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MEASURING MULTIDIMENSIONAL POVERTY: AN EMPIRICAL COMPARISON OF VARIOUS APPROACHES

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This paper makes a systematic comparison of four approaches to multidimensional poverty analysis based respectively on the theory of fuzzy sets, information theory, efficiency analysis and axiomatic derivations of poverty indices. The database was the 1995 Israeli Census that provided information on the ownership of various durable goods. There appears to be a fair degree of agreement between the various multidimensional poverty indices concerning the identification of the poor households. The four approaches have also shown that poverty decreases with the schooling level of the head of the household, first decreases and then increases with his/her age and with the size of the household. Poverty is higher when the head of the household is single and lower when he/she is married, lowest when the head of the household is Jewish and highest when he/she is Muslim. Poverty is also higher among households whose head immigrated in recent years, does not work or lives in Jerusalem. These observations were made on the basis of logit regressions. This impact on poverty of many of the variables is not very different from the one that is observed when poverty measurement is based only on the income or the total expenditures of the households.

1. INTRODUCTION

> "What goods do to people is identical neither with what people are able to do with them nor with what they actually do with them . . . To be sure, it is usually true that a person must do something with a good (take it, put it on, go inside it, etc.) in order to be benefited by it, but that is not always true, and, even, where it is true, one must distinguish what the good does for the person from what he does with it . . ." (from G. A. Cohen, "Equality of What? On Welfare, Goods, and Capabilities," 1993)

The above citation emphasizes quite clearly that the information that one may have on the types and amount of goods with which various individuals are endowed does not necessarily allow us to draw conclusions as to their standard of living or quality of life. Conceptualizing the idea of quality of life is in fact not a simple task. Sen (1985) made such an attempt when he introduced the notions of "capabilities" and "functionings." To translate empirically Sen's ideas, Lovell *et al.* (1994) advocated the use of efficiency analysis and Deutsch *et al.* (2003) repeated their attempt using more detailed and recent data. However, as stressed by Cohen

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(1993), not everybody will agree with Sen's emphasis on the concepts of "capabilities" and "functionings."

Beyond the philosophical difficulty stressed by Cohen, there are also practical problems since the informational basis necessary to implement Sen's notions is often not available. Census data are a good illustration of the type of data that give an exhaustive but very incomplete picture of the way members of a given society live. Not only do they generally not include any information on the "capabilities" or even the "functionings" of an individual. They even do not provide us with enough information on the goods at the disposal of an individual since in many countries Census questionnaires do not include an interrogation on the income of the households. They provide however quite detailed information on the standard of living of the households, in terms of the durable goods that are available to them. This type of information may in certain respects be more reliable than income data because they allow one to overcome difficulties such as unrecorded income sources or illegal transactions.

Using information on the ownership of durable goods raises however a new issue, that of deriving measures of standard of living that are of a multidimensional nature. Most studies of inequality and poverty are based on a unidimensional approach relying mainly on the income or the expenditures of a household. Taking a multidimensional point of view requires therefore the devising of new measures of inequality, poverty and more generally of the standard of living and the quality of life.

Several attempts have been made in this direction in recent years and the purpose of this paper is to compare the various techniques that have been proposed to estimate the standards of living and quality of life. The emphasis will be mainly on multidimensional approaches to poverty measurement and to compare them, the same data set will be used, the 1995 Israeli census. Such a systematic use of the same type of data will allow us to compare the results obtained and find out whether selecting a specific multidimensional approach to poverty measurement has an impact on the extent of poverty and on its determinants. As will be seen the database that is used includes only information on the durable goods available to the various households. This, on one hand, may represent an important constraint but, on the other hand, it prevents us from trying to go beyond the "goods," a limitation that may have some advantages from a philosophical point of view, as shown in the citation given previously.

The paper is organized as follows. In Section 2 various approaches to multidimensional poverty measurement are presented while Section 3 describes the data that are used and the available variables. In Section 4 a logit-type of analysis is presented that allows one to analyze the determinants of poverty, for each of the approaches adopted. Section 5 compares the results obtained and checks whether the same households are identified as poor under the various approaches while concluding comments are given in Section 6.

2. Theoretical Background

2.1. *The "Fuzzy Set" Approach to Poverty Analysis*

The theory of "Fuzzy Sets" was developed by Zadeh (1965) on the basis of the idea that certain classes of objects may not be defined by very precise criteria of membership. In other words there are cases where one is unable to determine which elements belong to a given set and which ones do not. Zadeh himself (1965) characterized a fuzzy set (class) as "a class with a continuum of grades of membership."

Let there be a set X and let x be any element of X. A fuzzy subset A of X is defined as the set of the couples $A = \{x, \mu_A(x)\}$ for all $x \in X$ where μ_A is an application of the set X to the closed interval [0, 1], which is called the membership function of the fuzzy subset A. In other words a fuzzy set or subset A of X is characterized by a membership function $\mu_A(x)$ which will link any point of X with a real number in the interval [0, 1], the value of $\mu_A(x)$ denoting the degree of membership of the element x to the set A.

If A were a set in the sense in which this term is usually understood, the membership function which would be associated to this set would take only the values 0 and 1. One would then write that

$$\mu_A(x) = 1 \quad \text{if } x \text{ belongs to the subset } A$$

$$\text{and } \mu_A(x) = 0 \quad \text{if } x \text{ does not belong to the set } A.$$

But if A is a fuzzy subset, we will say that $\mu_A(x) = 0$ if the element x does not belong to A and that $\mu_A(x) = 1$ if x *completely* belongs to A. But if $0 < \mu_A(x) < 1$, x belongs only partially to A and the closer to 1 the value of $\mu_A(x)$, the greater the degree of membership of x to A.

These simple ideas may be easily applied to the concept of poverty. Thus in some cases individuals are in such a state of deprivation that they certainly should be considered as poor while in others their level of welfare is such that they certainly should not be classified as poor. There are however also instances where it is not clear whether a given person is poor or not. This is specially true when one takes a multidimensional approach to poverty measurement, because according to some criteria one would certainly define him as poor whereas according to others one should not regard him as poor. Such a fuzzy approach to the study of poverty has taken various forms in the literature.

The Totally Fuzzy Approach (TFA)

Cerioli and Zani (1990) were the first to apply the concept of fuzzy sets to the measurement of poverty. Their approach is called the Totally Fuzzy Approach (TFA) and the idea is to take into account a whole series of variables that are supposed to measure each a particular aspect of poverty. When defining the membership function three cases should be distinguished.

Dichotomous variables

The typical case is that of variables which indicate whether an individual owns a given durable good or not. Let D_l be the subset of individuals (households) deprived of good l, with $l = 1, \ldots, k_d$. Let d_l be the set of dichotomous variables $d_{1l}, \ldots, d_{il}, \ldots, d_{nl}$ representing the ownership status of the various n individuals with respect to good l. In such a case the subset D_l will not be a fuzzy set because the membership function may be defined as

$$\mu_{D1}(i) = 1 \quad \text{if } d_{il} = 0$$
$$\text{and } \mu_{D1}(i) = 0 \quad \text{if } d_{il} = 1$$

where d_{il} takes the value zero when individual i does not possess good l and the value 1 in the opposite case. In other words when the membership function takes the value 1 it indicates a condition of absolute deprivation whereas a value of zero shows the absence of deprivation. The membership function is hence defined here as in the case of traditional sets.

Polytomous variables

When analyzing poverty there may be qualitative variables that may take more than two values. Let us assume that one may rearrange these values by increasing order, where higher values denote a higher risk of poverty.

Let O_l be the subset of individuals (households) who are in a situation of deprivation with respect to the indicator l, with $l = 1, \dots, k_O$. Let also o_l be the set of polytomous variables $o_{1l}, \dots, o_{nl}$ measuring the state of deprivation of the various individuals with respect to indicator l.

Let θ_l represent the set of the various states $\theta_{1l}, \dots, \theta_{ls}$ that indicator 1 may take and let $\psi_{1l}, \dots, \psi_{ml}, \dots, \psi_{sl}$ represent the scores corresponding to these various states, assuming that $\psi_{1l} < \dots < \psi_{ml} < \dots < \psi_{sl}$.

A good illustration of the use of polytomous variables would be that in which individuals are asked to evaluate in subjective terms the state of their health or of the physical conditions of the apartment or house they live in, the possible answers being very good, good, medium, bad, very bad. Following Cerioli and Zani (1990) one would define the membership function $\mu_{ol}(i)$ of individual i as

(1)
$$\mu_{ol}(i) = 0 \quad \text{if } \psi_{1l} < \psi_{1min}$$
$$\mu_{ol}(i) = ((\psi_{1l} - \psi_{1min})/(\psi_{1max} - \psi_{1min})) \quad \text{if } \psi_{1min} < \psi_{1l} < \psi_{1max}$$
$$\mu_{ol}(i) = 1 \quad \text{if } \psi_{1l} > \psi_{1max}$$

where ψ_{1min} and ψ_{1max} denote respectively the lowest and highest values taken by the scores ψ_{1l}.

