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ACCOUNTING FOR RESOURCE DEPLETION: A MICROECONOMIC APPROACH

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The theoretical basis of a practical method of accounting for depletion of mineral resources is presented. Rent rises at the rate of interest, but depletion does not. Rent is equal to the sum of depletion and depreciation less any opportunity cost of present production as compared to waiting. Depletion follows a path which is dependent on the depreciation formula chosen by the accountant. The approach is compared to the methods proposed by the BEA in 1994.

1. INTRODUCTION

In its system of satellite accounts, the U.S. Department of Commerce's Bureau of Economic Analysis (BEA, 1994) proposes methods to integrate natural-resource use into the national accounts. The BEA's approach is possibly the most comprehensive of, but can be viewed as representative of a number of, such proposals.

It is widely perceived that the shadow prices (user costs) obtained through the r-percent rule can be used to value stocks of minerals. For example, the BEA (1994: 54) observes of current rent estimates: "The simplest assumption ... is based on Harold Hotelling's observation that in equilibrium, the price of the marginal unit net of extraction costs ... should increase ... at ... the nominal rate of interest ... [T]he value of the stock of the resource is independent of when it is extracted and is equal to the *current* per-unit rent of the resource times the number of units of the resource ... [Two methods] use the current per-unit rent to value the resource and depletion." This seems to imply that the present value of depletion is equal to the value of reserves.

As befits a subject related to the national accounts, most theorists discuss the appropriate measure of depletion in the context of macroeconomic models. In practice, the national accounts are aggregates of values obtained for sub-units such as firms. Although the BEA estimates aggregate values for the U.S. domestic extractive industry, the accounting approach reflects a microeconomic view of investment and extraction.

The present paper utilizes a microeconomic model to gain insights into how to evaluate depletion in practice. Our results are reminiscent of a proposal by El Serafy (1989), but one difference is that we utilize an explicit optimization model. The user cost of the resource, which obeys the r-percent rule, is the depreciation, not of the resource, but of the mine. Separate values of the resource and capital

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cannot be uniquely determined once capital is sunk. Rather, given a depreciation schedule, an undepreciated value of capital can be calculated. The remainder of the present value of the mine is the undepleted value of the resource, the rate of change of which is depletion. Our findings are compared to the BEA's proposals.

2. Asset Values and Depreciation

Consider a mine at which capacity is determined at time $t=0$ and is not changed thereafter. Let the cost of installing capital, K, be $\phi(K)$, with $\phi'(K)>0$ and $\phi''(K)\leq 0$.[1] Also let the firm take known prices $p(t)$ for its output $q(t)$ and have variable extraction costs $C(q, K)$.[2] Let T be the time that mining ceases and $S(t)$ be the reserve at time t. The problem of the firm is to choose K, T and the production path to maximize the net present value of the mine,[3]

$$(1) \qquad R[S(0), K] = -\phi(K) + \int_0^T [p(t)q(t) - C(q(t), K)]\, e^{-rt}\, dt,$$

subject to

$$q(t) = -\dot{S}(t) \quad \text{and} \quad q(t) \leq K.$$

Following Campbell (1980), Cairns (1998, 1999) shows that, notwithstanding the fact that $\phi(K)$ is concave, this problem can be solved by an extension to control theory. In one stage, present value is maximized as in standard control theory but conditional on K. In a second stage, the optimal value of K is determined through a constrained maximization.

The current-value Lagrangean in the first stage is

$$L = pq - C(q, K) - uq + v(K - q),$$

where u is the user cost of the resource, $v \geq 0$ is the shadow value of the capacity constraint, and $v(K-q)=0$ by complementary slackness. The following hold:

$$(2) \qquad p - C_q - u - v = 0;$$

$$(3) \qquad \dot{u} - ru = 0;$$

Condition (3) is the r-percent rule. It is related to Hotelling's rule, but is closer to the decisions on production and investment. Most importantly, it allows for

[1]Increasing returns to scale are common in extractive industry, and this last assumption is a simple way to represent them. Mase Westpac, Ltd. (1990: Ch. 6) depicts strictly concave investment costs for gold mining, for example. Bradley (1985: 322) cites elasticities of investment outlays to capacity in copper mining and milling in British Columbia of 0.85 and 0.60, respectively. Camm (1991) estimates that, for various types of mining and milling, $\phi''<0$.

