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A GUIDE TO U.S. CHAIN AGGREGATED NIPA DATA

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In 1996, the U.S. Department of Commerce began using a new method to construct all aggregate "real" series in the National Income and Product Accounts (NIPA). This method is based on the so-called "ideal chain index" pioneered by Irving Fisher. The new methodology has some extremely important implications that are unfamiliar to many practicing empirical economists; as a result, mistaken calculations with NIPA data have become very common. This paper explains the motivation for the switch to chain aggregation, and then illustrates the usage of chain-aggregated data with three topical examples, each relating to a different aspect of how information technologies are changing the U.S. economy.

1. INTRODUCTION

In 1996, the U.S. Department of Commerce began using a new method to construct all aggregate "real" series in the National Income and Product Accounts (NIPA). This method, now used to create the real aggregates for flows such as GDP, consumption, and investment, as well as for stocks such as inventories and fixed capital, employs the so-called "ideal chain index" pioneered by Irving Fisher (1922).

The new methodology has some extremely important implications for calculations with real U.S. NIPA series, which if ignored, can lead to incorrect conclusions concerning many important economy-wide phenomena. For example, suppose you are interested in how the "high-tech" sector of the economy has influenced aggregate output and investment as well as the average age of the capital stock. Some basic manipulations of U.S. NIPA data could lead one to the following conclusions[1]:

(1) Computer production has been the dominant factor behind recent U.S. economic growth: While real GDP expanded at the brisk pace of 3.8 percent per year over the period 1996–98, real output excluding the computer sector grew only 2.1 percent per year. (Calculation: Take the sum of real consumption, real investment, real government spending, and real net exports of computing equipment, subtract this sum from real GDP and then calculate the percentage change. See the upper panel of Figure 1.)

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[1]All data in this paper pre-date the October 1999 benchmark revision to the NIPAs. Data for 1960–94 can be found in *National Income and Product Accounts of the United States*, data for 1995–97 can be found in the August 1998 issue of the *Survey of Current Business*, while data for 1998 are from the April 1999 issue. The capital stock data are from Department of Commerce (1998). Unless otherwise specified, the base year for all real expenditure series is 1992.

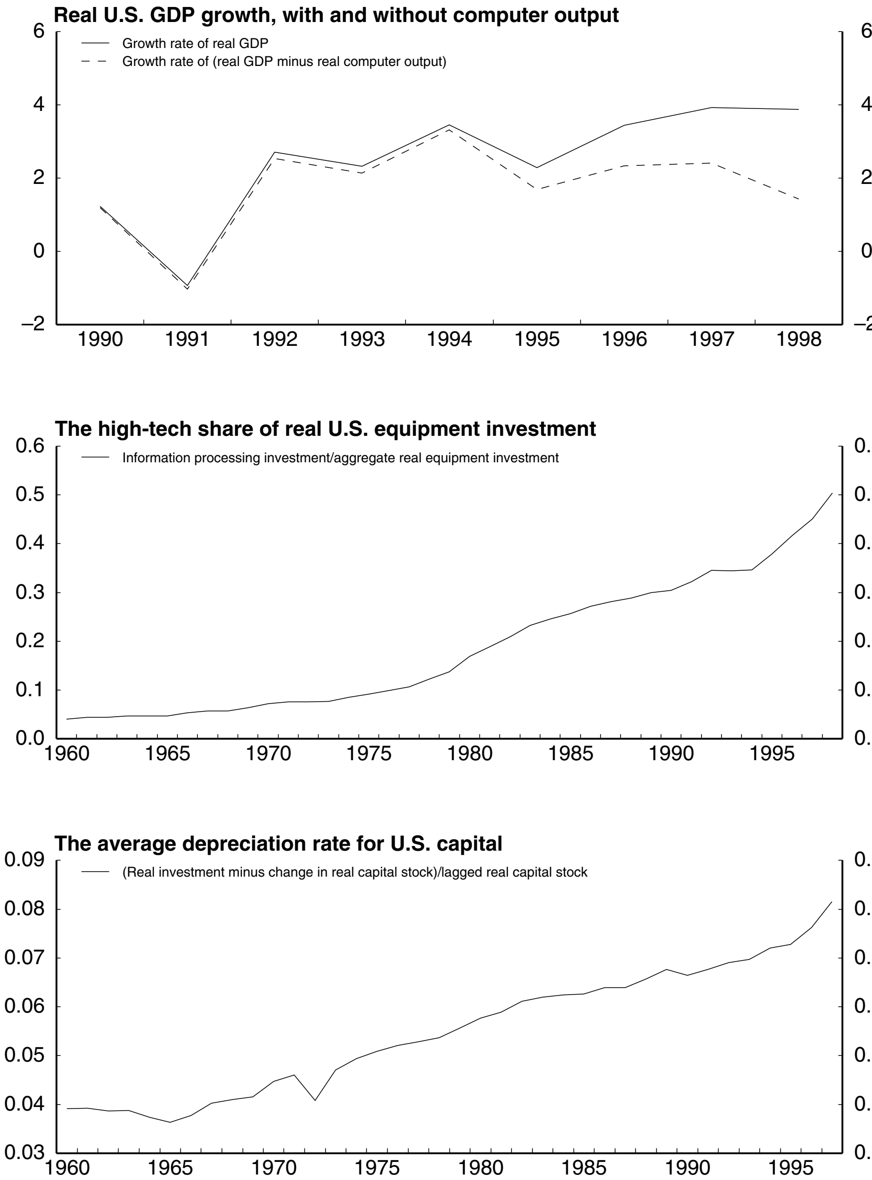

Figure 1. Three misleading calculations.

