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# AGGREGATION OF UTILITY AND EQUIVALENCE SCALES: A SOLUTION TO THE PANGLOSS CRITIQUE

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Definitions of equivalence scales are usually based on a household utility function. This may be founded on an assumption of the household maximizing a welfare function of individual utilities. Basing inter-household comparisons of welfare on this approach is fallacious because households put different weight on the utility of the various household members, a weighting that does not necessarily correspond to an ethically sound aggregation of utility. This is called the Pangloss critique. To solve the problem, I suggest keeping the model of household behavior, but to introduce a new function to aggregate the household members' utilities. Equivalence scales based on this approach are shown to have desirable properties.

## 1. INTRODUCTION

Different households require different incomes to obtain the same level of material well-being because individuals have different needs and because there are returns to scale in large households. For comparisons of welfare for both descriptive studies and practical policy making, we need to take this into account. The standard approach is to adjust incomes by an equivalence scale. For almost any measure of inequality or poverty, the results depend crucially upon the scaling of income. It is therefore important to construct equivalence scales that correspond closely to households' real needs.

Although the most common scales are based on perceived needs and budgets, some scholars estimate equivalence scales from observed household behavior. They postulate that the household maximizes a utility function and then use observed market behavior to recover the expenditure function. Since equivalence scales are defined as the ratio of the expenditure functions, it may seem straightforward to estimate the equivalence scales once the expenditure functions are known. However, there are well-known problems.

First, as only ordinal utility functions are recoverable from marked behavior, Blundell and Lewbel (1991) show that almost any equivalence scale is compatible with any observed market behavior. Furthermore, the definition of equivalence scales takes the structure of households as given. This is a strong assumption since both marriage and fertility are largely both controllable and voluntary, so the

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household should be modeled as not only choosing between consumption bundles but combinations of consumption bundles and household compositions (Pollak and Wales, 1979). Banks *et al*. (1994) adopt an explicit intertemporal framework and use this to solve a part of the problem. I limit my attention to a simple static model. Finally, most parents' major expenditure on their children is time (see Bittman and Goodin (2000) for an approach taking this explicitly into account). Consequently, the labor supply decision, and hence income, should not be seen as exogenous. The short run view of the present paper makes this problem less severe.

In what follows, the term "utility" is used to denote individual utility interpreted as material well-being. Utility is assumed to be interpersonally comparable ("ordinal level comparability" in the words of Sen (1977); see Blackorby and Donaldson (1991) for an extended discussion). This is a controversial assumption that I do not discuss further. See, among others, Hammond (1991) and the references therein for details. The aggregate utility of groups of people is denoted by the term "welfare" since it is unclear how a group can feel utility. An exception is the "household utility function," a term that appears so frequently in the literature that I prefer to keep it. The term is given a more precise meaning below.

Because inter-group comparability of welfare does not follow from interpersonal comparability of utility, we cannot generally compare levels of the household utility function. This is done without any discussion in much of the literature. An exception is the work of Blackorby and Donaldson (1993). Throughout most of their paper, they assume that households maximize a maximin welfare function of individual utilities. However, empirical studies indicate that the income share of different household members influences aggregate consumption (Lundberg *et al*., 1997; Duflo, 2003). Furthermore, several studies reveal that boys frequently get more resources than girls (Behrman, 1992). Hence, it is unrealistic that the household welfare function is maximin.

Equivalence scales are readily defined by more general welfare functions. This could reflect an unequal distribution of power by giving household members different weights. Especially children, who cannot choose where to live, may be given small weights. A utility function defined this way is useful for analyses of household demand. However, if we use the function for comparisons of welfare, we implicitly agree that the household's own weighting of individual utilities is the correct way to aggregate. As the household may have a highly unequal weighting of individuals, this is ethically unacceptable. Hence there is a serious flaw with the conventional approach. I label this the Pangloss critique of equivalence scales as we accept that "whatever is, is best."

Ignoring this problem may lead to absurd conclusions. In the extreme where only a single person matters for the household utility function, the cost of additional persons is zero so the equivalence scale would be unity. Generally, equivalence scales would be decreasing in the degree of inequality. Consequently, there is a downward bias in equivalence scales if intra-household distribution is unequal.

