Review of Income and Wealth
Series 45, Number 4, December 1999

ON OUTPUT OF THE SWEDISH EDUCATION SECTOR: ADDITIONAL REMARKS

BY THOMAS ARONSSON, KARL-GUSTAF LÖFGREN AND PER-OLOV MARKLUND

Umeå University

The main purpose of this paper is to relate the empirical attempt of measuring output from the education sector to theoretical results about the welfare significance of an extended net national product (NNP) measure. We show that economic theory provides a more focused way of interpreting such output estimates, which has not been recognized in previous studies. The paper also contains new estimates of the output from the Swedish education sector.

INTRODUCTION

In a recent paper in this journal, Ahlroth et al. (1997) introduce estimates of the true output from the educational sector in Sweden. Their study is based on a method developed in two papers by Jorgenson and Fraumeni (1992a, b), where it is applied to U.S. data. While we sympathize with the general idea of measuring output in the manner suggested in these papers, we feel that economic theory suggests an alternative and more focused way of presenting the material, which adds an important welfare dimension.

The purpose of the present paper is to relate the empirical attempts to fairly recent theoretical developments on the welfare relevance of an extended Net National Product (NNP) measure. This analysis shows that if you want to give the income from the educational sector a strict aggregate welfare interpretation, none of the measures suggested by Jorgensen and Fraumeni (1992a) and Ahlroth *et al.* (1997) are directly relevant. They introduce diffrent kinds of gross output measures, while the welfare relevant measure can be shown to be related to a net income concept. The theory also shows how possible externalities from human capital should be handled. Finally, and contrary to what is implied by the paper of Ahlroth *et al.*, we show that the empirical method is unable to handle the output from the whole educational sector in Sweden. At best we can measure the output from higher education. We close by estimating the net output (income) from higher education in Sweden. Our estimates show that although education is only 2 percent of current GDP, the real rate of return on the investment is 8.6 percent.

The paper is structured as follows: We start by introducing the theoretical considerations. Next we introduce our empirical estimates of the income from higher education in Sweden for the year 1990. The empirical analysis allows us to estimate the rate of return on investments in higher education in Sweden. We end the paper by introducing results from a numerical computation done in order to gauge the magnitude of possible externalities from investment in human capital.

The idea of measuring the output from the educational sector as the present value of the future income it generates has an intuitive appeal to an economist, although the fundamental reason is much deeper. In a now famous article by Martin Weitzman (1976), it is shown that in a perfect foresight intertemporal economy with no externalities, the net national product, measured as consumption plus the value of investments in all capital stocks (including human, natural and man-made capital), is a static equivalent of (is proportional to) the present value of future utility.

Later research has shown how externalities and exogenous technological progress should be handled in the framework introduced by Weitzman. The papers include Kemp and Long (1982), Löfgren (1992), Aronsson and Löfgren (1993, 1995). In modern growth theory, knowledge, which is an endogenous variable in the system, is viewed as an important promotor of growth. In the seminal papers by Lucas (1988) and Romer (1986), knowledge, or human capital, has a direct effect, as well as an indirect effect on the growth path, and the direct effect is modeled as a positive externality. For the present purpose, this means that the present value of future income may not gauge all welfare consequences from human capital formation.

The instantaneous utility function facing the consumer is $u(c, z)$, where c is consumption and z leisure time. The utility function is assumed to be increasing in its arguments, and strictly concave. Leisure is defined as $z = T - l - x$, i.e. as a time endowment, T, less time spent in market work, l, and time spent in education, x. Firm technology is given by the smooth production function $f(k, a(h)l, h)$, where k is the man-made or physical capital stock, h is the stock of human capital, while $a(h)$ is a function such that $a(0) = 1$, and with the derivative $a'(h) > 0$. The term $a(h)l = \bar{l}$ measures labor input in efficiency units, i.e. effective labor. We assume that $f(\cdot)$, which measures net output (after depreciation of physical capital), is increasing in effective labor and human capital. Our formulation of the production function implies that the stock of human capital, in addition to being a separate argument in the production function, will also affect the marginal product of labor. This means that the total effect on output of additional human capital can be written:

$$\frac{df}{dh} = f_{\bar{l}}(\cdot)a'(\cdot)l + f_h(\cdot)$$

where the first term on the right hand side measures the marginal effect of h via effective labor, and the second term measures the direct effect of h.