Continuous variables

Income or consumption expenditures are good examples of deprivation indicators which are continuous. Cerioli and Zani (1990) have proposed to define two threshold values x_{min} and x_{max} such that if the value x taken by the continuous indicator for a given individual is smaller than x_{min} this person would undoubtedly be defined as poor whereas if it is higher than x_{max} he certainly should be considered as not being poor.

Let X_l be the subset of individuals (households) who are in an unfavorable situation with respect to the l-th variable with $l = 1, \dots, k_x$. Cerioli and Zani (1990) have then proposed to define the membership function $\mu_{xl}(i)$ for individual i as

(2)
$$\mu_{x1}(i) = 1 \quad \text{if } 0 < x_{il} < x_{1,min}$$
$$\mu_{x1}(i) = ((x_{1,max} - x_{il})/(x_{1,max} - x_{1,min})) \quad \text{if } x_{il} \in [x_{1,min}, x_{1,max}]$$
$$\mu_{x1}(i) = 0 \quad \text{if } x_{il} > x_{1,max}$$

Some authors have proposed to modify Cerioli and Zani's (1990) Totally Fuzzy Approach (TFA) and suggested what they have called the Totally Fuzzy and Relative Approach (TFR). This approach was originally suggested by Cheli *et al.* (1994) and Cheli and Lemmi (1995).

The Totally Fuzzy and Relative Approach

Let Ξ_j represent the subset of individuals (households) who are deprived with respect to indicator j with $j = 1, \ldots, k$. Let ξ_j be the set of dichotomous, polytomous or continuous variables $\xi_{1j}, \ldots, \xi_{nj}$ which measure the state of deprivation of the various n individuals with respect to indicator j and let F_j be the cumulative distribution of this variable. One may then define the membership function in two ways, depending on whether the degree of deprivation increases or decreases with the value taken by the variable ξ_j. In the first case the membership function $\mu_{\Xi j}(i)$ will be defined as

$$\mu_{\Xi j}(i) = F_j(\xi_{ij}) \tag{3}$$

whereas in the second case it will be defined as

$$\mu_{\Xi j}(i) = 1 - F_j(\xi_{ij}) \tag{4}$$

Cheli and Lemmi (1995) consider that such a formulation is less arbitrary than the one originally proposed by Cerioli and Zani (1990), especially for polytomous and continuous variables because in both cases one has to define critical threshold values. Moreover the TFR approach has the advantage of taking a relative approach to poverty (the one which is taken in most developed countries), according to which one is usually poor with respect to some other individuals.

These authors have however stressed that when the risk of poverty is very low, that is, a high proportion of individuals will not be considered as poor, the value taken by the indicator of poverty may be too high for those who turn out not to be poor. They therefore proposed the following solution.

Let $\xi_{j(m)}$ with $m = 1$ to s refer to the various values, ordered by increasing risk of poverty, which the variable ξ_j may take. Thus $\xi_{j(1)}$ represents the lowest risk of poverty and $\xi_{j(s)}$ the highest risk of poverty associated with the deprivation indicator j. The authors propose then to define the degree of poverty of individual (household) i as

$$\mu_{\Xi j}(i) = 0 \quad \text{if } \xi_{ij} = \xi_{j(1)}$$

and

$$\mu_{\Xi j}(i) = \mu_{\Xi j}(\xi_{j(m-1)}) + ((F_j(\xi_{j(m)}) - F_j(\xi_{j(m-1)}))/(1 - F_j(\xi_{j(1)})) \\ \text{if } \xi_{ij} = \xi_{j(m)}, m > 1 \tag{5}$$

where $\mu_{\Xi j}(\xi_{j(m-1)})$ denotes the membership function of an individual for which the variable ξ_j takes the value m and F_j is the distribution function of the variable ξ_j.

The next step in the analysis is to decide how to aggregate the various deprivation indicators. Let $\mu_{\Xi j}(i)$ refer as before to the value taken by the membership function for indicator j and individual i, with $j = 1$ to k and $i = 1$ to n. Let w_j

represent the weight one wishes to give to indicator j. The overall (over all indicators j) membership function $\mu_P(i)$ for individual i is then be defined as

(6) $$\mu_P(i) = \sum_{j=1 \text{ to } k} w_j \mu_{\Xi j}(i)$$

For the choice of the weight w_j, Cerioli and Zani (1990) as well as Cheli and Lemmi (1995) have proposed to define w_j as

(7) $$w_j = \ln(1/\mu b_{\Xi j}) \Big/ \sum_{j=1 \text{ to } k} \ln(1/\mu b_{\Xi j}) = \ln(\mu b_{\Xi j}) \Big/ \sum_{j=1 \text{ to } k} \ln(\mu b_{\Xi j})$$

where $\mu b_{\Xi j} = (1/n)\ \Sigma_{i=1 \text{ to } n} \mu_{\Xi j}(i)$ represents the fuzzy proportion of poor individuals (households) according to the deprivation indicator ξ_j. One may observe that the weight w_j is an inverse function of the average degree of deprivation in the population according to the deprivation indicator ξ_j. Thus the lower the frequency of poverty according to a given deprivation indicator, the greater the weight this indicator will receive. The idea, for example, is that if owning a refrigerator is much more common than owning a dryer, a greater weight should be given to the former indicator so that if an individual does not own a refrigerator, this rare occurrence will be taken much more into account in computing the overall degree of poverty than if some individual does not own a dryer, a case which is assumed to be more frequent.

Having computed for each individual i the value of his membership function $\mu_{\Xi j}(i)$, that is his "degree of belonging to the set of poor," the Totally Fuzzy and Relative Approach (TFR), following in fact Cerioli and Zani (1990), defines the average value P of the membership function as

(8) $$P = (1/n) \sum_{i=1 \text{ to } n} \mu_P(i)$$

The Vero and Werquin Approach (VWA)

Another "fuzzy approach" to poverty measurement has been recently suggested by Vero and Werquin (1997). They noted that one of the serious problems one faces when taking a multidimensional approach to poverty measurement, such as the fuzzy approach which has just been described, is that some of the indicators one uses may be highly correlated. To solve this problem, Vero and Werquin (1997) have proposed the following solution.

Let again k be the number of indicators and n the number of individuals. Let f_i represent the proportion of individuals who are at least as poor as individual i when taking into account all the indicators. The deprivation indicator $m_P(i)$ for individual i will then be defined as

(9) $$m_P(i) = \ln(1/f_i) \Big/ \sum_{i=1 \text{ to } n} \ln(1/f_i)$$

The membership function $\mu_P(i)$ for individual i is then expressed as

(10) $$\mu_P(i) = [m_P(i) - \text{Min}\{m_P(i)\}] / [\text{Max}\{m_P(i)\} - \text{Min}\{m_P(i)\}]$$

Finally the average value of the membership function P, over all individuals, is, as in the TFR approach, defined as

(11) $$P = (1/n) \sum_{i=1 \text{ to } n} \mu_P(i)$$

2.2. *The Distance Function Approach*

The distance function is a concept widely used in Efficiency Analysis (see Coelli *et al*., 1998, for an introduction to this topic). It has however been only rarely applied to the analysis of household behavior. Lovell *et al*. (1994) were the first to make such an attempt and we summarize here their approach, giving first some general information on efficiency analysis.

On the Concept of Distance Functions

In the literature a distinction has been made between input and output distance functions (see Coelli *et al*., 1998) but since we use here only input distance functions we limit our presentation to this concept.

Let $L(y)$ represent the input set of all input vectors x which can produce the output vector y, that is,

$$L(y) = \{x\colon x \text{ can produce } y\}.$$

The input distance function $D_{in}(x, y)$ involves then the scaling of the input ector and will be defined as

$$D_{in}(x, y) = \text{Max}\{\rho\colon (x/\rho) \in L(y)\} \tag{12}$$

It may be proven (see Coelli *et al*., 1998) that:

(1) The input distance function is increasing in x and decreasing in y.
(2) It is linearly homogeneous in x.
(3) If x belongs to the input set of y (i.e. $x \in L(y)$) then $D_{in}(x, y) \geq 1$.
(4) The input distance function is equal to unity if x belongs to the "frontier" of the input set (the isoquant of y).

In Figure 1 let q' be the input vector corresponding to OB and q be that corresponding to OA. Let ρ be equal to the ratio OB/OA. In other words q' is obtained by a proportional change ρ in the input quantities defined by q. Assume the prices of the inputs are given by a vector p_0. Nothing guarantees then that the input contraction defined by the distance function ρ will yield the cheapest cost, at input prices p_0, of producing the output level y_0 defined by the isoquant BC. There exists however at least one vector price p for which this distance function ρ = OB/OA will yield the cheapest cost of producing this output level y_0. There is therefore a clear link between the concepts of distance and cost functions because $D_{in}\,(q', y_0) = Min_p\, pq'$ such that the cost function $c(y_0, p) = 1$.

