Review of Income and Wealth
Series 49, Number 4, December 2003

POVERTY REDUCING REFORMS AND SUBGROUP CONSUMPTION DOMINANCE CURVES

BY PAOLO LIBERATI*

University of Urbino, ISE

One vexed question of anti-poverty strategies is that of setting a reasonable poverty line. To escape its specification, recent developments by Yitzhaki and Slemrod (1991) have introduced the correspondence between non-intersecting concentration curves and poverty reducing directions of reforms. Makdissi and Wodon (2002) have derived consumption dominance curves for any order of restricted stochastic dominance. In this paper, consumption dominance curves are extended to subgroups of population. Empirical evidence of the approach will be shown using the 1997 data from Belarus, considering public subsidies on rents and utilities, health care and public transport in six groups of population.

1. INTRODUCTION

One vexed question of anti-poverty strategies is that of setting a reasonable poverty line and to test whether results are robust to different specifications of it. To escape the specification of the poverty line, recent developments by Yitzhaki and Slemrod (1991) have introduced the correspondence between non-intersecting concentration curves and poverty reducing directions of reforms, by extending previous results from Yitzhaki and Thirsk (1990). More recently, Makdissi and Wodon (2002) have derived consumption dominance curves for any order of restricted stochastic dominance.

None of these papers, however, investigates poverty by population subgroups. Yet, the understanding of how the aggregate poverty effect is distributed among groups might be of crucial importance for the political economy of anti-poverty policies. The crucial point is that a policy that is poverty reducing in aggregate may well have poverty increasing effects for some population groups.

This paper highlights this issue by extending the correspondence between poverty reducing reforms and consumption dominance curves to population subgroups. This is done by deriving Sub-Group Consumption Dominance (SGCD) curves.

An illustrative example of the approach is developed using data on consumption subsidies on rents and utilities (RU), public transport (PT) and health care (HC) in Belarus. Reforming subsidies is likely to cause non-negligible effects on poor households. Yet, these effects might be asymmetric among population groups. The approach here used allows us to investigate whether revenue-neutral

Note: I wish to thank David Newbery for his comments and supervision on an extended version of my work on poverty in Belarus. Very stimulating comments on this paper from Shlomo Yitzhaki are also gratefully acknowledged. Comments from an anonymous referee have been of great help in clarifying the issue. The usual disclaimer applies.

*Correspondence to: Paolo Liberati, University of Urbino, Faculty of Economics, Istituto di Scienze Economiche, Via Saffi 42, 61029 Urbino (PU), Italy (liberati@econ.uniurb.it).

changes of consumption subsidies are in fact poverty reducing for all population groups exploiting the minimal informational requirement implicit in SGCD curves.

The paper is organized as follows. Section 2 will set the theoretical framework, showing conditions for sub-group consumption dominance curves. Section 3 will briefly describe the data and some statistical figures. Section 4 will discuss the results. Section 5 concludes.

2. Theoretical setting

2.1. *Poverty Reducing Directions of Reform: Total Population*

The problem here considered is how to choose, with a minimal informational set, which good should be either taxed or subsidized more in order to reduce a general form of the poverty index depending on a measure of individual welfare. Let us define the quantity consumed of commodity i as $x_i(y, \mathbf{q})$ $\forall i = 1, \ldots, s$, where y is exogenous income, $\mathbf{q}$ is the price vector and s is the number of goods. Assume that all goods can be either taxed or subsidized or both. Define the net tax rate n_i as $n_i = t_i - b_i$ $\forall_i = 1, \ldots, s$, where t_i indicates the non-negative tax rate and b_i indicates the non-negative subsidy rate. To this purpose, let us recall the definition of the indirect utility function:

$$v = v(y, \mathbf{q}) \tag{1}$$

where the price vector $\mathbf{q}$ may be distorted by either taxes or subsidies. More generally, $\mathbf{q}$ represents the vector of prices before the reform. This vector is by assumption obtained by summing the net tax rate n to the producer price vector:[1]

$$q_i = p_i + n_i \quad \forall i = 1, \ldots, s \tag{2}$$

For our purposes, $\mathbf{q}$ can also be assumed as the reference price vector. Accordingly, define a poverty index in a general form:

$$P = -\int_0^{\hat{z}} D(v(y, \mathbf{q})) f(y) dy \tag{3}$$

where the upper integration limit $\hat{z}$ is such that all admissible poverty lines are below it, i.e. $z < \hat{z}$ $\forall z$. In expression (3), D is an individualistic social welfare function depending on individual utilities up to the poverty line.[2] The poverty index P may therefore be thought of as the negative of the social welfare function. With regard to the behaviour of D with respect to y, we make the following assumptions:

A.1 $\qquad D_y > 0;$

A.2 $\qquad D_{yy} < 0.$

Assumption A.1 means that an increase in income increases welfare. When individual welfare increases, social welfare among the poor increases and, by (3),

[1]In a partial equilibrium setting, producer prices $\mathbf{p}$ can be assumed constant and invariant to changes in $\mathbf{n}$. This implies $dq_i = dn_i$.