The stock of human capital accumulates according to the equation

$$\dot{h} = g(x) - \gamma h \tag{1}$$

where $g(x)$, the net output from the education sector, is increasing in x, i.e. $g'(x) > 0$. The parameter γ denotes depreciation of human capital. This does not necessarily mean that knowledge is forgotten, rather that knowledge becomes obsolete and looses part of its value. The latter interpretation of γ will be made clear below.

In the decentralized economy, firms maximize profit by solving the maximization problem

$$(2) \qquad \max_{k,l} \int_0^\infty [f(k(t),\, a(h(t))l(t), h(t)) - r(t)k(t) - w(t)l(t)] \exp\left(-\int_0^t r(s)\, ds\right) dt$$

where r is the interest rate and w the wage rate. Worth noting here is that the firm has no control over the time dependent argument $h(t)$ in the production function, and since the consumer is only paid for his input of labor, there are no reasons to believe that she will supply the socially optimal level of human capital. The most likely outcome is under-accumulation of human capital, and there will be a positive externality via the last argument in the production function.

The consumer in the decentralized economy maximizes

$$(3) \qquad \max_{c,l,x} \int_0^\infty u(c(t),\, T - l(t) - x(t))\, e^{-\theta t}\, dt$$

subject to

$$(3a) \qquad \dot{k} = \pi(t) + r(t)k(t) + w(t)l(t) - c(t) - d(x(t))$$

$$(3b) \qquad \dot{h} = g(x(t)) - \gamma h(t)$$

$$(3c) \qquad h(0) = h_0$$

$$(3d) \qquad k(0) = k_0$$

where the income of the consumer is the sum of labor income, wl, capital income, rk, and possible pure profit π (the consumer owns the firm). The term $d(x)$ denotes the operational cost of the education sector in terms of lost output. Since $\pi = f(\cdot) - rk - wl$, the accumulation of physical capital will be

$$(4) \qquad \dot{k} = f(k(t),\, a(h(t))l(t),\, h(t)) - c(t) - d(x)$$

in the general equilibrium.

We will not delve into first order conditions and other mathematical details, just state the relevant results to be used in the empirical part of the paper. The reader is referred to Aronsson and Löfgren (1996) for further details. Along the general equilibrium path, the current value Hamiltonian implicit is the consumers optimization problem can be written

$$(5) \qquad H^0 = u(c^0, z^0) + \lambda^0[f(k^0,\, a(h^0)l^0, h^0) - c^0 - d^0] + \mu^0[g(x^0) - \gamma h^0]$$

where the top index denotes the general equilibrium of the market economy. The variables λ^0 and μ^0 are the shadow prices of man-made and human capital, respectively. They can be interpreted as the present value in utility units of an additional unit of capital. By differentiating the Hamiltonian with respect to time (all variables are functions of time), and using the equilibrium properties of the market economy, one can show that this results in a differential equation which, if solved forwards, yields

$$(6) \qquad \theta \int_t^\infty u(c^0(s),\, z^0(s))\, e^{-\theta(s-t)}\, ds = H^0(t) + \int_t^\infty \mu^0(s) f_h(s) \dot{h}^0_{(s)}\, e^{-\theta(s-t)}\, ds.$$

The interpretation of equation (6) is the following: interest on the present value of future utility at time t, equals the value of the Hamiltonian at time t, plus the present value of the marginal external effect; all entities are measured along the market equilibrium path. The function $f_h(s)$ is the direct effect of the human capital stock on firm output. The intuitive reason why information at time t can be used to say something about future welfare is that the market economy has efficiency properties, due to utility and profit maximization, that inject envelope properties into the value function. The last term in (6) appears because the input of human capital is not optimally chosen by the consumer.

