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POLARIZATION BY SUB-POPULATIONS IN SPAIN, 1973–91

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We use an extension of the Esteban and Ray (1994) approach to polarization in order to analyze the role of different household characteristics in the formation of groups in Spanish expenditure distribution, e.g. educational level, position in the labor market, and region. According to a first approach we assume that groups are determined by a characteristic that their members share, and we study which gives rise to a higher level of polarization. In a second approach we can also investigate which characteristics better explain an observed level of polarization, assuming that income proximity determines the group to which one belongs. In both cases we take into account the effect of social stratification on polarization.

1. INTRODUCTION

The measure of the extent to which distribution is polarized has been the issue of different contributions in economic analysis. Wolfson (1994, 1997) and Esteban and Ray (1994)—*ER* hereafter—have independently conceptualized the notion of polarization and proposed the corresponding indices. This notion is closely related to inequality; however its development emerged as a result of some dissatisfaction in the use of standard inequality measurement to deal with the formation of groups in a society. The reason is that, as Esteban and Ray stressed: *the axioms of inequality measurement (or equivalently second-order stochastic dominance for mean normalized distributions) fail to adequately distinguish between "convergence" to the global mean and "clustering" around local means* (Esteban and Ray, 1994, p. 821). While the first aspect mentioned by Esteban and Ray reduces inequality and polarization, the second augments polarization in a society displaying decreasing inequality. The divergence between both notions lead Wolfson to maintain that *in order to capture the concerns of the general public, summary measures based on concepts like polarization should be given equal space along with Lorenz-consistent inequality measures when describing trends in income distribution* (Wolfson, 1994, p. 356).

A recent paper by Esteban, Gradín and Ray (1999)—henceforth, *EGR*—has addressed a statistical approach to measuring polarization which extends *ER* in such a way as to deal with distributions not necessarily pre-arranged in groups. In these approaches it is implicitly assumed that groups are defined as income classes. In this paper we explore two distinct ways of extending *EGR* sharing a similar motivation. The starting point is that every individual, together with his own income is endowed with a vector of other characteristics or attributes, such

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as his educational level, race, labor category among others. There exist at least two reasons for which these attributes may be relevant for polarization.

On the one hand, we can assume that despite polarization occurring in the income space, groups in the distribution are the result of similarities with respect to a relevant attribute other than income, such as, for instance, race or education. Thus, we treat the distribution as if it were the aggregate result of more than one stochastic process. Different assumptions on which these stochastic processes are based will lead to different levels of polarization. On the other hand, even if we assume groups as income classes, we may concern ourselves with the extent to which their members are similar not only on the basis of income but also on the basis of other relevant attributes, as it is possible that there exists a high correlation between income and other characteristics. Then again, several individuals might fall into an income class precisely because they share a given similarity according to another attribute. In this case we would say that this attribute explains the observed level of polarization or at least is highly correlated with its true causes.

The first approach will be referred to as *group polarization* and the second as *explained polarization*. Both approaches identify possible divisions. The consequences for political purposes of a given level of group polarization depend on the extent to which the characteristic is socially or politically relevant. One can find a distribution increasingly polarized with respect to one characteristic but not to another. Neither of these two approaches allows us to isolate the impact on polarization of any characteristic, as they will exhibit a high degree of correlation, but they both allow us to identify which are the most relevant as well as to describe particular trends for each characteristic.

In what follows we will present both extensions and provide the analysis of Spanish expenditure distribution between 1973 and 1991. The following section presents the approach in *EGR*. Group polarization will be presented in Section 3, and explained polarization in Section 4. Section 5 deals in more detail with the identification term and the relationship between polarization and stratification. Section 6 provides the analysis of the Spanish case and the last section recapitulates the main conclusions.

2. The Measurement of Polarization in *EGR*

Let us consider a particular distribution defined by a density f or its respective cumulative density F over the bounded support $[a, b]$ with mean income normalized to be $\mu = 1$. As in the *ER* approach we assume that each member of the distribution with income x feels some degree of *group identification* with members of his own group and *alienation* from those with income y belonging to a different group. The interaction between both feelings gives rise to the *effective antagonism* that individual y feels towards x, increasing in both terms such that a higher degree of intra-group identification reinforces the effect of alienation. Polarization is defined as the addition of all effective antagonism in the distribution. The approach in *ER* furthermore states a set of axioms restricting the functional forms of the identification and alienation functions, leading to their measure of polarization which is described below. However, this characterization of *ER* needs the

distribution to have been previously pre-arranged into groups, raising the question of a substantial loss of information about the intra-group distribution of income.

The approach proposed in *EGR* is an extension of *ER* which preserves the same notion of polarization but makes it more operative regardless of how data is presented, and in any case incorporates the information about the intra-group distribution of income. Therefore it is assumed that the attribute which determines the area where antagonism occurs, income in our case, determines also the allocation of individuals to a particular group. Then the groups should be understood as income classes.

Let the collection of numbers $\rho = (z_0, z_1, \ldots, z_k;\ y_1, \ldots, y_k;\ p_1, \ldots, p_k)$ be any non-intersecting partition of F with $a = z_0 < \cdots < z_k = b$, and y_i and p_i indicating respectively groups' conditional means and population shares. This partition defines groups as intervals of incomes $[z_{i-1}, z_i]$ for $i = 1, \ldots, k$.

