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LIFETIME AND VERTICAL INTERTEMPORAL INEQUALITY, INCOME SMOOTHING, AND REDISTRIBUTION: A SOCIAL WELFARE APPROACH

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Between and within-households intertemporal inequality indices are proposed to highlight the vertical and lifetime (i.e. cyclical) components of overall intertemporal inequality. Comparison with the classical static inequality indices is made. Income redistribution and smoothing (i.e. stabilization) are conveniently defined as the public policy impact on welfare, by means of the relative increase in intertemporal vertical and cyclical equity, respectively. The issue is important as many public policies are aimed at both (vertical and cyclical equity) objectives. Our approach provides a more appropriate evaluation of the desirability of public reforms aimed at achieving a greater vertical and cyclical equity, within a social welfare framework.

1. INTRODUCTION

This paper addresses the issues of intertemporal inequality and public sector redistribution and income smoothing within a social welfare framework. We establish a microeconomic social welfare framework to consider these objectives in terms of efficiency, vertical intertemporal inequality reduction and lifetime inequality reduction. We hope to contribute to a more appropriate simultaneous evaluation of the beneficial effects and desirability of public policies aimed not only at achieving a greater vertical equity but also a greater life-cycle stability. It is important to distinguish both elements as many public policies pursue these two objectives separately.

Within the overall intertemporal inequality we distinguish between the so-called vertical intertemporal inequality (among different households) and the lifetime or cyclical inequality (among the same households over their life cycle). We propose the use of the between and within-households intertemporal inequality indices to highlight these vertical and cyclical components. In particular, we provide a framework where the overall intertemporal inequality index is decomposed into "between-households vertical intertemporal" and "within-households cyclical" inequality indices. Redistribution and income smoothing (or stabilization) are measured as reductions of these indices respectively.

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Similarly, overall intertemporal inequality can also be decomposed into an aggregate "between-periods cyclical" and static "within-periods vertical cross-sectional" inequality indices. We find a set of results relating all the indices. We find some bias in the use of either the aggregate between-periods or the static within-periods indices in empirical work.

Most papers on intertemporal inequality focus only on the between-households vertical intertemporal inequality indices and so they tend to forget the importance of the so-called lifetime inequality. However they tend to point out that conclusions based on cross-sectional (or static) income data can give a misleading picture of the inequality of a more permanent or lifetime notion of income.[1] We prove the general result that within-periods vertical cross-sectional (or static) inequality indices overestimate real between-households vertical intertemporal inequality indices. However some empirical results tend to minimize the possible discrepancies [e.g. Shorrocks (1978) and Slemrod (1992)].

The traditional view of looking at the evolution of vertical redistribution by using a sequence of cross-sectional vertical inequality reducing indices is again very likely to be biased. This approach does not capture the real redistribution effects between rich and poor (since it does not capture the fundamental distinction between rich and poor on the basis of lifetime income). A relevant result is that the sign of the bias is ambiguous. Using annual income measures, Caspersen and Metcalf (1994) find VAT to be quite regressive and using lifetime income they find VAT only to be modestly regressive.

The importance of the lifetime or cyclical inequality is clear as many public policies are not specifically aimed at achieving a greater vertical equity but a greater within-households lifetime equity. Much of the welfare state schemes, such as public pensions or unemployment or health insurance schemes, are oriented to smoothing out or stabilizing the level of income over the life cycle rather than to achieving vertical redistribution. Macroeconomic stabilization policy can be seen as pursuing some kind of aggregate cyclical equity objective.[2] This can be traced back to Pigou who implicitly mentioned the importance of this element when stating the three objectives of government: growth, equity and stabilization.

We also prove that between-periods cyclical inequality indices subestimate real within-households cyclical inequality indices. We can now compute stabilization effects of public policy through the relative increase of the within-households cyclical equity indices.

These results hold for the two alternative income discounts considered. We believe that the choice of the discount is a separate problem that only concerns the definition of the permanent income and the benchmark upon which inequality is measured, but this choice does not affect the general propositions stated here.

[1]Caused by volatility of transitory income or due to life-cycle income effects. Alternative concepts of lifetime income are instead adopted in the literature. Annual income is replaced by average real income over the period [e.g. Slemrod (1992) and Harding (1994)] or by present values of future income [e.g. Creedy (1982) and Creedy, Disney and Whitehouse (1993)] or by real income-equivalent annuity that could be financed over life with the individual's wealth or utility-equivalent annuity. See Creedy (1992) for a survey of alternative concepts.