Bojer (1998) discusses some of the same problems, but the scope of her analysis is more limited. Haddad and Kanbur (1990) also discuss related problems when they show that ignoring intra-household inequality gives a downward bias in measures of inequality. Moreover, Apps and Rees (1988) give a thorough discussion of the intra-household distribution of resources in a similar model of the household decision making process, but for a different purpose.

If there is a social planner with an ethically sound way of aggregating the utilities of the household members, we can construct better equivalence scales. To calculate the welfare level a household obtains from a given income, we first study how the household distributes resources internally by their own welfare function. Then we use the social planner's welfare function to aggregate the individual utilities to obtain a measure of household welfare. These equivalence scales have more reasonable properties. Especially, households with an unequal distribution of resources have higher equivalence scales than more egalitarian households.

These equivalence scales also allow differentiating between the welfare effects of money transfers and transfers in kind. If, for instance, children get a low weight in a household's welfare function, money transfers would not increase the children's utility much. This might have a smaller impact on household welfare as judged by the social planner than a transfer in goods heavily consumed by children, thus creating a rationale for subsidies.

The present paper concentrates on equivalence scales estimation based on market behavior, although I do not go all the way to develop an estimator. A competing approach is the use of subjective data, often known as the Leyden school. See, for example, van Praag and van der Sar (1988) for an application of this technique and Charlier (2002) for an extension to an intertemporal setting. The critique I present in this paper may also apply to this approach albeit in a modified form. If the person asked to state the welfare of the household also has a dominating position in the household's welfare function, he or she may induce the same bias in welfare judgments that we would get from market behavior, so such scales may also be susceptible to the Pangloss critique, although it may be improved upon by interviewing all household members.

## 2. From Household Utility to Household Welfare

To formulate the Pangloss problem in a precise manner, I shall begin by outlining a model of household behavior closely related to Blackorby and Donaldson's (1993) framework. Households are assumed to behave as if they maximize a welfare function depending on the individual members' utilities. Although the aggregate behavior of the household may be described by a household utility function, the present framework clarifies the relationship between individual utility and household welfare.

We consider an economy populated with agents belonging to one out of a finite number of types or groups. These types may be simply adults and children or a more detailed decomposition. A household consists of individuals each belonging to one of the groups. The demographic composition of a household is described by a vector $z$, where the $g$-th element of $z$ denotes the number of household members belonging to type $g$. Let $\psi$ denote a vector of other household characteristics. These may be characteristics of the household members, such as cultural attributes and education, and other characteristics such as the area of residence. We shall assume that $\psi$ is observable. For our purposes, a household is now completely described by a vector $\phi := (z', \psi')' \in \Phi$ where $\Phi$ is the space of household types.

In the traditional approach to the definition and estimation of equivalence scales from household expenditure data, the household is assumed to behave as if it maximizes a household utility function:

$$U : \mathcal{Q} \times \Phi \to \mathbb{R}. \tag{1}$$

Here, $\mathcal{Q}$ is the consumption set, which for simplicity is assumed to be identical for all households and individuals. When the household faces a price vector $p \in \mathcal{P}$, duality theory gives us the household expenditure function $C$, which is the lowest income necessary for a household $\phi$ facing prices $p$ to reach a welfare level $\mathcal{W}$. An equivalence scale gives the ratio between the income a household $\phi$ needs to reach a welfare level $\mathcal{W}$ relative to that of a reference household $\phi_0$. Most of the time, $\phi_0$ is assumed to be a household consisting of a single agent, but it may be any household. Formally, an equivalence scale is a function $L$ defined as:

$$L(p, \mathcal{W}, \phi, \phi_0) := \frac{C(p, \mathcal{W}, \phi)}{C(p, \mathcal{W}, \phi_0)}. \tag{2}$$

If the reference household $\phi_0$ derives welfare $\mathcal{W}$ from an income $y_0$, then $y_0 L(p, \mathcal{W}, \phi, \phi_0)$ is said to be the equivalent income of household $\phi$ given prices $p$.

The existence of the household utility function (1) is not trivial. First, as mentioned, it is only meaningful to say that individuals experience utility. This implies that it is dubious to compare the value of the function $U$ for different households consisting of more than one person. Furthermore, the aggregate demand of a group of individuals each maximizing their own utility function is generally not rationalizable by a common utility function. Chipman and Moore (1979) show that unless all individuals have preferences satisfying Gorman's (1953) polar form, the distribution of income among the individuals has to be rationalizable as the maximum of a Bergson–Samuelson welfare function (see also the seminal paper by Samuelson, 1956).