[2]A possible generalization would allow cost to be a function of cumulated extraction, reflecting depth within the mine, etc. It is well known that in this case rent does not rise at the rate of interest. We abstract from such effects to focus on implications of capacity constraints. Davis and Moore (1998) discuss this case in a context comparable to that of the present paper.

[3]In equation (1), $R[S(0), K, 0]$ is the capitalized value of Diaz and Harchaoui's (1997) "natural commodity," and $\phi(K)$ is cash expenditure related to preparing that resource for exploitation. It includes development expenditure. We abstract from exploration, which produces information of value to the economy as well as pinpointing a deposit for development. To avoid issues related to recycling we can assume (as is common, implicitly, in resource economics) that the resource is extinguished as soon as used. Examples would be uranium, coal or industrial diamonds. A dot over a variable represents the derivative with respect to time. Subscripts are used to denote partial derivatives.

determining resource values even if (real) prices do not rise. The BEA (1994: 56) has to devise a special method to deal with this common occurrence.

In the second stage, the first-order condition is

$$(4)\qquad -\phi'(K)+\int_0^T (v-C_K)\,e^{-rt}\,dt=0.$$

Combining equations (1) through (4) yields that

$$\begin{aligned}R[S(0),K] &= -\phi(K)+\int_0^T q(u+v+C_q-C/q)\,e^{-rt}\,dt\\ &= -\phi(K)+K\int_0^T (v-C_K)\,e^{-rt}\,dt\\ &\quad+\int_0^T [q(u+C_q-C/q)+KC_K]\,e^{-rt}\,dt\\ &= S(0)u(0)+\left\{[K\phi'(K)-\phi(K)]+\int_0^T [q(C_q-C/q)+KC_K]\,e^{-rt}\,dt\right\}.\end{aligned}$$

Only if the term in braces is zero does the initial value of the resource equal the resource stock valued at initial user cost. If $\phi(K)$ is strictly concave, it may be that $R[S(0),K]<u(0)S(0)$. Moreover, the remaining value at time $t<T$ is

$$\begin{aligned}V[S(t),K,t] &= \int_t^T (pq-C)\,e^{-r(s-t)}\,ds\\ &= u(t)S(t)+\int_t^T q(v+C_q-C/q)\,e^{-r(s-t)}\,ds.\end{aligned}$$

Unless $v+C_q-C/q=0$ for all $s>t$, $V[S(t),K,t]\neq u(t)S(t)$. The BEA applies the so-called Hotelling valuation principle (Miller and Upton, 1985), by which $V=uS$ at any time. Cairns and Davis (1998) provide empirical evidence that the principle is not a valid guide to valuing gold reserves.

The following formula is derived in Appendix I in a way inspired by Lozada (1995).

$$(5)\qquad \dot V=rV-[pq-C]=-uq+\int_t^T \frac{\partial[pq-C(q,K)]}{\partial s}\,e^{-r(s-t)}\,ds.$$

Total cost, not output valued at marginal cost, appears in equation (5). In many models, much is made of the difference between so-called Hotelling rents (at the margin, due to resource scarcity) and Ricardian rents (inframarginal, due to diminishing returns). The distinction is not pertinent in our present values of net cash flows. Rearranging equation (5) yields

$$uq=-\dot V+\int_t^T \frac{\partial[pq-C(q,K)]}{\partial s}\,e^{-r(s-t)}\,ds.$$

User cost must cover depreciation of the program value, $-\dot{V}$, plus a second term, which we interpret as the gain (opportunity cost) of not waiting an instant *dt* before pursuing the same path at a possibly different level of net cash flow. For convenience of discussion, suppose that net cash flow is a function of time because of a (sufficiently long) trend in price. If $\dot{p} > 0$, the opportunity cost is that of *not* waiting for higher (real) prices. If $\dot{p} < 0$, then there is an opportunity cost of waiting. User cost uq is the depreciation of the program plus the opportunity cost.