The distance and cost functions are clearly dual to one another: just as the cost function seeks out the optimal input quantities given y_0 and p_0, the distance function finds the prices that will lead the consumer to reach the output level y_0 by acquiring a vector of quantities proportional to q.

Estimation Procedures

Let us take as a simple illustration the case of a Cobb–Douglas production function. Let $\ln y_i$ be the logarithm of the output of firm $i = 1$ to I and x_i *a* vector, whose first element is equal to one and the others are the logarithms of the N inputs used by the firm. We may then write that

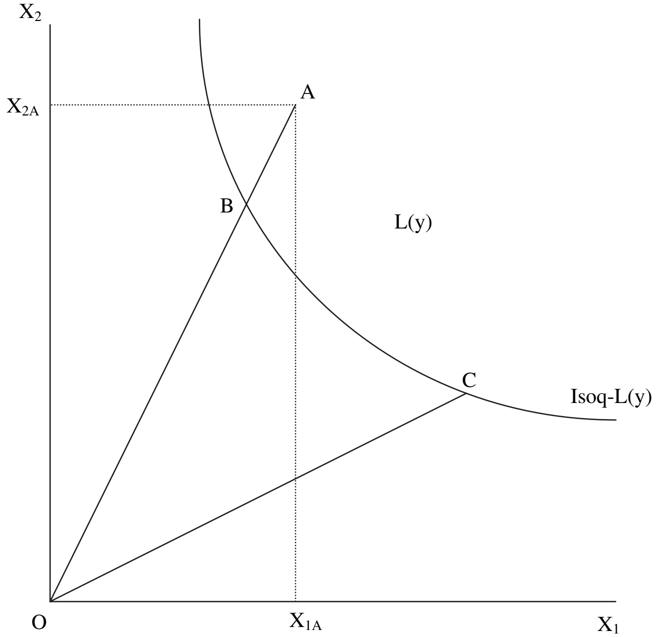

Figure 1. The Concept of Distance Function

$$\ln(y_i) = x_i \cdot \beta - u_i, \quad i = 1, \ldots, I. \tag{13}$$

where β is *a* $(N + 1)$ vector of parameters to be estimated and u a non-negative random variable, representing the technical inefficiency in production of firm *i*.

The ratio of the observed output of firm *i* to its potential output will then give a measure of its technical efficiency T_i so that

$$T_i = y_i / \exp(x_i \cdot \beta) = \exp(x_i \cdot \beta - u_i) / \exp(x_i \cdot \beta) = \exp(-u_i) \tag{14}$$

One of the methods allowing the estimation of this output-oriented Farrell measure of technical efficiency T_i (see Farrell, 1957) is to use an algorithm proposed by Richmond (1974) which has become known as corrected ordinary least squares (COLS). This method starts by using ordinary least squares to derive the (unbiased) estimators of the slope parameters. Then in a second stage the (negatively biased) OLS estimator of the intercept parameter β_0 is adjusted up by the value of the greatest negative residual so that the new residuals have all become non-negative. Naturally the mean of the observations does not lie any more on the estimated function: the latter has become in fact an upward bound to the observations.

One of the main criticisms of the COLS method is that it ignores the possible influence of measurement errors and other sources of noise. All the deviations from the frontier have been assumed to be a consequence of technical inefficiency. Aigner *et al*. (1977) and Meeusen and van den Broeck (1977) independently suggested an alternative approach called the stochastic production frontier method in which an additional random error v is added to the non-negative random variable u.

(15) $$\ln(y_i) = x_i \cdot \beta + v_i - u_i$$

The random error v is supposed to take into account factors such as the weather, the luck, etc. i is assumed to be i.i.d. normal random variables with mean zero and constant variance σ_v^2, independent of u, the latter being taken generally to be i.i.d. exponential or half-normal random variables. In the latter case where u is assumed to be i.i.d truncations (at zero) of a normal variable $N(0, \sigma)$, Battese and Corra (1977) suggested to proceed as follows. Calling σ_s^2 the sum $(\sigma^2 + \sigma_v^2)$, they defined the parameter $\gamma = (\sigma^2/\sigma_s^2)$ (so that γ has a value between zero and one) and showed that the log-likelihood function could be expressed as

(16) $$\ln(L) = -(N/2)\ln(\pi/2) - (N/2)\ln(\sigma_s^2) + \sum_{i=1}^{I}[1 - \Phi(z_i)] - [1/(2\sigma_s^2)]\sum_{i=1}^{I}(\ln y_i - x_i\beta)^2$$

where $z_i = [(\ln y_i - x_i \cdot \beta)/\sigma_s] \cdot \sqrt{(\gamma/(1-\gamma))}$ and $\Phi(\cdot)$ is the distribution function of the standard normal random variable.

The Maximum Likelihood estimates of β, σ_s^2 and γ are obtained by finding the maximum of the log-likelihood function defined previously where this function is estimated for various values of γ between zero and one. More details on this estimation procedure are available in programs such as FRONTIER (Coelli, 1992) or LIMDEP (Green, 1992). The same methods (COLS and Maximum Likelihood) may naturally be also applied when estimating distance functions.

Applying These Ideas to the Measurement of Poverty With Respect to the Standard of Living and the Quality Of Life

On the concepts of resources and "functionings"

Economists have traditionally identified well-being with market command over goods, thus, confounding the "state" of a person—well-being—with the extent of his or her possessions—being well-off. To some extent, such an "opulence-focused approach" (Sen, 1985) could be empirically justified by the scarcity of (individual) data. From a theoretical point of view, however, "economics has not been very interested in the plurality of focus in judging a person's states. In fact, often enough, the very richness of the subject matter has been seen as an embarrassment. There is a powerful tradition in economic analysis that tries to eschew the distinctions and make do with one simple measure of a person's interest and its fulfillment" (Sen, 1985).

To make a distinction between the standard of living and the quality of life notions we adopt Sen's "capability approach," which views individual well-being as a combination of various functionings. A functioning is an achievement of a person: what he or she manages to do or to be, and reflects a part of the "state" of that person. These functionings are then the constituents of an individual's quality of life, and the evaluation of the latter must take the form of valuing the functioning vectors.[1] In other words, according to Sen, the mere command over commodities cannot determine the valuation of the goodness of the life that one can lead for "the need of commodities for any specified achievement of living conditions may vary greatly with various physiological, social, cultural and other contingent features" (Sen, 1985). Commodity command is a means to the end of well-being.

In accordance to Sen's "capability approach" we see the standard of living primarily as a basket of multiple resources—commodities—and the quality of life as a basket of functionings. To evaluate these two vectors one needs a numerical representation in the form of an index. The few papers that have attempted in the literature to aggregate individual resources into an index of standard of living and individual functionings into a measure of quality of life have usually adopted a technique originally suggested by Lovell *et al*. (1994) which is based on the concept of distance function that was described previously.

Given that in the 1995 Israeli Census we had only information on the ownership of durable goods (considered here as "resources") we will only be able to estimate the standard of living of the households. In the estimation technique that we will use, we will however assume conceptually that the households convert "resources" into "functionings," as will now be shown.

Estimating the standard of living index on the basis of information on the ownership of durable goods

Let $x = (x_1, \ldots, x_N) \in \mathbb{R}^N_{++}$ denote the resources vector and $u = (u_1, \ldots, u_M) \in \mathbb{R}^M_{++}$ denote the functionings vector. Then an individual's resources and functionings are denoted by the pair (x^i, u^i), $i = 1, \ldots I$.

A theoretical standard of living index SL can be estimated using a Malmquist input quantity index (see Coelli *et al*., 1998):

$$SL(u, x^s, x^t) = D_{in}(u, x^s) / D_{in}(u, x^t) \tag{17}$$

where x^s and x^t are two different resource vectors and D_{in} is an input distance function. The idea behind the Malmquist index is to provide a reference set against which to judge the relative magnitudes of the two resource vectors. That reference set is the isoquant $L(u)$ and the radially farther x^i is from $L(u)$ the higher its standard of living, for x^i must be shrunk more to move back onto the reference set $L(u)$.

There is, however, a difficulty because the Malmquist index depends generally on u. One could use an approximation of this index such as the Tornquist index, but such an index requires price vectors as well as behavioral assumptions.[2] Since we do not have prices for resources we have to adopt an alternative strategy.

[1]More precisely, in his "capability approach," Sen proposes to evaluate quality of life in terms of capabilities, defined as "the alternative combinations of functionings individuals can achieve, and from which they can choose one collection" (Sen, 1993, p. 31). The notion of capability is thus conceptually superior to that of functioning in that it reflects the freedom individuals have in terms of the choice of functionings. Due to the difficulty in measuring such freedom to pursue different functionings, one often cannot use capabilities but tries to focus on achieved functionings. According to Sen (1999, pp. 38–40), such an exercise constitutes then an "elementary evaluation" of the capability set.