[2]In fact, concave intertemporal preference models, normally used in macroeconomics, implicitly assume beneficial cyclical inequality reduction.

The paper is organized as follows. In Section 2 the intertemporal social welfare framework is built and the main definitions, theorem and corollaries are stated. In Section 3 an alternative discounted income-based setting is developed. In Section 4 a decomposition of within-households cyclical inequality in population subgroups is proposed. In Section 5 we analyze the effects of redistribution and income smoothing on intertemporal welfare. Finally in Section 6 we illustrate the main postulates with an application to the Spanish Personal Income Tax and in Section 7 we offer some conclusions.

2. The Intertemporal Social Welfare Framework

We assume an individualistic social welfare function of the after-tax adult-equivalent real income levels Y^e_{ht} of all the households $h=1, 2, \ldots, H$ and for all periods over the life cycle $t=1, \ldots, T$:[3]

$$W(Y^e_{11}, \ldots, Y^e_{1T}; \ldots; Y^e_{H1}, \ldots, Y^e_{HT}). \tag{1}$$

We assume this function to be symmetric, increasing and concave in the household incomes of any period Y^e_{ht}. We define the *overall intertemporal inequality index* $I(A)$ as the Atkinson (1970) index of Y^e_{ht};

$$I(A)=1-\frac{Y^*}{Y^M}, \tag{2}$$

where Y^M is the aggregate mean income and Y^* is the *overall intertemporal equally-distributed equivalent income*, which is the level of income that, if equally distributed among all households and all periods, provides the same level of social welfare as the actual distribution. Given the concavity of $W(\cdot)$ in Y^e_{ht}, Y^* is no greater than Y^M. $I(A)$ indicates the proportion of total intertemporal income that it would be willing to lose in order to eliminate all the existing intertemporal inequality.

We define the *between-households vertical intertemporal inequality index* as the between-groups Atkinson index $I(A)^B_{h\cdot}$ derived from the distribution which assigns each person his mean income across time, that is:

$$I(A)^B_{h\cdot}=1-\frac{Y^{*B}_{h\cdot}}{Y^M}, \tag{3}$$

being $Y^{*B}_{h\cdot}$ the *between-households vertical equally-distributed equivalent income* that, if equally distributed among all households and all periods, is socially indifferent to a distribution which assigns to each household its average income across time:

$$\begin{aligned} W(Y^{*B}_{h\cdot}, \ldots, Y^{*B}_{h\cdot}; \ldots; Y^{*B}_{h\cdot}, \ldots, Y^{*B}_{h\cdot}) \\ = W(Y^M_{1\cdot}, \ldots, Y^M_{1\cdot}; \ldots; Y^M_{H\cdot}, \ldots, Y^M_{H\cdot}), \end{aligned} \tag{4}$$

[3]It can be derived from a more general setting where the social evaluation function is $W=w[V(\mathbf{P}, Y_1, \mathbf{a}_1), \ldots, V(\mathbf{P}, Y_H, \mathbf{a}_H)]$, being $\mathbf{P}$, the vector of commodity prices over time, $\mathbf{Y}_h$, the vector of current real income levels and $\mathbf{a}_h$, the vector of personal characteristics over time; where prices are held constant throughout the analysis, and Y^e_h is the vector that makes $V(\mathbf{P}, \mathbf{Y}_h, \mathbf{a}_h)=V(\mathbf{P}, \mathbf{Y}^e_h, \mathbf{a}_0)$ or alternatively $\mathbf{Y}^e_h=e(\mathbf{P}, V, \mathbf{a}_h)$, being $e(\cdot)$ the expenditure function. In Section 3 an analogous analysis is made for discounted income levels.

being $Y^M_{h\cdot}$ the household h mean income over time. Given the concavity of $W(\cdot)$ in Y^e_{ht}, $Y^{*B}_{h\cdot}$ is no greater than Y^M. $I(A)^B_{h\cdot}$ indicates the proportion of total households intertemporal income that it would be willing to lose in order to eliminate all between-households vertical intertemporal inequality.