From a statistical view, the k-spike distribution ρ is a representation of F which induces an error of approximation $\varepsilon(F, \rho)$, representing the lack of identification within the groups, as there exists some internal dispersion. Polarization in ρ as measured by the index $ER(\alpha, \rho)$, is:[1]

$$ER(\alpha, \rho) = \sum_{i=1}^{k} \sum_{j=1}^{k} p_i^{1+\alpha} p_j |y_i - y_j| \tag{1}$$

where α is the sensitivity to polarization and falls—in order to be consistent with a set of axioms—in the interval $[1, 1.6]$[2]. Then the measure proposed in *EGR* defines polarization given by F subtracting the error from (1):

$$P(F; \alpha, \beta, \rho) = ER(\alpha, \rho) - \beta\varepsilon(F; \rho) \tag{2}$$

where $\beta \geq 0$ indicates the weight assigned to the error in the representation. The case $\beta = 0$ leads to the particular case of *ER*.

The crucial aspect for the implementation of (2) is to obtain the accurate k-spike representation ρ choosing both, the *number of groups* and their *locations*. In *EGR* given an exogenous k, their locations are endogenously determined so as to minimize the error, which is expressed as the average of income distances within all groups. Thus, it can be expressed as the inequality in F minus the inequality in its representation ρ as measured by the Gini coefficient G:

$$\varepsilon(F; \rho) = G(F) - G(\rho). \tag{3}$$

Expressing (2) for the optimal collection $\rho^* = (z_0^*, z_1^*, \ldots, z_k^*;\ y_1^*, \ldots, y_k^*;\ p_1^*, \ldots, p_k^*)$ that solves this problem and leads to the measure *EGR* finally proposed to account for polarization in F:

$$P(F; \alpha, \beta, \rho^*) = ER(\alpha, \rho^*) - \beta[G(F) - G(\rho^*)]. \tag{4}$$

[1]For a different solution to account for intra-group heterogeneity, see D'Ambrosio (2000), who proposed using *ER* applied to a different notion of distance, the Kolmogorov measure of variation distance between sub-distributions.

[2]The smaller the sensitivity to polarization, the closer the notion of polarization to inequality. Indeed, if $\alpha = 0$ the index is a scalar transformation of Gini.

In the next two sections we will explore two distinct ways of extending *EGR* in such a way as to take into account the fact that different individual or households characteristics might influence the way they are inserted in a society, and then explain polarization.[3]

3. First Approach to the Analysis by Sub-Populations: Group Polarization

Consider now any characteristic, e.g. race, region, or occupation, yielding an exhaustive partition of the whole population into n groups or sub-populations, where the number depends on the nature of the characteristic. This partition can be described by a collection of numbers $\rho^c = (q_1, \ldots, q_n; m_1, \ldots, m_n)$, where q_i indicates the population share in group i and $m_1 \leq m_2 \leq \cdots \leq m_n$ indicate respective groups' average incomes. The distribution of each sub-population is given by the share of people in the i-th group with income at least y, $F_i(y)$, $i = 1, \ldots, n$, such that for any income y:

$$F(x) = \sum_{i=1}^{n} F_i(x) q_i \tag{5}$$

and for each i:

$$m_i = \int_a^b x dF_i. \tag{6}$$

In order to obtain the level of polarization associated with the characteristic, we define, in parallel to (2), the notion of *group polarization GP* to be the level of polarization encountered when we use the exogenous partition ρ^c to represent F. The collection ρ^c varies from ρ in as much that groups are now exogenously conformed according to whether their members share the same category for a given characteristic regardless of their income proximity, so that groups are not necessarily income intervals. Since the representation varies from the optimal, we now expect higher intra-group dispersion and, as a likely consequence, lower between-groups heterogeneity. It will be possible to find negative values in (2). This should not be a problem as the index still allows us to order distributions and to identify the intensity of differences in polarization levels. In order to make the interpretation of the results easier, we normalize the index P to have a non-negative range.[4] The measure is defined to be:

$$GP(F;\ \alpha, \beta, \rho^c) \equiv P(F;\ \alpha, \beta, \rho^c) - (-\beta) = ER(\alpha, \rho^c) - \beta[\varepsilon(F; \rho^c) - 1]. \tag{7}$$

Where the error term is expressed in parallel to (3) as:[5]

$$\varepsilon(F; \rho^c) = G(F) - G(\rho^c) \tag{8}$$

[3]We refer to *EGR* for details. In particular it is interesting to note that in the case of 2 groups this approach leads to Wolfson's measure as a particular case.

[4]The minimum in the case of exogenous groups is obtained when there is no polarization between the groups together with maximum intra-group inequality, then in that case $P = -\beta$.

[5]We adopt this view, rather than being consistent with the original definition in terms of the average distance within the groups, which would lead to a weighted sum of Gini within all groups, as we claim that overlapping is relevant for identification within a group.

and accounts for both intra-group inequality as well as overlap between groups as Section 5 describes in more detail. As in *EGR*, the case $\beta = 0$ results in *ER* being applied to the grouped distribution.

Expression (7) resembles a decomposition of an inequality index in inter-group and intra-group terms (see Shorrocks, 1980 and subsequent literature). However, the index deviates from that literature not only in the sign of the effect of intra-group dispersion, but also in the fact that it is not a decomposition. The final level of polarization is indeed different regarding the rule attaching individuals to groups. A careful specification of the categories for each characteristic is needed for a correct implementation of the index. It will be sensitive to the number of categories for which the characteristic is expressed. In particular, the smaller the number, the larger we expect both terms in the index, so the net effect is not clear at all. The most relevant characteristics will be those showing at the same time much polarization between the groups and homogeneity within them.

4. Second Approach to the Analysis by Sub-Populations: Explained Polarization

There is no reason to assume that we know which is the most relevant characteristic yielding the groups in the distribution. Indeed, several may be interacting, and income may be viewed as a proxy of all those which are relevant. Furthermore it is usual in economic analysis to split population into income intervals according to some income threshold and speak of the poor and rich for instance, or of the middle class vs. the bottom and rich ones. Therefore it still makes sense to assume that income proximity is what determines the group to which one is ascribed. Then we want to know which characteristics better explain antagonism in society, given that individuals may fall into a particular interval-group precisely because they are similar according to any relevant characteristic.