It seems reasonable to believe that trends in real metals prices have been nonpositive in recent decades and will remain so for the foreseeable future.[4] As a *constant* price is usually assumed by mining engineers in their decision-making (see any mining-engineering textbook), we take the case $\dot{p} = 0$ as a benchmark. In this case, the resource rent is equal to the depreciation of the value of the mine.

Let $\kappa(t)$ be what Baumol *et al.* (1982) call the payment to capital at time t. The present value of the payments to capital must equal the investment cost:

$$\phi(K) = \int_0^T \kappa(t)\, e^{-rt}\, dt.$$

Since investment does not take place for $t > 0$, many paths $[\kappa(t)]$ satisfy this condition. Once a schedule is chosen, the undepreciated value of capital assets at time $t > 0$ is

$$A[S(t), K, t] = \int_t^T \kappa(s)\, e^{-r(s-t)}\, dt.$$

and the corresponding depreciation $D[S, K, t]$ is the rate of decline of the asset value:

$$D[S(t), K, t] = -\dot{A}[S(t), K, t] = \kappa(t) - rA[S(t), K, t]. \tag{6}$$

Then,

$$\phi(K) = A[S(0), K, 0] = \int_0^T D[S, K, t]\; dt: \tag{7}$$

the integral of the *undiscounted* depreciation is equal to the value of the invested capital. This fact is realized by the BEA (1994: 51, 52).

Equivalently, a depreciation schedule satisfying condition (7) can be defined, from which $A(S, K, t)$ can be obtained by integration, and then $\kappa(t)$ from equation (6). This is the usual procedure in practice.

We can define the payment to the resource at any time to be the net cash flow less the payment to capital, so that the value of the resource stock remaining at time t is

$$R[S(t), K, t] = \int_t^T [p(s)q(s) - C(q(s), K) - \kappa(s)]\, e^{-r(s-t)}\, ds.$$

[4]Clark and Dunlevy (1996) provide some empirical evidence of the non-increasing trend. Such a trend does *not* contradict the theory of exhaustible resources, which predicts rising real prices only toward the time of exhaustion of world reserves. We are not near that point with respect to any commonly used mineral.

Thus, $V[S(t), K, t] = A[S(t), K, t] + R[S(t), K, t]$ at any time. Since $\kappa(t)$ can, within limits, be chosen, there is no necessary link between $R[S(t), K, t]$ and $u(t)$. Depletion $\Delta[S(t), K, t]$ at time t is the rate of decline of the resource value, or

$$(8) \qquad \Delta[S, K, t] = -\dot{R}[S, K, t] = [pq - C(q, K) - \kappa] - rR[S, K, t].$$

Comparably to the result in equation (6), the term $-rR[S, K, t]$, interest on the stock of the resource, appears in the expression for depletion. Neglecting it is one oversight in the BEA's (and others') approach. Postponing extraction means that there is no depletion currently, but also that the current interest on the resource value is forgone. Equation (8) can be rearranged to show that

$$pq - C = \kappa + rR + \Delta = D + \Delta + rA + rR.$$

Net cash flow is equal to the sum of depreciation, depletion and interest on the (undepreciated) values of capital *and* the resource. We stress that the relative sizes of these four variables depend on the chosen depreciation schedule.

Using equation (5), we can write

$$\Delta = -\dot{R} = -\dot{V} + \dot{A} = uq - \int_t^T q\dot{p}\, e^{-r(s-t)}\, ds - D,$$

or

$$uq = \Delta + D + \int_t^T q\dot{p}\, e^{-r(s-t)}\, ds.$$

Contrary to many models, the user cost uq measures, not the depletion alone, but the sum of depletion, depreciation, and the opportunity cost of producing now rather than later. The reason is that it is the *project* (resource *with* capital) which produces economic value; once the capital is sunk, the value of the resource can be separated only artificially from the value of the capital. When $\dot{p} \neq 0$, the opportunity cost $\int_t^T \dot{p}q\, e^{-r(s-t)}\, ds$ is a part of the resource value $V(S, K)$. In the benchmark case, in which $\dot{p} = 0$, the user cost is the sum of depreciation and depletion.