[2]This is also the case of other indices that are usually used to approximate the Malmquist index such as the Paasche index, the Laspeyres index or the Fisher index.

The idea is to get rid of u by treating all individuals equally and assume that each individual has the same level of functionings: one unit for each functioning. Let e represent such a vector of functionings—an M-dimensional vector of ones. Thus, the reference set becomes $L(e)$ and bounds the sample resource vectors form below. Individuals with resource vectors onto $L(e)$ share the lowest standard of living, with an index value of unity, whereas individuals with large resources vectors will then have higher standards of living, with index values above unity.

To estimate the distance function, let $\lambda = (1/x_N)$ and define a $(N - 1)$ dimensional vector z as $z = \{z_j\} = \{x_j/x_N\}$ with $j = 1, \ldots, N - 1$. Then $D_{in}(z, e) = (1/x_N) \cdot D_{in}(x, e)$ and, since $D_{in}(x, e) \geq 1$, we have

(18)
$$(1/x_N) \leq D_{in}(z, e).$$

This implies that we may also write

(19)
$$(1/x_N) = D_{in}(z, e) \cdot \exp(\varepsilon), \varepsilon \leq 0.$$

By assuming that $D_{in}(z, e)$ has a translog functional form, we have

(20)
$$\ln(x_N^{-1}) = \alpha_0 + \sum_{j=1}^{N-1} \alpha_j \ln z_j + \frac{1}{2} \sum_{j=1}^{N-1} \sum_{k=1}^{N-1} \alpha_{jk} \ln z_j \ln z_k + \varepsilon$$

Estimates of the coefficients α_i and α_j may be obtained using COLS (corrected ordinary least squares) or Maximum Likelihood methods while the input distance function $D_{in}(z^i, e)$ for each individual i is provided by the transformation

(21)
$$D_{in}(z^i, e) = \exp\{\max(\varepsilon_I) - \varepsilon_i\}.$$

This distance will, by definition, be greater than or equal to one (since its logarithm will be positive) and will hence indicate by how much an individual's resources must be scaled back in order to reach the resource frontier. This procedure guarantees that all resources vectors lie on or above the resource frontier $L(e)$. The standard of living for individual i will then be obtained by dividing $D_{in}(z^i, e)$ by the minimum observed distance value—which by definition equals 1.

2.3. *The Information Theory Approach*

Basic Concepts

Information theory was originally developed by engineers in the field of communications. Theil (1967) was probably the first one to apply this theory to economics. One of the basic concepts used in such an approach is the logarithm of a probability. Here is a summary of the basic ideas.

Let E be an experience whose result is x_i with $i = 1$ to n. Let $p_i = \text{Prob}\{x = x_i\}$ be the probability that the result of the experience will be x_i with evidently $0 \leq p_i \leq 1$. When we receive the information that a given event x_i occurred, we will not be surprised if the a priori probability that such an event would occur was high. In other words in such a case the information included in the message is not very important. On the other hand if the a priori probability that an event x_i will occur is very low, knowing that this event did indeed occur, will indeed surprise us and such a message will contain a significant amount of information.

We may hence define the information included in a message as a function of the probability a priori p that a result will occur. Let $h(p)$ be such an information function. Several axioms have been proposed in the literature on information theory to characterize and derive ideal information functions. One of the most popular forms however is to assume that

(22) $$h(p) = \log(1/p) = -\log(p)$$

Let us now define the concept of information expectancy. Since for each event x_i whose a priori probability of occurrence is p_i the information content of a message confirming the occurrence of such an event is $h(p_i)$, we may define the expected information, written as $H(p)$, as

(23) $$H(p) = \sum_{i=1 \text{ to } n} p_i h(p_i)$$

with $p = (p_1, \ldots, p_n)$.

Often the term entropy is used to refer to this expected information. Note that $H(p) \geq 0$ given the properties of the information function. Combining (22) and (23) we derive

(24) $$H(p) = \sum_{i=1 \text{ to } n} p_i \log(p_i)$$

and this is called the Shannon entropy (cf. Shannon, 1948).

Note (see Maasoumi, 1993) that this entropy may be interpreted as a measure of the uncertainty, the disorder or the volatility associated with a given distribution. It will be minimal (and equal to 0) when a specific result x_i is known to occur with certainty since in such a case a message informing us that the event x_i did indeed occur will not provide us with any information. To derive the maximal value of entropy, we have to maximize $H(p)$ subject to the constraint that $\Sigma_{i=1 \text{ to } n} p_i = 1$. In such a case uncertainty will be maximal because we have no idea a priori as to which event will occur. Imposing some restrictions on the function $h(p)$, it turns out that entropy will be maximal when all the events have the same probability, that is when $p_i = (1/n)$ for all $i = 1$ to n. This is, for example, the case when we adopt Shannon's entropy where $h(p) = -\log(p)$. We may then derive that

(25) $$0 \leq H(p) \leq \log(n)$$

Measuring the Distance or the Divergence Between Distributions

When we make a given experiment E which may end up in one of the n potential results $x_1, \ldots, x_n$, we often know the a priori probabilities $p_1, \ldots, p_n$ that these events will occur. It happens however sometimes that we receive some information that implies a modification of these a priori probabilities. In other words we have now received a message that transformed the a priori probabilities $p_1, \ldots, p_n$ into a posteriori probabilities $q_1, \ldots, q_n$ with $\Sigma_{i=1 \text{ to } n} q_i = 1$.

We may thus define what may be called the supplement of information $D(q, p)$ that is obtained when shifting from the distribution of a priori probabilities $\{p_1, \ldots, p_n\}$ to that of the a posteriori probabilities $\{q_1, \ldots, q_n\}$. $D(q, p)$ will be expressed as

(26) $$D(q, p) = \sum_{i=1 \text{ to } n} q_i \log(q_i / p_i)$$

$D(q, p)$ represents actually the expected information of a message transforming the a priori probabilities $\{p_1, \ldots, p_n\}$ into the a posteriori probabilities $\{q_1, \ldots, q_n\}$. Note that this supplement of information $D(p, q)$ may also be considered as a measure of the divergence between the distributions $\{p_1, \ldots, p_n\}$ and $\{q_1, \ldots, q_n\}$ or as the difference between the entropy corresponding to the distribution $\{p_1, \ldots, p_n\}$ and that relative to the distribution $\{q_1, \ldots, q_n\}$, assuming the weights to be chosen are those corresponding to the latter distribution.

This measure of divergence $D(p, q)$ is generally positive and will be equal to zero only in the very specific case where $p_i = q_i$ for all $i(i = 1$ to $n)$, that is when the new message does not modify any of the a priori probabilities.

$D(p, q)$ will be maximal when there is a result x_i such that $q_i > p_i = 0$ because in such a case the probability a priori that the event x_i would occur was nil whereas now, after reception of the correcting message, the probability that it will occur is not nil any more and thus the degree of surprise may be considered as infinite.

In the other extreme case where a priori all events x_i had the same probability p_i of occurring, with $p_i = (1/n)$ for all i, the divergence function $D(q, p)$ will be expressed as

$$D(q, p) = \log(n) - H(q) \tag{27}$$

An interesting measure of divergence is the Kullback–Leibler–Jeffreys measure $J(q, p)$ (see Kullback and Leibler, 1951; Jeffreys, 1967) which is defined as

$$J(q, p) = D(q, p) + D(p, q) = \sum\nolimits_{i=1 \text{ to } n} (q_I - p_i)[\log(q_i) - \log(p_i)] \tag{28}$$

Maasoumi (1986) mentions two additional classes of measures.

The first one $D_k(q, p)$ is defined as

$$D_k(q, p) = (1/(k-1))\left[\sum\nolimits_{i=1 \text{ to } n} \left\{\left[\left((q_i)^k\right) / \left((p_i)^{k-1}\right)\right] - 1\right\}\right. \tag{29}$$

with $k \neq 1$. Note that when $k \to 1$, $D_k(q, p) \to D(q, p)$.

The other class of generalized divergence measure mentioned by Maasoumi is $D_\gamma(q, p)$ with

$$D_\gamma(q, p) = [1/(\gamma(\gamma+1))]\left\{\sum\nolimits_{i=1 \text{ to } n} q_i \left[\left((q_i / p_i)^\gamma\right) - 1\right\}\right. \tag{30}$$

with $\gamma \neq 0, -1$. Note that as $\gamma \to 0$, $D_\gamma(q, p) \to D(p, q)$. One may also observe that as $\gamma \to 0$, $D_\gamma(q, p) \to D(p, q)$.

