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INCOME REDISTRIBUTION EFFECT OF PUBLIC PENSIONS BETWEEN DYNASTIC FAMILIES

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This paper investigates income transfers between dynastic families caused by a public pension system. Using Japanese data, we present simulation results based on a model in which intergenerational altruism works, and income distribution exists between and within generations. The growth rates of income and population, as well as the formulation for the determination of the contribution rate and the payment rate, are crucial to determine both the qualitative and quantitative effects. Especially, under negative income growth over generations, pay-as-you-go public pensions can cause negative income redistribution.

1. INTRODUCTION

Two kinds of income redistribution effects are plausible when the pay-as-you-go system prevails in a social security system, namely intergenerational and intragenerational income redistribution effects. Intergenerational redistribution has received considerable attention in the literature. One of the most important and controversial contributions is the proposition of Barro (1974), who concluded that a pay-as-you-go public pension system, as well as a tax reduction financed by government debt, would have no net effect on intergenerational income distribution so long as an altruistic bequest motive is operative in the overlapping generations model. This argument triggered the well-known fiscal neutrality issue in both theoretical and empirical works.

For studies on intragenerational income transfer in the public pension system, it has been most common to employ the framework based on the life cycle hypothesis; in particular, the transfer of lifetime income has been investigated by Creedy (1980, 1996), Shimono and Tachibanaki (1985), Nelissen (1987), Wolfson (1988), and Kennedy (1990) among others. While it is not our current objective to discuss the validity of specific behavioral models, lifetime income transfers within a generation would not measure the social security transfers appropriately when each generation is altruistically linked as in Barro (1974). The welfare of individuals having intergenerational altruism depends on the sum of not only their own lifetime income but also their child's and those of future generations.

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The problem may be argued in the following way. It would be quite valid to assume that the sum of social security contributions (or public pension premiums) in a particular year is equal to the sum of social security payments (or public pension benefits) to retirees under the pay-as-you-go system. However, it would not be valid to assume that the amount of the public pension premium for the child of a family is equal to the amount of public pension benefit for the parent of the family because the public pension system is normally either progressive or regressive. This particularly matters when altruistic bequests are observed. The above consideration leads us to predict that the disposable income (i.e. after-insurance premium income) of a family would be affected by any change in a public pension system, if we define disposable income of a family as the sum of current and future generations' disposal incomes in a family. This is equivalent to the disposable income of a dynastic family, or an altruistically linked household, which includes all generations. The present study attempts to investigate the income redistribution effect of public pensions on such a family or household in the case in which an altruistic bequest motive is operative.

The organization of the paper is as follows. In Section 2 we present an overlapping generations model with an altruistic bequest motive and income distribution between and within generations. In Section 3 we investigate the effect of changes in a pay-as-you-go social security system on the expected income of dynastic families based on the theoretical model, and provide the basis for simulation analyses. In Section 4 we present the simulation results using the *Family Expenditure Survey* (*Kakei Chosa*) in Japan. Finally, in Section 5, we summarize the results.

2. Theoretical Model

The starting point for the model developed here is the Barro-type overlapping generations model with an altruistic bequest motive. The budget constraint for the first generation and the j-th individual is given by

$$y_{1j} - S_{1j} + \frac{B_{1j}}{1+r} + \frac{b_{0j}}{1+n} = c_{1j}^{y} + \frac{c_{1j}^{o} + b_{1j}}{1+r}, \tag{1}$$

where y is income, S is social security contribution, B is social security payment, b is bequest, c^y is consumption during the working period, c^o is consumption during the retirement period, n is the growth rate of population, and r is the interest rate. This equation assumes that a person's life may be divided into two periods: a working period and a retirement period. Consumption during the retirement period is paid by social security payments and personal savings. If the altruistic bequest motive is operative and positive bequests are chosen by all generations, then the budget constraint for the j-th family is given by

$$\sum_{i=1}^{\infty}\left(\frac{1+n}{1+r}\right)^{i-1} y_{ij} - S_{1j} + \sum_{i=1}^{\infty}\left(\frac{1+n}{1+r}\right)^{i}\left(\frac{B_{ij}}{1+n} - S_{i+1,j}\right) + \frac{b_{0j}}{1+n} = \sum_{i=1}^{\infty}\left(\frac{1+n}{1+r}\right)^{i-1}\left(c_{ij}^{y} + \frac{c_{ij}^{o}}{1+r}\right). \tag{2}$$