(2) Because of the rapid growth in investment in high-tech equipment, this category now accounts for more than half of all real equipment investment (Calculation: Take the ratio of real investment in information-processing equipment to total real equipment investment. See the middle panel of Figure 1.)

(3) The shift in the composition of investment away from structures and towards faster-depreciating equipment such as computers has radically

changed the average service life of the capital stock: The average depreciation rate for U.S. capital has more than doubled over the past 40 years, going from 4 percent in 1960 to over 8 percent in the late 1990s. (Calculation: Take the real aggregate capital stock, K_t, and real aggregate investment, I_t, and re-arrange the perpetual inventory equation, $K_t = (1 - \delta_t)K_{t-1} + I_t$, to give $\delta_t = (I_t - \Delta K_t)/(K_{t-1})$. See the lower panel of Figure 1.)

Each of these statements, if true, should be of great interest. They paint a dramatic picture of how information technologies affected the U.S. economy in the 1990s, and they appear to provide confirmation for the notion that technology has truly been creating a "New Economy." However, the statements are, in fact, quite misleading because the calculations on which they are based fail to take account of the methodology used to construct the relevant aggregates. A crucial feature of this chain aggregation methodology—which is the method recommended by the System of National Accounts, 1993—is that the real aggregate of X and Y will generally not be the arithmetic sum of the real series for X and Y. It is this lack of additivity that invalidates each of the calculations described above. Moreover, this feature applies most noticeably when we are dealing with categories undergoing large changes in relative prices, making it particularly tricky to assess the role of high-tech output, which has declined dramatically in price relative to other components of GDP.

In this paper, I briefly describe the U.S. NIPA's chain aggregation methodology and then discuss some of its implications for empirical calculations. Perhaps because of its roots in the relative obscurity of index-number theory, or perhaps because of the usual scientific diffusion lags, many practicing empirical economists have had little or no exposure to the implications of chain aggregation for the analysis of U.S. macroeconomic data. As a result, mistaken calculations based on real NIPA data have become common in the work of academic, policy, and business economists. This paper is intended to help researchers avoid some of the common pitfalls of empirical work with U.S. chain-aggregated data and also to suggest some practical alternative strategies when using such data. Of course, some other countries have also adopted chain aggregation, and many others are intending to do so in the future, so the examples in this paper have relevance beyond their implications for use of U.S. data.

The paper begins with a brief general discussion of the reasons why the chain aggregation methodology was introduced into the U.S. national accounts, and why it is superior to the more traditional fixed-weight approach. I then illustrate some important implications of the chain aggregation procedure by re-visiting the three calculations just described.

2. Why Chain Aggregation?

Estimates of nominal GDP for the United States are constructed by the Commerce Department's Bureau of Economic Analysis (BEA) according to the expenditure method, using the textbook identity $Y = C + I + G + X - M$. In practice, BEA does not directly calculate the major expenditure series (such as C and

I) but rather builds each of them up from a large number of disaggregated component series.

The purpose of the series known as *real* GDP is to tell us what part of the increase in nominal GDP is due to higher *quantities*, rather than higher *prices*. Naturally, the estimation of real GDP starts with a set of price indexes ($P_i(t)$) for a disaggregated set of goods and services. These price indexes are set equal to 1 in some base year, b. They are then combined with the series on nominal expenditures ($Y_i(t)$) to construct a set of quantity or real expenditure series, $Q_i(t) = (Y_i(t))/(P_i(t))$. The interpretation of $Q_i(t)$ is "the dollar value of year-t's expenditures on category i had its price remained at its year-b level"; this series is usually termed real year-b-dollar expenditures on category i.[2]

These first steps are simple and intuitive. To use an obvious metaphor, they involve simply counting the quantity of apples and oranges produced. The nontrivial part of calculating real GDP is creating a summary statistic that combines the quantity of apples and the quantity of oranges in an appropriate manner.

Fixed-Weight Real GDP

The traditional way to define real GDP has been to sum the real year-b dollar expenditures for each category. The resulting series has the interpretation of "the value of period t's output had all prices remained at their year-b level." Because this method values all quantities in terms of a fixed set of prices, as in a traditional Laspeyres index, it is known as a "fixed-weight" measure of real output. Until 1996, U.S. real GDP was constructed according to this method.

While the fixed-weight methodology has the advantage of simplicity and ease of interpretation, it also has a number of undesirable features. The most important practical drawback is that the growth rate of a fixed-weight measure of real GDP depends on the choice of base year. Take 1998 as an example: The growth rate of fixed-weight real GDP for the U.S. in this year was 4.5 percent if we use 1995 as the base year; using 1990 prices it was 6.5 percent; using 1980 prices it was 18.8 percent; and using 1970 prices, it was a stunning 37.4 percent![3]

The explanation for the pattern of higher growth rates for real GDP when using earlier base years is relatively simple. Categories with declining relative prices tend to have faster growth in quantities; the farther back the base year used to choose price weights, the larger is the weight on these fast-growing categories, and so the faster is the growth rate of real output. Similarly, for a given base year, the growth rate of a fixed-weight quantity index tends to increase over

[2]In this paper, I have adopted the standard terminology in the U.S. NIPAs by refering to real series or quantity indexes. However, I should note that the SNA and statistical agencies in many other countries often refer instead to GDP in "volume terms" or "volume indexes."