[2]This amounts to give zero weight to those above the poverty line.

poverty falls. Assumption A.2 means that a transfer from richer to poorer individuals increases welfare (and therefore, reduces poverty). This implies that D satisfies the Pigou–Dalton principle of transfers. This level of generality is maintained throughout the paper in order to develop our results for all poverty indices satisfying the principle of transfers.[3]

Let us now define government tax revenue as:

$$R = \int_0^m \sum_{k=1}^{s} n_k x_k(y, \mathbf{q}) f(y) dy \tag{4}$$

where m is the maximum level of income, and individual demands for s goods depend on income y and price vector $\mathbf{q}$. Now, select a pair of commodities, say n_1 and n_2, and suppose that the government's fiscal rule is to change the net tax rate n_1 with revenue neutrality. This strategy implies changing n_2 in an opposite direction. By (2) it is known that $dq_i = dn_i \; \forall i$. Therefore, the total effect of the revenue neutral change on the poverty index would be:

$$dP = -\int_0^{\hat{z}} \left[\left(\frac{\partial D}{\partial v} \frac{\partial v}{\partial n_1} \right) dn_1 + \left(\frac{\partial D}{\partial v} \frac{\partial v}{\partial n_2} \right) dn_2 \right] f(y) dy \tag{5}$$

For revenue neutrality, from (4) it must be:

$$dn_2 = -\frac{\partial R / \partial n_1}{\partial R / \partial n_2} dn_1 \tag{6}$$

Substituting Roy's identity $\frac{\partial v}{\partial q_i} = -\frac{\partial v}{\partial y} x_i(y, \mathbf{q})$ and the revenue neutrality condition (6) into (5), one can yield:

$$dP = -\int_0^{\hat{z}} \left[\frac{\partial D}{\partial v} \left(-x_1 \frac{\partial v}{\partial y} \right) dn_1 + \frac{\partial D}{\partial v} \left(-x_2 \frac{\partial v}{\partial y} \right) \left(-\frac{\partial R / \partial n_1}{\partial R / \partial n_2} \right) dn_1 \right] f(y) dy \tag{7}$$

In order to simplify (7), let us define the following term:

$$\xi_i = \frac{\int_0^m \phi_i f(y) dy}{X_i} \tag{8}$$

where $\phi_i = \sum_k n_k \frac{\partial x_k}{\partial n_i}$ and X_i is the aggregate consumption of good i. Using this notation, the total tax revenue change due to a change of the i-th net tax rate may be written as:

$$\frac{\partial R}{\partial n_i} = \int_0^m (x_i + \phi_i) f(y) dy \tag{9}$$

Using (8) and (9) and substituting back in (7), after slight manipulation one can yield:

[3]It is worth mentioning that one of the most popular poverty indexes, the headcount ratio, does not respect this principle, as long as individuals involved in transfers do not cross the poverty line.

(10) $$\frac{dP}{dn_1} = -X_1 \int_0^{\hat{z}} \beta(y,\mathbf{q}) \left[\frac{(1+\xi_1)}{(1+\xi_2)} \frac{x_2}{X_2} - \frac{x_1}{X_1} \right] f(y)dy$$

where $\beta = \frac{\partial D}{\partial v}\frac{\partial v}{\partial y}$ is the social weight, which is positive and decreasing with income according to A.1 and A.2.

In order to highlight the main point, let us assume that $\xi_i = 0\ \forall_i$ (either because initial net tax rates are zero or because own and cross-price elasticities are negligible). In this particular case equation (10) could be rewritten as:

(11) $$\frac{dP}{dn_1} = -X_1 \left[\frac{\int_0^{\hat{z}} \beta(y,\mathbf{q}) x_2 f(y)dy}{X_2} - \frac{\int_0^{\hat{z}} \beta(y,\mathbf{q}) x_1 f(y)dy}{X_1} \right]$$

If one is ready to interpret β as a social weight, and normalizing its mean to one, it is easily seen that each term of the square bracket is in fact the distributional characteristic (DC) of the good *up to the poverty line*.[4] In this case, directions of reforms could be easily inferred. A revenue neutral increase of n_1 will be poverty reducing only if the truncated DC of that good is lower than the corresponding truncated DC of good 2. This is quite in line with the general meaning of the use of the distributional characteristics in redistributive analysis.