We define the *within-household h cyclical inequality index* as the Atkinson (1970) index, $I(A)_{h\cdot}$:

$$I(A)_{h\cdot} = 1 - \frac{Y^*_{h\cdot}}{Y^M_{h\cdot}}, \tag{5}$$

being $Y^*_{h\cdot}$ the *cyclical equally-distributed equivalent income of household h* that, if lifetime equally-distributed, is socially indifferent to his actual distribution. Given the concavity of $W(\cdot)$ in Y^e_{ht}, $Y^*_{h\cdot}$ is no greater than $Y^M_{h\cdot}$. $I(A)_{h\cdot}$ indicates the proportion of total household h intertemporal income that it would be willing to lose in order to eliminate h cyclical inequality.[4]

Aggregation into the *overall within-households cyclical inequality index*, $I(A)^W_{h\cdot}$ is made as:

$$\begin{aligned} &W(Y^e_{11}, \ldots, Y^e_{1T}; \ldots; Y^e_{H1}, \ldots, Y^e_{HT}) \\ &\quad = W(Y^M_{1\cdot}(1 - I(A)^W_{h\cdot}), \ldots, Y^M_{1\cdot}(1 - I(A)^W_{h\cdot}); \ldots; \\ &\qquad Y^M_{H\cdot}(1 - I(A)^W_{h\cdot}), \ldots, Y^M_{H\cdot}(1 - I(A)^W_{h\cdot})), \end{aligned} \tag{6}$$

that indicates the proportion of total households intertemporal income that it would be willing to lose in order to eliminate all within-households cyclical inequality, that is moving from the actual distribution to the distribution which assigns to each household its average income over time.

Similarly, we define the *overall between-periods cyclical inequality index* as the between-groups Atkinson index $I(A)^B_{\cdot t}$ derived from a distribution which assigns each person the period mean income:

$$I(A)^B_{\cdot t} = 1 - \frac{Y^{*B}_{\cdot t}}{Y^M}, \tag{7}$$

where $Y^{*B}_{\cdot t}$ is the *overall between-periods cyclical equally-distributed equivalent income* that guarantees a level of social welfare equal to what would be obtained if all the households would have the period mean income level across time:

$$\begin{aligned} &W(Y^{*B}_{\cdot t}, \ldots, Y^{*B}_{\cdot t}; \ldots; Y^{*B}_{\cdot t}, \ldots, Y^{*B}_{\cdot t}) \\ &\quad = W(Y^M_{\cdot 1}, \ldots, Y^M_{\cdot T}; \ldots; Y^M_{\cdot 1}, \ldots, Y^M_{\cdot T}) \end{aligned} \tag{8}$$

being $Y^M_{\cdot t}$ the period t mean income across households. Given the concavity of $W(\cdot)$ in Y^e_{ht}, $Y^{*B}_{\cdot t}$ is no greater than Y^M. $I(A)^B_{\cdot t}$ indicates the proportion of total households intertemporal income that it would be willing to lose in order to eliminate all between-periods cyclical inequality.

[4]This approach differs from that in Creedy and Wilhelm (1995) as we consider social and not individual aversion to lifetime inequality.

We define the *within-period t vertical static inequality index* as the Atkinson index $I(A)_{\cdot t}$:

(9) $$I(A)_{\cdot t} = 1 - \frac{Y^*_{\cdot t}}{Y^M_{\cdot t}},$$

being $Y^*_{\cdot t}$ the *vertical* (*static*) *equally-distributed equivalent income* of period t which is the equally distributed income level for period t that provides the same level of social welfare as the actual distribution. Given the concavity of $W(\cdot)$ in Y^e_{ht}, $Y^*_{\cdot t}$ is no greater than $Y^M_{\cdot t}$. $I(A)_{\cdot t}$ indicates the proportion of total period t income that it would be willing to lose in order to eliminate the existing period inequality.

We aggregate into the *overall within-periods vertical static inequality index* $I(A)^W_{\cdot t}$ as follows:

(10) $$\begin{aligned} &W(Y^e_{11}, \ldots, Y^e_{H1}; \ldots; Y^e_{1T}, \ldots, Y^e_{HT}) \\ &= W(Y^M_{\cdot 1}(1 - I(A)^W_{\cdot t}), \ldots, Y^M_{\cdot 1}(1 - I(A)^W_{\cdot t}); \ldots; \\ &\qquad Y^M_{\cdot T}(1 - I(A)^W_{\cdot t}), \ldots, Y^M_{\cdot T}(1 - I(A)^W_{\cdot t})), \end{aligned}$$

that indicates the proportion of total households intertemporal income that it would be willing to lose in order to eliminate all within-periods vertical inequality, that is moving from the actual distribution to the distribution which assigns to each household the period average income.

