of return. This could be taken from financial market data. Diewert (2001) suggests using a constant real interest rate of 4 percent per annum (which he argues suits the OECD experience) plus the actual rate of consumer price inflation. However, there are advantages in not having to resort to extraneous data, especially if we are trying to develop methods that could be applied in practice by statistical agencies. An alternative, two step procedure requires only the data used in the ex post method. First, define the weighted average actual rate of return ($\bar{r}_t$) as:

$$\bar{r}_t = \frac{\sum_{i=1}^{m}(T_{it}p_{i,t-1}A_{i,t-1})r_{it}}{\sum_{i=1}^{m}T_{it}p_{i,t-1}A_{i,t-1}}$$

where the weights are asset values (adjusted for tax). Then we can solve for this average rate of return using equations (7) and (10):

$$\text{(21)} \qquad \bar{r}_t = \frac{S_t - \sum_{i=1}^{m}T_{it}p_{i,t-1}A_{i,t-1}[\delta_i(1+\pi_{it}) - \pi_{it}]}{\sum_{i=1}^{m}T_{it}p_{i,t-1}A_{i,t-1}}.$$

Note that this is *not* the same as assuming that the ex post rates of return are the same for all assets. However, were we to make that assumption (i.e. $r_{it} = r_t$,

$i = 1, \ldots, m$), then the common rate of return would also be given by the right hand side of (21): see below. The second step is to form a forecast of this average rate. We then have all the information needed to calculate the ex ante user costs. The justification of this procedure is as follows. The average, ex post rate of return must bear some relation to the rate of return required ex ante. Firms would not go on investing indefinitely if they expected the actual rate to be below the required one. Hence an econometric estimate of the required rate should be extractible from the average rate experienced in practice.[6]

Ex Post User Costs

As noted above, if there were only one asset then calculating the ex post user cost would present no difficulty. We could just use total profit as the weight for capital in the growth accounting calculation. But in practice there are many assets, which leaves the problem of how to in effect divide up profit amongst the assets. The usual way to calculate ex post user costs is to assume that the rate of return is the same for all assets; this common rate can then be found from the relationship $S_t = \Sigma_{i=1}^{m} q_{it} K_{it}$, equation (10), where the q_{it} are given by equation (7). The solution is given by the right hand side of equation (21), but now with a different interpretation. Using this solution together with the other known data, the ex post user costs can then be calculated. This method has been used by Christensen and Jorgenson (1969) and Jorgenson *et al.* (1987), who have been followed by many other researchers, e.g. Oulton and Srinivasan (2003, 2005a) and O'Mahony and van Ark (2003); it is also used by the Bureau of Labor Statistics and the Australian Bureau of Statistics to produce official estimates on an ongoing basis. The difficulty here is that the assumption of a common rate of return ex post cannot be justified in general. In fact we can prove a proposition:

> *Proposition*: In the model set out above, and in the absence of a numerical fluke, the ex post rate of return will be the same for all assets if and only if all expectations (for asset prices, output price and output) are satisfied, in which case the ex post and ex ante rates of return are equal.

The proof is in the Appendix. The intuition behind the proof is as follows. We have just shown that ex ante and ex post user costs are proportional to each other. But the rate of return element as a proportion of the user cost varies between assets. So there is no way that the ex post rates of return can all be equal and so proportional to the ex ante rate unless there are offsetting differences between the expected and actual rates of asset price inflation. But if such differences occurred they would be just a numerical fluke.[7]

[6]Harper *et al.* (1989) use an ARIMA model to estimate expected asset price inflation in some of their variants, as recommended by Diewert (1980). But no-one seems to have used any such method to estimate the rate of return.

[7]The earlier version of this paper (Oulton, 2005), gives numerical examples based on a CES production function to show how ex post rates of return can differ across assets.

TABLE 1

Asset Types and Depreciation Rates Used

Asset	Depreciation Rate (% per annum)
Structures	2.5
Plant and machinery (excluding ICT)	13.0
Vehicles	25.0
Intangibles (excluding software)	13.0
Computers	31.5
Software	31.5
Communications equipment	11.0

Note: For the transport industries, depreciation rates for vehicles were assumed to be as follows: rail transport (5.89 percent), water transport (6.11 percent), and air transport (8.25 percent).

4. How Much Difference Does Method Make? A Check Using U.K. Data

4.1. *Data and Methods*

I employ the Bank of England Industry Dataset (BEID) to estimate the growth of capital services and the contribution of capital by three different methods. The BEID provides data on real and nominal investment in seven types of asset, three ICT and four non-ICT, for each of 34 industries covering the whole economy over the period 1948–2000 inclusive: see Oulton and Srinivasan (2005a, 2005b) for full details. The asset types together with the depreciation rates that I used are given in Table 1.

These depreciation rates are comparable to those employed by the U.S. Bureau of Economic Analysis and by most U.S. researchers. I exclude three industries that are largely in the public sector, accounting for around 20 percent of GDP: Public Administration & Defense, Education, and Health & Social Work. For these the measures of gross operating surplus are not very meaningful.[8] The 31 remaining industries constitute what I call the market sector.