An alternative interpretation of (11) is possible. Social weights do not depend on goods; rather they depend on income and prices. If households are ranked according to their income, social weights may be thought of as a decreasing function of income (i.e. poor people weight more). Therefore, if it happened that

$$\left[\frac{\int_0^{\hat{z}} \beta(y,\mathbf{q}) x_2 f(y)dy}{X_2} - \frac{\int_0^{\hat{z}} \beta(y,\mathbf{q}) x_1 f(y)dy}{X_1} \geq 0; \quad \forall y < \hat{z} \right]$$

the following must also be true:

(12) $$\left[\frac{\int_0^{\hat{z}} x_2 f(y)dy}{X_2} - \frac{\int_0^{\hat{z}} x_1 f(y)dy}{X_1} \right]$$

As $\int_0^{\hat{z}} xf(y)dy = X_P$ is the aggregate consumption of x by poor people, i.e. *up to the poverty line* z, a revenue-neutral increase in n_1 will be poverty reducing if the share of consumption by the poor (over total consumption of X_1) is lower than the corresponding share for the good X_2. Alternatively, it means that the good to be subsidized is identified by the poor consuming a larger fraction of it.[5]

This result, however, is not independent of the specific poverty line chosen, as total consumption by poor individuals must be calculated up to a given $z < \hat{z}$, which is arbitrarily chosen. Changing the poverty line might require iterating the

[4]See Feldstein (1972).
[5]This interpretation has been suggested by Reutlinger (1985) and Besley and Kanbur (1988).

analysis in order to verify the robustness of poverty prescriptions to alternative specifications.

Equation (11) can instead give information on the *direction* of the change of the poverty index for any $z < \hat{z}$. Let us maintain the assumption $\xi_i = 0 \; \forall_i = 1, \ldots, s$ and define, as in Makdissi and Wodon (2002), consumption dominance (CD) curves as follows: $C_k^1(y) = \frac{x_k}{X_k}$ and $C_k^2(y) = \int_0^y C_k^1(u) f(u) du$ where the superscript indicates the order of dominance. Substituting these definitions back to (10), one can write:

$$\text{(13)} \qquad \frac{dP}{dn_1} = -X_1 \int_0^{\hat{z}} \beta(y, \mathbf{q})[\lambda C_2^1(y) - C_1^1] f(y) dy$$

where $\lambda = \frac{1+\xi_1}{1+\xi_2}$ is the efficiency factor. Makdissi and Wodon (2002) proved necessary and sufficient conditions to have poverty reducing reforms for any order of dominance. In particular, for the second order of dominance and when $\xi_i = 0$ $\forall i$ (i.e. $\lambda = 1$), a necessary and sufficient condition for poverty reducing reforms is the dominance of the cumulated share of consumption of good 2 by the individuals whose income is less than y on the cumulated share of consumption of good 1 by the same individuals.[6] When $\xi_i \neq 0$, at least for some i, the difference between CD curves of order two must be corrected by an efficiency factor, giving rise to efficiency augmented CD curves (EACD curves).[7]

2.2. *Sub-Group Consumption Dominance Curves*

In this section, the decomposable property of many poverty indices is exploited in order to derive dominance conditions for subgroups of population. To this purpose, let us re-define equation (3), assuming G groups of population:

$$\text{(14)} \qquad P = \sum_{g=1}^{G} \omega_g P_g$$

where ω is the share of population belonging to group g. Let us also redefine the poverty index of the g-th group in the usual way:

$$\text{(15)} \qquad P_g = -\int_0^{\hat{z}} D(v^g(y, \mathbf{q})) f(y) dy \quad \forall g = 1, \ldots, G$$

where the superscript g on the indirect utility function indicates that we are now dealing with members of group g. The other terms have the usual meaning. As $\hat{z}$

[6]Proof of this result has been given in Makdissi and Wodon (2002), on the basis of the results shown by Besley and Kanbur (1988) and Yitzhaki and Slemrod (1991).

[7]Incidentally, it is worth noting that the efficiency factor λ in (13) is equivalent to $\gamma \frac{X_1}{X_2}$, i.e. the efficiency factor used by Yitzhaki and Slemrod (1991). If one knew aggregate reactions, it would be possible to infer poverty reducing directions of tax/subsidy reforms by applying a scaling factor to mean consumption and then calculating EACDs. If the information is only partial, e.g. aggregate behavioral reactions are known for, say, good 1, by calculating EACDs, it would be possible to infer the critical level of the factor $(1 + \xi_2)$ preserving a poverty reducing direction of tax/subsidy reform. Efficiency issues are not dealt with in this paper.

is the maximum admissible poverty line for total population, it must also be separately taken for each group.