Given the cut-offs $z_0, z_1, \ldots, z_k$ and the n-spike representation ρ^c, let $\phi_j \equiv \{i/m_i \in [z_{j-1}, z_j]$ indicate for each $j = 1, \ldots, k$ those members of groups in ρ^c with average income m_i lying in the interval $[z_{j-1}, z_j]$. Then we define a new k-spike representation of F according to the given characteristic as the collection of numbers $\rho^+ = (z_0, z_1, \ldots, z_k; r_1, \ldots, r_k; g_1, \ldots, g_k)$ such that:

$$r_j = \sum_{i \in \phi_j} q_i, \qquad g_j = \frac{1}{r_j} \sum_{i \in \phi_j} q_i m_i, \tag{9}$$

stand respectively for the j-th population share and conditional mean.

For a given ρ, we will say that a characteristic explains the level of polarization $P(F; \alpha, \beta, \rho)$, so long as the alternative partition ρ^+—based on the characteristic and having the same number of groups determined by the choice on the *ER* polarization measure—generates the same level of polarization. Let us denote by EP^+ the level of polarization recorded when we represent F by ρ^+:

$$EP^+(z, \alpha, \beta) \equiv ER(\alpha, \rho^+) - \beta\varepsilon(F; \rho^+) \tag{10}$$

where

$$\varepsilon(F; \rho^+) = G(F) - G(\rho^+). \tag{11}$$

To account for the share of observed polarization that the characteristic actually explains, we can compare how well EP^+ performs compared to $P(\cdot)$ which we want to explain. Denoting by $EP^{+\min}$ the minimum level possible given F, this gives rise to the index EP defined in the following form:

$$EP(z, \alpha, \beta) \equiv \frac{EP^+ - EP^{+\min}}{P(F; \alpha, \beta, \rho) - EP^{+\min}}. \tag{12}$$

The higher the EP the larger the share a given characteristic explains, but the sum of EP for different characteristics exaggerates their influence so long as there exists some correlation between them. EP will equal 1 whenever $\rho^+ = \rho$ and it will equal 0 when there is no polarization between the groups by the characteristic, $m_i = \mu$ for every i. Given that $EP^{+\min} = -\beta G(F)$, the index can be rewritten as:

$$EP(z, \alpha, \beta) \equiv \frac{EP^* + \beta G(F)}{P(F; \alpha, \beta, \rho) + \beta G(F)} = \frac{ER(\alpha, \rho^+) + \beta G(\rho^+)}{ER(\alpha, \rho) + \beta G(\rho)}. \tag{13}$$

For the special case of bipolarization, given $p = F(z)$ and r the share of population in groups with average below z:

$$EP(z, \alpha, \beta) = \frac{[r^\alpha + (1-r)^\alpha + \beta] G(\rho^+)}{[p^\alpha + (1-p)^\alpha + \beta] G(\rho)}. \tag{14}$$

Figure 1 provides an example for the extreme cases. And for $\alpha = 1$ this expression becomes simplified to:

$$EP(z, \alpha = 1) = \frac{G(\rho^+)}{G(\rho)}, \tag{15}$$

which is independent of β. In this case, the index EP will always fall in the interval [0, 1].[6]

Furthermore, for $D(\,)$ being the relative mean deviation for the corresponding distribution, if we focus on the optimal partition ρ^* then as $z^* = \mu$, $G(\rho^*) = D(\rho^*) = D(F)$ and $G(\rho^+) = D(\rho^c)$. As a consequence, the index is expressed in

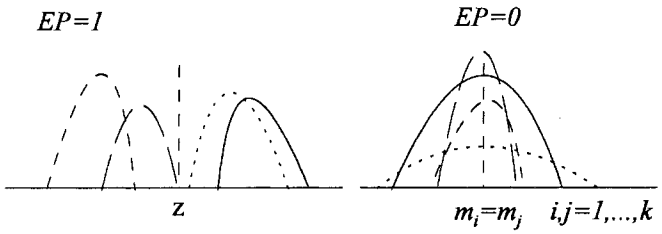

Figure 1. Extreme Cases of EP for Bipolarization and $n = 4$

[6]This is true because in this case minimizing the error term is equivalent to maximizing polarization in (10). In the other cases, though improbable, it is possible to find EP^+ higher than P, and then EP higher than 1.

terms of the D, comparing inter-group inequality and overall inequality for that index:

$$EP(z^* = \mu, \alpha, \beta) = \frac{[r^\alpha + (1-r)^\alpha + \beta]D(\rho^c)}{[p^\alpha + (1-p)^\alpha + \beta]D(F)}. \tag{16}$$

$$EP(z^* = \mu, \alpha = 1) = \frac{D(\rho^c)}{D(F)}. \tag{17}$$

In order to identify whether a given change in EP was due to the effect of changes in the population shares, "distribution effect," or, in the average incomes, "mean effect," we can compute the index fixing m_i and q_i respectively at their values for the reference distribution.

The following example will clarify the differences between explained polarization and the other notion of polarization, group polarization, introduced in the previous section. A population exhibits incomes $T = \{2, 2, 6, 6, 6, 8\}$ such that it may be partitioned into three sub-populations: white $W = \{2, 6, 6\}$, Asian $A = \{8\}$ and black people $B = \{2, 6\}$. Now implement a distributive change transferring 4 units of income from the black 6 towards the white 2. The new racial distribution is given by $B = \{2, 2\}$, $A = \{8\}$ and $W = \{6, 6, 6\}$.