Under homotheticity of $W(\cdot)$ we obtain:[5]

(11) $$\begin{aligned} W(Y^M(1 - I(A))) &= W(Y^M(1 - I(A)^B_{h\cdot})(1 - I(A)^W_{h\cdot})) \\ &= W(Y^M(1 - I(A)^B_{\cdot t})(1 - I(A)^W_{\cdot t})). \end{aligned}$$

Overall welfare can be decomposed into an efficiency measure Y^M and into overall intertemporal inequality, which in turn can be doubly decomposed, on the one hand, into overall within-households cyclical inequality and between-households vertical intertemporal inequality:

(12) $$1 - I(A) = (1 - I(A)^W_{h\cdot})(1 - I(A)^B_{h\cdot})$$

and, on the other hand, into the overall within-periods vertical static inequality and the between-periods cyclical inequality:

(13) $$1 - I(A) = (1 - I(A)^W_{\cdot t})(1 - I(A)^B_{\cdot t}).$$

These decompositions are relevant as many public policies pursue these redistributive objectives separately. Moreover, we find a set of general results which relates these indices and which gives some ideas for their appropriate use in empirical work.

[5]In Appendix 1 we derive the explicit aggregation for the traditional Atkinson inequality index, derived from the more restrictive additive separable homothetic $W(\cdot)$. Analogous but not equal results can be found in von Weizsäcker (1978) and Blackorby *et al.* (1981).

Theorem 1. For any convex inequality index:[6]

$$(14) \qquad I(Y^e_{11}, Y^e_{12}, \ldots, Y^e_{1T}; \ldots; Y^e_{H1}, Y^e_{H2}, \ldots, Y^e_{HT}).$$

Define the between-periods cyclical inequality $I^B_{\cdot t}$ and the between-households vertical intertemporal inequality $I^B_{h\cdot}$ as the inequality index of all households having the same period mean income and the same personal mean income across time, respectively. If $T, H \geq 2$, then:

$$(15) \qquad I \geq I^B_{h\cdot} + I^B_{\cdot t}.$$

Proof can be found in Appendix 2.

Corollary 1. Define the overall within-households cyclical inequality $I^W_{h\cdot}$ as a generalized mean of within-household inequalities $I_{h\cdot}$ so that:

$$(16) \qquad I^{\max}_{h\cdot} \geq I^W_{h\cdot} \geq I^{\min}_{h\cdot},$$

where $I^{\max}_{h\cdot}$ and $I^{\min}_{h\cdot}$ are the highest and lowest values of $I_{h\cdot}$ for all t, and given[7]

$$(17) \qquad I = I^W_{h\cdot} + I^B_{h\cdot} - f(I^W_{h\cdot}, I^B_{h\cdot}),$$

where $f(\cdot)$ is a non-negative, lower than $I^B_{h\cdot}$, non-decreasing function in the arguments and being zero if any argument is zero. Then:

$$(18) \qquad I \geq I^W_{h\cdot} \geq I^B_{\cdot t}.$$

The overall within-households cyclical inequality $I^W_{h\cdot}$ is not lower than the between-periods cyclical inequality $I^B_{\cdot t}$. A weighted average of households cyclical inequalities is not lower than the cyclical inequality of the mean income levels over time.

Corollary 2. Define the overall within-periods vertical static inequality $I^W_{\cdot t}$ as a generalized mean of within-period inequalities $I_{\cdot t}$ so that:

$$(19) \qquad I^{\max}_{\cdot t} \geq I^W_{\cdot t} \geq I^{\min}_{\cdot t}$$

where $I^{\max}_{\cdot t}$ and $I^{\min}_{\cdot t}$ are the highest and lowest values of $I_{\cdot t}$ for all h, and given[8]

$$(20) \qquad I = I^W_{\cdot t} + I^B_{\cdot t} - f(I^W_{\cdot t}, I^B_{\cdot t})$$

where $f(\cdot)$ is a non-negative, lower than $I^B_{\cdot t}$, non-decreasing function in the arguments and being zero if any argument is zero. Then:

$$(21) \qquad I \geq I^W_{\cdot t} \geq I^B_{h\cdot}.$$

The overall within-periods vertical static inequality $I^W_{\cdot t}$ is not lower than the between-households intertemporal vertical inequality $I^B_{h\cdot}$. A weighted mean of