Consider now a revenue-neutral change of two net tax rates maintaining the obvious hypothesis that revenue-neutrality must be obtained over the total population, and not within each group. Using again Roy's identity for individuals in the g-th group would yield the following total change of the poverty index, which is the subgroup equivalent of equation (10):

$$\frac{dP_g}{dn_1} = -X_1 \int_0^{\hat{z}} \beta(y, \mathbf{q}) \left[\frac{(1+\xi_1)}{(1+\xi_2)} \frac{x_2^g}{X_2} - \frac{x_1^g}{X_1} \right] f(y) dy \tag{16}$$

where variables have the usual meaning, but now consumption of the two goods is specific to the members of the g-th group.

To simplify notation and in order to highlight the main point, let us assume again $\xi_i = 0 \ \forall i$. Define now sub group consumption dominance (SGCD) curves:

$$SGCD_i^1 = \frac{x_i^g}{X_i} \quad \text{and} \quad SGCD_i^2 = \int_0^y SGCD_i^1 f(u) du \quad \forall i = 1, \ldots, s.$$

Substituting back in (16), one can rewrite:

$$\frac{dP_g}{dn_1} = -X_1 \int_0^{\hat{z}} \beta(y, \mathbf{q}) [\lambda SGCD_2^1 - SGCD_1^1] f(y) dy \tag{17}$$

Now, one can formally state the following proposition:

> *Proposition 1*. With a revenue-neutral change in n_i and n_j a necessary and sufficient condition for $dP_g/dn_i \leq 0$ for all P_g satisfying the principle of transfers and for all $z < \hat{z}$ is: $\lambda SGCD_j^d(y) \geq SGCD_j^d \quad \forall y \in [0, \hat{z}]$ and $d \in \{1, 2\}$.
>
> *Proof*. See Appendix.

Obviously, a revenue-neutral change may well be poverty reducing for all subgroups. However, this is not necessary to have a poverty reducing *total* effect, as it is perfectly conceivable to have $dP/dn_i \leq 0$ with some $dP_g/dn_i \geq 0$. This makes clear how important it is to break the poverty dominance criterion by subgroups using SGCD curves. The following sections empirically illustrate the case.

3. Data

Data used in this paper come from the Yearly Personal File (YPF) and the Yearly Household File (YHF) of Belarus for 1997.[8] Surveys contain information on socio-demographic characteristics of individuals and households and on the main income sources (including in-kind incomes) and state transfers. Detailed information on households' expenditures is also available. Table 1 reports basic information on the main variables.

According to official methodological notes, households are surveyed 17 times by an interviewer and data collection consists of two stages. In the first, the family

[8]Both surveys are from the Ministry of Statistics and Analysis of the Republic of Belarus, whose kindness is gratefully acknowledged as well as that of the Department of Applied Economics, University of Cambridge for allowing me to elaborate data. Obviously, they do not bear any responsibility for the analysis and the interpretation of the data.

TABLE 1

SUMMARY INFORMATION*

Number of households in YHF	4,916			
Number of households in total population	3,414,177			
Modal worker position	Blue collar			

	Average per Household	Standard Deviation	Min	Max
Household size	2.73	1.35	1.00	13.00
Total number of children	0.75	0.95	0.00	8.00
Number of earners	1.25	0.95	0.00	3.00
Age of the head of household	50.91	15.64	18	94
Net wages	1,308	1,359	0	11,666
Total monetary income	2,140	1,622	74	57,034
In kind income from plots	527	450	0	3,087
Food expenditures	1,194	711	54	5,949
Expenditures on rents and utilities	102	98	0	1,712
Expenditures on health care	36	57	0	1,130
Expenditures on public transports	55	70	0	1,498
Total expenditures	2,428	1,804	168	43,699

*Monetary values are in thousands of rubles and monthly averages for 1997. Weighted figures.
Source: Author's elaboration from YHF (1997).

keeps track of the expenditures it incurs every day during 14 days of each quarter, including food produced on the individual land plot or received as a gift. The second type of data collection is an interview during which data on non-food expenditures and incomes are collected. The statistical data are monthly averages for a year and are adjusted to consumer price index.

4. AN EXAMPLE: DIRECTIONS OF REFORMS FOR CONSUMPTION SUBSIDIES IN BELARUS

4.1. *Total Population*

To illustrate the approach and the informational advantage of using SGCD curves, the population has been divided into six groups: workers with children (34.7 percent of total population); adults without children (32.2 percent); singles (21.1 percent); single parents (4.8 percent); pensioners with children (3.3 percent); and other households with children (3.9 percent). Three subsidized goods have been selected: rents and utilities (RU); health care (HC); and public transport (PT). Expenditures on each item have been equivalized by the OECD equivalence scale to take into account different household sizes.