We verify that

$$\int_0^T (D + \Delta)\, dt = \int_0^T \left[uq - \int_t^T q\dot{p}\, e^{-r(s-t)}\, ds \right] dt$$
$$= -\int_0^T \dot{V}\, dt = V[S(0), K, 0].$$

Therefore,

$$\int_0^T \Delta\, dt = V[S(0), K, 0] - \phi(K)$$
$$= \int_0^T (pq - C)\, e^{-rt}\, dt - \int_0^T \kappa\, e^{-rt}\, dt$$
$$= R[S(0), K, 0].$$

The integral of *undiscounted* depletion is equal to the value of the resource stock. This discussion puts the two types of assets, capital and reserves, on the same theoretical footing consistently with accounting theory, as the BEA (1994: 51, 52) seeks to do.

If $C(q, K) = cq$, then $q = K$ throughout the interval $[0, T]$ (Crabbé, 1982). Let there be straight-line depreciation of capital on the interval $[0, T]$. Then,

$$D(t) = \phi(K)/T;\ A(t) = (T-t)\phi(K)/T;\ \dot{A}(t) = -D(t);$$

$$\kappa(t) = D(t) + rA(t) = [1 + r(T-t)]\phi(K)/T;$$

$$V[S(t),K] = \int_t^T (p-c)K\,e^{-r(s-t)}\,ds = (p-c)K[1 - e^{-r(T-t)}]/r;$$

$$R(t) = V[S(t), K] - A(t) = (p-c)K[1-e^{-r(T-t)}]/r - (T-t)\phi(K)/T;$$

$$\Delta(t) = -\dot{R}(t) = (p-c)K\,e^{-r(T-t)} - \phi(K)/T.$$

Depletion does not grow at the rate of interest. Finally,

$$\Delta(t) + D(t) + r[R(t) + A(t)] = (p-c)K.$$

The usefulness of this case in practice is that engineers assume a constant price and constant marginal costs in their planning. In the Appendix, some further examples are presented. We concur with Diaz and Harchaoui (1997: 472) that the two types of assets have different depreciation (depletion) patterns. The different conventions for the rate of depreciation can have complicated effects on the implied rate of depletion. Depletion rises at the rate of interest only if depreciation falls at the rate of interest.

3. Comparison with BEA's proposals

The BEA's (1994: 58–59) charts show wide ranges in values arising in their proposed methods. The largest spread is between Current Rent Methods I and II, the ones which most directly attempt to apply the theory of exhaustible resources. We now evaluate the BEA's methods. Recall that in the notation of our model:

$$\text{payment to capital} = \kappa;$$

$$\text{undepreciated capital value} = A = \int_t^T \kappa(s)\,e^{-r(s-t)}\,ds;$$

$$\text{payment to resource} = pq - C(q, K) - \kappa;$$

$$\text{undepreciated resource value} = R = \int_t^T [pq - C(q, K) - \kappa]\,e^{-r(s-t)}\,ds;$$

$$\text{depletion} = -\dot{R} = pq - C(q, K) - \kappa - rR;$$

$$\text{resource value per unit of reserves} = \frac{R}{S} = \frac{\int_t^T [pq - C(q, K)]\,e^{-r(s-t)}\,ds}{S} - \frac{A}{S}.$$

I. Current Rent Method I (p. 55):

$$\text{per-unit resource rent} = \frac{pq - C(q, K) - \kappa}{q};$$

$$\text{depletion} = pq - C(q, K) - \kappa;$$

$$\text{value of resource} = S\left[\frac{pq - C(q, K) - \kappa}{q}\right].$$

The proposed expression for depletion does not subtract the return on the resource value and hence is an overestimate of depletion. The expression for the rent assumes that $[pq - C(q, K) - \kappa]/q = u$ (the rent, or the value which rises at the rate of interest) at all times, which does not hold in general. Rather, the value of κ depends on the chosen depreciation method. The expression for the resource value applies the (incorrect) Hotelling valuation principle.