The actual period of the analysis was 1970–2000, since the data for value added and gross operating surplus are for this period. Investment data for the years 1948–69 were used to generate the capital stocks at the beginning of 1970, in conjunction with estimates of the starting stocks at the beginning of 1948. For two of the ICT assets—computers and software—nominal investment was deflated by official U.S. price indices, adjusted for exchange rate changes. For the other assets, I employed official U.K. price indices as deflators.

The starting point for each method is estimates of capital stocks for each of the seven asset types. These are generated from the investment data by assuming that depreciation is geometric as in equations (2).

Capital services were then estimated by three methods: (1) ex post; (2) ex ante; and (3) hybrid. Under the ex post method we solve for the unknown common rate of return. The returns to the assets then by definition sum to observed gross

[8]Obviously, most output in the public sector is not sold to consumers at a market price. So GOS cannot be calculated as revenues minus labor and intermediate costs. Instead in the national accounts GOS is calculated as depreciation on the estimated stocks of assets, with no allowance for a return to capital.

TABLE 2
Comparison of Methods of Estimating Capital Services and the Contribution of Capital

Method	Rate of Return	Prices	Weights in Capital Services Index	Weight for Contribution of Capital
Ex post	Ex post, same for all assets; differs across industries	Actual	Returns to assets estimated using common, *ex post* rate of return and actual prices; returns sum to actual, observed GOS	Observed GOS
Ex ante	Derived *a priori*, same for all assets	Forecast by ARMA model	Returns to assets estimated using *ex ante* rate of return and *predicted* prices; returns do *not* sum to observed GOS	Predicted GOS
Hybrid	Derived *a priori*, same for all assets	Forecast by ARMA model	As for *ex ante* method, except that returns sum to observed GOS	Observed GOS

operating surplus (GOS).[9] Recall however that, as shown in the last section, the ex post rate of return will only be equalized across assets when all expectations are realized. Under the ex ante method, we use an estimate of the required rate of return in the user cost formula. We forecast prices using ARMA models. And we employ predicted, not actual, GOS to calculate capital contributions. In the hybrid method we use the ex ante method to estimate capital services but actual, ex post profit to calculate the capital contribution. See Table 2 for more details.

The hybrid method still gives only an approximation to the true weights suggested by the theory. It differs from the latter to the extent that there are errors in the estimates of the required rate of return or of price expectations. A second qualification to this result is that, when we come to empirical work, all assets that contribute to profit should be identified. If some assets, e.g. land or inventories, contribute to profit but are not included amongst the capital stocks, then some of the return assigned to the included assets should really be assigned to the excluded ones.[10] So the ex post rate of return to the included assets will be overstated as will the overall weight to be assigned to capital. Also, investment in some assets, e.g. some types of intangibles, may be misclassified as current rather than capital expenditure (Corrado *et al.*, 2006). If so, some part of measured profit may again be a return to these omitted assets.

Ex Post Method

The first step was to estimate the nominal rate of return and the user costs for each of the seven assets, for each of the 31 industries, using the ex post method. It is also helpful for analytical purposes to calculate real rates of return, defined as the nominal rate for each industry minus the growth rate of an appropriate price

[9]Gross operating surplus was adjusted to remove mixed income and the portion estimated to be the return to holding inventories.

[10]In the empirical work reported in Section 4, I make allowance for the contribution of inventories to profit, but not for the contribution of land.

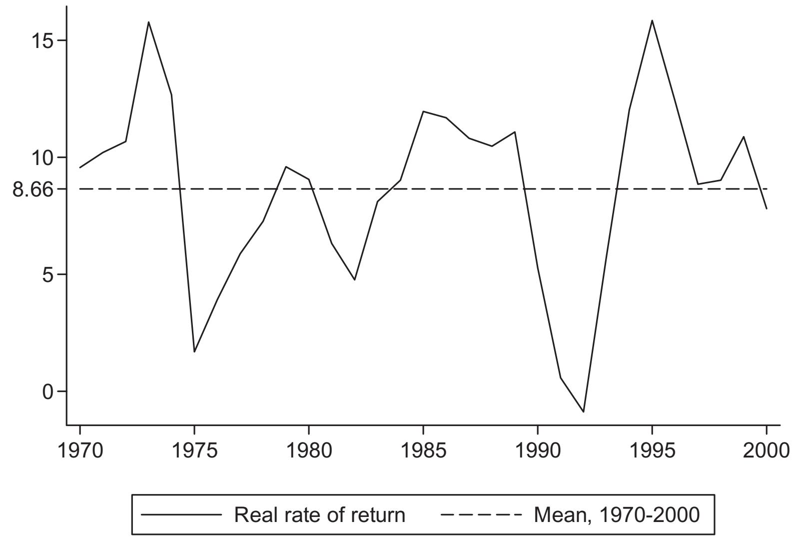

Figure 1. Real rate of return in the market sector, % p.a.
Source: Bank of England Industry Dataset. Market sector is aggregate of 31 industries.

index. For the latter I used the same price index for each industry, namely the implicit deflator for GDP in the market sector. This was estimated as a Törnqvist index of the implicit deflators for value added in each industry (the weights were nominal value added); data again came from the BEID.