Before proceeding any further, it is worth noting the distribution of expenditures on subsidized goods across deciles of total income, as reported in Figure 1.

The highest decile consumes proportionally more of all subsidized goods. It would mean that the efficiency score of the corresponding subsidies might be quite low, with a greater degree of leakage to richer households. From the point of view of the policy-maker, therefore, subsiding consumption is not a very effective policy, as consumption of many expenditure items is correlated with income. Yet, from the point of view of the social impact, eliminating subsidies may be a bad

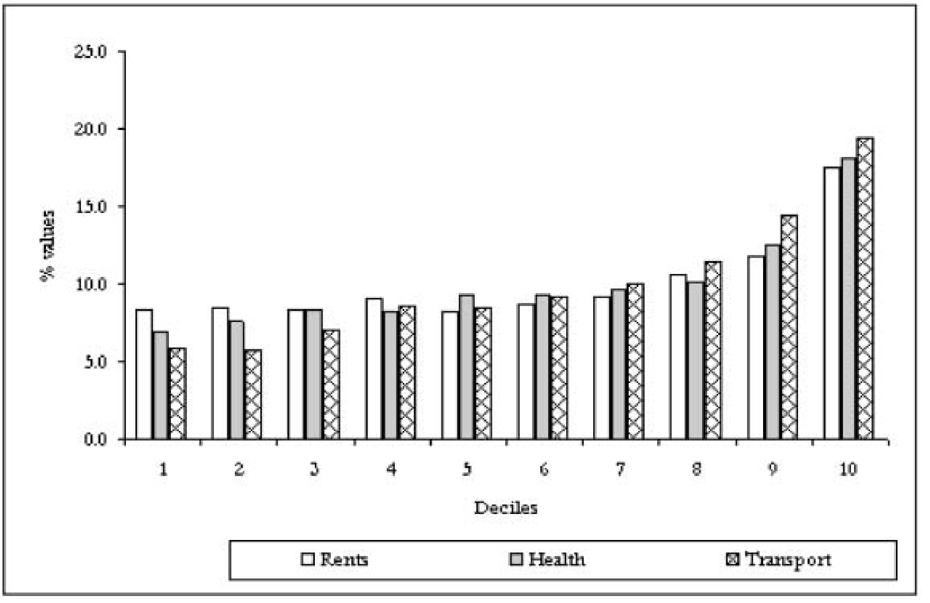

Figure 1. Distribution of Expenditures, by Deciles

Source: Author's elaborations from YHF (1997).

strategy; rather, there might be margins to redirect them in order to increase their anti-poverty effect.

In Figure 1, the most disproportionate distribution is from public transport, with slightly more than 11 percent on the first two deciles, while the most concentrated *on* low-income households is rents-utilities with 17 percent on the first two deciles. With this information, one can now perform comparisons of the three hypothetical revenue-neutral reforms:

(1) Reducing subsidies on HC and increasing subsidies on RU (RU-HC).

(2) Reducing subsidies on PT and increasing subsidies on HC (HC-PT).

(3) Reducing subsidies on PT and increasing subsidies on RU (RU-PT).

First, it is worth considering how the three hypothetical reforms perform with respect to total population. Figure 2 reports these results. Equivalent income is the variable chosen for y. Therefore, the x-axis reports equivalent income normalized to mean equivalent income. The y-axis plots the difference between SGCD curves over the total income distribution. For illustrative purposes, we set $\lambda = 1$.

As can be easily seen, there are no welfare improving reforms. In the case of reform 1 (RU-HC), this is clearly visible, as the two curves intersect many times for levels of equivalent income above 95 percent of mean equivalent income. In the case of reform 2 (HC-PT), violation of dominance occurs at very low levels of equivalent income, while in the case of reform 3 (RU-PT) that same violation occurs at the very top of the income distribution. Yet, at least for reforms 1 and 3 there may be a wide range of poverty lines $z < \hat{z}$ for which those reforms are poverty reducing. In particular, reform 1 is poverty reducing for all z not greater than 95 percent of mean equivalent income, while reform 3 is poverty reducing for