[6]The inequality index is convex if $I(\cdot)$ is a convex function in income levels. The traditional Atkinson index, see Appendix 1, the traditional Theil index, $I(T) = (1/H)\sum(Y_i/Y^M)\,\mathrm{Ln}\,(Y_i/Y^M)$, and the Gini coefficient are convex.

[7]Traditional Atkinson and Theil index decompositions satisfy this requirement. Following equation (12), for traditional Atkinson index, $I(A_\varepsilon) = I(A_\varepsilon)^W_{h\cdot} + I(A_\varepsilon)^B_{h\cdot} - I(A_\varepsilon)^W_{h\cdot} I(A_\varepsilon)^B_{h\cdot}$. For the traditional Theil index, $I(T) = I(T)^W_{h\cdot} + I(T)^B_{h\cdot}$, where $I(T)^W_{h\cdot} = \sum(Y^M_{h\cdot}/HY^M)\, I(T)_{h\cdot}$. See Theil (1967) and Shorrocks (1980). The Gini coefficient does not satisfy this requirement.

[8]Traditional Atkinson and Theil index decompositions satisfy this requirement. See previous note for analogy.

cross-section vertical inequalities is not lower than the vertical inequality of mean intertemporal income levels across households.

Corollary 3. A set of sufficient conditions for relative inequality indices that any average of cross-section vertical inequalities is equal to the vertical inequality of average intertemporal income levels across households

$$I \geq I^{W}_{\cdot t} = I^{B}_{h\cdot} \tag{22}$$

is that household's incomes increase at the same rate within each period, although this rate may differ across periods. For relative indices, in this case, $I^{W}_{\cdot t} = I^{B}_{h\cdot} = I_{\cdot 1}$. In this case it follows that for relative inequality indices satisfying conditions given by expressions (15) and (18):

$$I \geq I^{W}_{h\cdot} = I^{B}_{\cdot t}. \tag{23}$$

3. Discounted Income-based Intertemporal Inequality

We redefine the social welfare function in terms of the after-tax adult-equivalent discounted income levels Y^{d}_{ht} of all the households $h = 1, 2, \ldots, H$ and for all the periods over the life cycle $t = 1, \ldots, T$:

$$W(Y^{d}_{11}, Y^{d}_{12}, \ldots, Y^{d}_{1T}; \ldots ; Y^{d}_{H1}, Y^{d}_{H2}, \ldots, Y^{d}_{HT}). \tag{24}$$

We assume this function to be symmetric, increasing and concave in the discounted income Y^{d}_{ht} across households and periods. We discount household's income by the mean annual income rate of growth over the whole period, r^{M}.[9] Therefore, we make the following transformation:

$$Y^{d}_{ht} = \frac{Y^{e}_{ht}}{(1 + r^{M})^{t-1}} \tag{25}$$

being

$$r^{M} = \left[\frac{Y^{e}_{hT}}{Y^{e}_{h1}}\right]^{1/T-1} - 1. \tag{26}$$

All previous definitions and propositions remain analogous. The main changes are qualitative changes and affect the concept of dynamic inequality. We change the benchmark from which inequality is measured. The main changes due to this new definition are summarized below.

(I) Under discounted income inequality indices, zero cyclical inequality is attained when all households increase their incomes at the same rate within and between periods, in this case, $I^{d} = I^{dW}_{\cdot t} = I^{dB}_{h\cdot} = I_{\cdot 1}$. With no discounted income inequality indices, zero cyclical inequality is attained when all households have same income levels across periods, so household's income growth rates should be zero in order to satsify $I = I^{W}_{\cdot t} = I^{B}_{h\cdot} = I_{\cdot 1}$.

As a consequence, under discounted income inequality indices, cyclical inequality is referred to as the divergence over the mean growth cycle, instead of

[9] Any other alternative of income discount can be applied, for example the mean income growth rate of any period.

over the mean income level of the period, as in the case of no discounted inequality indices. The new version of discounted cyclical inequality may be more appropriate and more intuitive in relation with the implicit concept of cyclical inequality.