Based on the Japanese public pension system,[1] we specify the functional form of S and B as

$$S_{ij} = sy_{ij}, \tag{3}$$

$$B_{ij} = \bar{B}_i + py_{ij}, \tag{4}$$

where s is the rate of social security contribution, p is the rate of social security benefit, $\bar{B}_i$ is a fixed part of the benefit, and py_{ij} is a proportional part.[2]

Under these institutional particularities, there is no guarantee that $B_{ij} = (1+n)S_{i+1,j}$ is satisfied for all families, because the social security system has an intragenerational income redistribution mechanism. We define the left-hand side of (2) as the family income expected by the first generation:

$$yp^e_{1j} \equiv \sum_{i=1}^{\infty}\left(\frac{1+n}{1+r}\right)^{i-1} y^e_{ij} - S_{1j} + \sum_{i=1}^{\infty}\left(\frac{1+n}{1+r}\right)^{i}\left(\frac{B^e_{ij}}{1+n} - S^e_{i+1,j}\right) + \frac{b_{0j}}{1+n}, \tag{5}$$

where the superscript e indicates the expected value, and we investigate the effects of changes in the public pension system, i.e. changes in s, p, and/or $\bar{B}_i$, on yp^e_{1j}.

3. Effects of Changes in Public Pension System

The public pension system given by (3) and (4) may be inserted into (5) in the following way:

$$yp^e_{1j} = \sum_{i=1}^{\infty}\left(\frac{1+n}{1+r}\right)^{i-1} y^e_{ij} - S_{1j} + \sum_{i=1}^{\infty}\left(\frac{1+n}{1+r}\right)^{i}\left(\frac{\bar{B}_i + py^e_{ij}}{1+n} - sy^e_{i+1,j}\right) + \frac{b_{0j}}{1+n}. \tag{6}$$

Assuming the government budget for the pay-as-you-go social security system is balanced, we have

$$\bar{B}_i + p\bar{y}_i = (1+n)s\bar{y}_{i+1}, \qquad i = 1, 2, \ldots, \infty, \tag{7}$$

where $\bar{y}_i$ is the average income of the i-th generation. Totally differentiating (6) and (7) with respect to yp^e_{1j}, $\bar{B}_i$, p, and s yields

$$dyp^e_{1j} = \sum_{i=1}^{\infty}\left(\frac{1+n}{1+r}\right)^{i}\left(\frac{d\bar{B}_i}{1+n} + \frac{y^e_{ij}}{1+n}\,dp - y^e_{i+1,j}\,ds\right), \tag{8}$$

$$d\bar{B}_i = (1+n)\bar{y}_{i+1}\,ds - \bar{y}_i\,dp, \qquad i = 1, 2, \ldots, \infty. \tag{9}$$

Substitution (9) into (8) yields

$$dyp^e_{1j} = \sum_{i=1}^{\infty}\left(\frac{1+n}{1+r}\right)^{i}\left[(\bar{y}_{i+1} - y^e_{i+1,j})\,ds + (y^e_{ij} - \bar{y}_i)\frac{dp}{1+n}\right]. \tag{10}$$

[1] Although the actual public pension system of Japan has maximum limits for both contributions and benefits (see Shimono and Tachibanaki, 1985), these limits are ignored in this paper to simplify the analysis. The introduction of these limits would produce weaker income redistribution effects than those presented here, but would not affect the qualitative results.