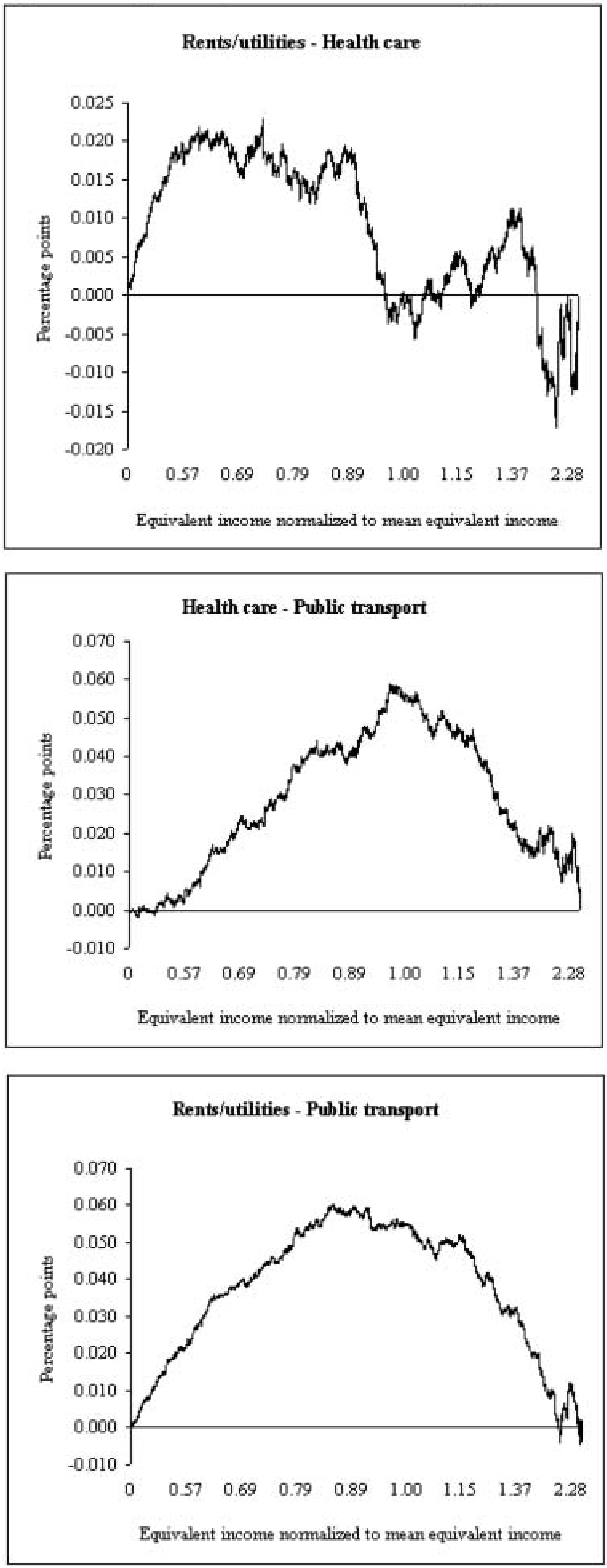

Figure 2. Povety Reducing Directions of Reform; Total Population

all reasonable z, as violation of dominance occurs at the very top of the income distribution. Reform 2, instead, may be poverty increasing, as repeated violations of dominance occur for z below 48 percent of mean equivalent income.

The following subsection will extend these conclusions to the analysis by subgroups.

4.2. *Population Subgroups*

Results from the previous section can now be decomposed by population groups. This is done in Figure 3, where each graph reports the outcome of the three reforms for each subgroup. The range of the x-axis has been limited to a level $\hat{z}$ corresponding to mean equivalent income.

The aim of the graphs is to verify how population groups may lose and gain from the revenue-neutral change of consumption subsidies.

Let us start again from reform 1 (RU-HC). In this case, we have already observed that considering total population the reform cannot be poverty reducing for poverty lines above 95 percent of equivalent income. Decomposition by subgroups shows that this reform is instead definitely poverty reducing for "workers with children" and "single parents," while for the other groups there are at least some $z \in [0, \hat{z}]$ for which the reform cannot be considered poverty reducing.

With regard to reform 2 (HC-PT), we observed that in the case of the total population, violation of dominance occurred at very low levels of z. Decomposition by subgroups reveals that a definite answer cannot be achieved for any of the subgroups included in the analysis, as violation of dominance occurs very frequently in the income range analyzed.

The case of reform 3 (RU-PT) is more interesting. This reform was poverty reducing for all admissible z in the case of total population. When decomposing by subgroups, one can realize that the same prescription holds only for "workers with children" and "single parents" (about 40 percent of total population); while for the other groups there are violations of dominance at various points for z below 40 percent of mean equivalent income.