[3]These calculations actually understate the true pattern. BEA still calculates fixed-weight estimates of real GDP; this series was last published as Table 8.27 in Department of Commerce (1998). BEA's (unpublished) estimate of 1992-based fixed-weight real GDP growth for 1998 was 6.6 percent. The method I used to do these calculations gives 5.5 percent when using 1992 weights and shows less acceleration. The reason for the discrepancy is the level of disaggregation. In constructing the figures reported here, I divided output into a large number of disaggregated expenditure categories and constructed real series for each according to various base years. However, I did not use as fine a level of disaggregation as that used by BEA to construct the real GDP so, for reasons that will become apparent later (some of these disaggregated series are actually chain aggregates) these figures actually underestimate the tendency of fixed-weight GDP to accelerate.

time, as the output bundle becomes increasingly expensive when measured in terms of the base year's prices.[4]

Because of the tendency for growth rates to increase simply because we are moving away from the base year, BEA used to periodically "re-base" its fixed-weight measures of U.S. GDP by moving the base year forward. However, this practice leads to a pattern of predictable revisions to real output growth, which can cause problems of interpretation for users of national income data. To quote SNA93 (page 392): "When the base year for a time series of fixed weight Laspeyres type volume indices is brought forward, the underlying trend rate of growth may appear to slow down if the previous base has become very out of date. This slowing down is difficult to explain to users and may bring the credibility of the measures into question."

An Alternative: *Chained Indexes*

In the United States, the problems with fixed-weight measures of real GDP became more severe after the mid-1980s because of BEA's decision to measure computer prices according to the hedonic method pioneered by Zvi Griliches in 1961 (Griliches, 1971).[5] This approach revealed enormous declines in the quality-adjusted price of computing power, and thus rapid increases in the real computer spending. While the hedonic method certainly improved the measures of real computer output, placing them on a more sound economic basis, this decision led to a substantial category of output being measured as having rapidly declining prices and rapidly increasing quantities, which, as we just noted, is the factor underlying the tendency of fixed-weight GDP to accelerate over time.

To address the problems with its fixed-weight measures, in 1996 BEA began calculating real GDP and all other published real aggregates according to a chain index formula. Instead of using a fixed set of relative price weights, chain indexes continually update the price weights used to calculate the *growth rate* of real output. The *level* of real GDP associated with the chain-index is calculated by setting it equal to nominal GDP in some base year, and then "chaining" forward and backward from the base year using these growth rates. The specific chain index employed by the BEA is the so-called "ideal" chain index popularized by Irving Fisher (1922):

$$Q(t) = Q(t-1)\sqrt{\frac{\sum_{i=1}^{n} P_i(t)Q_i(t)}{\sum_{i=1}^{n} P_i(t)Q_i(t-1)} \times \frac{\sum_{i=1}^{n} P_i(t-1)Q_i(t)}{\sum_{i=1}^{n} P_i(t-1)Q_i(t-1)}} \tag{1}$$

The gross growth rate of the real aggregate at time t is calculated as a geometric average of the gross growth rates of two separate fixed-weight indexes, one a

[4]In addition to current growth rates looking higher when using old base years, we also find that growth rates from years in the past look lower when using current base years. In general, however, the latter pattern is weaker than the former. For example, a fixed-weight measure of GDP grew 2.2 percent if 1995 is used as the base, and 2.8 percent if 1960 is used. For 1960, the growth rates are 2.4 percent if 1960 is used as the base, and 2.0 percent if 1995 is used. The discrepancy between growth rates is smaller in the 1960 case because the categories for which there have been the biggest relative price declines—such as computing equipment—are a very small part of nominal expenditures in 1960, so that year's growth rate is not very sensitive to the prices used to deflate these categories.

[5]See Silver (1999) for a review of the use of hedonic regression techniques.

Paasche index (using period t prices as weights) and the other a Laspeyres index (using period $t-1$ prices as weights).[6]

It is important to distinguish the general advantages associated with chain indexes from the specific properties of the Fisher approach. The first advantage of chain aggregates is that they do not tend to accelerate over time. For example, the tendency of the computer sector to grow at very fast average rates in real terms causes it to become an ever more important share of fixed-weight GDP, making this series accelerate, but the declining relative price for computers provides an offsetting effect on chain-aggregates, because this growth in real computer output receives less relative weight over time.

This advantage of stable growth rates associated with chain-aggregates was particularly important in the 1990s because the combination of rapidly declining computer prices and large increases in nominal spending on computers would have caused substantial acceleration in fixed-weight measures of GDP growth. Prior to the adoption of the chain aggregation procedure, BEA's practice had been to move the base year forward every five years; as our example comparing 1990-based and 1995-based fixed-weight measures showed, such a procedure would have resulted in predictable revisions to published real GDP growth for recent years of over two percentage points. By preventing the need for such large revisions, the move away from a fixed-weight approach has avoided a problem that would have greatly complicated the interpretation of the recent macroeconomic performance of the U.S. economy.