(II) Empirically we obtain in most of the cases that $I^B_{\cdot t} \geq I^{dB}_{\cdot t}$, being equal when household's income growth rates are zero. We lose between-periods cyclical intertemporal inequality due to the loss of some of the intertemporal variability in the income levels.

(III) If households increase their incomes at the same rate within each period, although this rate may differ across periods, overall discounted income inequality is not higher than no discounted income inequality. Under this condition, $I^W_{\cdot t} = I^{dW}_{\cdot t} = I_{\cdot 1}$, and for relative indices that satisfy expression (18), the following condition is satisfied:

$$I = g(I^B_{\cdot t}, I^W_{\cdot t}) \geq I^d = g(I^{dB}_{\cdot t}, I^{dW}_{\cdot t}). \tag{27}$$

4. Decomposition of Within-households Cyclical Inequality in Population Subgroups

Define the overall within-households cyclical inequality $I^W_{h\cdot}$ as the generalized mean of within-household inequalities $I_{h\cdot}$ (as traditional Atkinson indices case, see Appendix 1):

$$1 - I^W_{h\cdot} = \phi^{-1}\left[\sum_{h=1}^{H} w_h \phi(1 - I_{h\cdot})\right] \tag{28}$$

where weights w_h are between one and zero and sum up to one and $\phi(\cdot)$ is an increasing function; or as the weighted mean (as traditional Theil index case):

$$I^W_{h\cdot} = \sum_{h=1}^{H} w_h I_{h\cdot}. \tag{29}$$

Define the subgroup within-households cyclical inequality $I^W_{hs\cdot}$ as the generalized mean of within-household inequalities $I_{hs\cdot}$ in subgroup s:

$$1 - I^W_{hs\cdot} = \phi^{-1}\left[\sum_{hs=1}^{Hs} w_{hs} \phi(1 - I_{hs})\right] \tag{30}$$

or, respectively, as the weighted mean:

$$I^W_{hs\cdot} = \sum_{hs=1}^{Hs} w_{hs} I_{hs\cdot} \tag{31}$$

being hs the household h who belongs to subgroup s and Hs the number of households in subgroup s. In either case we obtain the following aggregation properties, S being the number of subgroups:

$$1 - I^W_{h\cdot} = \phi^{-1}\left[\sum_{s=1}^{S} w_s \phi(1 - I^W_{hs\cdot})\right] \tag{32}$$

or, respectively:

$$I_{h\cdot}^{W}=\sum_{s=1}^{S} w_s I_{hs\cdot}^{W}. \tag{33}$$

5. Intertemporal Vertical Redistribution and Income Smoothing

Traditionally intertemporal vertical redistribution is observed as an evolution of a sequence of cross-sectional vertical inequality reduction indices. That is the reduction of inequality caused by the public sector. This approach does not capture the real lifetime redistribution effects between rich and poor, as it does not capture the fundamental distinction between rich and poor on the basis of the lifetime income, but on the basis of annual income. Comparisons of redistribution among groups with same annual income levels can be misleading, as we observe redistribution among groups which are formed by individuals who are fundamentally different (with different lifetime incomes) although they have the same income because they are in different stages of their life-cycle. An interesting result is that the sign of the bias is ambiguous. We can now compute the more appropriate intertemporal vertical redistribution effect through the vertical between-households redistribution effects associated with public policy. Empirical analysis would then give us the sign of the bias.

However, the redistribution between rich and poor is not the only objective of public policy. Much of what public policy is aimed at is stabilization across the life cycle, namely, redistribution between higher income levels to lower income level periods. We can now compute these life cycle effects through the cyclical within-households redistribution effects associated with public policy. Yet interpretation of empirical results tell us more about the effects of a series of systems under the period rather than the effect of a particular change, which is difficult to isolate. It would require a simulation model to compute the steady-state effects of a particular reform in a particular period over lifetime income. Specifically, we define the concepts of intertemporal vertical redistribution and income smoothing as the relative effect of the policy on social welfare, by means of the relative increase in the intertemporal vertical or cyclical equity indices, respectively. Given social welfare before the policy as the following, derived in a natural way from equation (11):

$$W=Y^{M}(1-I(A)_{h\cdot}^{W})(1-I(A)_{h\cdot}^{B}) \tag{34}$$

the policy produces the following infinitesimal relative effect on social welfare:

$$\frac{dW}{W}=\frac{dY_M}{Y_M}+\frac{d(1-I(A)_{h\cdot}^{B})}{(1-I(A)_{h\cdot}^{B})}+\frac{d(1-I(A)_{h\cdot}^{W})}{(1-I(A)_{h\cdot}^{W})}. \tag{35}$$