As a diagnostic tool I first derive the nominal rate of return in the market sector as a whole (i.e. using aggregate data). The corresponding real rate is shown in Figure 1.[11] The mean of this rate over 1970–2000 was 8.66 percent per annum. The rate of return is clearly quite volatile (the standard deviation is 3.96) and shows steep declines in the major recessions (1975–76, 1981–82, and 1991–92). Also, it appears to show no trend. After some experimentation with ARMA models, an AR(2) model was found to fit well and displayed white noise errors (here R is the real rate of return in the market sector and z statistics are in parentheses):

$$R_t = \underset{[10.20]}{0.0830} + \underset{[6.83]}{0.9040}R_{t-1} - \underset{[-4.27]}{0.6202}R_{t-2} + \text{error}$$

Period: 1972–2000, N = 29, see = 0.0260
Tests that errors are white noise:
Bartlett's B statistic = 0.5605, probability > B = 0.91
Ljung – Box Q statistic = 7.8587, probability > Q = 0.80

[11]The real rate (R_t) is calculated by the formula: $R_t = [(1 + r_t)/(1 + \pi_t)] - 1$, where r_t is the ex post nominal rate of return and π_t is the (discrete) growth rate of the GDP deflator, both in the market sector.

I next calculated the ex post real rate of return in each industry separately and then for each industry the time mean of these rates. Due to differences in risk, one would not expect the mean return to be the same in all industries. But the standard deviation across the 31 industries of the time means is very high, 11.72 percent per annum, indicating persistent differences over a 31 year period that seem implausibly high.[12]

A check on the validity of the ex post method is given by looking at the number of negative user costs that the method generates. In the quasi-fixed capital model a negative user cost implies a negative marginal product, but this cannot occur with a neo-classical production function such as (1).[13] There were 116 negative user costs over the analysis period, out of a total number of user costs of 6727 (= 31 industries × 7 assets × 31 years) or 1.7 percent; 101 of the 116 occurred for buildings, mostly in the period 1970–81, and most of these in a small number of industries with negative rates of return (industries 3, 10, 22 and 24). The reason is that, particularly in the years 1970–81, the buildings price index rose particularly rapidly (at times at 20 percent per annum or more) while the depreciation rate for buildings is low (2.5 percent per annum).

Ex Ante Method

The first issue here is picking an ex ante rate of return. The simplest choice is the mean rate in the market sector as a whole, 8.66 percent per annum in real terms. As we have seen (Figure 1), there is no trend in this rate over our period. Another possibility is to use the rate predicted by the AR(2) model. However this would be quite variable and even at times negative. A justification for using the mean rate is that investments cannot be unwound over a horizon as short as a year, hence a one-year rate of return is not appropriate. After all, the theory is based on the concept of investors looking ahead one *period*, but the theory has nothing to say about how long such a period is in practice. A second possibility is to use the mean rate for each industry, since industries may be subject to different degrees of risk. Unfortunately, this founders on the finding above that some industry rates are implausibly low or high. So I use the mean rate in the market sector as a whole.

The next step is to use forecasts of prices in the user cost formula. I continue to assume that the required, real rate of return is the same for all industries and constant over time. And as before the required *nominal* rate varies over time (though not across industries) since firms have to estimate the rate of inflation. I model the rate of inflation as an ARMA process, but with year dummies for the oil price shocks. Arguably, the oil price hikes of 1974 and 1979 (which caused spikes in inflation one year later) led to firms anticipating more rapid inflation; without

[12]Excluding two outliers reduces the standard deviation to 6.30 percent per annum, still an implausibly large number. The wide variation in the mean rates of return could be due to a number of causes, such as inaccurate allowance at the industry level for mixed income and inventory holding, differences across industries in depreciation rates, or unmeasured intangible assets (e.g. R&D stocks).

[13]Negative user costs might indicate that the quasi-fixed capital model with a CES production function is not an adequate description of the data. Or it might indicate that the assumed length of the period, here annual, is too short for full adjustment of capital stocks. But however that may be, any empirical researcher still has to decide how to deal with negative user costs: can it really be the case that the marginal product of (say) buildings is negative in some industries?

these dummies an ARMA process tends always to lag behind actual inflation (whether in periods of rising or falling inflation). I found that an AR(2) model with year dummies for 1975 and 1980 produced satisfactory results for the market sector GDP deflator. "Satisfactoriness" was tested (a) by the significance of the coefficients and (b) by whether the errors were white noise using the Bartlett and Ljung–Box tests.[14]

I model asset price inflation as ARMA processes for the growth of *relative* asset prices, i.e. the growth of asset prices minus the growth of the GDP deflator for the market sector. Either an AR(2) or an AR(1) model worked quite well. In no case did an MA element turn out to play a significant role.

Using forecasts of prices I now find that there are no longer any negative user costs. So compared with the ex post method, the combined use of a fixed real rate of return and forecasted, not actual, prices has completely eliminated the problem of negative user costs.

Hybrid Method

I continue to use the AR models to forecast prices and assume the same required real rate of return, 8.66 percent per annum But in estimating the contribution of capital to growth I now use actual GOS, not predicted GOS.