[2] The type of benefit scheme with the form $B_{ij} = C_i + p(y_{ij} - C_i)$ is often used in other countries, where C_i is a parameter. Although this can be obviously reduced to (4) with $\bar{B}_i = C_i(1-p)$, the two ways of writing the benefit formula become relevant when changes in the parameters are considered (see Note 3).

While we consider the cases in which the contribution side and the benefit side are changed simultaneously with the government budget balanced, we distinguish two cases for the benefit side. The first case raises a fixed part, keeping a proportional part constant, i.e. $dp = 0$, while the second case raises a proportional part, keeping a fixed part constant, i.e. $d\bar{B}_i = 0$. This distinction is made to investigate the difference between the impact of a fixed part and that of a proportional part on income redistribution.[3] Needless to say, in these two cases the contribution rate is raised, and thus $ds > 0$ is always assumed.

(i) Case 1: $ds > 0$ and $dp = 0$

Equation (10) is reduced to

$$\frac{dyp^e_{1j}}{ds} = \sum_{i=1}^{\infty} \left(\frac{1+n}{1+r}\right)^i (\bar{y}_{i+1} - y^e_{i+1,j}). \tag{11}$$

This equation suggests that a change in expected family income by the policy change depends on whether future generations' income in a family is expected to be higher or lower than the same generations' average income. The public pension system has a positive redistribution effect in this case since it decreases the disposal income of a family with higher income and increases the disposal income of a family with lower income.

(ii) Case 2: $ds > 0$ and $d\bar{B}_i = 0$

Equation (10) is reduced to

$$\frac{dyp^e_{1j}}{ds} = \sum_{i=1}^{\infty} \left(\frac{1+n}{1+r}\right)^i \left(\frac{y^e_{ij}}{\bar{y}_i} - \frac{y^e_{i+1,j}}{\bar{y}_{i+1}}\right) \bar{y}_{i+1}. \tag{12}$$

A change in expected family income depends upon the transition of relative future-generation income (in comparison with the same generation's average income). If the relative position rises as the generation proceeds, the change in expected income is negative, whereas the change is positive if the relative position falls.

4. Simulation Results of Income Redistribution Effect

In this section, we adapt a simulation approach to analyze the income redistribution effects in the above two cases of changes in the public pension system. As shown in the previous section, the effects crucially depend on how future generations' income is expected. For treatment of uncertainty in future generations' income, we assume an income distribution function of the first generation and a transition from one generation to the next generation. The first generation's

[3]Results of these two cases can be used to examine the case in which the fixed and proportional components change simultaneously. At the same time, since a rise in p under the alternative formulation $B_{ij} = C_i + p(y_{ij} - C_i)$ is transformed into a rise in the proportional component and a fall in the fixed component under our formulation, the impact of such a change can also be investigated using our results.

income is supposed to be distributed log-normally with

$$x_1 \sim N(\mu, \sigma^2), \tag{13}$$

where $x_i \equiv \ln y_i$. Although Atoda and Tachibanaki (1991) show that the log-logistic function provides better fits for income distribution in Japan, we adopt the log-normal distribution for the following reasons. First, the log-normal and log-logistic functions are similar since their log-transformed variables are distributed symmetrically. Second, the log-normal is easier to be handled mathematically, and has been used frequently in the literature on income distribution. The transition of income over generations is also supposed to obey the log-normal distribution with the average $ax_i + \beta$ and the variance v^2:[4]

$$x_{i+1} \sim N(\alpha x_i + \beta, v^2). \tag{14}$$

In this framework the expected income and the average income are given by

$$y_{ij}^e = \exp\left[\alpha^{i-1}\left(x_{1j} - \frac{\beta}{1-\alpha}\right) + \frac{\beta}{1-\alpha} + \frac{1}{2}(1-\alpha^{2(i-1)})\frac{v^2}{1-\alpha^2}\right], \quad i = 2, 3, \ldots, \infty, \tag{15}$$

$$\bar{y}_i = \exp\left\{\alpha^{i-1}\left(\mu - \frac{\beta}{1-\alpha}\right) + \frac{\beta}{1-\alpha} + \frac{1}{2}\left[\alpha^{2(i-1)}\sigma^2 + (1-\alpha^{2(i-1)})\frac{v^2}{1-\alpha^2}\right]\right\}, \quad i = 2, 3, \ldots, \infty. \tag{16}$$