To study the household utility function, we need to introduce individual utility functions. Let an agent belonging to group $g$ derive utility from consumption given by a function $u_g : \mathcal{Q} \to \mathbb{R}$. These utility functions are assumed to be interpersonally comparable, continuous, and strictly concave. To construct a model that is useful for discussing equivalence scales, it is crucial to model returns to scale within the household in a reasonable way. To achieve this, I employ a model formulation from production theory. For a given household consumption, the set of possible intra-household allocations of resources is given by the allocation-possibility correspondence $\Omega_\phi : \mathcal{Q} \to \mathcal{Q}^{n_\phi}$, where $n_\phi$ denotes the number of household members in a household with characteristics $\phi$.

To clarify the model, consider a couple of particular cases. The case of no returns to scale implies that each unit of household consumption can be consumed by one and only one household member. Consequently, when we assume free disposal, the sum of individual consumption vectors has to remain below total household consumption, which means that $\Omega_\phi(q) = \{(q_1, \ldots, q_{n_\phi}) : \Sigma_i q_i \leq q\}$. In a world with only pure public goods, every household member has access to the aggregate consumption vector. In this case, we have $\Omega_\phi(q) = \{(q_1, \ldots, q_{n_\phi}) : q_i \leq q \text{ for all } i\}$. Obviously, some goods may be private, some goods may be purely public and some goods may be semi-public. We shall assume that there is no true production of goods within the households in the sense that if $(q_1, \ldots, q_{n_\phi}) \in \Omega_\phi(q)$, then $q_i \leq q$ for all $i$. This implies that returns to scale arises solely from the possibility of sharing certain goods. To assure the existence of an optimal intra-household allocation, we shall assume that $\Omega_\phi(q)$ is compact for all $q$.

The welfare functions are any class of increasing functions $W_\phi: \mathbb{R}^{n_\phi} \to \mathbb{R}$ for each $\phi \in \Phi$. $W_\phi$ takes as arguments the individual utilities of each household member and aggregates them to a common measure of household welfare. For a given household consumption $q \in \mathcal{Q}$, the individual household members have access to a consumption vector:

$$(3) \qquad (q_1,\ldots,q_{n_\phi}) = \arg\max_{q_i \in \mathcal{Q}} \{W_\phi(u_{\gamma(1)}(q_1),\ldots,u_{\gamma(n_\phi)}(q_{n_\phi})) : (q_1,\ldots,q_{n_\phi}) \in \Omega_\phi(q)\},$$

where $\gamma$ is the function that assigns to each household member its agent-type. We may now use this to give a rationale for the household utility function (1) by defining:

$$(4) \qquad U(q,\phi) := \max_{q_i \in \mathcal{Q}} \{W_\phi(u_{\gamma(1)}(q_1),\ldots,u_{\gamma(n_\phi)}(q_{n_\phi})) : (q_1,\ldots,q_{n_\phi}) \in \Omega_\phi(q)\}.$$

To assure continuity of $U$, we assume that $\Omega_\phi$ has a closed graph for all $\phi$. Now, we may also derive individual indirect utility functions given by $v_i(p, y, \phi) := u_{\gamma(i)}(q_i)$, where $q$ is determined as the maximum of $U$ given the budget constraint $p'q \leq y$ and $q_i$ is determined by (3) given $q$. $v_i$ gives the utility of individual $i$ in a household $\phi$ when the household has income $y$ and faces prices $p$.

The definitions of the cost function and the equivalence scales given above still make sense for this new interpretation of $U$. However, for (2) to be a meaningful expression, welfare levels between households have to be comparable in the sense that we are able to say that two households with different characteristics are at the same level of welfare. To compare the welfare of two households, it is necessary to normalize the welfare function in some way. One normalization of $W_\phi$ that may make these inter-household comparisons of welfare more meaningful is the concept of agreeing, which is based on Aczél and Roberts (1989). A household welfare function $W_\phi$ is said to satisfy the agreeing property (AG) if for all $u$ we have $W_\phi(u, \ldots, u) = u$. It is then seen that when AG holds, for all $\phi \in \Phi$ such that $n_\phi = 1$, $W_\phi$ is the identity function, so the welfare of a one-person household is simply the utility of the person. A welfare function satisfying the AG property corresponds to Blackorby and Donaldson's (1993, p. 338) concept of "equally-distributed-equivalent utility functions." Since we, for any intra-household distribution of utility $u_1, \ldots, u_{n_\phi}$, may find a utility level $\tilde{u}$ such that $W_\phi(u_1, \ldots, u_{n_\phi}) = W_\phi(\tilde{u}, \ldots, \tilde{u})$, inter-household comparisons of the level of $W_\phi$ is sensible when individual utility is interpersonally comparable.