Table 2 summarizes the outcome of dominance for each group and each reform. By rows, the table gives the outcome of each reform across groups, considering first total population (up to mean equivalent income) and then subgroups. It can easily be seen that the outcome of reform 2 strictly depends on the poverty line chosen, while the other two reforms are definitely poverty reducing only for "workers with children" and "single parents." By columns, the table gives the situation of each group across reforms. It is therefore easily seen, for example, that for "married couples and other adults," "singles," "pensioners with children" and "other households with children," there is no reform that is definitely poverty reducing.

The possibility of contrasting results across groups makes the analysis of dominance by subgroups through SGCD curves particularly important.

5. Conclusions

This paper has extended the correspondence between consumption dominance curves and poverty reducing directions of reforms for sub-groups of the

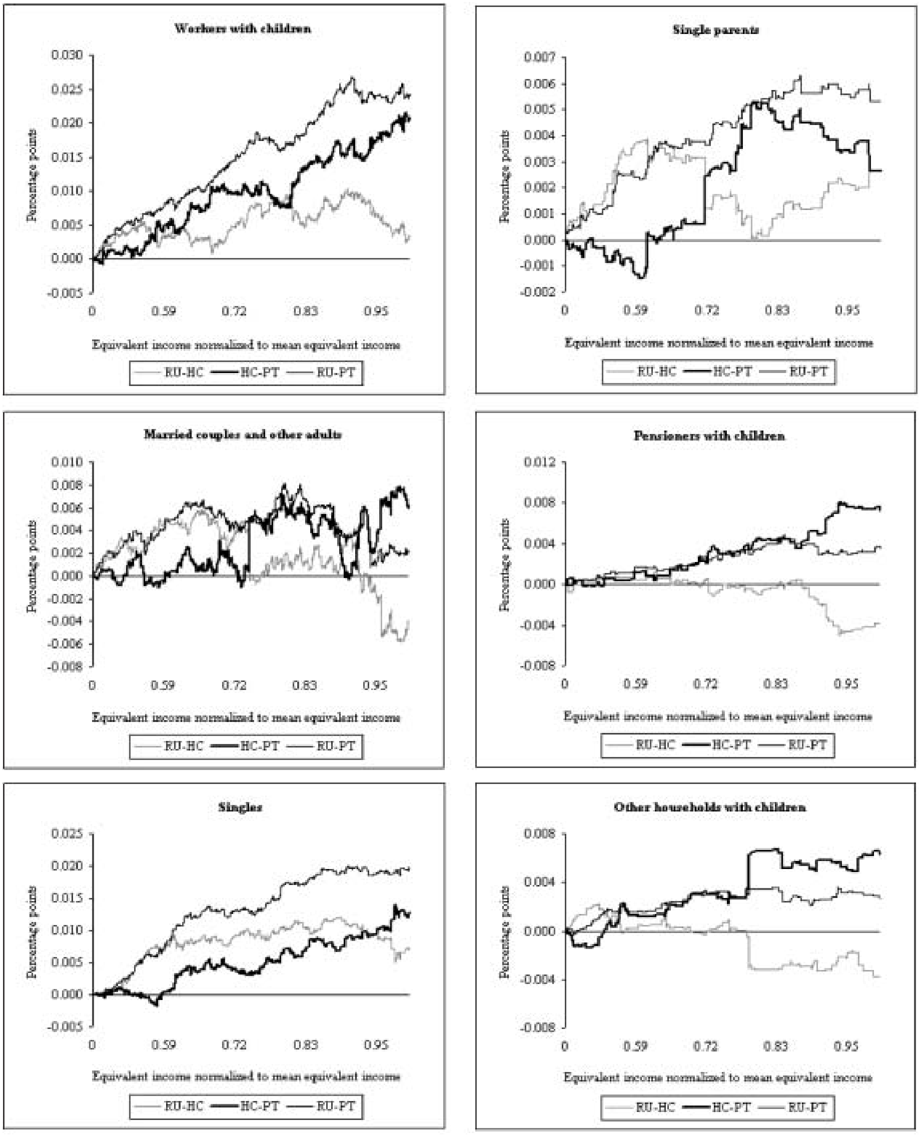

Figure 3. Poverty Reducing Directions of Reform; Population subgroups

population. We think that this decomposition has useful informational advantages, as it allows the policy-maker to get detailed information on poverty reducing strategies for groups of population, without being constrained to a specific poverty line. The empirical decomposition of poverty reducing reforms by sub-groups of the population in Belarus has revealed that different groups may suffer a poverty increase for some conceivable poverty lines even in the presence of clear-cut poverty reducing directions of reforms obtained when considering the total population.