Using (15) and (16), and given the values of μ, σ^2, α, β, v^2, n and r, we can evaluate (11) and (12) numerically. As to the method of obtaining these parameter values, some are estimated using statistical data, and some are borrowed from the results of other studies. We apply the moment method to estimate μ and σ^2, based on the formulation given in Johnson and Kotz (1970). Income in this paper is defined by the regular income of a household in the *Family Expenditure Survey* (*Kakei Chosa*) conducted by the Management and Coordination Agency of Japan. This data source includes two semiannual observations, in the first and second half of each year from 1985 to 1989, and contains about 1,200 households for each point in time. A major component of the regular income of a household is the wage income of a household head. To estimate the values of α, β and v^2,we use the results for Japanese micro data obtained in Tachibanaki and Takata (1991), and transform them into our parameter requirements since the authors estimated them not for generational but annual transitions. Since we assume that each period lasts for 20 years, and Tachibanaki and Takata obtained $a =$ 1.003495, $b = -0.005429$ and $V^2 = 0.003369$ for $x_{t+1} \sim N(ax_t + b, V^2)$, we have the solutions of α, β and v^2 from the following relationships: $\alpha = a^{20}$, $\beta = (1 - a^{20})b/(1 - a)$, and $v^2 = V^2(1 - a^{40})/(1 - a^2)$. While setting b and V^2 at the estimated values, we assume three values of a (0.999, 1.001, and 1.003) in order to consider

[4]This is an application of Creedy (1974, 1985), and Tachibanaki and Takata (1991), who assume distribution functions similar to (13) and (14) for annual incomes of an individual.

various income growth processes.[5] We consider two different values for n, 0.02 and –0.01. Since Japan is now experiencing an aging trend, the case of $x = -0.01$ enables us to investigate the situation. The rate of interest r is given by 0.02. To simplify the calculation, we replace ∞ in (11) and (12) with a finite integer T, which indicates how many future generations of a family are expected to exist by the current generation of the family. We consider two different values for T: 2 and 3.

(i) Case 1: $ds > 0$ and $dp = 0$

By plugging (15) and (16) into (11), we obtain the following equation:

$$\frac{dyp_{1j}^{e}}{ds} = \sum_{i=1}^{T}\left(\frac{1+n}{1+r}\right)^{i}\exp\left[(1-\alpha^{i})\frac{\beta}{1-\alpha}+\frac{1}{2}(1-\alpha^{2i})\frac{\nu^{2}}{1-\alpha^{2}}\right] \times\left\{\exp\left[\alpha^{i}\left(\mu+\frac{1}{2}\alpha^{i}\sigma^{2}\right)\right]-\exp(\alpha^{i}x_{1j})\right\}. \quad (17)$$

Equation (17) shows that the change in the expected family income for each income level due to the policy reform that simultaneously raises the rate of social security contribution and the fixed part of the payment. The numerical solutions under the parameter values are given in Tables 1 and 2. Table 1 shows that the change in the expected family income is negative for higher income classes, but positive for lower income classes. The change in absolute value increases as the income of the higher income class increases or as the income of the lower income class decreases. This suggests that, in this case, a public pension system generally has a positive redistribution effect.

Table 2 presents simulation results when the values of α, T and n are changed. As α is raised, the absolute value of the change in the expected family income increases. In other words, a higher growth rate economy encourages a stronger income redistribution effect. The income level of the break-even point increases as the value of α increases, and the break-even point is below the average income when $\alpha < 1$ holds while it is above the average income when $\alpha > 1$ holds. An increase in T strengthens the income redistribution effect of public pensions. The effect is very remarkable. For example, when it is changed from $T = 2$ to $T = 3$, the change in the expected family income in each cell is about 3–5 times higher. Furthermore, an increase in T raises the break-even point income. An increase in n also gives a stronger income redistribution effect. This is quite natural because the number of income earners increases, and so it is possible to anticipate an increase in the expected family income.