4.2. *Results*

How sensitive are the industry-level growth rates of capital to the method employed? The answer is, in most cases not very much (see Table 3). However, in the case of some important industries such as finance and business services the difference can be substantial.

Next let us consider results for the market sector as a whole (Table 4). These are derived by aggregating over the industry-level results:

$$\Delta \ln K_t = \frac{1}{2}\sum_{j=1}^{n}\left[\frac{GOS_{jt}}{\sum_{j=1}^{n} GOS_{jt}} + \frac{GOS_{j,t-1}}{\sum_{j=1}^{n} GOS_{j,t-1}}\right]\Delta \ln K_{jt}. \tag{22}$$

That is, the growth of capital services in the market sector ($\Delta \ln K_t$) is a Törnqvist index of the n industry-level capital growth rates ($\Delta \ln K_{jt}$, calculated in accordance with equation (16) above), where the weights are industry profits (GOS_{jt}) and $n = 31$. Both the weights and the industry-level indices vary with the method. For the ex post and hybrid methods the weights in each industry's capital services aggregate are actual GOS; for the ex ante method, they are predicted GOS. To obtain the contribution of capital to output growth in the market sector, I weight the growth of capital services in the market sector by the share of profit in market sector value added. The profit share is the actual one for the ex post and

[14]I used Stata's *arima* command, which does ML estimates of ARMA models. In many industries, changes in the relative prices of plant and machinery are insignificant, i.e. these prices rise at the same rate as the GDP deflator.

TABLE 3

Mean Growth Rates of Capital Services, 1970–2000 (% p.a., by method)

Number	Name	Ex Post	Ex Ante	Ex Post Minus Ex Ante
1	Agriculture	0.99	1.04	–0.05
2	Oil & gas	11.17	11.25	–0.08
3	Coal & other mining	–1.40	–1.05	–0.35
4	Manufactured fuel	–0.89	–0.86	–0.03
5	Chemicals & pharmaceuticals	1.46	1.38	0.08
6	Non-metallic mineral products	3.27	3.49	–0.22
7	Basic metals & metal goods	0.13	–0.04	0.17
8	Mechanical engineering	0.88	0.97	–0.09
9	Electrical equipment & electronics	4.05	4.30	–0.25
10	Vehicles	2.65	2.17	0.48
11	Food, drink & tobacco	2.06	2.11	–0.05
12	Textiles, clothing & leather	–0.72	–0.67	–0.05
13	Paper, printing & publishing	3.16	3.41	–0.25
14	Other manufacturing	2.40	2.55	–0.15
15	Electricity supply	–0.56	–0.51	–0.05
16	Gas supply	1.06	1.15	–0.09
17	Water supply	3.45	3.47	–0.02
18	Construction	1.56	1.65	–0.09
19	Wholesaling, vehicle repairs & sales	5.13	5.66	–0.53
20	Retailing	5.31	5.33	–0.02
21	Hotels & catering	5.44	5.22	0.22
22	Rail transport	0.05	–0.25	0.30
23	Road transport	2.01	1.99	0.02
24	Water transport	1.50	–0.21	1.71
25	Air transport	4.04	4.03	0.01
26	Other transport services	4.87	4.66	0.21
27	Communications	6.09	5.73	0.36
28	Finance	7.83	8.62	–0.79
29	Business services	8.47	9.88	–1.41
33	Waste treatment	5.11	4.46	0.65
34	Miscellaneous services	5.68	6.11	–0.43
	Cross-industry mean (unweighted)	3.10	3.13	–0.03

Notes: There were 31 industries in the market sector. The ex ante method imposes a common real rate of return of 8.66 percent per annum in each year and every industry.

Source: Bank of England Industry Dataset.

TABLE 4

Growth Rates and Contributions of Capital in the Market Sector, 1970–2000 (% p.a.)

	Mean	S.D.
Growth rates		
Ex post	4.52	1.44
Ex ante	3.81	1.41
Hybrid	4.84	1.60
Contributions		
Ex post	1.20	0.43
Ex ante	1.00	0.37
Hybrid	1.29	0.47

Source: Bank of England Industry Dataset. Market sector is aggregate of 31 industries.

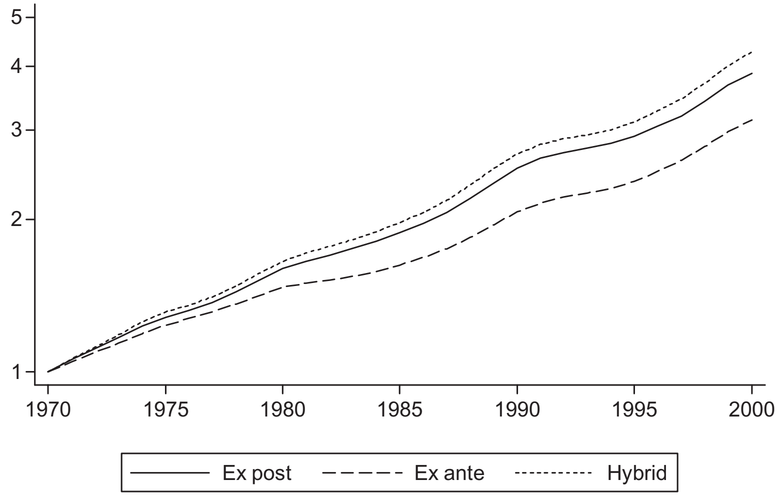

Figure 2. Capital services in the market sector, 1970 = 1.0 (log scale)
Source: Bank of England Industry Dataset. Market sector is aggregate of 31 industries.