It should also be noted that, beyond the practical problems for users associated with base-year dependence, chain aggregates are generally preferable to fixed-weight series when used as proxies for the theoretical concepts that economists are typically interested in. For example, real fixed-weight consumption per capita will generally be a poor measure of welfare, while real fixed-weight output per hour will be a poor measure of the state of technology. This is because the values assigned by agents to goods and services at each period in time are likely to be better summarized by the structure of relative prices prevailing at that time than by the relative price structure from some arbitrary base period. In contrast, because they are based upon continually-updated price weights, chain-indexes tend to weight quantities in a way that provides a better approximation to theoretical concepts.[7]

In addition to these general benefits of the chain index approach, the Fisher index has some other attractive features. For example, the GDP deflator (the ratio of nominal to real GDP) obtained from this real output series is itself a Fisher ideal index, based on moving quantity weights. Thus, this approach allows for a methodologically unified treatment of aggregate prices and quantities. However, the principal practical complication associated with chaining—the non-additivity feature—which is the focus of the rest of this paper, applies to all chain indexes.

[6]See Landefeld and Parker (1995, 1998) for BEA's official position on the change in methodology. See also Vavares, Prakken, and Guirl (1998) for an excellent discussion of many of the issues covered in this section.

[7]The classic work of Diewert (1976) covers many of these issues.

A Complication: Non-Additivity

Clearly, the chain aggregation approach greatly alleviates the interpretational problems associated with fixed-weight measures of real output *growth*. Nevertheless, few improvements come without some cost, and the principal problem with chain aggregation is that it makes the interpretation of the *level* of real output more complex. BEA's procedure has been to set real chain aggregates equal to their nominal counterparts in the same base year, b, used to define the published real series for the expenditure components (the $Q_i(t)$s). The published levels of real aggregates are then described as being in terms of "chained year-b dollars." These series need to be interpreted very carefully.

The level of chain-aggregated real GDP reflects the cumulation of period-by-period growth rates, where the growth rates are determined by continuously updated price weights. This level can be best thought of as simply an index number with a reference value that differs from one or one hundred. The "chained year-b dollar" terminology adopted by BEA merely reflects the fact that b was the year chosen to equate real and nominal output. However, this choice of base year is arbitrary and has no effect on the measure of the growth rate of real output. More importantly, this measure of the level of real output *cannot* be interpreted as the cost of output had all prices remained fixed at their year-b levels. This means that, by definition, "chained year-b dollar" real GDP does not equal the simple sum of the real year-b dollar series of its individual components.

This non-additivity may seem a little mysterious to those used to the fixed-weight approach, but its pattern is actually quite simple and intuitive. Note from equation (1) that the growth rate of a chain aggregate will be the same as that of a fixed-weight aggregate if relative prices do not change. But if relative prices are changing, then those products that decline in relative price will have a smaller impact on chained GDP growth after the base year and a larger impact prior to the base year. As quantities of these products tend to grow fastest this means that, in general, chain aggregates will grow slower than their fixed-weight counterparts after the base year, and faster prior to the base year. Because both methods equate real and nominal output in the base year, the difference between the levels of chain-weight and fixed-weight GDP (sometimes rather uninformatively labeled a "chain-weighting residual") follows an inverse-U shape, equalling zero in the base year and becoming more negative as we move away from the base year in both directions.

It is the lack of additivity associated with chain aggregates that invalidates each of the calculations discussed in our introduction. In the next three sections, we will consider each of these calculations in turn. However, before doing so, I should note that while the chain aggregates used in the rest of the paper will generally be of the Fisher ideal form, the non-additivity property holds for all chain index methods, so the key points made should be understood to be general across all chain aggregation methodologies.

3. Addition and Subtraction

Turn to our first example, the calculation of real output growth for the U.S. economy excluding the computer sector for 1996–98. Recall that we first calculated real computer output as the arithmetic sum of real consumption, real investment, real government spending, and real net exports of computing equipment and then subtracted this sum from real GDP.[8]

When using chain-aggregated data, both these steps are incorrect. The first step—simple addition of the separate real expenditure series for computers—is incorrect because the price deflators differ for the separate computer series, so straight addition of the real expenditure series will fail to account for the effect of relative price shifts within the computer bundle on chain-aggregated GDP. However, because the relative price movements between these different computer categories are small, this error is fairly harmless; in other cases, where relative price shifts are important, direct addition of real expenditure series will be a more significant mistake.

The second step—estimating the level of non-computer real output by subtracting real computer output from real GDP—is a far more serious error. Computer prices fell rapidly relative to other components of GDP after 1992 (the base year for these calculations), so that an additional unit of real computer output had a much smaller effect on chain-aggregated real GDP growth over 1996–98 than it would if a fixed-weight methodology been used. As a result, subtracting real computer output from chain-aggregated real GDP (as though this had been constructed from a fixed-weight method) will understate growth in the non-computer sector.

What is the correct way to do this calculation? If Y is a chain aggregate of n components $(X_1, X_2, \ldots, X_n)$ then Y will also be (arbitrarily close to) a chain aggregate of X_1 and the chain aggregate of $(X_2, \ldots, X_n)$. So, the only intuitive meaning of "the real aggregate for Y excluding X_1" is the chain aggregate of $(X_2, \ldots, X_n)$. This means that, in principle, since real GDP is a chain aggregate of a large number of categories, to calculate the real output of the non-computer sector we need to *re-aggregate* but this time excluding the components of computer output. In practice, however, most researchers will not have access to data for all the categories that BEA uses to construct real GDP. And ideally, even if Y is a chain aggregate of n components, we would like only to use information on Y and X_1 when calculating the real growth rate for Y excluding X_1.