(ii) Case 2: $ds > 0$ and $d\bar{B}_i = 0$

This case stipulates a policy that raises both the contribution rate and the proportional part of the payment with the fixed part constant. With equations

[5]There have been several studies assessing the extent of intergenerational income mobility, e.g. Solon (1992) and Zimmerman (1992) for the U.S., and Atkinson (1981) and Dearden, Machin, and Reed (1997) for the U.K., but none for Japan to our knowledge.

TABLE 1

CHANGES IN EXPECTED FAMILY INCOME FOR EACH INCOME CLASS: CASE 1 ($ds > 0,\ dp = 0$) (yen)

Annual income (yen millions)	1985 (2)	1986 (1)	1986 (2)	1987 (1)	1987 (2)	1988 (1)	1988 (2)	1989 (1)
0.5	2,808,005	2,612,788	2,594,151	2,656,521	3,168,752	3,843,308	5,730,515	6,358,462
1.0	1,675,042	1,479,825	1,461,188	1,523,558	2,035,789	2,710,345	4,597,552	5,225,498
1.5	504,937	309,720	291,082	353,452	865,683	540,239	3,427,446	4,055,393
2.0	−689,961	−885,178	−903,816	−841,445	−329,215	345,342	232,548	2,860,495
2.5	−1,903,640	−2,098,857	−2,117,494	−2,055,124	−1,542,893	−868,337	1,018,870	1,646,816
3.0	−3,132,497	−3,327,714	−3,346,352	−3,283,981	−2,771,751	−2,097,194	−209,988	417,959
4.0	−5,626,786	−5,822,003	−5,840,640	−5,778,270	−5,266,039	−4,591,483	−2,704,277	−2,076,330
5.0	−8,160,278	−8,355,495	−8,374,133	−8,311,762	−7,799,532	−7,124,975	−5,237,769	−4,609,822
6.0	−10,725,456	−10,920,673	−10,939,311	−10,876,940	−10,364,710	−9,690,153	−7,802,947	−7,175,000
7.0	−13,317,288	−13,512,505	−13,531,142	−13,468,772	−12,956,541	−12,281,985	−10,394,778	−9,766,832
8.0	−15,932,157	−16,127,374	−16,146,012	−16,083,642	−15,571,411	−14,896,855	−13,009,648	−12,381,701
10.0	−21,220,694	−21,415,911	−21,434,548	−21,372,178	−20,859,947	−20,185,391	−18,298,185	−17,670,238
B-E point	1,712,377	1,630,000	1,622,640	1,648,843	1,863,100	2,142,983	2,914,949	3,168,846
$E(y)$	1,704,113	1,620,747	1,611,982	1,637,304	1,853,136	2,133,158	2,908,800	3,163,466
$V(y)$	492,689	561,762	618,372	684,788	651,736	730,322	599,018	566,306
C.V.	0.411897	0.462446	0.487826	0.505416	0.435641	0.400621	0.266076	0.237882

Notes: $T = 2$, $n = 0.4859$ (0.02), $r = 0.4859$ (0.02), and $\alpha = 1.0617$ (1.003), where figures in parentheses are the parameters for annual bases. 1986 (1) means the first half in 1986, and 1986 (2) means the second half in 1986. B-E point is the income level of the break-even point (i.e. $dyp^e_{1j}/ds = 0$). $E(y)$, $V(y)$ and C.V. indicate the average, variance and coefficient of variation under the log-normal distribution, and $V(y)$ in yen millions.