II. Current Rent Method II (p. 56):

$$\text{value of project} = S\left[\frac{pq - C(q, K)}{q}\right];$$

$$\text{value of resource} = S\left[\frac{pq - C(q, K)}{q}\right] - A;$$

$$\text{rent per unit} = \left[\frac{pq - C(q, K)}{q}\right] - \frac{A}{S};$$

$$\text{depletion} = pq - C(q, K) - \frac{q}{S}A.$$

Current rent method II takes a current-period approximation of R/S to be the rent per unit, as opposed to u. The formula for depletion involves a value, A/S, which appears to be intended to be a measure of depreciation rather than the full payment to capital, κ. Furthermore, rR is not subtracted. Therefore, depletion is overestimated.

Current Rent Methods I and II's inconsistency can be shown by subtraction:

$$R_{\text{II}} - R_{\text{I}} = S\left[\frac{pq - C(q, K)}{q}\right] - A - S\left[\frac{pq - C(q, K) - \kappa}{q}\right] = \frac{S}{q}\kappa - A.$$

$$\Delta_{\text{II}} - \Delta_{\text{I}} = pq - C(q, K) - \frac{q}{S}A - [pq - C(q, K) - \kappa] = \kappa - \frac{q}{S}A.$$

Suppose, for the sake of illustration, that q and κ are constant. Then

$$A = \int_0^T \kappa\, e^{-rt}\, dt = \kappa \frac{(1 - e^{-rT})}{r}.$$

Therefore, $R_{\text{II}} - R_{\text{I}} = \kappa(rT - 1 + e^{-rT})/r$. The bias is positive, as depicted in the BEA's charts.

III. Present-Discounted-Value Estimates (p. 56). Our model indicates what to do when prices do not rise according to Hotelling's rule. The BEA's proposed method incorrectly introduces a "discount factor" to be applied to the value of rent per unit obtained by the approximation in Current Rent Method II. In fact, the method assumes that $q = K$ (which is usually planned by engineers) and that $r = 0$. The overestimate leads the BEA to discount the figure determined for R/S in an *ad hoc* way. The BEA seems to realize that the estimate is high, but not why it is.

IV. Replacement-Cost Estimates (p. 56). This method is an application of Adelman's (1990) perspective on the depletion of oil reservoirs. Suppose the rate of depletion of oil in a reservoir under natural drive is α. Then the proposal is to apply a so-called barrel factor, $\alpha/(\alpha + r)$, to the gross rent per unit, $[pq - C(q, K)]/q$, and then to subtract per-unit exploration and development cost. Under natural exponential drive with a constant net price,

$$V = \frac{\alpha}{\alpha + r}(p - C/q)S,$$

from which, at any time, undepreciated exploration and development cost can be subtracted to give a value for the resource. But then,

$$D + \Delta = -\dot{V} = rV - (pq - C),$$

and resource depletion would be given by this value less current depreciation of exploration and development expenses.

V. Transaction-Price Method (p. 57):

$$\text{per-unit resource rent} = \frac{V}{S} - \frac{A}{S} = \frac{R}{S}.$$

Provided the undepreciated capital stock, A, is equal to the industry-wide average, this gives an estimate of the resource value divided by the stock, which is equal to the rent only in special circumstances. The formula for $\dot{V}$ must be applied to obtain the depletion.

4. Conclusion

A forward-looking valuation is needed to determine depletion. First, an accounting depreciation schedule is chosen, from which a payment to capital is calculated. That payment is subtracted from projected net cash flow to yield a payment to the resource. The value of the resource is the present value of these projected payments. Accounting depletion is the negative of the rate of change of that value. Only if depreciation rules are contrived to allow it do unit depletion values rise at the rate of interest.

In making the projections, the assumptions of mining engineers should be used. Usually, this will mean projecting a fixed, normal price of output and production at capacity for the life of the mine.