hybrid methods; for the ex ante method it is the expected share (i.e. expected profit over actual value added).[15]

Both the growth rate of capital services and the contribution of capital are strongly affected by the method used (Table 4 and Figures 2, 3 and 4). The largest difference is between the ex ante (AR) and the hybrid methods, more than one percentage point per annum averaged over 1970–2000, though since 1992 the three methods show similar growth rates (Figure 2). Why is the difference so large? The answer is that in calculating the market sector index for capital services the different methods apply different weights to the industry-level indices. The ex post and hybrid methods use *actual* profits, the ex ante uses *expected* profits. It turns out that the industry growth rates are more strongly correlated with *actual* profit than they are with *predicted* profit. The cross-industry correlation coefficient of the ex ante (AR) measure of capital services growth with actual profit is 0.26 but with predicted profit it is only 0.19. The economic reason for this is that the ex post rate of return (and hence ex post profits) and the growth rate of capital are positively correlated. If we exclude two outliers, industries 2 (Oil & Gas) and 18 (Construction), the correlation is +0.31 (Figure 5). So use of *actual* rather than predicted profit tends to increase the weight on fast-growing industries, thus raising the average.

[15]In order to make the comparison at all, it is necessary to eliminate the negative user costs. Otherwise it is not possible in some cases to calculate the industry-level index of capital services. In any case, negative user costs make no sense economically. I eliminate negative user costs when they occur by setting the real rate of return equal to the market sector average and setting the real capital gain part of the user cost formula to zero.

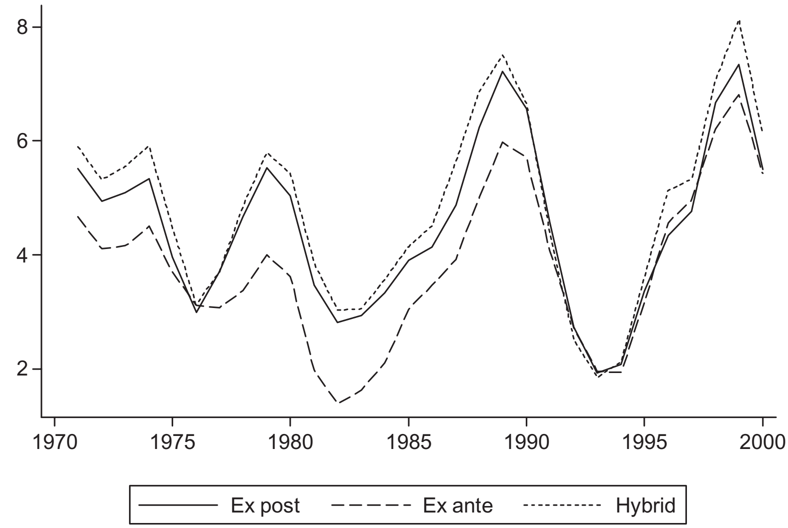

Figure 3. Growth rate of capital services in the market sector, % p.a.
Source: Bank of England Industry Dataset. Market sector is aggregate of 31 industries.

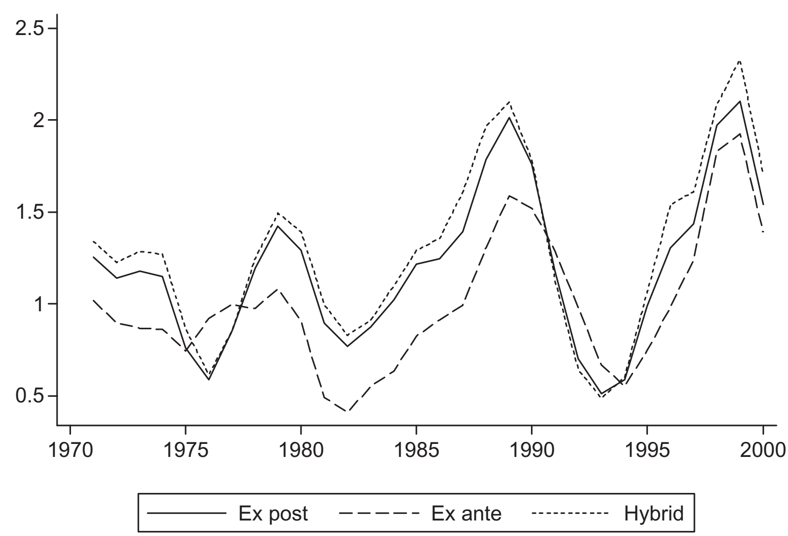

Figure 4. Contribution of capital services in the market sector, % p.a.
Source: Bank of England Industry Dataset. Market sector is aggregate of 31 industries.

In summary, when we aggregate up from the industry-level estimates to the market sector as a whole, the method used makes a substantial difference. The main reason is that industries with high ex post rates of return tend to have high growth rates of capital.