The relevance of this result for national accounting will be more transparent if we approximate the instantaneous utility by a linear function and use the fact that, along an optimal path, the marginal utility of the last unit consumed equals what would have been created in future utility by investing it, i.e. the shadow price of capital. Similarly, the marginal utility of leisure time, along an optimal path, equals what the unit would have generated in terms of utility had it been market work or invested in human capital. The linear approximization of the right hand side of (6) can, therefore, be written

$$(7)\quad \text{LWM} = u_c c^0 + u_z z^0 + \lambda^0 \dot{k}^0 + \mu^0 \dot{h}^0 + mee^0 = \lambda c^0 + \lambda w z^0 + \lambda^0 \dot{k}^0 + \mu^0 \dot{h}^0 + mee^0$$

where mee denotes the current value of the marginal externality, i.e., the last term on the right hand side of equation (6). Moreover, wz is the market value of leisure time. The linear welfare measure, LWM, is expressed in utility units. To transfer it to real terms, one divides by λ, the marginal utility of consumption at time t, to obtain

$$(7a)\qquad \frac{\text{LWM}}{\lambda^0} = c^0 + wz^0 + \dot{k}^0 + \frac{\mu^0}{\lambda^0}\dot{h}^0 + \frac{mee^0}{\lambda^0}.$$

The term we intend to estimate on a national level is $(\mu^0/\lambda^0)\dot{h}^0$, the future real value of net investment in human capital at time t. Provided that the externality term is non-negligible, and under the incomplete human capital investment concept, the externality would be hidden in the Solow residual.

Solving for the shadow price μ^0 from the first order conditions for optimum consumer behavior in the market economy yields

$$(8)\qquad \mu^0(t) = \int_t^\infty \lambda^0(s) a'(h^0(s)) f_l(s) l^0(s)\, e^{-(\theta+\gamma)(s-t)}\, ds.$$

The term $a'(h^0(s))f_l(s)$ measures the extra wage paid by the firm at time s for an extra unit of human capital. Worth noting is that the shadow price of human capital in the market economy does not contain any information from the marginal externality of human capital $f_h(s)$. On the other hand, this information would affect the shadow price if the consumer obtained a subsidy equal to the marginal product of the human capital externality, and this indicates how the externality could be internalized. The reader is again referred to Aronsson and Löfgren (1996) for details.

The following points are relevant for the empirical analysis

(i) The value of leisure at time t enters the welfare measure, not the present value of the future increase in the value of leisure time. The latter entity is added in by Jorgenson and Fraumeni (1992) as well as Ahlroth *et al.* (1997).

(ii) The value of leisure time should be added irrespective of whether the the model contains human capital or not, i.e. the second term of the welfare measure in equation (7a) remains, even if $h = 0$. The reason is that leisure time is an argument in the utility function.

(iii) The welfare measure given by equation (7a) contains the present value of the future increase in income—measured by the product wage—following an extra unit invested in human capital (term 4 on the right hand side of equation (7a)). This is also measured by Ahlroth *et al.* in Table 7. It is not measured by Jorgensen and Fraumeni, who deduct both the employers' wage tax and the income tax. This means that the latter provides an estimate of the private rather than the social value of education.

(iv) The cost of the educational sector should be deducted. Note that $\dot{k} = f(\cdot) - c^0 - d^0$, where $d(x)$ is the output lost in the education sector. Neither of the two previous research groups deduct the costs of the sector.

Data, The Empirical Model and the Results

An estimate of future extra income generated by human capital is, of course, always difficult to obtain, although some useful information is stored in historical data. However, the importance of education for the level of income may not be constant over time. In this paper, we will use data from the Swedish Standard of Living Survey, LNU, which is a household panel-data base containing data collected in 1968, 1974, 1981, and 1991. It contains individual data on e.g. wages, number of hours worked, and the number of years of schooling. These data are used to estimate a wage equation, where the number of years of schooling measures human capital. This equation is combined with a "numbers of hours worked" equation, and a probabilty to work equation in order to estimate the yearly gain in future income from the education system. The details are available in Ahlroth *et al.* (1997) and Löfgren and Marklund (1996). We nevertheless restate the key equation, which is an estimate of the fourth term on the right hand side of equation (7a).

$$(9) \qquad E^g\left(\frac{\mu^0}{\lambda^0}\dot{h}^0\right) = \sum_a \sum_e \sum_s \phi(a, e, s)[E\{LY(a, e+1, s, t)\} - E\{LY(a, e, s, t)\}]$$

where we interpret the sum over gender, s, to include all individuals with the age a and years of schooling e at time t. Since the probability to enroll in schooling, ϕ, is zero, by assumption, for all individuals over 40 and equal to one for all individuals under 17, we will, in the final calculation, only account for the expected income gains for the population between 16–40. In other words:

(v) We can, at best, measure the output from the high school and university systems. This point seems to be overlooked by Ahlroth *et al.* (1997).