Thankfully, some low-cost methods are available that will usually provide good approximations to the results that would be obtained using BEA's full level of detail. Consider, for example, the following "chain-subtraction" procedure that constructs a Fisher index using the real series for Y and $-X_1$ as the quantities and the deflators for Y and X_1 (denoted P and P_1) as the prices

(2)
$$Q(t) = Q(t-1)\sqrt{\frac{(P(t)Y(t) - P_1(t)X_1(t)}{P(t)Y(t-1) - P_1(t)X_1(t-1)}\,\frac{P(t-1)Y(t) - P_1(t-1)X_1(t)}{P(t-1)Y(t-1) - P_1(t-1)X_1(t-1)}}$$

This procedure will generally produce a new series that is almost identical to that obtained from chain aggregating $X_2, \ldots, X_n$. A simpler, although somewhat less precise, method uses the Divisia index approximation to the Fisher formula. The

[8]Technically, this series is an attempt to measure real final sales of the computer industry rather than real output because the investment series does not include the inventory investment of the computer industry, which is not published by the BEA.

growth rate of a Divisia index is a weighted average of the growth rates of its components, where the weights are averages of nominal shares in the current and base periods.[9]

This index provides a very close approximation to the Fisher index. Suppose, then, that we know the real and nominal series for a Fisher chain-aggregate, Y, and one of its components, X, and we want to construct a time series for real Y excluding X (call this series Z.) Using the Divisia formula, we know that the following is approximately true:

$$\frac{\Delta Y_t}{Y_{t-1}} = \theta_t \frac{\Delta X_t}{X_{t-1}} + (1 - \theta_t) \frac{\Delta Z_t}{Z_{t-1}} \tag{3}$$

where θ_t is the average of the ratio of nominal X to nominal Y in periods t and $t-1$. This equation can be re-arranged to arrive at an estimate of the growth rate of Z that will be close to that obtained from re-aggregating the components of Z using the Fisher formula. The level of Z can then be constructed by setting it equal to the nominal series for Y minus the nominal series for X in the base year and chaining forward and back from the base year using the calculated growth rate.

Applying either of these methods reveals that real U.S. GDP excluding computer output grew 3.2 percent on average over 1996–98, compared to 3.8 percent growth for total real GDP. (See the series labeled "Correct calculation" in Figure 2.) Recall that the series obtained from summing the real components of computer output and subtracting from real GDP grew by only 2.1 percent per year over this period. Thus, the spectacular growth in real computer output over this period (60 percent per year) added 0.6 percent per year to aggregate output growth, not the 1.7 percent implied by the incorrect calculation.[10] The upper panel of Figure 2 displays the correctly-calculated growth rate.

Of course, this example was not chosen at random. Chain- and fixed-weight indexes differ because of relative price changes, so the mistake of treating chain-weighted series as though they are fixed-weight series will prove most misleading when the calculations involve categories with large relative price movements. This also means that the categories for which we have to be most aware of the implications of chain aggregation are those upon which the most attention is currently being focused, namely, computers and other types of high-tech equipment, which have declined dramatically in price relative to other types of output.

4. Real Shares

Consider now the second statement above, that the share of information-processing equipment in total real equipment spending was over one-half. This statement was based on the ratio of real 1992-dollar investment in information-processing equipment to the published (chain) aggregate series for real 1992-dollar

[9]Technically, the Divisia index weights log-differences as opposed to growth rates but the differences between these two types of series over short periods are usually very small. See Hulten (1973).

[10]See Tevlin and Whelan (2002) for a discussion of the computer investment boom in the United States in the 1990s.