So far as the first approach is concerned, *racial polarization* over this distribution should increase as we face a more stratified population with groups displaying less internal dispersion and increasing distance between the two largest.

For the relevance of the second approach, so long as we see the distribution split into rich and poor income classes, the level of bipolarization is kept unchanged by the permutation of incomes. However, note that after the permutation the poor are all black, while the non-poor are all non-black if we set the cut-off income between 2 and 6. It seems reasonable to say that race better explains the constant level of bipolarization. The way in which both approaches might diverge will be clarified in the empirical analysis.

5. Polarization, Inequality and Stratification

In this section we will show how overlapping enters polarization in (7) and (13), through the identification of groups when they overlap.[7] We can rewrite the error term in the following way:[8]

$$\varepsilon(F, \rho^c) = \sum_{i=1}^{n} s_i G(F_i) I_i \tag{18}$$

where for each group i, $G(F_i)$ represents Gini index and s_i the share of income for that group, I_i is an index measuring the degree to which sub-population i overlaps with the other groups. This overlapping index is very close to the index

[7]Sartori (1966) employs overlapping jointly with average distance to approach polarization of political parties. Deprivation theory in Yitzhaki (1982) also deals with the effect of overlapping or stratification on sustainable inequality.

[8]The same can be undertaken for $\varepsilon(F; \rho^+)$ where i would now indicate each subset of groups for each of the k income classes in ρ^+. If we reject the sensitivity of identification to the degree of overlap, one could replace (18) with half the average of all income distances within the groups.

proposed in Yitzhaki (1994) and is also obtained in a similar way but through the more classical breakdown of Gini index (Bhattacharia and Mahalanobis, 1967 or Pyatt, 1976 among others).[9] Then we define *Overall Overlapping of group i* as the weighted sum of overlapping with respect to every group in the population, weights being respective population shares:

$$I_i = \sum_{j=1}^{n} I_{ij} q_j \tag{19}$$

where I_{ij} is the *overlapping index* for the i-th group with respect to group j which represents a share q_j of population, and is expressed as follows:

$$I_{ij} = \frac{\int_0^\infty \int_0^\infty |x-y| dF_j(y) dF_i(x) - |m_i - m_j|}{\int_0^\infty \int_0^\infty |x-y| dF_i(y) dF_i(x)} \equiv \frac{d_{ij}}{d_{ii}} \equiv \frac{d_{ij}}{m_i G(F_i)}. \tag{20}$$

Note that $I_{ji} \neq I_{ij}$ unless both share the same absolute Gini. This index satisfies the following properties, given $m_j \geq m_i$: (1) I_{ij} and I_{ji} are non-negative and unbounded.[10] They are equal to 0 if and only if there is no overlap between both groups, and by definition $I_{ii} = 1$. (2) The larger the share of people in j with incomes below the richest person in i, the higher the I_{ij}. The larger the share of people in group i with incomes above the poorest person in j, the higher the I_{ji}. (3) I_{ij} is a decreasing function of the income of that fraction of population of group j with incomes below the richest person in i. I_{ji} is an increasing function of the income of that fraction of population of group i with incomes above the minimum in group j. (4) Given the distribution of i, I_{ij} reaches its maximum if all income in the group j is concentrated on one individual. For a given characteristic, we can aggregate the extent to which different sub-populations overlap in the distribution defining *Overall Overlapping I* as the weighted average of overlapping indices for all sub-populations:

$$I = \sum_{i=1}^{n} I_i q_i. \tag{21}$$

6. Polarization in Spain, 1973–91

The present study uses data from the Spanish Household Expenditure surveys, *Encuestas de Presupuestos Familiares* conducted by the Instituto Nacional de Estadística (INE) for the years 1973–74, 1980–81 and 1990–91. Each of these surveys, the best data source for Spanish income distribution, contains information from more than 20,000 households about their earnings, expenditure and relevant characteristics.

[9]Its interpretation in terms of the Lorenz curve is described in Lambert and Aronson (1993). This term is given by the intensity of permutations from the original distribution F—with individuals ranked by their income—which we need to consider to reach that distribution where people are first ranked by groups' mean incomes, and then by their own income. This can be represented graphically by the area between the concentration curve of this permuted distribution and the Lorenz curve for F.

[10]See Gradín (1999a) for details.

To represent the standard of living we will refer to expenditure including imputation for housing rents, self-consumption, and others.[11] Using individual household price indices and OECD equivalence scales we build *household equivalent spending at constant prices of the winter of 1981.*[12,13]

The characteristics of households considered in this study are the following. The size of the municipality where the household lives according to the number of inhabitants: less than 2,000; between 2,000 and 10,000; between 10,000 and 50,000 and more than 50,000. For less than 50,000 the municipality is classified as rural and for more it is considered as urban. The *comunidades autónomas* (autonomous regions) and province of residence, excluding the very small ones of Ceuta and Melilla, which are not in the first survey, comprise 17 autonomous regions and 49 provinces. Household size measured as the number of members and household composition broken down into 8 categories according to the number of adults and children. For the householder we consider the educational level attained, 8 categories from illiterate to advanced education, the relationship with economic activity, distinguishing between working, unemployed and type of inactivity—retired, rentiers or others—professional position according to the last occupation, employer, entrepreneur without employees, employees and inactive—or looking for first job—and the socio-economic condition broken down into 13 categories according to the occupation and work category.[14] We also consider the gender of the householder, ten categories according to age and gender, and the migration condition based on whether the household has lived in the same municipality for more than 5 years.