TABLE 2

CHANGES IN EXPECTED FAMILY INCOME FOR VARIOUS VALUES OF α, T AND n: CASE 1 ($ds > 0, dp = 0$) (yen)

Annual Income (yen millions)	α / T / n	1.003 / 2 / 0.02	1.001 / 2 / 0.02	0.999 / 2 / 0.02	1.003 / 2 / –0.01	1.003 / 3 / 0.02
0.5		2,808,005	1,510,660	830,731	1,545,605	9,704,798
1.0		1,675,042	889,190	482,630	921,990	5,874,870
1.5		504,937	261,123	138,121	277,931	1,822,641
2.0		–689,961	–371,267	–204,077	–379,774	–2,381,616
2.5		–1,903,640	–1,006,891	–544,565	–1,047,816	–6,702,951
3.0		–3,132,497	–1,645,104	–883,698	–1,724,214	–11,120,112
3.5		–4,374,123	–2,285,479	–1,221,708	2,407,640	–15,618,710
4.0		–5,626,786	–2,927,710	–1,558,759	–3,097,140	–20,188,307
4.5		–6,889,178	–3,571,568	–1,894,974	–3,791,996	–24,820,966
5.0		–8,160,278	–4,216,875	–2,230,499	–4,491,645	–29,510,433
5.5		–9,439,263	–4,863,488	–2,565,258	–5,195,634	–34,251,646
6.0		–10,725,456	–5,511,291	–2,899,464	–5,903,591	–39,040,418
7.0		–13,317,288	–6,810,091	–3,566,264	–7,330,207	–48,747,060
8.0		–15,932,157	–8,112,656	–4,231,173	–8,769,504	–58,607,729
10.0		–21,220,694	–10,727,325	–5,556,231	–11,680,462	–78,725,945
B-E point		1,712,377	1,706,811	1,701,469	1,712,377	1,718,687

Notes: Period = 1985 (2) and $r = 0.4859$ (0.02). See also Notes in Table 1.

(15) and (16), (12) is rewritten as follows:

$$\text{(18)} \quad \frac{dyp_{1j}^{e}}{ds} = \sum_{i=1}^{\infty}\left(\frac{1+n}{1+r}\right)^{i} \exp\left[(1-\alpha^{i})\frac{\beta}{1-\alpha} + \frac{1}{2}(1-\alpha^{2i})\frac{v^{2}}{1-\alpha^{2}}\right] \times \left\{\exp\left[\alpha^{i-1}\left(x_{1j} - \left(\mu + \frac{1}{2}\alpha^{i-1}\sigma^{2}\right)\right)\right]\exp\left[\alpha_{i}\left(\mu + \frac{1}{2}\alpha^{i}\sigma^{2}\right)\right] - \exp(\alpha^{i}x_{1j})\right\}.$$

Table 3 shows simulation results for changes in income transfers when the values of α are changed. We note the following observations, which suggest the difference between a change in the fixed part and a change in the proportional part of the payment. First, we see a positive income redistribution effect of public pensions when $\alpha > 1$ holds. However, the effect in Case 2 is much weaker than in Case 1. Second, the maximum income gain for $\alpha > 1$ is obtained at the income level (approximately 0.7 million yen) above the lowest in Case 2, while it was achieved by the lowest income level in Case 1. Also, the income loss is progressive for the higher income brackets. Third, for $\alpha > 1$, the break-even point is raised in Case 2 in comparison with Case 1. It is noted also that the break-even point is raised as α increases.