Information Theory and Multidimensional Measures of Inequality

The idea of using concepts borrowed from information theory to define multidimensional measures of inequality was originally proposed by Maasoumi (1986). He suggested proceeding in two steps. First a procedure would be defined that would allow to aggregate the various indicators of welfare to be taken into account. Second an inequality index would be selected to estimate the degree of multidimensional inequality.

Assume n welfare indicators have been selected, whether they be of a quantitative or qualitative nature. Call x_{ij} the value taken by indicator j for individual (or household) i, with $i = 1$ to n and $j = 1$ to m. The various elements x_{ij} may be represented by a matrix $X = [x_{ij}]$ where the i-th line will give the welfare level of individual i according to the various m indicators, while the j-th column the distribution among the n individuals of the welfare level corresponding to indicator j.

Maasoumi's idea is to replace the m pieces of information on the values of the different indicators for the various individuals by a composite index x_c which will be a vector of n components, one for each individual. In other words the vector $(x_{i1}, \ldots x_{im})$ corresponding to individual i will be replaced by the scalar x_{ci}. This scalar may be considered either as representing the utility that individual i derives from the various indicators or as an estimate of the welfare of individual i, as an external social evaluator sees it.

The question then is to select an "aggregation function" that would allow derivation of such a composite welfare indicator x_{ci}. Maasoumi (1986) suggested finding a vector x_c that would be closest to the various m vectors x_{ij} giving the welfare level the various individuals derive from these m indicators. To define such a "proximity" Maasoumi proposes a multivariate generalization of the generalized entropy index $D_\gamma(q, p)$ that is expressed as

(31) $$D_\gamma(x_c, X; \alpha) = (1/(\gamma(\gamma+1)))\sum_{j=1 \text{ to } m} \alpha_j \left\{\sum_{i=1 \text{ to } n} x_{ci}\left[(x_{ci}/x_{ij})^\gamma - 1\right]\right\}$$

with $\gamma \neq 0, -1$, and where α_j represents the weight to be given to indicator j.

When $\gamma \rightarrow 0$ or -1, one obtains the following indicators:

(32) $$D_0(x_c, X; \alpha) = \sum_{j=1 \text{ to } m} \alpha_j \left[\sum_{i=1 \text{ to } n} x_{ci} \log(x_{ci}/x_{ij})\right]$$

and

(33) $$D_{-1}(x_c, X; \alpha) = \sum_{j=1 \text{ to } m} \alpha_j \left[\sum_{i=1 \text{ to } n} x_{ij} \log(x_{ij}/x_{ci})\right]$$

The minimization of the "proximity" defines a composite index x_{ci} in each of the three cases corresponding to expressions (34), (35) and (36).

In the first case x_{ci} is defined as

(34) $$x_{ci} \propto \left[\sum_{j=1 \text{ to } m} \delta_j (x_{ij})^{-\gamma}\right]^{-(1/\gamma)}$$

In the second case, when $\gamma \rightarrow 0$, one gets

(35) $$x_{ci} \propto \left[\prod_{j=1 \text{ to } m} (x_{ij})^{\delta_j}\right]$$

Finally in the case where $\gamma \rightarrow -1$, one obtains

(36) $$x_{ci} \propto \left[\sum_{j=1 \text{ to } m} \delta_j (x_{ij})\right]$$

In expressions (34) to (36) δ_j is defined as the normalized weight of indicator j, that is $\delta_j = \alpha_j / \Sigma_{j=1 \text{ to } m} \alpha_j$.

Thus it turns out that the composite indicator x_c is a weighted average of the different indicators. In the general case (34) it is an harmonic mean; in the case where $\gamma \rightarrow 0$, it is a geometric mean while in that where $\gamma \rightarrow -1$, it is an arithmetic mean. Moreover it is easy to interpret this composite welfare indicator as a utility function of the CES type with an elasticity of substitution $\sigma = 1/(1 + \gamma)$ when $\gamma \neq 0, -1$, as a Cobb–Douglas utility when $\gamma \to 0$, and as a linear utility function when $\gamma \to -1$.

Having derived a composite index x_{ci} for each individual i, one may measure inequality by applying generalized entropy inequality indices that were defined by Shorrocks (1980) and applied to the multidimensional case by Maasoumi (1986).

Information Theory and a Multidimensional Approach to Poverty Measurement

Although Information Theory has been applied several times to the analysis of multidimensional inequality (see the survey by Maasoumi, 1999), it seems to have been used only once in the study of multidimensional poverty (see Miceli, 1997). Miceli has suggested derivation of the measurement of multidimensional poverty from the distribution of the composite index X_C whose definition is given in expressions (34) to (36). Such a choice implies evidently that a decision has to be made concerning the selection of the weights δ_j to be given to the various indicators x_{ij} (the subindex i referring to the individual while the subindex j denotes the indicator) as well as to the parameter γ. We have examined two possibilities. In the first case we decided to give to each indicator a weight proportional to its mean, this implying in fact that the more diffused the durable good is, the higher its weight. This was already the point of view adopted previously when summarizing the fuzzy approaches to poverty measurement, the idea being that if a household does not have a durable good, more weight should be given to this information, the higher the percentage of households who have this durable good. In the second case we simply decided to give an equal weight ($1/m$) to all the indicators j (where m refers to the total number of indicators). In both cases we assumed that the parameter γ was equal to 1.

Once the composite indicator X_C is defined, one still has to define a procedure to identify the poor. Here again we will follow Miceli (1997) and adopt the so-called "relative approach" which is commonly used in the unidimensional analysis of poverty. In other words we will define the "poverty line" as being equal to some percentage of the median value of the composite indicator X_C. More precisely we have chosen as cutting points a "poverty line" assumed to be equal to 70 percent of the median value of the distribution of the composite index X_C. In other words any household i for which the composite index X_{Ci} will be smaller than the "poverty line" will be identified as poor.

2.4. *Axiomatic Derivations of Multidimensional Poverty Indices*

Very few studies have attempted to derive axiomatically multidimensional indices of poverty. Tsui (2002) made recently such an attempt, following his earlier work on axiomatic derivations of multidimensional inequality indices (see Tsui, 1995, 1999) but it seems that Chakravarty *et al.* (1998) were the first to publish an article on the axiomatic derivation of multidimensional poverty indices.

The basic idea behind Chakravarty *et al.* (1998) as well as Tsui's (2002) approach is as follows. Both studies view a multidimensional index of poverty as an aggregation of shortfalls of all the individuals where the shortfall with respect to a given need reflects the fact that the individual does not have even the minimum level of the basic need. Let $z = (z_1, \ldots, z_k)$ be the k-vector of the minimum levels of the k basic needs and $x^i = (x_{i1}, \ldots, x_{ik})$ the vector of the k basic needs of the i-th person. Let X be the matrix of the quantities x_{ij} which denote the amount of the j-th attribute accruing to individual i.

Chakravarty *et al*. (1998) then defined the following list of desirable properties of a multidimensional poverty measure.

(1) *Symmetry*: This property assumes that the multidimensional poverty index depends only on the various attributes j that the individuals have and not on their identity.

(2) *Focus*: If for any individual i an attribute j is such that $x_{ij} > z_j$, $P(X; z)$ does not change if there is an increase in x_{ij}.

(3) *Monotonicity*: If for any individual i an attribute j is such that $x_{ij} \leq z_j$, $P(X; z)$ does not increase if there is an increase in x_{ij}.

(4) *Principle of Population*: An m-fold replication of X will not affect the value of the poverty index.

(5) *Continuity*: An index of multidimensional poverty $M(X)$ should be a continuous function, that is, it should be only marginally affected by small variations in x_{ij}.

(6) *Non-Poverty Growth*: If the matrix Y is obtained by adding a rich person to the population defined by X, then $P(Y; z) \leq P(X; z)$.

(7) *Non-decreasingness in Subsistence Levels of Basic Needs*: If z_j increases for any j, $P(X; z)$ does not decrease.

(8) *Scale Invariance*: This implies that the ranking of any two matrices of attributes is preserved if the attributes are rescaled according to their respective ratio scales.

(9) *Normalization*: $P(X; z) = 1$ whenever $x_{ij} = 0$ for all i and j.

(10) *Subgroup Decomposability*: Assume n_i is the population size of subgroup $i(i = 1 \text{ to } m)$ with $n = \Sigma_{i=1 \text{ to } m} n_i$ representing the total size of the population. Then the poverty index for the whole population (where the data on each subpopulation is represented by a matrix X^i) may be expressed as

$$P(X^1, \ldots, X^m) == \sum\nolimits_{i=1 \text{ to } m} (n_i/n) P(X^i; z).$$

(11) *Factor Decomposability*:

$$P(X; z) == \sum\nolimits_{j=1 \text{ to } k} a_j P(x_j; z_j)$$

where x_j; is the j-th column of X, a_j is the weight attached to attribute j such that $\Sigma_{j=1 \text{ to } k} a_j = 1$.