Another concept that will prove useful is the notion of anonymity (AN) introduced by May (1952). It states that the name (and hence agent type) of an individual is irrelevant for her weight in the household welfare function. AN is a strong property that is not necessarily satisfied by all household welfare functions. Formally, a household welfare function $W_\phi$ satisfies the anonymity property if for every vector of utility levels $\mathbf{u} \in \mathbb{R}^{n_\phi}$ and every permutation of $\mathbf{u}$, $\mathbf{u}^*$, we have $W_\phi(\mathbf{u}) = W_\phi(\mathbf{u}^*)$.

A related property may hold for the allocation-possibility correspondences. A household allocation-possibility correspondence $\Omega_\phi$ will be said to satisfy permutational symmetry (PS) if for every $q$ and every $(q_1, \ldots, q_{n_\phi}) \in \Omega_\phi(q)$ we also have $(q_1^*, \ldots, q_{n_\phi}^*) \in \Omega_\phi(q)$, where $(q_1^*, \ldots, q_{n_\phi}^*)$ is any permutation of $(q_1, \ldots, q_{n_\phi})$. PS implies that the feasibility of an intra-household allocation does not depend on the recipient of each consumption bundle.

Before embarking on a discussion of the Pangloss problem, I prove a result that will become handy in what follows and which shows the usefulness of the concepts introduced above.

> *Proposition 1*: If a household $\phi$ maximizes a quasi-concave AN welfare function $W_\phi$, all household members have an identical concave utility function $u$ and $\Omega_\phi$ is PS and convex for all $q$, then the set of solutions to the welfare maximization problem contains the case in which all household members get the same utility level for any household consumption vector $q \in Q$.

*Proof*: Since $W_\phi$ is increasing and quasi-concave and $u$ is concave, $(q_1, \ldots, q_{n_\phi}) \rightarrow W_\phi[u(q_1), \ldots, u(q_{n_\phi})]$ is quasi-concave. Furthermore, if $(q_1, q_2, q_3, \ldots, q_{n_\phi}) \in \Omega_\phi(q)$, then it follows from PS that $(q_2, q_1, q_3, \ldots, q_{n_\phi}) \in \Omega_\phi(q)$. Hence, if we define $\tilde{q} := \frac{1}{2}(q_1 + q_2)$, convexity of $\Omega_\phi$ implies that $(\tilde{q}, \tilde{q}, q_3, \ldots, q_{n_\phi}) \in \Omega_\phi(q)$. Assume now that $(q_1, q_2, \ldots, q_{n_\phi})$ is a solution to the maximization problem for a given household consumption. Then $(\tilde{q}, \tilde{q}, q_3, \ldots, q_{n_\phi})$ is also feasible. Furthermore, since $(q_1, \ldots, q_{n_\phi}) \rightarrow W_\phi[u\,(q_1), \ldots, u(q_{n_\phi})]$ is quasi-concave and AN,

$$\begin{aligned} W_\phi[u(q_1),u(q_2),u(q_3),\ldots,u(q_{n_\phi})] &= W_\phi[u(q_2),u(q_1),u(q_3),\ldots,u(q_{n_\phi})] \\ &\leq W_\phi[u(\tilde{q}),u(\tilde{q}),\ldots,u(q_{n_\phi})]. \end{aligned}$$

Consequently, if we have $q_1 \neq q_2$ in one optimum, then there is another optimum where $q_1 = q_2$. By a renaming of individuals, an analogue line of reasoning implies that there is an optimum where $q_1 = q_2 = \ldots = q_{n_\phi}$.■

*Remark*: If either $W_\phi$ is strictly quasi-concave or $u$ is strictly concave and $W_\phi$ is strictly increasing in at least one of its arguments, then there is a unique solution to the welfare maximization problem where everybody gets the same consumption vector.

## 3. The Pangloss Critique

Although the household utility function as defined in (4) may be suitable for a positive analysis of household behavior, it is not suitable for a normative analysis. Particularly, the function $W_\phi$ is not generally appropriate to aggregate the utility levels of the household members, and ethically questionable conclusions arise when using this function uncritically.