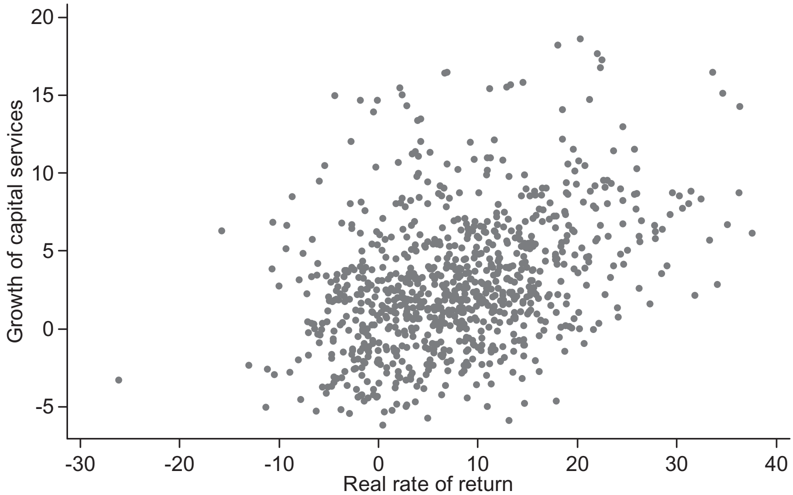

Figure 5. Growth of capital services versus real rate of return

Note: 29 industries, 1970–2000: 870 annual observations. Industries 2 and 18 have been excluded as outliers. Correlation coefficient is 0.31.

Source: Bank of England Industry Dataset.

5. Conclusions

We have established the following results:

1. Ex post user costs are in principle the correct weights to employ in constructing an index of capital services.
2. The usual method of estimating ex post user costs, which assumes a common ex post rate of return, is not in principle correct, since ex post the rates of return typically differ between assets, unless expectations are fully realized.
3. Ex ante user costs are proportional to *true* ex post user costs and the factor of proportionality is the same for all assets, under the assumption that the production function is CES. Hence ex ante user costs can in this case be validly employed as the weights in the index of capital services.
4. Whether ex post or ex ante user costs are used to construct the index of capital services, the growth of this index should be weighted by the actual (ex post), observed share of profits in output, when we come to calculate the contribution of capital to output growth.
5. The components of the ex ante user costs that are not known in advance with certainty or not directly observed by the econometrician—future asset and output prices and the required rate of return—can all be estimated from the same data as are typically used to estimate ex post user costs, namely actual prices plus an estimate of the ex post *average* rate of return (though extraneous data from financial markets could also be used for estimating the required rate of return).

6. Once ex ante user costs have been estimated, ex post user costs (consistent with the ex ante ones) can also be derived and the latter will by definition add up to observed aggregate profit (gross operating surplus).

Using investment data for seven assets for 31 U.K. industries in the market sector, 1970–2000, I compared estimates of capital services by three methods: ex post, ex ante and hybrid. I found that the ex post method (where the realized rate of return is assumed the same for all assets) produces a significant number of economically impossible, negative user costs, while the ex ante and hybrid methods produce none. At the industry level the growth rates of capital services are insensitive to the method employed, except in a few cases. But when we aggregate up from the industry-level estimates to the market sector as a whole, the method used makes a substantial difference. Now the hybrid method produces substantially higher estimates of the growth rate of capital, and also of the contribution of capital to growth, than does the ex ante method (the difference with the ex post method is less stark). The main reason is that industries with high ex post rates of return tend to have high growth rates of capital.

In summary, both theory and empirical tests favor the hybrid method which requires three steps. First, estimate the unobserved elements of the ex ante user costs using observed data on the average, ex post rate of return, and asset and output prices. Estimation could be by single equation methods (as here) or conceivably by a reduced form VAR. Second, calculate the growth of capital services using these estimated ex ante user costs as weights. Finally, calculate the contribution of capital to output growth by the growth of capital services weighted by the observed, ex post profit share. This hybrid method has the advantage of employing exactly the same data as is required for the usual ex post measures and hence could be readily implemented by national statistical agencies.

Appendix: Proofs of Propositions in the Text

A.1 *Proof that the Ex Post Rate of Return Varies Across Assets Unless Expectations Are Realized*

We prove the proposition stated in the text:

Proposition: In the model set out above, and in the absence of a numerical fluke (to be defined below), the ex post rate of return will be the same for all assets if and only if all expectations (for asset prices, output price, TFP, and output) are satisfied, in which case the ex post and ex ante rates of return are equal.

Proof

To speed up the notation, set $x_{it}^e = \delta_i(1+\pi_{it}^e) - \pi_{it}^e$, $x_{it} = \delta_i(1 + \pi_{it}) - \pi_{it}$, and let $z_{it} = q_{it}^e/q_{it} = E_{t-1}[p_{Yt}(\partial Y_t/\partial K_{it})]/p_{Yt}(\partial Y_t/\partial K_{it})$. Then from (4), (5) and (7):

$$\frac{r_t^* + x_{it}^e}{r_{it} + x_{it}} = z_{it}$$

and solving for the rate of return:

$$\text{(A1)} \qquad r_{it} = \frac{r_t^* + x_{it}^e - x_{it}z_{it}}{z_{it}}, \quad i = 1, \ldots, m$$

Sufficiency: If all expectations are realized, then the ex post rates of return are all the same, and are equal to the ex ante rate

In this case, $z_{it} = 1$ and $x_{it}^e = x_{it}, \forall i$. So plugging these values into (A1): $r_{it} = r_t^*, \forall i$.