TABLE 2

DOMINANCE, UP TO THE MEAN EQUIVALENT INCOME

		Groups					
Reforms	All Population	Workers with Children	Married Couples and Other Adults	Singles	Single Parents	Pensioners with Children	Other Households with Children
RU-HC	No	Yes	No	No	Yes	No	No
HC-PT	No	No	No	No	No	No	No
RU-PT	Yes	Yes	No	No	Yes	No	No

Source: Author's elaboration from YHF (1997).

APPENDIX

For $d = 1$, the proof follows very simply from considering that: (a) $X_1 \geq 0$; (b) $\beta(y, \mathbf{q}) \geq 0\ \forall y \in [0, \hat{z}]$. For $d = 2$, the proof goes along the lines used by Makdissi and Wodon (2002). To prove sufficiency, first integrate by parts:

$$\int_0^{\hat{z}} \beta(y,\mathbf{q})[\lambda SGCD_2^1(y)]f(y)dy = \beta(y,\mathbf{q})SGCD_2^2(y)\Big|_0^{\hat{z}} - \int_0^{\hat{z}} \beta_y SGCD_2^2(y)f(y)dy$$

The first term on the r.h.s. is zero as $SGCD_2^2(0) = 0$ and $\beta(\hat{z}, \mathbf{q}) = 0$. Therefore, the following must hold:

$$\text{(a.1)} \qquad \int_0^{\hat{z}} \beta(y,\mathbf{q})[\lambda SGCD_2^1(y)]f(y)dy = -\int_0^{\hat{z}} \beta_y SGCD_2^2(y)f(y)dy$$

Analogously, one can prove that:

$$\text{(a.2)} \qquad -\int_0^{\hat{z}} \beta(y,\mathbf{q})[\lambda SGCD_1^1(y)]f(y)dy = \int_0^{\hat{z}} \beta_y SGCD_1^2(y)f(y)dy$$

Therefore:

$$\text{(a.3)} \qquad \frac{dP_g}{dn_1} = -X_1\Big[-\int_0^{\hat{z}} \beta_y(\lambda SGCD_2^2(y) - SGCD_1^2(y))f(y)dy\Big]$$

If the round brackets are positive, the square brackets are positive (as $\beta_y \leq 0$ from assumption A.2 in the text). Then $dP_g \leq 0$. To prove the necessity for $d = 2$, let us rewrite the integral in (a.3) as follows:

$$\int_0^{\hat{z}} \beta_y(\lambda SGCD_2^2(y) - SGCD_1^2(y))f(y) = \int_0^{\hat{z}} \beta_y(\lambda SGCD_2^2(y) - SGCD_1^2(y))f(y)dy$$

$$\text{(a.4)} \qquad + \int_{y_1}^{\hat{z}} \beta_y(\lambda SGCD_2^2(y) - SGCD_1^2(y))f(y)dy$$

Now, suppose there exists $y < \hat{z}$, with $0 < y < y_1$, such that $\lambda SGCD_2^2(y) < SGCD_1^2(y)$. This prevents the reform to be poverty reducing for $\forall y \in \hat{z}$. Therefore, (a.3) must hold for any arbitrary small interval $[y, y + \varepsilon]$, with ε arbitrarily close to zero. The necessity for $d = 1$ can be proved analogously from equation (17) in the text.

References

Besley, Thomas and Ravi Kanbur, "Food Subsidies and Poverty Alleviation," *Economic Journal*, 98, 701–19, 1988.

Duclos, Jean Yves, Paul Makdissi, and Quentin Wodon, *Socially Efficient Tax Reforms*, Université de Sherbrooke, mimeo, 2002.

Feldstein, Martin S., "Distributional Equity and the Optimal Structure of Public Prices," *American Economic Review*, 62, 32–6, 1972.

Foster James, Joel Greer, and Erik Thorbecke, "A Class of Decomposable Poverty Measures," *Econometrica*, 52, 761–76, 1984.

Kanbur, Ravi, "Measurement and Alleviation of Poverty," IMF Staff Papers, 60–85, 1987.

Makdissi, Paul and Quentin Wodon, "Consumption Dominance Curves: Testing for the Impact of Indirect Tax Reforms on Poverty," *Economics Letters*, 75, 227–35, 2002.

Reutlinger, Shlomo, "Food Security and Poverty in LDCs," *Finance & Development*, 22, 7–11, 1985.

Yitzhaki, Shlomo and Joel Slemrod, "Welfare Dominance: An Application to Commodity Taxation," *American Economic Review*, 81, 480–96, 1991.

Yitzhaki, Shlomo and W. Thirsk, "Welfare Dominance and the Design of Excise Taxation in the Cote d'Ivoire," *Journal of Development Economics*, 33, 1–18, 1990.