To see the fundamental problem, assume that $W_\phi$ satisfy AG for all $\phi \in \Phi$ and that the reference household $\phi_0$ consists of a single agent, say of type one. If this agent receives an income $y_0$, she reaches a level of utility $v_1(p, y_0, \phi_0) =: w_0$. A household $\phi$'s equivalent income is $y_\phi^* = y_0 L(p, w_0, \phi, \phi_0)$, giving agent $i$ a level of utility $v_i(p, y_\phi^*, \phi) =: w_i$. From the definition of equivalent income, it follows that:

(5) $$W_\phi(w_1,\ldots,w_{n_\phi}) = w_0.$$

Furthermore, from the AG property, we have:

(6) $$W_\phi(w_1,\ldots,w_0) = w_0.$$

Consequently, the intra-household allocation of utility ($w_1, \ldots, w_{n_\phi}$) gives the same total household welfare as an allocation giving every household member a utility level $w_0$. However, this statement is certainly non-trivial. Its validity depends crucially on $W_\phi$ being in some sense the correct way of aggregating the individual utility levels to an aggregate measure of welfare. Even for AG welfare functions, there is a myriad of possible welfare functions giving different results when the intra-household distribution of utility is unequal. To claim that the function the household itself uses to evaluate different intra-household distributions is the best, is certainly "Panglossian," referring to Voltaire's (1990) character Dr. Pangloss, whose doctrine was "tout est au mieux dans ce meilleur des mondes possibles" (everything is best in the best of all possible worlds). I refer to this as the Pangloss-problem, a label first suggested by Muellbauer (quoted in Pollak, 1981) within a resembling framework. To rephrase the problem, if we consider $W_\phi$ to be the best way of aggregating utility levels into an aggregate measure of welfare, we claim that whatever is, is best. However, individual household members or an independent observer could very well object to this aggregation. As pointed out by a referee, to maximize political support, the head of household may be judged as the most important person in the household. This would imply a difference between politically and ethically correct equivalence scales. However, as some of the other household members may have the right to vote, the difference is not likely to be large. I disregard this possibility for the rest of the paper.

If power is unequally distributed within the household, it is likely that the utility of some of the household members influences the objective function $W_\phi$ more than that of other household members. The utmost case is probably a household welfare function equaling the utility level of one household member. In this case, the utility allocations ($u$, 0, . . . , 0) and ($u$, $u$, . . . , $u$) would be judged equal, which is rather unintuitive. In this case, it is probable that a social planner would aggregate the utilities of the household members by another welfare function than $W_\phi$.

To see an implication of the Pangloss problem, consider the case where all agents have the same utility function and the regularity conditions of Proposition 1 hold. If the one-person reference household $\phi_0$ receives $y_0$, then the equivalent income of a household $\phi$ with a AN and AG welfare function is $y_\phi^*$ defined implicitly as

(7) $$v_1(p, y_0, \phi_0) = v_i(p, y_\phi^*, \phi)$$

since Proposition 1 implies that every household member gets the same level of utility, and this is equal to that of the reference household by definition of equivalent income. Hence the equivalence scale is $y_\phi^*/y_0$. This equal intra-household distribution will generally not take place if $W_\phi$ is not AN, since this normally implies that some individuals get a higher share of total income than others. Nevertheless, since $W_\phi$ is AG, the equal utility-allocation will still give a welfare level equal to that of the reference household. Since this allocation is not chosen, it implies that the household manages to reach a higher level of welfare with a different allocation of resources. Consequently, the equivalent income to $y_0$ is less than $y^*_\phi$, and a household that does not pursue equality in the distribution of income will have an equivalence scale below $y^*_\phi/y_0$. Furthermore if for some $\phi \in \Phi$, we have for all $\mathbf{u} \in \mathbb{R}^{n_\phi}$ that $W_\phi(\mathbf{u}) = u_i$ for some $i$, that is, the utility of agent $i$ determines the welfare of the household, then maximizing household welfare equals maximizing agent $I$'s utility. If agent $i$ and the one-person reference household has the same utility of money for some price-regime $p \in \mathcal{P}$, then the equivalence scale equals unity.