Necessity: If all ex post rates of return are the same, then all expectations are realized (in the absence of a numerical fluke)

Suppose that $r_{it} = r_{jt}$, $\forall i, j$. Then from (A1),

$$\text{(A2)} \qquad x_{it}^e - x_{it}z_{it} = \frac{(z_{it} - z_{jt})r_t^*}{z_{jt}} + \frac{z_{it}(x_{jt}^e - x_{jt}z_{jt})}{z_{jt}}$$

One way this equation can be satisfied is if $z_{it} = z_{jt} = 1, \pi_{it}^e = \pi_{it}$, and $\pi_{jt}^e = \pi_{jt}, \forall i, j$, i.e. all expectations are realized. Can it be satisfied if some or all expectations are not realized? Only by a numerical fluke. The difference between the forecast and outturn of capital gains on asset i (the latter weighted by z_{it}) must be a constant multiple z_{it}/z_{jt} of the same difference for asset j, plus a constant that depends on z_{it} and z_{jt} again. But z_{it} and z_{jt} (the ratios of the expected real value to the actual real value of the marginal products) depend on expectations about TFP, the industry's output price, and the level of demand, which are largely independent of expectations of asset prices (see (4)). And the outturns for real capital gains (reflected in x_{it} and x_{jt}) are not directly related to the actual marginal products, which (given the pre-determined asset stocks) depend only on the labor input levels. In other words, x_{it} (x_{jt}) is largely independent of z_{it} (z_{jt}).[16] So if equation (A2) were satisfied when expectations were incorrect, it would be a numerical fluke.

Note that this proof does not require that the production function be CES.

A.2 *The Translog Case*

The translog production function with m inputs X_i and constant returns to scale can be written in log form as:

$$\ln Y = \alpha_0 + \sum_{i=1}^m \alpha_i \ln X_i + (1/2)\sum_{i=1}^m \sum_{j=1}^m \beta_{ij} \ln X_i \ln X_j + \alpha_t t + \sum_{i=1}^m \beta_{it} \ln X_i t + \beta_{tt} t^2,$$
$$\sum_{i=1}^m \alpha_i = 1, \quad \sum_{j=1}^m \beta_{ij} = 0, \quad \beta_{ij} = \beta_{ji}, \quad \sum_{i=1}^m \beta_{it} = 0$$

The elasticity of output (Y) with respect to the i-th input (X_i), ε_i, is:

[16]Outcomes for marginal products and for real capital gains might be correlated: a better than expected outcome for the marginal product (lower than expected z_{it}) might be associated with a higher than expected real capital gain (lower than expected x_{it}), if a common demand shock raises the prices of capital goods more than those of other goods; in this case x_{it} and z_{it} would be positively correlated. But a negative correlation between x_{it} and z_{it} is also possible.

$$\varepsilon_i = \alpha_i + \sum_{j=1}^{m} \beta_{ij} \ln X_j + \beta_{it} t, \quad i = 1, \ldots, m$$

These elasticities, if we could measure them, are what we would like to use in a growth accounting analysis. Now suppose that only the m-th input can be varied in the short run, inputs 1 to $m - 1$ being fixed (this can be easily generalized to any number of freely variable inputs). Then the expected elasticity of output with respect to the i-th fixed input, after the levels of the fixed inputs have been chosen ($X_j = \bar{X}_j$, $j = 1, \ldots, m - 1$), is:

$$E(\varepsilon_i) = \alpha_i + \sum_{j=1}^{m-1} \beta_{ij} \ln \bar{X}_j + \beta_{im} E[\ln X_m] + \beta_{it} t$$

We can estimate the expected elasticity from estimates of expected user costs. The actual elasticity is

$$\varepsilon_i = \alpha_i + \sum_{j=1}^{m-1} \beta_{ij} \ln \bar{X}_j + \beta_{im} \ln X_m + \beta_{it} t$$

The marginal product of the i-th input is $\varepsilon_i Y/X_i$. The ratio of the expected to the actual marginal product of the i-th fixed input is $E(\varepsilon_i Y)/\varepsilon_i Y$. Now if $\beta_{im} = 0$, all i, then from the last two equations $E(\varepsilon_i) = \varepsilon_i$ and hence $E(\varepsilon_i Y)/\varepsilon_i Y = E(Y)/Y$. That is, the ratio of the expected to the actual marginal product is the same for all fixed inputs just as in the CES case. If the β_{im} are sufficiently small, even if not zero, then this result will still hold approximately.