Fourth, the change in the expected family income is positive for higher income brackets, but negative for lower income ones, when a is equal to 0.999. As mentioned in the previous section, the change in the public pension system transfers income from a family whose relative income rises as the generation proceeds to a family whose relative income falls as the generation proceeds. When $\alpha < 1$ holds, since the variance of income decreases over generations, the relative income of a family with higher current-generation income falls and the relative income of a family with lower current-generation income rises as the generation

TABLE 3

CHANGES IN EXPECTED FAMILY INCOME FOR VARIOUS VALUES OF α, T AND n: CASE 2 ($ds > 0$, $d\bar{B} = 0$) (yen)

Annual Income (yen millions)	α / T / n	1.003 / 2 / 0.02	1.001 / 2 / 0.02	0.999 / 2 / 0.02	1.003 / 2 / –0.01	1.003 / 3 / 0.02
0.5		87,725	16,152	–9,120	48,286	293,417
1.0		84,340	15,267	–8,478	46,423	292,767
1.5		43,813	7,784	–4,245	24,116	158,322
2.0		–21,506	–4,021	2,300	–11,837	–69,076
2.5		–105,606	–19,061	10,554	–58,128	–368,799
3.0		–204,885	–36,690	20,163	–112,774	–728,179
3.5		–316,932	–56,481	30,896	–174,448	–1,138,570
4.0		–440,016	–78,127	42,587	–242,197	–1,593,662
4.5		–572,830	–101,401	55,114	–315,302	–2,088,630
5.0		–714,351	–126,123	68,382	–393,199	–2,619,659
5.5		–863,757	–152,153	82,315	–475,436	–3,183,644
6.0		–1,020,372	–179,371	96,851	–561,641	–3,778,012
7.0		–1,353,046	–237,003	127,536	–744,754	–5,049,510
8.0		–1,708,758	–298,399	160,112	–940,548	–6,420,138
10.0		–2,478,980	–430,730	230,023	–1,364,500	–9,418,168
B-E point		1,851,948	1,845,929	1,840,000	1,851,948	1,865,235

Notes: Period = 1985 (2) and $r = 0.4859$ (0.02). See also Notes in Table 1.

proceeds. This result is in marked contrast with Case 1 in which the positive redistribution effect is observed regardless of α. Moreover, the break-even point is always higher than the average income in Case 2. These results suggest that the growth rate of the average income is crucial in determining the redistribution effects of public pensions, especially in Case 2.

Table 3 also presents a simulation with various values of T and n. We find that an increase in either T or n gives a stronger income redistribution effect. At the same time, the break-even point is raised with a higher value of T, while the effect of n is zero. The case of $\alpha < 1$ gives effects symmetrical to the case of $\alpha > 1$.

5. CONCLUDING REMARKS

The present study examined the impact on the income distribution between dynastic families from public pension programs which work on the pay-as-you-go basis. The main findings of this paper may be summarized as follows. The degree of income redistribution caused by public pensions critically depends on the value of α which signifies the growth rate of income, and thus that of the economy. Public pension programs always have a positive effect on income redistribution when $\alpha > 1$. This is true whether an increase occurs in the fixed part in payment, or in the proportional part. It is emphasized, however, that the positive effect is not achieved for $\alpha < 1$ when the proportional part is increased. This is in direct contrast to the proposition by Creedy (1980, 1996) and Shimono and Tachibanaki (1985) that a funded scheme of public pensions always has a positive redistribution effect, and implies a clear distinction between a funded scheme and a pay-as-you-go scheme. Moreover, even when the positive redistribution effect is observed in the case for $\alpha > 1$, some households with below-average income may receive a negative income transfer, depending upon the values of α and T.

Finally, several subjects for further studies are suggested. First, since changes in expected family income would affect the consumption level of the current generation, the present paper would provide a starting point for a theoretical framework to estimate impacts on consumption and welfare when the public pension system is changed. This would also enable us to evaluate the effects of the public pension system on a macroeconomy and on economic growth. Second, we assume that a population growth rate is constant because our data source does not cover the number of children in a household. However, fertility may be endogenously determined as in Becker and Barro (1988) and Barro and Becker (1989), and the number of children may differ by income class. It would be interesting to examine the effect of public pensions on fertility of each income group using a dynasty model with endogenous fertility. Third, we considered only the altruistic bequest motive in the present study. It would be worth taking into account the strategic bequest motive (Bernheim, Shleifer, and Summers, 1985) alternatively in an investigation on public pensions and other intergenerational government transfers.

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