(12) *Transfer Axiom*: Let X_p be the submatrix of X corresponding to the poor. If Y is derived from X by multiplying X_p by a bistochastic matrix (not a permutation matrix), then $P(Y; z) \leq P(X; z)$ given that the bundles of attributes of the rich remain unaltered.

(13) *Nondecreasing Poverty under Correlation Increasing Arrangement*: This property refers to switches of some attribute(s) between individuals that increase the correlation of the attributes.

Chakravarty *et al*. (1998) then derive the following propositions.

First Proposition

The only non constant focused poverty index that satisfies the properties of subgroup decomposability, factor decomposability, scale invariance, monotonicity, transfer axiom, continuity and normalization is defined as

(37) $$P(X;z)=(1/n)\sum_{j=1 \text{ to } n}\sum_{j=1 \text{ to } k} a_j f(x_{ij}/z_j)$$

where f is continuous, nonincreasing and convex with $f(0) = 1$ and $f(t) = c$ for all $t \geq 1$ and $c < 1$ is a constant. The parameters a_j are positive and constant with $\Sigma_{j=1 \text{ to } k} a_j = 1$.

Define now the function $g(t)$ as $g(t) = (f(t) - c)/(1 - c)$. This allows Chakravarty *et al.* (1998) to derive their second proposition.

Second Proposition

The poverty measure $P(X; z) = (1/n)\ \Sigma_{i=1 \text{ to } n}\Sigma_{j=1 \text{ to } k} a_j g(x_{ij}/Z_j)$ satisfies the properties of Symmetry, Population Replication, Non-Poverty Growth and Non-Decreasingness in Subsistence Levels of Basic Needs. If g is twice differentiable on (0, 1) P, the poverty index, satisfies also the property of Nondecreasing Poverty under Correlation Increasing Arrangement.

The following multidimensional poverty index may be considered

(38) $$P(X;z)=(1/n)\sum_{j=1 \text{ to } k}\sum_{i\in Sj} a_j\left[1-(x_{ij}/z_j)^e\right]$$

where s_j is the set of poor people with respect to attribute j.

This index is a multidimensional extension of the subgroup decomposable index suggested by Chakravarty (1983).

When $e = 1$ we get

(39) $$P(X;z)=(1/n)\sum_{j=1 \text{ to } k}\sum_{i\in Sj} a_j[(z_j-x_{ij})/z_j]=\sum_{j=1 \text{ to } k} a_j H_j I_j$$

where $H_j = (q_j/n)$ and I_j are respectively the head-count ratio and the poverty-gap ratio for attribute j ($I_j = \Sigma_{i\in Sj}[(z_j - x_{ij})/(q_j z_j)]$).

Another possible index is

(40) $$P_\alpha(X;z)=(1/n)\sum_{j=1 \text{ to } k}\sum_{i\in Sj} a_j[1-(x_{ij}/z_j)]^\alpha$$

This index is a multidimensional generalization of the Foster, Greer and Thorbecke (1984) subgroup decomposable index.

3. The Information Basis for the Derivation of Multidimensional Poverty Indices in Israel in 1995

As indicated earlier, the database we used was the 1995 Israeli Census. This Census however provides only information on the ownership of durable goods. It does not include, for example, any question on the satisfaction of the household members with respect to their standard of living, their work, their health, etc. The estimation of the multidimensional poverty indices previously defined will hence be based only on the ownership of the various durable goods. Although in many cases the available information was of the binary type in the sense that we knew for example whether the household had a washing machine or not, some variables were polytomous. This was the case of the variables indicating the period in which the apartment or house was built, the number of rooms in the dwelling and whether there was a bath or a shower in the dwelling (see Appendix 1 for the exact listing of the categories distinguished). There was even a purely quantitative variable, that giving the number of cars available for household use. Moreover we have actually used as indicator not the number of rooms or cars in the household but the number of rooms or cars per individual. Note that the ownership of the dwelling is defined as a dichotomous variable, taking the value 1 when the apartment (or house) is owned by the household.

To analyze the impact on multidimensional poverty of variables such as the gender, the household size, the age, the marital status, the year of immigration, the level of schooling, the number of months worked during the last twelve months, the status at work, the place of residence and the religion of the head of the household, we have estimated logit type regressions. In these regressions the dependent variable is the probability of being poor while the variables previously mentioned are the explanatory variables.

In order to use the information on the ownership of durables in a compact fashion, we had to summarize the information available in the case of polytomous variables, which are categorical variables that may take many values (e.g. the period of construction of the dwelling). In order to do so, we have borrowed a technique used by Cheli and Lemmi (1995) in their work on the fuzzy approach to poverty measurement that was mentioned previously. Their idea is as follows. Rank the households by increasing degree of ownership of the durable good (or of its quality, in the case for example of the year of construction). Compute the distribution function of such a variable and define the degree of satisfaction of a household as being equal to the ratio of the percentage of households that are worse off than this household over the percentage of households that are worse off than the richest[3] household. Compute then the average level of satisfaction of the households in the population by summing the satisfaction derived by all households and dividing by the total number of households. One may easily notice that, in the case of a dichotomous variable, such a definition will give us precisely the proportion of households owning this durable good. The indicator of ownership of a durable good we adopted in the case of a polytomous variable is thus consistent with the very intuitive indicator of ownership that would give us in the case of a binary variable the proportion of households owning the durable good. Let us now examine the results obtained on the basis of these logit regressions.

4. Results of the Logit Regressions

The following exogenous variables have been taken into account: the size of the household and its square, the age of the head of the household and its square, the number of years of schooling, the gender, the religion (three dummy variables), the marital status (three dummy variables) and the status at work (working or not) of the head of the household, the area of residence of the house-

[3]Richest with respect to the ownership of this durable good.

hold (three dummy variables corresponding to the three big cities) and a variable indicating whether the head of the household immigrated to Israel after 1989. In addition we introduced interaction variables between the gender and the marital status and between the gender and the working status.

Results of the Logit Regressions

These results are given in Tables 1 to 4, giving successively the results of the estimations derived on the basis of the fuzzy approach to poverty measurement (Table 1), the distance function approach (Table 2), information theory (Table 3) and the axiomatic approaches to poverty measurement (Table 4).

In all the four cases it appears that the explanatory variables that have been introduced have generally a significant impact. Thus households whose head has a higher educational level have, ceteris paribus, a lower probability of being poor. This probability decreases and then increases again with the size of the household as well as with the age of the head of the household.

Other things constant we also observe that the probability that a household is considered as poor is highest among heads of household that are Muslims and lowest among those who are Jewish. This probability is also lowest when the head of the household is married and highest when he/she is single. It is higher when he/she is a new immigrant, is highest when he/she lives in Jerusalem and lowest when he/she lives outside the three main cities.

As far as the combined effect of the gender, the marital and the working status is concerned, we usually observe, ceteris paribus, that whatever their gender or working status, divorced individuals have the highest probability of being poor and married individuals the lowest probability. As expected, whatever their gender and marital status, non working individuals have generally a higher probability of being poor. Finally in most cases, once the interactions are taken into account, for a given marital and working status, males seem to have a higher probability of being poor. Note however that some results indicate that among divorced individuals females have a higher probability of being poor.

One should also stress that when the information theory approach is adopted, the results of the logit regressions are better (in the sense that their predictive power is better) when the weights of the indicators are proportional to their mean than when equal weights are given to all the indicators.

Finally for the axiomatic approach to multidimensional poverty measurement, the estimations are based on the definition of the poverty indices given in equations (38) to (40). One should observe that the values of the coefficient of a given variable in these regressions are very similar in the three cases examined (two illustrations of the Chakravarty *et al*. index and one of the generalization of the FGT index).