To investigate polarization in Spain we first compute the level of polarization when the proximity in expenditure levels is the sole criterion to allocate households in a group; thus we obtain those groups—two or three—such that we minimize the error we make given by expression (3). This level of polarization is given by the index $P(F; \alpha, \beta, \rho^*)$ in (4) where we have considered incomes in logs for the index ER.[15] In a second stage we look for the determinants of these levels of polarization, identifying which characteristics give rise to these groups, and computing the normalized index of explained bipolarization EP, applied also to incomes in logs. In a third stage we inspect an alternative way to build the groups,

[11]In these surveys expenditure is more reliable than earnings. Indeed, aggregate savings are negative and earnings are misrepresented, especially in higher incomes. Consider that according to our results and others, income performs a more equal distribution than spending, while savings are expected to be allocated to richer households. A study conducted to check the consistency of aggregate expenditure and income with National Accounts such as that of Sanz (1996) confirms this view.

[12]These price indices are borrowed from J. Ruiz-Castillo and M. Sastre.

[13]OECD equivalence scales assign a weight 1 to the first adult, 0.7 to each additional adult, and 0.5 to each child—here less than 14 years in order to allow comparability between years.

[14]The categories are: agrarian entrepreneurs with employees; agrarian entrepreneurs without employees and members of agrarian co-operatives; agrarian managers and executives of agrarian firms and qualified agrarian staff; the remaining agrarian workers; non-agrarian entrepreneurs with employees, and professionals with/without employees; non-agrarian entrepreneurs without employees, and self-employed persons; non-agrarian managers and executive staff; non-agrarian employees, sellers and intermediate staff; non-agrarian foremen and similar; non-agrarian workers and the remaining service workers; professionals in armed forces; inactive and finally non-classified.

[15]The reason is that in our view in the two-spike distribution for a given distance $y_2 - y_1$ polarization should be at a maximum when both groups have the same size, $p = 1 - p = 1/2$. This property is satisfied by log-incomes and not by the ratio of incomes to the global mean.

using directly the information regarding characteristics. We compute polarization for each characteristic so as to describe how polarization has evolved had each one been the relevant one, as well as identifying which of them gives rise to the highest polarized distribution.

6.1. *EGR polarization*

Using the optimal partition ρ^*, polarization according to *EGR* has substantially decreased in Spain for the period under consideration 1973–91, with more

TABLE 1

POLARIZATION AND INEQUALITY IN SPAIN, 1973–91
Index $P(F;\ \alpha=1,\ \beta=1, \rho^*)$

	2 Groups			
Year	P	Confidence Interval (95%)	ER	ε
1973–74	0.138	[0.136, 0.140]	0.242	0.104
1980–81	0.130	[0.128, 0.132]	0.225	0.095
1990–91	0.124	[0.122, 0.125]	0.215	0.091
	3 Groups			
Year	P	Confidence Interval (95%)	ER	ε
1973–74	0.160	[0.158, 0.162]	0.208	0.048
1980–81	0.147	[0.145, 0.148]	0.191	0.044
1990–91	0.139	[0.137, 0.140]	0.181	0.042
Year	Gini	Confidence Interval (95%)		
1973–74	0.359	[0.355, 0.363]		
1980–81	0.332	[0.329, 0.336]		
1990–91	0.316	[0.312, 0.319]		

Source: Own construction using EPFs (INE).
Note: Confidence intervals according to biased corrected bootstrap estimates.

intensity for the 1970s, as Table 1 shows for the case $\alpha=\beta=1$ with 2 and 3 groups.[16] The table also provides confidence intervals estimated by bootstraps.[17] This result is statistically significant at the level of 95 percent, and furthermore, is robust to the use of a different number of groups together with different values of α and β so long as we do not put too much weight on the error term, and is due to decreasing polarization between the groups—*ER*—despite their becoming more identified—lower ε.[18]

[16]Note that the 1970s exhibit growth rates in average expenditure (9 percent) which is lower than in the 1980s (30 percent).

[17]It requires resampling the original sample a large number of times (500 in this case). This allows us to obtain the distribution of the indices, using the "percentile method" and correcting for the bias of the estimated average with respect to the observed value. We refer to Efron and Tibshirani (1993) for details.

[18]Gradín (1999b) further extends the analysis, showing the trends in polarization in Spain in greater detail.

TABLE 2

OPTIMAL PARTITION IN THE EXPENDITURE DISTRIBUTION IN SPAIN, 1973–91

2 Groups	Population		Means		Cut-off
Year	p_1	p_2	y_1	y_2	z
1973–74	0.634	0.366	0.60	1.70	1.00
1980–81	0.630	0.370	0.62	1.64	1.00
1990–91	0.620	0.380	0.64	1.59	1.00

3 Groups	Population			Means			Cut-offs	
Year	p_1	p_2	p_3	y_1	y_2	y_3	z_1	z_2
1973–74	0.430	0.385	0.184	0.48	1.00	2.21	0.73	1.40
1980–81	0.418	0.381	0.201	0.50	0.99	2.04	0.74	1.36
1990–91	0.416	0.385	0.200	0.53	1.00	1.99	0.75	1.34

Source: Own construction using EPFs (INE).
Note: Incomes expressed as the ratio to the global mean.

Table 2 shows the optimal partition in both cases, two and three groups, so that, looking at the three-group case, we find that there was a transfer of population from the bottom and middle of the distribution to the top during the 1970s while during the 1980s only a small transfer from both extremes to the middle is found. If we focus on the case of two groups we find that population at the bottom progressively shifted to the top. In each case the distance between extreme groups has shrunk continuously, which explains the decreasing polarization between the groups. Note that the cut-offs defining these groups are endogenous, so, in the case of three groups, the interval of spending—relative to the mean—which defines the middle group, has narrowed. In the case of two groups the cut-off is always the mean of the distribution. In the case in which we keep the cut-off expenditure fixed in terms of the mean income from 1973 onwards, then we find increasing size of the middle group from 28.5 percent of households to 42.9 percent in 1991 [see Gradín (1999b) for details].