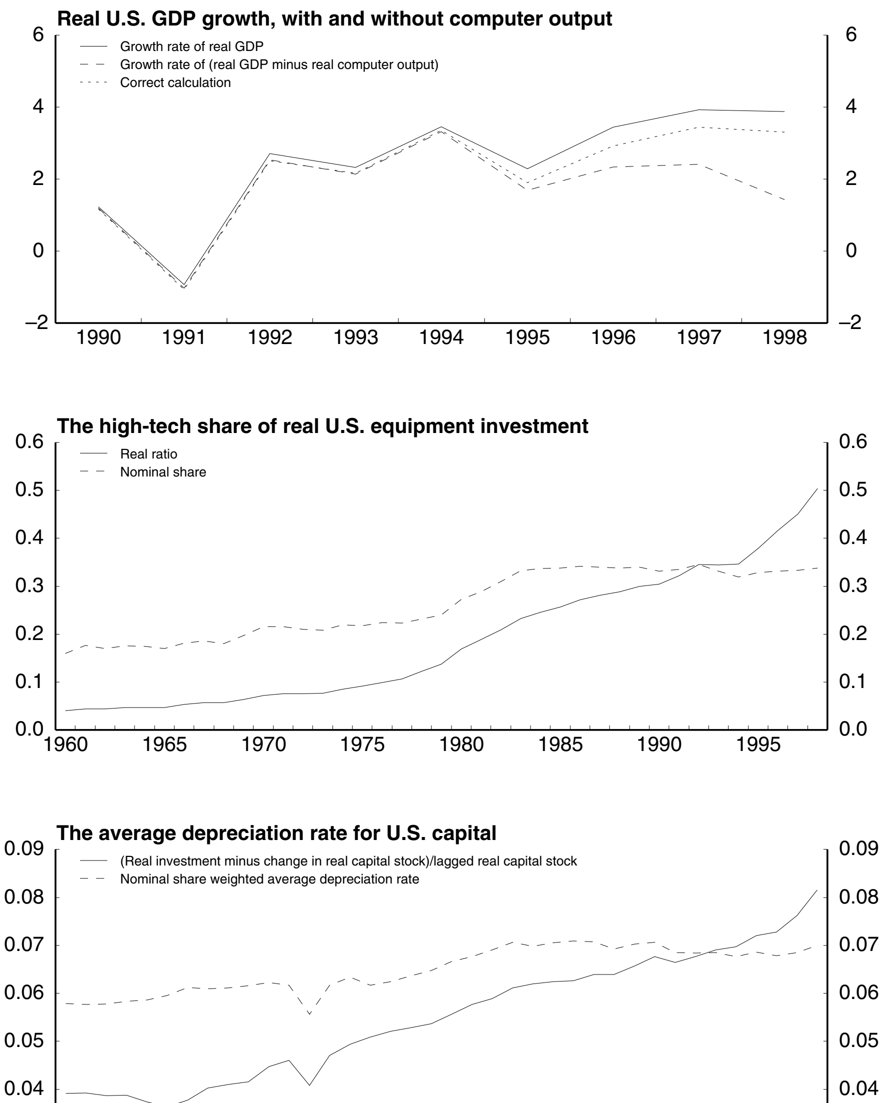

Figure 2. Three misleading (and three correct) calculations.

equipment investment.[11] The first problem here is a simple one: This "share" is not a share at all because the sum of these ratios across all expenditure categories does not equal one.

In this example, the series for aggregate real equipment investment is a chain aggregate of 24 component series for different types of equipment. Summing the

[11]Greenwood and Jovanovic (1999) cite the increase over time in this ratio as evidence of the increasing importance of high-tech investment.

1998 values for the 24 real 1992-dollar equipment series we get a figure that is 15 percent larger than the chain-aggregate. The two series are equal in the base year of 1992, but moving back in time prior to the base year, the sum of the 24 components again gets continually larger relative to the chain-aggregate: For 1960, the sum of the component series is 46 percent higher than the chain-aggregate. This non-additivity explicitly invalidates the interpretation of the ratio of the component series to the chain-aggregate as being a share.

Beyond the simple point that these ratios cannot be interpreted as shares, caution must be generally applied when interpreting these calculations. Obviously, any ratio can be used simply to illustrate how the numerator has grown relative to the denominator. However, if the purpose of the calculation is to show the increasingly important role the numerator (in this case, information-processing investment) plays in the determination of the denominator (the chain aggregate), then it can give a misleading impression to the extent that it fails to capture the changes over time in the weight that the component receives in the calculation of the aggregate.

The inability to calculate real shares with chain-aggregated data could be viewed as a disadvantage. It is important to note, though, that even when using the fixed-weight methodology, real shares are an elusive concept, particularly when they involve components whose relative prices are changing rapidly. The ratio of real year-b dollar output of product i to real year-b dollar fixed-weight GDP answers the following question: "Suppose all prices had remained at year-b's level; what proportion of the total value of this year's output would have been accounted for by the output of product i?" Clearly, the answer depends on the base year chosen. For example, in 1998, investment in information-processing equipment constituted 48 percent of a 1992-based fixed-weight measure of U.S. real equipment investment; because this type of equipment was very expensive in 1960, the corresponding share of a 1960-based measure was 85 percent.

The problems with real share calculations involving categories with fast changing relative prices are likely to be particularly serious when making cross-country comparisons. For example, it is not uncommon to see international comparisons of the share of information technology output in real GDP at a point in time. Such calculations require great care, however. If the U.S. is included, then the simple share calculation is invalid because of the chain-aggregation methodology used to construct U.S. real GDP; for the other countries, even if a share interpretation can be applied, one must be careful to use the same base year for all countries, otherwise it may appear that one country is more IT-intensive than another simply because an earlier base year was used.

That real shares are a problematic concept, particularly with chain-weighted data, may be a little frustrating for those used to performing such calculations. Usually, however, one can answer the question at hand by using one of the following two methods.

Compute Nominal Shares

While inflation has an adverse effect on the use of nominal time series for certain tasks, that doesn't mean they can't ever be used. In fact, if the question is about resource allocation, then nominal shares usually give an intuitive answer. For instance, suppose we want to know what proportion of output is being allocated towards capital investment. The ratio of nominal investment to nominal GDP gives a much cleaner answer than the corresponding real ratio: the nominal ratio tells us very simply what fraction of each dollar spent is allocated to purchasing investment goods.

Nominal shares can also help correct some misleading impressions that real ratios may give about the changing role of information technology in the economy. While the ratio of real 1992-dollar information-processing investment to aggregate real equipment investment goes from 0.07 in 1970 to 0.50 in 1998, the corresponding nominal ratio only changes from 0.22 to 0.34 over the same period (see the middle panel of Figure 2). This shows that, in terms of actual dollars spent, the increase in the role of information technology has been more modest than one might think.[12] Information technologies may have been extremely expensive in 1970, with large nominal expenditures buying small amounts of computing power; nevertheless, many firms were aware of the use of these technologies and were willing to allocate significant fractions of their capital spending budgets to them.