In practice the β_{ij} coefficients do appear to be quite small and not to vary very much between industries. For example, Jorgenson *et al.* (1987, Table 7.3 and Appendices B–D) report estimates of the coefficients of a translog production function with three inputs (capital (K), labor (L) and intermediate (X)) fitted to 21 U.S. manufacturing industries. Interpreting this as a system with one quasi-fixed input (capital) and two freely variable ones (labor and intermediate), we are then interested in the values of the β_{KL} and β_{KX} coefficients. The β_{KL} coefficient averaged 0.05 with standard deviation of 0.08; the β_{KK} coefficient averaged 0.002 with standard deviation of 0.007. This suggests that the approximation implied by use of a CES production function is not too bad in this context.

References

Berndt, E. R., "Comments on Hulten," in E. R. Berndt and J. Triplett (eds), *Fifty Years of Economic Measurement*, Chicago University Press, Chicago and London, 1990.

Berndt, E. R. and M. A. Fuss, "Productivity Measurement with Adjustments for Variations in Capacity Utilisation and Other Forms of Temporary Equilibrium," *Journal of Econometrics*, 33, 7–29, 1986.

Christensen, L. and D. W. Jorgenson, "The Measurement of U.S. Real Capital Input, 1929–1967," *Review of Income and Wealth*, 15(4), 293–320, 1969.

Corrado, C. A., C. R. Hulten, and D. E. Sichel, "Intangible Capital and Economic Growth," NBER Working Paper, No. 11948, 2006.

Diewert, W. E., "Aggregation Problems in the Measurement of Capital," in D. Usher (ed.), *The Measurement of Capital*, Studies in Income and Wealth No. 45, The University of Chicago Press, Chicago, 433–528, 1980.

———, "*Measuring the Price and Quantity of Capital Services Under Alternative Assumptions*," Department of Economics, University of British Columbia, Discussion Paper No. 01-24, 2001.

Hall, R. E. and D. W. Jorgenson, "Tax Policy and Investment Behaviour," *American Economic Review*, 57, 391–414, 1967.

Harper, D. J., E. R. Berndt, and D. O. Wood, "Rates of Return and Capital Aggregation Using Alternative Rental Prices," in D. W. Jorgenson and R. Landau (eds), *Technology and Capital Formation*, The MIT Press, Cambridge, MA, 1989.

Intersecretariat Working Group on National Accounts, "The Full Set of Consolidated Recommendations," U.N. Statistical Commission, http://unstats.un.org/unsd/statcom/doc07/BG-SNA.pdf, 2007.

Jorgenson, D. W., "Capital as a Factor of Production," in D. W. Jorgenson and R. Landau (eds), *Technology and Capital Formation*, The MIT Press, Cambridge, MA, 1989.

Jorgenson, D. W. and Z. Griliches, "The Explanation of Productivity Change," *Review of Economic Studies*, 34, 249–83 (reprinted in D. W. Jorgenson, *Productivity: Volume 1: Postwar U.S. Economic Growth*, The MIT Press, Cambridge, MA), 1967.

Jorgenson, D. W., F. M. Gollop, and B. M. Fraumeni, *Productivity and U.S. Economic Growth*, Harvard University Press, Cambridge, MA, 1987.

OECD, *OECD Productivity Manual: A Guide to the Measurement of Industry-Level and Aggregate Productivity Growth*, OECD, Paris, 2001a.

———, *Measuring Capital: A Manual on the Measurement of Capital Stocks, Consumption of Fixed Capital and Capital Services*, OECD, Paris, 2001b.

O'Mahony, M. and B. van Ark, *EU Productivity and Competitiveness: An Industry Perspective*, European Commission, Luxembourg, 2003.

Oulton, N., "Ex Post Versus Ex Ante Measures of the User Cost of Capital," Centre for Economic Performance, Discussion Paper No. 698. http://cep.lse.ac.uk/pubs/download/dp0698.pdf, 2005.

Oulton, N. and S. Srinivasan, "Capital Stocks, Capital Services, and Depreciation: An Integrated Framework," Bank of England Working Paper No. 192, http://www.bankofengland.co.uk/workingpapers/wp192.pdf, 2003.

———, "Productivity Growth in UK Industries, 1970–2000: Structural Change and the Role of ICT," Bank of England Working Paper No. 259, http://www.bankofengland.co.uk/publications/workingpapers/wp259.pdf, 2005a.

———, "Data Appendix (Annex A) and Annexes B–D," Technical Appendix to Bank of England Working Paper No. 259, http://www.bankofengland.co.uk/publications/workingpapers/wp259techannex.pdf, 2005b.

Schreyer, P., "Measuring Multi-Factor Productivity When Rates of Return are Exogenous," OECD, mimeo, 2004.

Schreyer, P., P.-E. Bignon, and J. Dupont, "OECD Capital Services Estimates: Methodology and a First Set of Results," OECD Statistics Working Paper 2003/6, http://www.olis.oecd.org/oils/2003doc.nsf/LinkTo/std-doc(2003)6, 2003.

Schreyer, P., E. Diewert, and A. Harrison, "Cost of Capital Services and the National Accounts," Issues Paper for the July 2005 Meeting of the Experts Advisory Group on National Accounts (SNA/M1.05/04), http://unstats.un.org/unsd/nationalaccount/AEG/papers/m3CapitalServices.pdf, 2005.