As mentioned above, this is just part of the story. On the one hand it would be useful to be able to explain the described trends in polarization. On the other hand we should explore what would change if we assume that groups in society share certain attributes other than the level of expenditure. We undertake the analysis of both aspects using the approaches proposed here.

6.2. *Polarization by Characteristics: Group Polarization*

For the partition raised by each characteristic we compute the index GP as expressed in (7) and both indices of overlapping I and O described in Section 5. According to the results, shown in Table 3 and Table 4 respectively, we conclude that polarization decreased generally for the 1973–80 period except where the groups are given by the educational level and the socio-economic condition, which have remained stable with a slight increase, increasing in the case of household size and especially the relationship with economic activity. During the 1980s polarization declines for education and occupational variables, is stable in the case of town size and gender/sex, but increases especially for autonomous regions and

TABLE 3

GROUP POLARIZATION IN SPAIN, 1973–91
($\alpha = \beta = 1$)

Characteristic	Group Polarization *GP*			Decomposition: Polarization Between *ER*			Decomposition: Lack of Identification ε		
	1973	1980	1990	1973	1980	1990	1973	1980	1990
Town size	0.835	0.809	0.808	0.074	0.056	0.050	0.239	0.246	0.242
Urban–rural	0.850	0.825	0.819	0.104	0.079	0.068	0.254	0.254	0.248
Autonomous region	0.791	0.775	0.795	0.025	0.018	0.019	0.234	0.243	0.223
Province	0.791	0.773	0.795	0.014	0.009	0.011	0.223	0.236	0.216
Household composition	0.749	0.727	0.739	0.028	0.014	0.012	0.278	0.288	0.273
Education	0.891	0.894	0.877	0.084	0.069	0.049	0.192	0.175	0.172
Relationship with economic activity	0.730	0.795	0.785	0.044	0.061	0.047	0.314	0.266	0.261
Socioeconomic condition	0.852	0.853	0.825	0.046	0.044	0.035	0.194	0.191	0.209
Number of members	0.697	0.736	0.752	0.013	0.016	0.016	0.316	0.281	0.264
Gender		0.674	0.695		0.003	0.005		0.329	0.310
Gender/age		0.752	0.751		0.025	0.018		0.273	0.267
Migration condition		0.691	0.701		0.011	0.008		0.320	0.307
Professional position		0.789	0.779		0.050	0.041		0.261	0.261

Source: Own construction using EPFs (INE).

TABLE 4

OVERLAPPING IN SPAIN, 1973–91

Characteristic	*I*			*O* (Yitzhaki)		
	1973–74	1980–81	1990–91	1973–74	1980–81	1990–91
Town size	0.712	0.775	0.790	0.939	0.962	0.969
Urban–rural	0.759	0.798	0.813	0.948	0.965	0.972
Autonomous region	0.710	0.764	0.752	0.942	0.964	0.958
Province	0.683	0.747	0.728	0.933	0.959	0.951
Household composition	0.810	0.882	0.882	0.975	0.986	0.985
Education	0.666	0.637	0.648	0.883	0.880	0.900
Relationship with economic activity	0.887	0.821	0.845	0.983	0.968	0.981
Socioeconomic condition	0.661	0.677	0.749	0.896	0.907	0.937
Number of members	0.900	0.864	0.856	0.987	0.981	0.978
Gender		0.993	0.986		0.999	0.999
Gender/age		0.842	0. 864		0.982	0.988
Migration condition		0.971	0.978		0.996	0.998
Professional position		0.816	0.848		0.972	0.983

Source: Own construction using EPFs (INE).

provinces together with household composition and size, gender and migration condition.

The educational level appears to be the element which generates the highest polarized expenditure distribution. Despite this, the level of polarization became smaller in the 1980s. The breakdown of the index, also reported in Table 3, shows that the distribution progressively exhibited groups less polarized despite becoming internally more identified. This increasing identification within educational

groups in the case of $\beta = 1$ overcompensates decreasing polarization between the groups between 1973 and 1980, but not that between 1980 and 1990. The indices of overlapping reveal that groups by education became on average more stratified during the first decade, but less during the second. Education gives rise to the most stratified distribution for all three years according to both indices. The apparent contradiction of finding that education during the 1980s exhibits lower polarization while increasingly explaining bipolarization is basically due to the fact that here we use all eight educational groups while there they were aggregated into just two big groups.[19] Subsequently here the resulting shape of the bimodal distribution is less relevant to determine polarization.

After education, the socio-economic condition presents the highest level of polarization. The distribution shows a slightly increasing trend in polarization between 1973 and 1980, due to the larger identification of people within the socio-economic groups despite the fact that between themselves they were less polarized. Now, the contradiction with the decreasing contribution to bipolarization during the 1970s is caused because, here, group polarization is sensitive to the intra-group distribution. For the 1980s it performs quite a different path, showing less polarization regardless of β, because groups became less identified within themselves and less polarized among themselves. Regarding the indices of overlapping, the degree of stratification persistently declined across socio-economic groups, but, after education, it still is the most important.

There are two characteristics close to the socio-economic condition: relationship with economic activity and professional position. The former shows increasing polarization between groups together with these groups becoming more identified, and stratified, in the 1970s. This leads to strongly higher polarization during that period regardless of β. The reason is the large increase in the size of unemployed, from 1 to 5 percent of all householders, and retired, from 17 to 23 percent, while spending grew moderately. The professional position decreases its level of polarization because the groups are less polarized, while identification is unchanged.