Equation (9) is the empirical counterpart of the fourth term on the right-hand side of equation (7a). The difference is that (7a) is a net measure and (9) is gross measure, since we do not know how to handle the "depreciation" of human capital. To see this, note that the discount factor in equation (8) not only contains the time preference; it also contains the rate of depreciation of human capital, γ. The term E is the expectations operator and $E\{LY(a, e, s, t)\}$ the expected lifetime income at time t of an individual at age a, schooling e, and gender s. To estimate lifetime product wage income we have assumed, following Jorgenson and Fraumeni (1992a) and Ahlroth *et al.* (1997), that (exogenous) income growth is 1.9 percent and the rate of time preference is set at 5.4 percent. The employers tax is set at 40 percent.

To be complete, however, we have to deduct the opportunity costs of the resources used as inputs in the system from the gross benefits obtained from equation (9). We would also wish to add possible external effects from human capital accumulation, which are not accounted for in the above wage equation based calculation. The externalities, which we unfortunately will not be able to measure, correspond to the last term in equations (6) and (7a), respectively.

The cost data to be deducted from the gross measures have been collected from Statistics Sweden and the Swedish Association of Municipalities. These data sets contain information about the yearly operating costs of the local school systems up to (and including) high school. To obtain the costs of the university system at the local level, we have deducted the aggregate cost of the school system up to high school from the aggregate cost of the total education system, excluding the cost of labor market education. By dividing the result by the total number of students in tertiary education, we obtain an estimate of the average cost per student. Finally, by multiplying this average cost with the number of registered students, we obtain the cost of the university system.

Table 1 lists the operating costs of the different levels of the Swedish education system in 1990. Since we do not fully include the opportunity cost of capital, we are conceivably underestimating the costs.

TABLE 1
OPERATING COSTS OF SECONDARY AND TERTIARY EDUCATION
(millions of 1990 SEK).

Area	High School	University/College
Sweden	13,586	10,049

The net output is computed in Table 2 below:

TABLE 2
THE OUTPUT OF THE SWEDISH EDUCATIONAL SYSTEM—SECONDARY AND TERTIARY LEVELS IN 1990
(millions of 1990 SEK).

Area	P Population	C Costs	E^g Gross Output	E^g/P SEK	$E^g - C$ Net Output	$(E^g - C)/P$ (SEK)
Sweden	8,558,833	23,636	51,130	5,970	27,494	3,212

The gross output figure in column 3 is approximately the same level as the corresponding figure in Ahlroth *et al.* (1997). However, this is not obvious at first.

This is because, in the same way as Jorgensen and Fraumeni (1992a, b), they routinely measure total lifetime income as including both wage income and leisure income evaluated at current wages. Therefore, most of their output measures include changes in future leisure income induced by education. As should be clear from the theoretical section above, such an output measure has no clear welfare interpretation.

In a social cost-benefit analysis the wage rate should, to reflect the marginal product of labor, be measured before income tax and include pay-roll taxes. The value of future leisure time should be excluded from the future benefits of human capital. Current leisure time enters the measure of welfare, because it is an argument in the utility function (see equation 7). This means, that to correctly reflect the gross output from the educational sector, the Ahlroth *et al.* figure, before taxes and excluding the value of leisure time, should be scaled to account for the effect of the payroll tax. If we do this for their 1990 estimate, we obtain 54.6 billions SEK, which corresponds favorably with our estimate of gross output for the same year, 51.3 billions. The difference may, to some extent, be due to the fact that they use 75 years as "retirement age," while we have chosen the actual retirement age, which is 65 years. This level is less than the total cost of the Swedish education sector in 1990 (66 billions), i.e. adding the costs of the primary school system to the operating cost in in column two in Table 2. A researcher not aware of the fact that the method does not enable us to measure the output from the primary school system would be inclined to conclude that the Swedish school system is over-dimensioned.