Rate of Change of $V[S(t), K, t]$

This derivation simplifies and extends to constrained optimization Lozada's (1995: 140–41, 151–2) derivation of a formula for the depreciation of the program. We make use of the necessary conditions that (i) $v(K-q)=0$; (ii) $L|_T e^{-rT} = 0$; (iii) $(d/dt)[e^{-rt}L|_t] = (\partial/\partial t)[e^{-rt}L|_t]$ on intervals on which q is continuous; and (iv) $\dot{u} = ru$.

$$
\begin{aligned}
\dot{V} &= rV - [pq - C] \\
&= rV - uq - e^{rt}(L\,e^{-rt}) \\
&= rV - uq - e^{rt}\left[L|_T e^{-rT} - \int_t^T \frac{d}{ds}(L|_s e^{-rs})\,ds\right] \\
&= rV - uq + e^{rt}\int_t^T \frac{\partial}{\partial s}(L|_s e^{-rs})\,ds \\
&= r\int_t^T [pq - C]\,e^{-r(s-t)}\,ds - uq \\
&\quad + \int_t^T e^{-r(s-t)}\left\{-r[pq - C - uq] + \left[\frac{\partial}{\partial s}(pq - C) - ruq\right]\right\} ds \\
&= -uq + \int_t^T \frac{\partial}{\partial s}(pq - C)\,e^{-r(s-t)}\,ds.
\end{aligned}
$$

Depreciation Conventions

Assume that $C(q, K) = cq$ and that $p(t) = p$ is constant. Let $k = p - C(K, K)/K$.

I. Exponential Depreciation. Since the project has a finite life, the formula for exponential depreciation must be modified. For some value, $B(t)$, then, $D(t) = \delta B(t) = -\dot{B}(t)$. Hence, $B(t) = b\,e^{-\delta t}$, with boundary condition $\phi(K) = \int_0^T D(t)\,dt$, so that $B(t) = \phi(K)\,e^{-\delta t}/(1 - e^{-\delta T})$. Let

$$A(t) = \phi(K) - \int_0^t D(s)\,ds = \phi(K)(e^{-\delta t} - e^{-\delta T})/(1 - e^{-\delta T}).$$

The payment to capital is

$$\kappa(t) = rA(t) + D(t) = [(r+\delta)\,e^{-\delta t} - r\,e^{-\delta T}]\phi(K)/(1 - e^{-\delta T}),$$

or

$$\kappa(t) = -\alpha_E + \beta_E\,e^{-\delta t},$$

say, where

$$\alpha_E,\ \beta_E > 0.$$

Therefore,

$$R(t) = (kK + \alpha_E) - \beta_E e^{-\delta t}.$$

The implied depletion is

$$\Delta(t) = e^{rt}[(r-1)\phi(K)\frac{e^{-\delta T}}{1-e^{-\delta T}} + kK]\, e^{-rT} - \frac{\delta\phi(K)}{1-e^{-\delta T}} e^{-\delta t}.$$

II. Exponential Depletion. Suppose that $\Delta(t) = \Delta(0)\, e^{rt}$, so that

$$R(t) = \int_t^T \Delta(s)\, ds = \frac{\Delta(0)}{r}(e^{rT} - e^{rt}).$$

Then

$$\dot{R}(t) = rR(t) + \Delta(t) = \Delta(0)\, e^{rT},$$

and

$$\kappa(t) = kK - \Delta(0)\, e^{rT} = \beta_R,$$

say, where

$$\phi(K) = \int_0^T \beta_R\, e^{-rt}\, dt,$$

so that

$$\beta_R = \frac{r\phi(K)}{1-e^{-rT}}.$$

Then

$$A(t) = \int_t^T \beta_R\, e^{-r(s-t)}\, ds = \frac{\beta_R}{r}(1 - e^{-r(T-t)}),$$

and

$$D(t) = -\dot{A}(t) = \beta_R\, e^{-r(T-t)}.$$

III. Amortizing Capital by an Annuity. In this case

$$\kappa(t) = \frac{r\phi(K)}{1-e^{-rT}} = \alpha_A \quad \text{and} \quad R(t) = kK - \alpha_A,$$

both constants. Then

$$D(t) = \alpha_A\, e^{-r(T-t)} \quad \text{and} \quad \Delta(t) = (kK - \alpha_A)\, e^{-r(T-t)}.$$

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