This shows that when using $W_\phi$ to evaluate household welfare, there is a tendency that it is more expensive to run a household where resources are distributed evenly than one with unequal intra-household distribution. This is because a welfare function putting much weight on a small group of individuals will result in a high household welfare if that particular group receives most of the income. If all agents count equally, on the other hand, the resources have to be spread among a larger number of individuals.

## 4. A Solution

One way to solve the problem above is to retain the model of household behavior presented above, but aggregate individual utilities by a different welfare function. Since only individuals experience utility, a social planner should take the individual utility levels of all the citizens as a starting point for social welfare calculations. For an evaluation of a policy change, this approach is certainly desirable. Nevertheless, such a welfare function gets complicated unless severe simplifying assumptions are imposed. Furthermore, construction of an aggregate measure of a household's welfare normally requires less data than calculation of all the individual utilities. Particularly, most statistical surveys use the household as the unit of analysis. Hence it is occasionally convenient to consider a social welfare function depending on the welfare level of the individual households, or at least to determine which of two households is "best off." Although it may be doomed nonsensical to state that two households are "equally well off" in this case, it appears frequently in everyday speech. This indicates that we are able to perform such judgements. Furthermore, to claim that no comparison of household welfare is possible is almost as absurd as the claim that all interpersonal comparisons of utility are impossible. Nevertheless, the construction of a complete transitive ranking of the welfare of every household in a society may be more complicated. In any case, the computation of a household's welfare level should be in accordance with the social planner's evaluation of the household's welfare for public policy purposes. One approach may be to assume that the social planner assigns a welfare level to the household equal to the utility level of the least well off member. This may be sensible in some cases, but may underestimate the total welfare of a household where resources are distributed unequally. Consequently, it may be cases where other welfare functions are more sensible. If, as above, the household is assumed to behave as if it maximizes a welfare function, this particular function is only usable if it corresponds to the social planner's welfare function. In the general case, these functions are not equal. Consequently, a new set of equivalence scales corresponding to the new measure of welfare has to be derived.

Consider a new class of welfare functions $W_\phi^S$: $\mathbb{R}^{n_\phi} \to \mathbb{R}$ for each $\phi \in \Phi$ that corresponds to the social planner's aggregation of the individual utilities. To make inter-household comparisons of welfare, we normally require $W_\phi^S$ to satisfy AG. Further, it is normally ethically appealing to let everybody count equally, which implies that the welfare function should be AN in most cases. The social planner's expenditure function for a household $\phi$ is a function $C^S$: $\mathcal{P} \times \mathbb{R} \times \Phi \to \mathbb{R}$ that gives the minimum income the household requires to obtain a given level of welfare as measured by $W_\phi^S$ when income is distributed within the household according to (3). The cost of reaching some welfare level $W$ given prices $p$ is:

$$C^S(p,W,\phi) := \min\{y \in \mathbb{R} : W_\phi^S[v_1(p,y,\phi),\dots,v_{n_\phi}(p,y,\phi)] \geq W\}. \tag{8}$$

From this definition, we may also define the social planner's equivalence scales:

$$L^S(p,W,\phi,\phi_0) := \frac{C^S(p,W,\phi)}{C^S(p,W,\phi_0)}. \tag{9}$$

It was argued above that in most cases, $L$ decreases when more weight is put on a small group of household members. This conclusion seems rather unethical. Assume as above that all agent types have identical utility functions and that the regularity conditions of Proposition 1 hold. Moreover, assume that $W_\phi^S$ is AN. If the household's own welfare function $W_\phi$ is also AN, then every household member get the same level of utility, and the aggregate welfare is identical whether measured by $W_\phi$ or $W_\phi^S$. Let $y_\phi^*$ denote household $\phi$'s equivalent income to the reference household $\phi_0$ receiving $y_0$. If $W_\phi$ is not AN, then the intra-household allocation of resources is not optimal when measured by $W_\phi^S$, which implies that:

$$W_\phi^S[v_1(p,y_\phi^*,\phi),\dots,v_{n_\phi}(p,y_\phi^*,\phi)] < v_1(p,y_0,\phi_0). \tag{10}$$

Consequently, the equivalent income corresponding to $y_0$ is above $y_\phi^*$ when judged by the social planner's welfare function.

Hence, when using a AG-AN function $W_\phi^S$ to aggregate individual utilities, a household distributing resources equally within the household is the "cheapest" household to run. Distributing resources unevenly is inefficient since the marginal gain to increasing household welfare of making an agent that is well off is less than that of an agent that is less well off.