Compute Contributions to Growth

Sometimes, we want to calculate a "real share" for a particular category to show that it has contributed more (or less) to the growth of a real aggregate than other categories. However, there are other ways to do this calculation for chain aggregates. For example, Ehemann, Katz, and Moulton (2001) have shown that the growth rate of a chain aggregate can be expressed as

$$\frac{\Delta Q(t)}{Q(t-1)}) = \sum_{i=1}^{n} \frac{(P_i(t-1) + P_i(t)/\Pi(t))\Delta Q_i(t)}{[\sum_{i=1}^{n} (P_i(t-1) + P_i(t)/\Pi(t))Q_i(t-1)]} = \sum_{i=1}^{n} c_i(t) \tag{4}$$

where $\Pi(t)$ is the aggregate Fisher price index in period t relative to period $t-1$ (that is, the gross growth rate of the GDP deflator at time t). This equation decomposes the growth rate of a chain aggregate into the contributions due to the change in the quantity of each component (the $c_i(t)$s).

These contributions to growth are very useful because they can correct the potentially misleading impressions given by ratios of real expenditure series; for instance, if the relative price of a product is falling, then the ratio of real expenditures for the product to real GDP could be accelerating while its contribution to real GDP growth is not changing. Contribution series are also easily available because BEA now publishes them for all major categories in the *Survey of Current Business*. So, while we may not be able to add up the $Q_i(t)$s to obtain the *level* of the real aggregate, we can easily add the $c_i(t)Q(t-1)$ terms to obtain the *change* in the aggregate.

Because we know that $\Delta Q(t) = \sum_{i=1}^{n} c_i(t)Q(t-1)$, this method can also be used to calculate the contribution of a particular category to the change in a real chain

[12]As a result of the introduction of software as a capital asset in the October 1999 comprehensive revision, the most recent NIPA data show this share increasing from 0.24 in 1970 to 0.44 in 1998.

aggregate over longer periods of time. For instance, returning to our information-processing example, one can use this method to show that increases in real investment in information-processing equipment accounted for 51 percent of the increase in real aggregate equipment investment between 1960 and 1980, and that this contribution moved up to 63 percent for 1980 to 1998. So, while the simple real ratio quoted above is somewhat misleading, it is still true that increased real spending on information-processing equipment has accounted for most of the growth in aggregate real equipment investment since 1960.

5. A More Involved Example

Our first two examples involved assessing the influence of component series on the behavior of real expenditure aggregates. But these are not the only calculations requiring care when using chain-aggregated data. Often, calculations involving aggregate data rely implicitly on an assumption of additivity that does not hold for chain-aggregates, and this can result in misleading inferences.

Consider the third statement above, about the average depreciation rate for the U.S. capital stock. This parameter features in most empirical work on dynamic general equilibrium models in macroeconomics, and is usually estimated as described above by re-arranging a "law of motion" or perpetual inventory equation with real aggregates for investment and the capital stock.[13] In other words, given the chain-aggregates for investment and capital, I_t^{cw} and K_t^{cw}, the aggregate depreciation rate, δ_t^{cw}, is calculated as the series that makes the following equation hold:

$$K_t^{cw} = (1 - \delta_t^{cw})K_{t-1}^{cw} + I_t^{cw} \tag{5}$$

Specifically, this is

$$\delta_t^{cw} = \frac{I_t^{cw}}{K_{t-1}^{cw}} - \frac{\Delta K_t^{cw}}{K_{t-1}^{cw}} \tag{6}$$

Using this method, we concluded that the average depreciation rate for capital more than doubled over the past 40 years, going from 4.0 percent in 1960 to 8.3 percent in 1997 (see the bottom panel of Figure 1). However, it turns out that this calculation gives a misleading picture.

The NIPA capital stock used in this calculation is a chain aggregate of stocks of a large number of underlying categories of equipment and structures. These are each constructed according to a perpetual inventory equation ($K_t^j = (1 - \delta_j)K_{t-1}^j + I_t^j$) with separate constant depreciation rates for each category.[14] Nominal stocks are defined for each category on a "current-dollar replacement value" basis and are obtained by inflating each real stock by the relevant current-period investment deflator.

[13]For instance, Cooley and Prescott (1995) is a commonly cited paper on calibration of dynamic stochastic general equilibrium models that uses this method. Of course, the capital stocks used in that paper were fixed-weight aggregates. The specific problem discussed here concerns applying this method to the current published chain-aggregated capital stocks.

[14]See Katz and Herman (1997) for a full discussion of the NIPA capital stock methodology.

Obviously, one can create fixed-weight aggregates from these real capital stock and investment series by simple addition ($K_t^{fw} = \sum_{j=1}^{n} K_t^j$ and $I_t^{fw} = \sum_{j=1}^{n} I_t^j$). And the depreciation rate backed out from a perpetual inventory equation using K_t^{fw} and I_t^{fw} is a weighted average of the underlying depreciation rates, where the weight for each category is its share in the fixed-weight capital stock. In other words,

$$K_t^{fw} = (1 - \delta_t^{fw}) K_{t-1}^{fw} + I_t^{fw} \tag{7}$$

where

$$\delta_t^{fw} = \sum_{j=1}^{n} \frac{\delta_j K_{t-1}^j}{K_{t-1}^{fw}} \tag{8}$$

When using chain-aggregated investment and capital stocks, however, this intuition does not hold. The series δ_t^{cw} cannot be interpreted as a weighted average of the underlying rates. To see why, consider a simple steady-state example.

Suppose there are two types of capital, *A* and *B*, which depreciate at the same rate, δ. Type-*A* capital falls in price relative to type-*B* capital at a steady pace: This can occur, for example, if we have a two-sector model in which the sector producing good *A* has a faster pace of technological progress than the sector producing good *B*.[15] Now also assume that the price elasticity of demand for *A* and *B* is one, and that the elasticity of demand relative to all other variables is the same for both *A* and *B*. These assumptions imply that the real capital stock and real investment for *A* grow faster than their type-*B* counterparts, but that *A* and *B*'s shares in the nominal capital stock and in nominal investment are constant.