5. A Comparison of the Various Multidimensional Approaches to Poverty Measurement

The previous section has indicated that in most cases there were no big differences between the various multidimensional poverty indices that have been used,

TABLE 1

RESULTS OF THE LOGIT REGRESSIONS FOR THE FUZZY APPROACH TO MULTIDIMENSIONAL POVERTY (NUMBER OF OBSERVATIONS: 204,098)

	Totally Fuzzy Approach (TFA)		Totally Fuzzy and Relative Approach (TFR)		Vero-Werquin Approach	
Variable	Coefficient	t-value	Coefficient	t-value	Coefficient	t-value
Intercept	5.12354	67.00	5.27915	70.30	3.77452	51.21
Number of years of schooling	–0.07740	–51.70	–0.08117	–55.94	–0.08924	–61.92
Household size	–0.67555	–53.23	–0.58880	–47.15	–0.30789	–21.45
Square of household size	0.05833	45.60	0.05526	43.24	0.06435	40.14
Age of head of household	–0.14186	–66.52	–0.14365	–68.85	–0.12610	–60.51
Square of age of head of household	0.00112	54.94	0.00114	56.69	0.00102	50.58
Head of household is male	0.07551	2.02	0.04078	1.11	0.12881	3.29
Head of household is Jewish	–0.50158	–10.51	–0.55112	–11.69	–0.68580	–14.88
Head of household is Muslim	1.05343	20.52	0.83768	16.54	0.46292	9.38
Head of household is Christian	0.33539	5.43	0.27644	4.59	0.32038	5.56
Head of household is Druze	0.40463	5.69	0.16961	2.43	–0.02362	–0.35
Head of household immigrated after 1989	1.28791	76.86	1.28038	78.93	1.03535	64.61
Head of household is married	–0.31403	–4.08	–0.27279	–3.62	–0.37975	–4.81
Head of household is divorced or separated	1.18369	11.47	1.14942	11.28	1.13499	10.74
Head of household is single	0.42661	4.68	0.44708	4.96	0.34113	3.67
Households lives in Jerusalem	0.50691	25.31	0.51921	26.65	0.67571	35.24
Household lives in Tel-Aviv	0.24042	12.07	0.28505	14.81	0.89813	49.52
Household lives in Haifa	0.17459	7.21	0.22050	9.48	0.66255	30.16
Head of household is working	–0.30852	–10.43	–0.35334	–12.36	–0.27144	–9.76
Interaction term: Head of household is male and married	0.09554	2.15	0.07160	1.65	0.25675	5.73
Interaction term: Head of household is male and divorced	–0.33449	–5.89	–0.28444	–5.08	–0.24362	–4.20
Interaction term: Head of household is male and single	0.26685	5.15	0.23932	4.66	0.33041	6.28
Interaction term: Head of household is male and working	–0.03024	–16.79	–0.02967	–17.06	–0.02522	–14.89

TABLE 1 (*Continued*)

Actual versus Predicted Values

Actual value	Identification of Poor Based on the Totally Fuzzy Approach (TFA)			Identification of Poor Based on the Totally Fuzzy and Relative Approach (TFR)			Identification of Poor Based on the Vero-Werquin Approach		
	Predicted Value		Total (in %)	Predicted Value		Total (in %)	Predicted Value		Total (in %)
	0	1		0	1		0	1	
0	71.6	4.6	76.2	68.2	5.3	73.6	67.5	5.0	72.5
1	15.9	8.0	23.9	16.8	9.6	26.4	18.0	9.5	27.5
Total (in %)	87.4	12.6	100.0	85.1	14.9	100.0	85.5	14.5	100.0

TABLE 2

RESULTS OF THE LOGIT REGRESSION BASED ON THE DISTANCE FUNCTION APPROACH (NUMBER OF OBSERVATIONS: 100,000)

Variable	Distance Function Approach: Coefficient	t-value
Intercept	2.26259	22.91
Number of years of schooling	–0.05763	–29.70
Household size	–0.35081	–22.09
Square of household size	0.02820	17.83
Age of head of household	–0.06560	–23.32
Square of age of head of household	0.00053	19.30
Head of household is male	0.13098	2.50
Head of household is Jewish	–0.49867	–7.97
Head of household is Muslim	0.36730	5.45
Head of household is Christian	0.06390	0.79
Head of household is Druze	0.22288	2.43
Head of household immigrated after 1989	0.71895	32.88
Head of household is married	–0.28071	–2.66
Head of household is divorced or separated	0.50478	3.54
Head of household is single	0.25393	2.03
Households lives in Jerusalem	0.32049	12.32
Household lives in Tel-Aviv	–0.02708	–1.02
Household lives in Haifa	–0.14881	–4.49
Head of household is working	–0.05427	–1.43
Interaction term: Head of household is male and married	0.19653	3.27
Interaction term: Head of household is male and divorced	–0.08135	–1.04
Interaction term: Head of household is male and single	0.06582	0.93
Interaction term: Head of household is male and working	–0.01172	–5.11

Actual versus Predicted Values (in percent)

Actual Value	Predicted Value: 0	Predicted Value: 1	Total (in %)
0	73.1	1.9	75.0
1	22.4	2.6	25.0
Total (in %)	95.5	4.5	100.0

TABLE 3

RESULTS OF THE LOGIT REGRESSION BASED ON THE INFORMATION THEORY APPROACH TO MULTIDIMENSIONAL POVERTY MEASUREMENT (NUMBER OF OBSERVATIONS IN BOTH CASES: 204,098)

	Weight of Indicators Proportional to their Mean		An Equal Weight is Given to All the Indicators	
Variable	Coefficient	t-value	Coefficient	t-value
Intercept	3.68344	46.43	3.88285	51.37
Number of years of schooling	–0.10745	–66.96	–0.12614	–84.70
Household size	–0.42329	–32.74	–0.33800	–26.86
Square of household size	0.03695	29.15	0.03420	26.66
Age of head of household	–0.11009	–48.49	–0.10369	–48.44
Square of age of head of household	0.00091	41.94	0.00093	45.47
Head of household is male	0.06110	1.52	–0.08370	–2.25
Head of household is Jewish	–0.67216	–14.05	–0.62680	–13.46
Head of household is Muslim	1.10967	21.49	1.24252	24.78
Head of household is Christian	0.39876	6.40	0.45153	7.59
Head of household is Druze	0.30046	4.09	0.45619	6.63
Head of household immigrated after 1989	1.44458	81.25	1.50131	90.64
Head of household is married	–0.42080	–5.07	–0.28625	–3.76
Head of household is divorced or separated	1.00272	9.04	0.78115	7.43
Head of household is single	0.09461	0.97	0.04017	0.44
Households lives in Jerusalem	0.44237	20.66	0.43212	21.38
Household lives in Tel-Aviv	0.26222	12.03	0.21494	10.81
Household lives in Haifa	0.28935	11.13	0.29318	12.50
Head of household is working	–0.27532	–8.75	–0.32324	–11.18
Interaction term: Head of household is male and married	0.21059	4.38	0.08329	1.90
Interaction term: Head of household is male and divorced	–0.11468	–1.88	–0.00525	–0.09
Interaction term: Head of household is male and single	0.52654	9.56	0.52172	9.99
Interaction term: Head of household is male and working	–0.03883	–19.85	–0.03579	–20.06

Actual versus Predicted Values

	Mean Weights			Equal Weights		
	Predicted Value			Predicted Value		
Actual value	0	1	Total (in %)	0	1	Total (in %)
0	78.2	3.1	81.3	68.1	5.7	73.7
1	14.0	4.7	18.7	15.8	10.5	26.3
Total (in %)	92.2	7.8	100.0	83.9	16.1	100.0

Notes: The poverty line is equal to 50% of the median of the distribution of the composite index.
Case A: The weight of the indicators is proportional to their mean.
Case B: An equal weight is given to all the indicators.

at least as far as the impact on poverty of various explanatory variables was concerned. Thus poverty was found to first decrease, then increase with the size of the household and the age of its head. Poverty is also lower when the head of the household has a higher level of education, works, is self-employed, married, Jewish, lives in a medium-sized city and has been for a longer period in Israel.

TABLE 4

RESULTS OF THE LOGIT REGRESSIONS BASED ON THE AXIOMATIC APPROACH TO MULTIDIMENSIONAL POVERTY MEASUREMENT (NUMBER OF OBSERVATIONS: 204,098)

	Identification of Poor based on the Chakravarty *et al.* Index, e = 0.5		Identification of Poor based on the Chakravarty *et al.* Index, e = 1		Identification of Poor based on the Generalization of the FGT Index	
Variable	Coefficient	t-value	Coefficient	t-value	Coefficient	t-value
Intercept	4.99136	64.47	4.99596	64.52	4.99097	64.47
Number of years of schooling	–0.09476	–63.31	–0.09481	–63.34	–0.09476	–63.31
Household size	–0.67226	–50.98	–0.67802	–51.29	–0.67202	–50.97
Square of household size	0.06529	48.11	0.06617	48.57	0.06525	48.09
Age of head of household	–0.13661	–63.23	–0.13652	–63.17	–0.13661	–63.23
Square of age of head of household	0.00119	57.54	0.00119	57.49	0.00119	57.54
Head of household is male	–0.07273	–1.97	–0.07358	–1.99	–0.07274	–1.97
Head of household is Jewish	–0.62188	–13.01	–0.62254	–13.02	–0.62180	–13.01
Head of household is Muslim	1.07601	20.92	1.07912	20.98	1.07599	20.92
Head of household is Christian	0.38044	6.17	0.38317	6.21	0.38049	6.17
Head of household is Druze	0.18692	2.57	0.19212	2.64	0.18714	2.57
Head of household immigrated after 1989	1.52821	89.91	1.52912	89.94	1.52820	89.91
Head of household is married	–0.27072	–3.56	–0.26896	–3.53	–0.27064	–3.56
Head of household is divorced or separated	0.93732	9.01	0.93963	9.03	0.93786	9.01
Head of household is single	0.04058	0.45	0.04259	0.47	0.04062	0.45
Households lives in Jerusalem	0.71835	35.75	0.72943	36.32	0.71846	35.76
Household lives in Tel-Aviv	0.44431	22.62	0.44573	22.69	0.44432	22.62
Household lives in Haifa	0.42512	18.02	0.42661	18.09	0.42512	18.02
Head of household is working	–0.32438	–10.93	–0.32470	–10.94	–0.32438	–10.93
Interaction term: Head of household is male and married	0.03891	0.88	0.03849	0.87	0.03882	0.88
Interaction term: Head of household is male and divorced	–0.10906	–1.91	–0.11027	–1.93	–0.10966	–1.92
Interaction term: Head of household is male and single	0.52391	10.12	0.52196	10.08	0.52381	10.12
Interaction term: Head of household is male and working	–0.03917	–21.49	–0.03913	–21.47	–0.03916	–21.49