The right-hand side term of equation (35) gives us the three components into which the infinitesimal relative effect on welfare can be divided: the rate of change in average intertemporal income (this is the contribution to efficiency of the policy); the rate of change in intertemporal vertical equity (the contribution to intertemporal redistribution of the policy) and, finally, the rate of change of cyclical equity (the contribution to stabilization of the policy). Under small discrete policy changes, the intertemporal vertical redistribution (IVR) and income smoothing (S) indices of the policy can be approximated be a relative increase in the respective equity index:[10]

(36)
$$IVR \simeq \frac{\Delta(1-I(A)^B_{h\cdot})}{(1-I(A)^B_{h\cdot})} = \frac{I(A)^B_{h\cdot} - I'(A)^B_{h\cdot}}{(1-I(A)^B_{h\cdot})}$$

(37)
$$S \simeq \frac{\Delta(1-I(A)^W_{h\cdot})}{(1-I(A)^W_{h\cdot})} = \frac{I(A)^W_{h\cdot} - I'(A)^W_{h\cdot}}{(1-I(A)^W_{h\cdot})}.$$

6. Application to the Spanish Personal Income Tax 1985–91

We use the database of the Spanish Personal Income Tax Panel Data, consisting of simple samples of individual Income Tax returns for the years 1985 to 1991. In 1988 and subsequent years the separate tax returns of married couples are added together to constitute one single item.

As a tax variable, we use of the net tax liability recorded in the tax returns. Since some tax returns have negative tax bases, these have been modified to one peseta following verification that this change, or elimination of these items, did not produce any significant differences in the indices. This is done in order to eliminate non-positive arguments from the logarithms, while at the same time keeping the maximum possible statistical information under uniform criteria for all the indices.

Tables 1.1 to 1.5 show the value of the $\varepsilon=1$ Atkinson, $I(A_\varepsilon)$, the standard Theil, $I(T)$, and the Gini indices, $I(G)$, for all intertemporal inequality indicators defined in the paper for the period 1985–91. First of all, all intertemporal indices, except from the Atkinson cyclical within-households (Table 1.3), show a lower inequality when computed over discounted rather than real income values, although the difference is not substantial. In the two last rows of Table 2.1 we present the increase of the real and discounted mean income for comparison.

Except from the Gini index there exists a multiplicative decomposition (Atkinson index case[11]) or an additive decomposition form (Theil index case) of overall indices in the within and between-periods or households indices. (Recall notes 6 and 7.)

Cyclical inequality is adequately measured by the within-households indices, which is infraestimated by the between-periods indices, satisfying

(38)
$$I \geq I^W_{h\cdot} \geq I^B_{\cdot t}.$$

The overall within-households cyclical inequality $I^W_{h\cdot}$, a weighted average of households cyclical inequalities, is no lower than the between-periods cyclical inequality $I^B_{\cdot t}$, the inequality of the mean income levels across periods.

[10] The classical absolute inequality reduction indices $\mathrm{IRI} = I(A)_{h\cdot} - I'(A)_{h\cdot}$ could also be proposed [e.g. Lambert (1993)], even though their interpretation would be somewhat different since they would approximate the increase in absolute welfare.

[11] $\varepsilon=1$ Atkinson index can be proved to be ordinal equivalent to Generalized Entropy Theil-zero index, also additively decomposable. See Cowell (1977), Shorrocks (1980) and Jenkins (1991) for details.