Re-arranging the perpetual inventory equations for capital of type *A* and *B*, we know that the faster capital stock growth for *A* implies the ratio of real investment to real capital stock is higher for *A* than for *B*:

$$\frac{I_t^A}{K_t^A} > \frac{I_t^B}{K_t^B} \tag{9}$$

By definition, this implies the ratio of nominal investment to nominal capital stock is greater for *A* than for *B*:

$$\frac{P_t^A I_t^A}{P_t^A K_t^A} > \frac{P_t^B I_t^B}{P_t^B K_t^B} \tag{10}$$

This also means the share of type-*A* capital in nominal investment is higher than its share in the nominal capital stock:

$$\frac{P_t^A I_t^A}{P_t^B I_t^B} > \frac{P_t^A I_t^A}{P_t^B K_t^B} \tag{11}$$

Recalling our discussion above about the Divisia index, we know that Fisher chain aggregates approximately weight the growth rates of their component series

[15]Whelan (2002a) develops a two-sector general equilibrium model of this type, and shows how it can be fitted to U.S. chain-aggregated data.

by their nominal shares. Because investment and capital grow faster for A than for B, and because the share of A in nominal investment is bigger than its share in the nominal capital stock, this implies that *the chain-aggregate for real investment will always grow faster than the chain aggregate for the real capital stock*.

Now, since K_t^{cw} grows at a constant rate in this example, equation (6) tells us that δ_t^{cw} will get larger every period because I^{cw} always grows faster than K^{cw}. This occurs despite the fact that both types of capital depreciate at the same constant rate δ. More generally, if we allowed the two types of capital to have different rates of depreciation, then δ_t^{cw} would only be a weighted average of the underlying depreciation rates in the base year; moving away from the base year this measure could eventually be higher or lower than each of the underlying depreciation rates.

Obviously, this example is somewhat artificial but it captures an important aspect of reality. The shares in nominal investment of those components that have grown fastest in real terms, such as computers, have indeed been significantly higher than their shares in the nominal capital stock, so the pattern described in this example is empirically important. An alternative way to measure the average depreciation rate is to take a weighted average of depreciation rates of the underlying categories using shares in the nominal capital stock as weights; this is appropriate because the growth in the aggregate real capital stock is approximately a nominal-share weighted average. This method confirms that the calculation using chain aggregates is fairly misleading: It shows a much more modest increase in the average pace of depreciation, from 5.8 percent in 1960 to 7.0 percent in 1997 (see the bottom panel of Figure 2).

This example illustrates a general principle: manipulating chain-aggregated series under the assumption that these series were constructed from traditional additive formulae can produce misleading conclusions. In this case, the problematic calculation came from the assumption that one could characterize the chain-weighted capital stock using the traditional additive law of motion, equation (5). While intuition may tell us that the aggregate real capital stock is the sum of aggregate real investment and depreciated aggregate real capital from last period, this identity actually only holds at a disaggregated level. In fact, under the stylized two-sector conditions of this example, the chain aggregate for investment can grow faster than the chain aggregate for the capital stock *ad infinitum*, with the level of aggregate real investment potentially becoming larger than the level of the aggregate real capital stock.

6. Conclusions

The adoption of the Fisher chain procedure for creating real aggregates has removed many of the anomalies previously associated with U.S. NIPA data on aggregate real stocks and flows. However, it has also introduced some complexities into macroeconomic data analysis, and these complexities have led to a proliferation of mistaken calculations by business and academic economists. This paper has used some simple examples to illustrate the problems that can arise when using chain-aggregated data and to suggest some alternative ways of manipulating these data.

One important conclusion is that researchers need to be particularly aware of the implications of chain aggregation when assessing the role of information technologies in the U.S. economy, and when making comparisons with other countries. U.S. price indices for high-tech products have fallen rapidly relative to other components of GDP and chain aggregates differ most from their traditional fixed-weight counterparts when there are large shifts in the relative prices of their components. Without taking care to handle aggregate series in a manner consistent with their construction, it is easy to mistakenly assign too important a role to the high-tech sector in the recent behavior of investment and output. This is not to say that this sector has been unimportant in the U.S. economy's recent performance. In fact, using growth accounting techniques, Jorgenson and Stiroh (2000), Oliner and Sichel (2000), and Whelan (2002) all concluded that the high-tech sector was central to the acceleration in productivity during the latter half of the 1990s.

Finally, I should note that the calculations in this paper using 1992-based series are a little out of date. These data were current in September 1999 but since then, BEA has published a comprehensive revision to the NIPAs. This revision incorporated some definitional changes such as the inclusion of software as capital asset and also updated the base year for all real series to 1996.[16] A consequence of moving the base year forward is that if one performs the incorrect calculations described in this paper using the most current NIPA data, the errors for years close to 1996 will now be smaller than those described in this paper using the 1992-based data. However, the inclusion of software (prices for which have been declining on average) and the ongoing attempts to better capture quality improvements in published price series make it likely that relative price movements will become more important in the NIPAs in future. Thus, as we move further away from the new base year, calculations using real NIPA data will have to account for chain aggregation because future mistakes may prove even more misleading than those discussed here.

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[16]See Moulton, Parker, and Seskin (1999) for discussion of the changes in the comprehensive revision.

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