TABLE 4 (*Continued*)

Actual versus Predicted Values

Actual value	Identification of Poor based on the Chakravarty *et al.* Index, e = 0.5			Identification of Poor based on the Chakravarty *et al.* Index, e = 1			Identification of Poor based on the Generalization of the FGT Index		
	Predicted Value		Total (in %)	Predicted Value		Total (in %)	Predicted Value		Total (in %)
	0	1		0	1		0	1	
0	69.6	5.3	74.9	69.6	5.3	74.9	69.6	5.3	74.9
1	15.1	10.0	25.1	15.1	10.0	25.1	15.1	10.0	25.1
Total (in %)	84.7	15.3	100.0	84.7	15.3	100.0	84.7	15.3	100.0

TABLE 5

Distribution of Households by the Number of Poverty Indices on the Basis of which they are Defined as Poor

Number of Indices	Number of Households	Percentage of Households	Cumulative Percentage of Households
0	53,246	53.2	53.2
1	15,390	15.4	68.6
2	3,119	3.1	71.8
3	2,842	2.8	74.6
4	3,329	3.3	77.9
5	2,280	2.3	80.2
6	3,892	3.9	84.1
7	4,884	4.9	89.0
8	11,018	11.0	100.0

The question remains to know to what extent these various indices overlap. In other words, although the overall picture given by these different indices is quite similar, do they really identify the same households as poor? The data provided in the following tables will attempt to answer such questions as: which percentage of the households defined as poor according to a given index will also be considered as poor when another poverty index is used.

In order to be able to make relevant comparisons, we assume in this section that, whatever poverty index is used, 25 percent of the households are poor. We have therefore in each case the same proportion of poor and what we want to check is to what extent we find the same poor households in each case.

In Table 5, for example, we give the distribution of the households according to the exact number of poverty indices that define them as being poor. One may observe that 53.2 percent of the households are never defined as poor while 15.4 percent of them are considered as poor according to one poverty index (and one only). Note that 11 percent of the households are defined as poor according to all the indices, which is not a small percentage.

In Table 6 we ask a somehow different question: according to at least how many poverty indices is a household considered as poor. It then appears that 31.4

TABLE 6

DISTRIBUTION OF HOUSEHOLDS BY THE MINIMAL NUMBER OF POVERTY INDICES ON THE BASIS OF WHICH THEY ARE DEFINED AS POOR

Household Defined as Poor on the Basis of at Least k Indices, with k =	Number of Households	Percentage of Households
1	46,754	46.8
2	31,364	31.4
3	28,245	28.2
4	25,403	25.4
5	22,074	22.1
6	19,794	19.8
7	15,902	15.9

TABLE 7

PERCENTAGE OF HOUSEHOLDS DEFINED AS POOR BY TWO INDICES

	TFR	VWA	Dist	Inf	Chak.;0.5	Chak.;1	Generalization of FGT[4] with $\alpha = 2$
Totally fuzzy absolute approach (TFA)	24	18	13	20	20	20	20
Totally fuzzy relative approach (TFR)	0	18	13	21	20	20	20
Vero and Werquin approach (VW)	0	0	14	18	18	18	18
Distance function approach (dist)	0	0	0	12	12	12	12
Information theory approach (inf)	0	0	0	0	22	22	22
Chakravarty *et al.* approach with e = 0.5 (Chak.;0.5)	0	0	0	0	0	25	25
Chakravarty *et al.* approach with e = 1 (Chak.;1)	0	0	0	0	0	0	25

percent of the households are defined as poor according to at least two indices, 25.4 percent according to at least four indices and almost 20 percent (19.8 percent) according to at least six indices.

Table 7 takes a look at all the possible binary comparisons of multidimensional poverty indices and gives in each case the percentage of households that are considered as poor according to the two indicators selected for the binary comparison. It appears that the higher percentage is observed either when the Chakravarty *et al.* (1998) indices are compared for two different values of the parameter e (25 percent of the households are poor in such a case according to both indicators so that the two indices obviously identify the same households as being poor), when the Chakravarty *et al.* index with the parameter e equal to 1 is compared with the generalization of the Foster *et al.* (1984) index (25 percent of poor

[4]FGT: Foster, Greer and Thorbecke index (see Foster *et al.*, 1984).

households also) or when the two fuzzy approaches (TFR and TFA) are compared (24 percent of households that turn out to be poor according to both indices).

The lowest percentage is observed when the distance function is combined either with one of the Chakravarty *et al.* indices or with the generalization of the FGT index (only 12 percent of the households are considered as poor according to the two indices compared).

6. Concluding Comments

In this paper we attempted to compare empirically the various approaches to multidimensional poverty measurement that have appeared in recent years in the literature. In order to do so we used in each case the same database, the 1995 Israeli Census.

What conclusions may be drawn on the basis of such a systematic comparison of the various methods leading to multidimensional measures of poverty? First, 46.8 percent of the households are considered as poor according to at least one approach while 22.1 percent of the households are classified as poor according to at least five approaches. Poverty is thus not a marginal feature of the Israeli society, assuming one is ready to accept a definition of poverty that is based only on the ownership of various durable goods, since this is the kind of data that have been used in this paper.

Second, it appears that there is a fair degree of agreement between the various multidimensional poverty indices. The index based on the concept of distance function and that based on the fuzzy approach suggested by Vero and Werquin are the only indices that seem, to a certain degree at least, to identify different households as poor, assuming that in all cases 25 percent of the households are poor. This does not imply that these two approaches should be considered as less attractive. One might on the contrary argue that since they are the only one that in a certain way take into account the redundancy of some of the indicators, they may ultimately be more reliable than the others that ignore this problem of "collinearity." Additional empirical illustrations are evidently necessary before firmer conclusions may be drawn.

Differences between the various methods are however much smaller as far as the determinants of multidimensional poverty are concerned. All the approaches have shown that poverty decreases with the schooling level of the head of the household, first decreases and then increases with his/her age and with the size of the household. Poverty was found to be higher when the head of the household is single and lower when he/she is married. Poverty is lowest when the head of the household is Jewish and highest when he/she is Muslim. Poverty is also higher among households whose head immigrated in recent years, does not work or lives in Jerusalem. These observations were made on the basis of the various logit regressions that were estimated.

As a whole the impact on poverty of many of the variables is not different from the one that is observed when poverty measurement is based only on the income or the total expenditures of the households. Sorin (1999) presents such an analysis on the basis of Israeli data (see also Atkinson (1998) for a study of

European data). Such a conclusion seems to indicate that when income data are not available or reliable, the extent and the determinants of poverty may still be measured on the basis of multidimensional poverty indices that aggregate the information available, for example, on the ownership of various durable goods.

Appendix 1: Information Available in the 1995 Israeli Census on Durable Goods; List of Variables

Number of Rooms

1: 1 room
2: 1.5 rooms
3: 2 rooms
4: 2.5 rooms
5: 3 rooms
6: 3.5 rooms
7: 4 rooms
8: 4.5 rooms
9: 5 rooms
10: 5.5 rooms
11: 6 or more rooms

Year of Construction of Dwelling

1: Before 1947
2: 1948–1954
3: 1955–1964
4: 1965–1974
5: 1975–1984
6: 1985–1989
7: 1990
8: 1991
9: 1992
10: 1993
11: 1994
12: 1995

Ownership of Dwelling

1: Family owned
2: Rented

Bath/Shower

1: Bath (with/without shower)
2: Shower only
3: No bath or shower

Telephone

1: Yes
2: No

Television

1: Yes
2: No

Videotape

1: Yes
2: No

Washing Machine

1: Yes
2: No

Microwave Oven

1: Yes
2: No

Dishwasher

1: Yes
2: No

Computer

1: Yes
2: No

Air-Conditioner

1: Yes
2: No

Solar Heating System

1: Yes
2: No

Drying Machine

1: Yes
2: No

Availability of Cars for Household Use

1: No car
2: One car
3: 2 cars
4: 3 cars or more

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