*Remark*: If utility functions differ among household members, the conclusions become less clear cut. If there are important differences in the "productivity of utility" between individuals, the distribution of resources in a household with an AN welfare function putting little weight on equality between household members may be highly unequal. This may result in distributions further away from the ideal distribution of a social planner putting much weight on equality than certain non-AN welfare functions would. Nevertheless, it is unclear what we should mean by large differences in productivity, so the conclusions above probably hold as an approximation under most reasonable circumstances.

Two households with identical demographic compositions, but with different $\psi$s, have different welfare functions. Then the household with the most uneven intra-household distribution of well-being needs a higher income than the one with a more even distribution for the two households to be at the same level of welfare as seen by the social planner. This may seem counter-intuitive and even unfair. Nevertheless, this is not as much a problem of the approach considered herein as a general problem of unfair decision mechanisms within the households. Furthermore, in cases where some households have an unfair way of distributing resources, a pure transfer of income may be a poor instrument. Instead, subsidies of particular goods may be judged superior.

It may also be argued that imposing $W_\phi^S$ to evaluate household welfare is paternalistic. Most normative studies have a non-paternalistic approach considering the agents' own utility functions as the "correct" way of calculating their utility in a given situation. It may very well be argued that this should apply to the welfare function as well. As mentioned in the introduction, households choose to some extent both consumption and composition. Especially, the formation of couples is normally based on voluntariness from both parties. This means that all household members should prefer $W_\phi$ to any other possible welfare function when they choose to enter a household of type $\phi$. However, all household members do not choose which household to belong to. Probably most important, children do not decide where to be born. Children may have a perceived individual utility function that differs from what is their own benefit, for instance putting much weight on sweets. Also the adults are the main income earners. Therefore, children's consumption is usually to a large extent determined by the parents. Consequently, it is natural to model the children's impact on household welfare through the adults' utility function. For instance, an adult may have an altruistic utility function depending on his or her own individual utility and the utility of his or her children. Then the household welfare function may depend on this function for each adult. From this, we may derive a household welfare function as in Section 2. Nevertheless, even if the welfare function is in some sense correct for the adults, the children's position is not necessarily correct as seen by the social planner. Public provision of, for example, schools and kindergartens indicates that the social planner does not agree with the intra-household allocation taking place, and hence provides subsidies to influence the household's behavior. A similar argument is made by Del Boca and Flinn (1995); see also, for example, Bojer (2000) or Levison (2000) and the references therein for further discussion of problems related to children in models of households.

## 5. Discussion

Hitherto, $W_\phi$ has been regarded as a "black box." This is probably not particularly useful to get a grasp of the welfare implications of the decision mechanism that are necessary for a discussion of the Pangloss problem. Some more intuitively appealing justifications of this particular household decision structure may include:

(1) A "household council" where all household members are present makes consensually the consumption and distribution decisions. This does not necessarily imply an equal intra-household distribution of utility. For instance, maximization of household income may imply inequality, cf. Pitt *et al*. (1990); see also Glaeser (1992) for a more amusing example.

(2) $W_\phi$ corresponds to the utility function of a more or less altruistic head of household who determines the intra-household distribution according to needs. This interpretation is in accordance with, for example, Becker's (1974) "Rotten kid"-theorem.
(3) The household's consumption and distribution decisions are in reality the outcome of a bargaining process. For instance, $W_\phi$ may be a Nash product where the outside opportunity is assumed to be independent of prices. In this case, different weights on the household members may correspond to differences in bargaining power. Such a bargaining procedure may be given an ethical content although the issue is controversial. Roemer (1996, ch. 2) provides a discussion of some of the points of view.

Evidently, most real-world household would be characterized by a mixture of these, where for instance the "household council" is only composed of the adults, and where there is some degree of bargaining, but also a large extent of consensus.

If the household is seen as a "locus of gender, class, and political struggle" (Hartmann, 1981), it is quite clear that the observed household welfare function $W_\phi$ is inappropriate to aggregate household utility in an ethically appealing way. On the other hand, if household decision making is seen as mainly based on agreement, it may seem superfluous to introduce a separate social planner's welfare function. Nevertheless, even though unanimity may seem to prevail, in the sense that there is no apparent use of power, there may be latent conflicts, cf. Lukes' (1974) view of power. Lukes presents a critique of the behavioral focus of more traditional definitions of powers, and argues in favor of including control of issues and potential uses, as well as a distinction between real and expressed interests.