The geographical variables constitute the third important group of characteristics according to the level of polarization to which they give rise. Polarization substantially decreases in every case, autonomous regions, provinces or municipalities, regardless of the weight β for the 1970s. The main reason is that though, in general, average spending grows faster in relatively poor provinces and autonomous regions together with small towns, at the time they are, on average, less identified, and also stratified. The process is completely inverted during the 1980s in the case of autonomous regions and provinces, showing more identified, and stratified, groups increasingly polarized among themselves. Especially there is an increasing gap between poor provinces in rich regions and rich provinces in poor regions, given that spending in the latter grows at a lower rate. All this leads to higher polarization regardless of the weight β. Polarization, increased by the size of the municipality and the urban–rural condition declines during this period, but

[19]Not only do we find the normalized index EP increasing, but also bipolarization between both major educational groups, $ER(\alpha, \rho^+)$.

with less intensity than in the previous one, and with groups becoming more identified, but with more overlaps among them.

Which characteristic generates more polarization is not independent of the weight we put on the identification term; this is partially the result of considering characteristics with a different number of groups. In the case of zero weight we focus exclusively on *ER*, finding that the urban–rural condition and even the size of the town yield a more polarized distribution than education. Given that those two geographical characteristics do not generate groups which are too much identified, they are rather less relevant as we incorporate the identification term into the index of polarization. Something similar happens when we compare the relationship with economic activity or professional position with the socio-economic condition. Note also that provinces and autonomous regions give rise to similar levels of polarization for the case of equal weights despite decomposition being quite different as the 17 autonomous regions generate more polarized but less identified groups than the 49 provinces. Both education and socio-economic conditions generate the most identified groups, far more than the rest. Education also shows the most stratified distribution, followed by provinces, the socio-economic condition and autonomous regions.

6.3. *Determinants Explaining EGR Polarization*

This approach identifies which characteristics might be causing the described trend in polarization in Spain. Focusing on one particular specification: two groups and $\alpha = 1$, regardless of β, we compute the index *EP* in (17), which is reported in Table 5 for each characteristic. According to the results in that table

TABLE 5

EXPLAINED BIPOLARIZATION IN SPAIN, 1973–91

	Contribution ($\alpha = 1$) *EP*						
				Keeping Fixed:			
	Population Shares and Average Expenditure Change			Mean Expenditure "Distribution Effect"		Population Shares "Mean Effect"	
Characteristic	1973–74	1980–81	1990–91	1980/81	1990/91	1980–81	1990–91
Town size	0.408	0.333	0.303	0.445	0.470	0.329	0.299
Urban–rural	0.408	0.333	0.303	0.445	0.470	0.329	0.299
Autonomous region	0.380	0.272	0.333	0.402	0.419	0.276	0.344
Province	0.388	0.305	0.336	0.427	0.439	0.299	0.338
Household composition	0.230	0.162	0.145	0.219	0.158	0.183	0.241
Education	0.345	0.389	0.450	0.500	0.758	0.289	0.233
Relationship with economic activity	0.185	0.290	0.240	0.265	0.317	0.218	0.159
Socioeconomic condition	0.429	0.385	0.330	0.394	0.401	0.451	0.401
Number of members	0.134	0.183	0.184	0.126	0.111	0.210	0.253
Gender		0.013	0.023		0.160		0.190
Gender/age		0.177	0.146		0.183		0.149
Migration condition		0.046	0.035		0.500		0.340
Professional position		0.257	0.211		0.281		0.204

Source: Own construction using EPFs (INE).

we can conclude that in 1973 the socio-economic condition was the main element explaining observed bipolarization, while it was later replaced by the educational level. That is, for the 1973–90 period the socio-economic condition substantially decreased its power to explain the division of the Spanish distribution of expenditure into two well-defined and distant groups, while the opposite was found for the educational level. After education and the socio-economic condition, the geographical variables appear as the best determinants of bipolarization. However, within these variables the rural–urban condition[20] was more important until 1990, after which the province of residence replaced it. Other characteristics such as sex, sex and age, etc. seem to be rather less relevant.

Explaining the trends found for overall bipolarization we can say that for the 1973–80 period the distribution was less bipolarized thanks to the geographical variables which have strongly contributed to this declining tendency. The contribution to bipolarization by the dichotomy between rural and urban municipalities decreased from 0.408 to 0.333, while the regions' contribution declined from 0.380 to 0.272 and that of provinces from 0.388 to 0.305. Table 5 also shows that these declines in contributions were due to the "mean effect." If the groups' average expenditure had been kept fixed from 1973 onwards, the contribution would have been substantially increased. The socio-economic condition decreased its contribution by a smaller amount, explained by the distribution effect, but even when we approach this condition via the relationship with economic activity, it has increased the share of bipolarization it explains. Education also contributes increasingly to bipolarization due to the distribution effect.

For the 1980s, the picture changes. In this decade all socio-economic variables significantly reduce their contribution to bipolarization, so they seem to explain declining bipolarization in this period. Now the rural–urban condition reduces its contribution again, but rather less than in the 1970s, while the autonomous regions and provinces reverse direction and increase their respective contributions to bipolarization. Education follows its rising trend for this period.

The index *EP* was shown to be determined by the level of bipolarization implicit in its two-spike representation. Table 6 describes the degenerated bimodal distribution underlying each characteristic. This table reveals that the decreasing role played by the socio-economic condition was due to a persistent decline in the distance between the extreme groups, the ratio of average spending of the rich to that of the poor falling for the whole period from 1.71 to 1.55, mostly during the 1980s. At the same time the share of population in the bottom group according to socio-economic condition increased from 71 to 79 percent of all households, which creates an even less symmetrical distribution. The reason is the strong increase in the proportion of unemployed and retired.