To give the reader a feeling for the magnitude of the net present value produced by higher education in Sweden, the gross output figure in Table 2 is approximately 2 percent of GDP. This may seem far from impressive, even if one acknowledges the fact that we are unable to measure the output from the mandatory part of the educational system. However, calculating the rate of return by finding the interest rate which makes net output equal to zero yields 8.6 percent. This figure corresponds reasonably well with the private rate of returns of higher education in Sweden in 1990, which is reported in Edin and Holmlund (1993).[1]

The possible external effects from the educational sector are not measured by our method. They correspond to the present marginal value of the externalities of knowledge along the future path of the economy [the term *mee* in equation (7)], and are extremely difficult to estimate. To obtain a within the ballpark estimate, one can solve a numerical version of the model. This is done in Aronsson, Johansson and Löfgren (1997), where a Cobb–Douglas technology, as well as a Cobb–Douglas utility function, are used to mimic real world conditions.[2] Sensitivity analyses show that the marginal externality along the market solution amounts to between 30–50 percent of current NNP, excluding the impact from externalities. In other words, it is not unlikely that the contribution to future welfare from the externalities of human capital are huge. Similar results are

[1]They calculate the rate of return both before and after taxes and subsidies. However, they do not calculate output at the product wage (including the employer's wage tax). This means that the figures are not directly comparable.
[2]See Chapter 5.

reached by Weitzman (1997) and Weitzman and Löfgren (1997), where the contribution from a forecast of future technological progress is estimated.

References

Ahlroth, S., A. Björklund, and A. Forslund, The Output of the Swedish Education Sector, *Review of Income and Wealth*, 43, 89–104, 1997.

Aronsson, T. and K. G. Löfgren, Welfare Consequences of Technological and Environmental Externalities in the Ramsey Growth Model, *Natural Resource Modeling*, 7(1), 1–14, 1993.

———, National Product Related Welfare Measures in the Presence of Technological Change, Externalities and Uncertainty, *Environment and Resource Economics*, 5, 321–32, 1995.

———, Social Accounting and Welfare Measurement in a Growth Model with Human Capital, *Scandinavian Journal of Economics*, 98, 185–201, 1996.

———, P. O. Johansson and K. G. Löfgren, *Welfare Measurement, Sustainability and Green National Accounting*, Edward Elgar, Cheltenham U.K., 1997.

Edin, P. A., P. Frediksson and B. Holmlund, Utbildningsnivå och utbildningsavkastning i Sverige, *Ekonomiska Rådets Årsbok*, 11–37, 1993.

Jorgenson, D. W. and B. Fraumeni, Investment in Education and U.S. Economic Growth, *Scandinavian Journal of Economics*, 94, Supplement, 51–70, 1992a.

———, The Output of the Education Sector, in Z. Griliches (ed.), *Output Measures in the Service Sectors*, The University of Chicago Press, Chicago, 1992b.

Kemp, M. C. and N. Long, On the Evolution of Social Income in A Dynamic Economy: Variations in the Samuelsonian Theme, in G. R. Feiwel (ed.), *Samuelson and Neoclassical Economics*, Kluwer–Nijhoff, Boston, 1982.

Lucas, R. E., On the Mechanics of Economic Development, *Journal of Monetary Economics*, 22, 3–42, 1988.

Löfgren, K. G., Comments on C. R. Hulten, "Accounting for the Wealth of Nations: The Net versus Gross Output Controversy and its Ramifications," *Scandinavian Journal of Economics*, 94, Supplement, 25–28, 1992.

——— and Marklund, P. O., The Regional Output from Human Capital: Do Universities Matter? *Umeå Economic Studies*, No 427, 1996.

Romer, P. M., Endogenous Technical Change, *Journal of Political Economy*, 94, 1002–37, 1986.

Weitzman, M. L., On the Welfare Significance of National Product in a Dynamic Economy, *The Quarterly Journal of Economics*, 90, 156–62, 1976.

———, Sustainability and the Welfare Significance of National Product Revisited, *Scandinavian Journal of Economies*, 99, 1–14, 1997.

——— and K. G. Löfgren, On the Welfare Significance of Green Accounting as Taught by Parable, *Journal of Environmental Economics and Management*, 32, 139–53, 1997.