The functions $u_i$ has implicitly been considered as known to all the household members as well as the social planner. The household being a close-knit group, it is probably reasonable to assume that the household members know each others' utility functions. The social planner, and indeed an economist, does not necessarily have such knowledge. Hence, the definition of equivalence scales in equation (9) is not operationalizable without further information making estimation of the individual utility functions possible. Since utility should be interpersonally comparable, it is far from obvious how to do this. Consequently, estimation of household equivalence scales should probably be based on a more restrictive definition taking the definition (9) as a basis. Bourguignon (1999) shows that from a collective approach to household behavior, it is possible to identify the sharing rules, which would correspond to the function $W_\phi$, under quite general conditions. However, he does not get equivalence scales from his approach. Pitt (1997) also discusses identification of individual consumption, but within a unitary model of household behavior.

In their discussion of equivalence scales, Blackorby and Donaldson (1991, 1993) advocate using the Equivalence scales exactness (ESE, also known as independence of base) procedure. If the households' and the social planner's welfare functions are similar, this is an excellent way to identify equivalence scales if one believes in the identifying restrictions. Nevertheless, if the social planner has a different way of aggregating individual utilities than the household, estimation is more complicated. If the household expenditure function satisfies the ESE conditions, this does not imply that the social planner's equivalence scales are independent of the household's welfare level. On the other hand, if we want the social planner's equivalence scales to satisfy ESE, then this does impose restrictions on household behavior, but these restrictions are different from the ordinary ESE-restrictions.

*Remark*: The cost function presented by (8) may be generalized. Among others, Alderman *et al*. (1995) argue that the unitary approach to household decision making is too restrictive both from a theoretical and an empirical point of view. Alternatives include the general approach of Bourguignon, Browning, Chiappori, and Lechêne (Browning *et al*., 1994; Browning and Chiappori, 1998), as well as a wide range of cooperative and non-cooperative game theoretic approaches (Lundberg and Pollak, 1996). Assume that we can construct a class of functions $\xi_i$: $\mathcal{P} \times \mathbb{R} \times \Phi \rightarrow \mathcal{Q}$, such that for all $i$, $\xi_i(p, y, \phi)$ gives agent $i$'s consumption vector. Such an allocation rule is compatible with a wide variety of household decision mechanisms, also the one presented above. It is reasonable that a particular agent's consumption vector depends on the income share of the different household members. To take this into account, we may let some of the elements of $\xi$ contain information on own income shares. Furthermore, $\psi$ may also contain information on the household members' bargaining power. Consequently, it may be restrictive to assume $\psi$ fixed across time. Since the present analysis is restricted to the static case, this will not imply any difficulties. Now individual $i$ has an individual indirect utility function:

$$\upsilon_i(p, y, \phi) = u_{\gamma(i)}(\xi_i(p, y, \phi)). \tag{11}$$

Consequently, the social planner's expenditure function is still given by (8) and the equivalence scales by (9) if we use the new definition of $\upsilon_i$.

## 6. Conclusion

In most approaches to estimating equivalence scales from household expenditure data, the household is modeled as maximizing a common household utility function. Because households normally consist of more than one individual, this approach is dubious, both in its positive and normative implications. The former may be resolved by considering the household utility function as a reduced form of a household welfare function. Nevertheless, the welfare significance of this function is indeed questionable. The Pangloss problem arises since the use of the household welfare function implies accepting that the observed intra-household distribution of resources, and hence of utility, is optimal from a social point of view. This implies among others that households distributing resources unevenly are judged more efficient in generating welfare than households distributing resources evenly.

The equivalence scales exactness-method to estimate equivalence scales as well as the modified techniques by Donaldson and Pendakur (2003) are clearly vulnerable to this criticism. Since both the Engel and Rothbarth methods are also derived from household utility functions, their validity is also doubtful. The subjective scales of the Leyden school are also affected if only the head of household is asked to evaluate household welfare.

To solve the Pangloss problem, it was suggested to keep the model of household behavior. A new welfare function corresponding to the ethical preferences of the social planner aggregates the utility level of the household members. This new measure has more appealing properties than the traditional measure. One difficulty is that we need some way to evaluate the utility individuals derive from consumption. An approach based on subjective data may serve this purpose. A second difficulty is that the social planner's welfare function would have to be determined politically. This may not be straightforward.

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