Paradoxically the increasing role played by the educational level was due to the improvement in the average educational level attained by householders leading to an increasing proportion of them belonging to more qualified groups which coincide with the relatively richest, so that a larger share of households was allocated to the top group of the distribution, with at least secondary education

[20]Since those municipalities with less than 50,000 inhabitants—considered as rural—coincide with those whose average spending falls below the global mean, the size of the municipality and the condition urban–rural are equivalent in order to explain bipolarization.

TABLE 6

BIMODAL PARTITIONS BY CHARACTERISTIC IN SPAIN, 1973–91

Characteristic	Mean Radio Rich/Poor (g_2/g_1)			Poor's Population Share (%) ($100r_1$)			Rich's Population Share (%) ($100r_2$)		
	1973	1980	1990	1973	1980	1990	1973	1980	1990
Town size	1.52	1.37	1.31	55.4	51.3	48.8	44.6	48.7	51.2
Urban–rural	1.52	1.37	1.31	55.4	51.3	48.8	44.6	48.7	51.2
Autonomous region	1.48	1.30	1.36	54.1	57.8	59.7	45.9	42.2	40.3
Province	1.50	1.34	1.36	57.6	53.0	58.5	42.4	47.0	41.5
Household composition	1.27	1.19	1.24	50.8	32.7	18.5	49.2	67.3	81.5
Education	2.10	1.78	1.55	86.2	80.0	64.2	13.8	20.0	35.8
Relationship with economic activity	1.34	1.38	1.25	20.1	30.8	40.7	80.0	69.2	59.3
Socioeconomic condition	1.71	1.69	1.55	71.3	77.5	78.6	28.7	22.5	21.4
Number of members	1.16	1.24	1.27	37.9	28.9	22.0	62.1	71.1	78.0
Gender		1.03	1.04		13.7	82.4		86.3	17.6
Gender/age		1.19	1.15		56.6	59.6		43.4	40.4
Migration condition		1.28	1.19		95.4	95.3		4.6	4.7
Professional position		1.29	1.21		40.3	46.8		59.7	53.2

Source: Own construction using EPFs (INE). Incomes expressed as the ratio to the global mean.

completed: 14 percent at the beginning but 36 percent at the end. This increasing symmetry in the bimodal distribution caused the greater bipolarization despite the distance between both groups declining substantially: the ratio of mean spending of the rich to the poor declined from 2.1 to 1.55. This ratio in 1990 coincides with that for the socio-economic condition, but what determines education more to explain bipolarization is the greater symmetry in the relative size of the groups in this case.

The autonomous regions and provinces show a contradictory path. For 1973–80 the economic advantage of the rich regions declines from a ratio of rich to poor means of 1.48 to 1.3 while for 1980–90 the opposite happens, finishing at 1.36. The share of households in the bottom group of the distribution by regions grows from 54 to 59 percent during the entire period. The size of the town shows a tendency to narrow the relative distance between rural households, the relatively poor group, and urban ones, the richer, while population moves from the former to the latter.[21]

7. CONCLUSIONS

In this paper we have extended the analysis in *EGR* to address sub-populations arising from the individual attachment to possibly overlapping groups according to any relevant characteristic. Two alternative approaches are possible.

To investigate the level of polarization that each characteristic generates, the same scheme was proposed as that in *EGR* but is now applied to new, possibly

[21]In the case of the variable sex, note that households with female householders were slightly richer by 1980 and slightly poorer by 1990. This small difference in expenditure levels, jointly with the small share of female householders (less than 18 percent in 1990), explains its slight relevancy. Dissociating sex by age leads to higher income differences, but these are still smaller than for other characteristics.

overlapping sub-distributions. This leads to different levels of polarization depending on the characteristic we use. Alternatively if our concern is to know the degree to which one characteristic helps to explain the level of polarization observed when groups are supposed to be determined by expenditure classes, a second approach is proposed. This approach consists of comparing polarization in the partition by income intervals with polarization in the partition by the relevant characteristic.

In both approaches the error term takes into account the effect that overlaps produce on polarization as it is expected it will affect the internal identification of groups. Given intra-group dispersion, as the group becomes more isolated in the attribute space it is assumed their members will feel more identified with each other.

An empirical application to Spanish data has shown the relevance of analysis to better understand polarization trends in Spain. It was shown how the educational level attained by the householders has replaced their socio-economic condition as the main determinant of observed levels in bipolarization when the groups are built according to proximity in spending levels. The geographical variables appear as the main factor explaining depolarization during the 1970s while the socio-economic condition played that role during the 1980s.

When each characteristic is the criterion in forming groups, then we find that educational level displays the highest levels of polarization, despite it having decreased during the 1980s. The socio-economic condition exhibits stable distribution for the 1973–80 period but strongly less polarized after the 1980s. Despite polarization declining in Spain for most characteristics there are several for which it has increased, as in the case of the autonomous regions and provinces during the 1980s, these becoming more polarized among themselves and internally more identified.

Some contradictory findings have arisen between both proposed approaches which justify the fact that they are complementary. The educational level of the householder has been shown to be increasingly contributory to bipolarization while the level of group polarization it generates was declining during the 1980s. The socio-economic condition was shown to persistently decrease in its contribution to bipolarization while its group polarization slightly rose for the 1970s. The first contradiction is due to the increasing symmetry in educational bimodal distribution, while the second was the result of sensitivity of group polarization to intra-group identification. In any case, it should be noticed that the number of groups is different in both approaches, in the case of explained polarization the groups considered are only two while in group polarization the number of groups depends on the characteristic analyzed.

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