TABLE 1

OVERALL, BETWEEN-WITHIN HOUSEHOLDS AND BETWEEN-WITHIN PERIODS INTERTEMPORAL INEQUALITY INDICES BEFORE TAX (PERIOD 1985–91)

Table 1.1: Overall Intertemporal Inequality Indices

Index	Real Incomes	Discounted Income
$I(A)$	0.23596	0.23270
$I(T)$	0.26462	0.25884
$I(G)$	0.36857	0.36479
Y^M	1,828,630	1,600,291

Table 1.2: Vertical Between-households Inequality Indices

Index	Real Incomes	Discounted Incomes
$I(A)^B_{h\cdot}$	0.18821	0.18059
$I(T)^B_{h\cdot}$	0.22572	0.22328
$I(G)^B_{h\cdot}$	0.34351	0.34235

Table 1.3: Overall Cyclical Within-households Inequality Indices

Index	Real Incomes	Discounted Incomes
$I(A)^W_{h\cdot}$	0.05882	0.06359
$I(T)^W_{h\cdot}$	0.03890	0.03556
$I(G)^W_{h\cdot}$	0.10876	0.10492

Table 1.4: Overall Vertical Within-periods Inequality Indices

Index	Real Incomes	Discounted Incomes
$I(A)^W_{\cdot t}$	0.23264	0.23264
$I(T)^W_{\cdot t}$	0.26030	0.25876
$I(G)^W_{\cdot t}$	0.36857	0.36579

Table 1.5: Cyclical Between-periods Inequality Indices

Index	Real Incomes	Discounted Incomes
$I(A)^B_{\cdot t}$	0.00432	0.0000743
$I(T)^B_{\cdot t}$	0.00432	0.0000743
$I(G)^B_{\cdot t}$	0.05259	0.0066035

Analogously, the vertical intertemporal inequality is adequately measured by the between-households and is overestimated by the within-periods indices, satisfying

(39) $$I \geq I^W_{\cdot t} \geq I^B_{h\cdot}.$$

The overall within-periods vertical static inequality $I^W_{\cdot t}$, a weighted average of the cross-section vertical inequalities, is no lower than the between-households intertemporal vertical inequality $I^B_{h\cdot}$, the inequality of average intertemporal income levels across households. In this case the difference is not substantially large.

TABLE 2

DECOMPOSITION OF THE WITHIN-PERIODS INTERTEMPORAL AND WITHIN-HOUSEHOLD INEQUALITY INDICES BEFORE TAX (PERIOD 1985–91)

Table 2.1: Cross-section within-periods vertical indices

Index	1985	1986	1987	1988
$I(A)_{\cdot t}$	0.20885	0.21085	0.22315	0.24090
$I(T)_{\cdot t}$	0.22846	0.23148	0.24826	0.27251
$I(G)_{\cdot t}$	0.34807	0.34857	0.35759	0.36905
$Y^M_{\cdot t}$	1,592,807	1,642,448	1,721,917	1,841,144
$Y^M_{\cdot t}$ disc.	1,592,807	1,573,256	1,579,892	1,618,118
Index	**1989**	**1990**	**1991**	**1985–91**
$I(A)_{\cdot t}$	0.24135	0.24599	0.25609	0.23264
$I(T)_{\cdot t}$	0.28022	0.27572	0.27339	0.25876
$I(G)_{\cdot t}$	0.37267	0.37565	0.38025	0.36579
$Y^M_{\cdot t}$	1,919,249	2,020,708	2,062,139	1,828,630
$Y^M_{\cdot t}$ disc.	1,615,703	1,629,450	1,592,807	1,600,291

Table 2.2: Cyclical Within-household Indices Per Decile

Decile	$I(A)^W_{hs\cdot}$	$I(T)^W_{hs\cdot}$	$I(G)^W_{hs\cdot}$
1	0.18825	0.08161	0.17296
2	0.08084	0.03822	0.11054
3	0.05402	0.03059	0.09982
4	0.05010	0.02934	0.09844
5	0.04130	0.02716	0.09468
6	0.03977	0.02610	0.09174
7	0.03787	0.02552	0.09089
8	0.03736	0.02658	0.09281
9	0.04264	0.02923	0.09661
10	0.05269	0.04875	0.12082
Total	0.06359	0.03556	0.10492

Moreover, the overall intertemporal inequality is approximately similar to the overall within-periods static vertical inequality.

In Table 2.1 we present the evolution of all the cross-section within-period vertical inequality indices, traditionally used to evaluate intertemporal changes in vertical inequality. This view is a correct one if within-households life-cycle income effects are not taken into account.

Table 2.2 decomposes the overall within-households cyclical inequality $I^W_{h\cdot}$ into the decile within-households cyclical inequality indices and shows that the highest volatility of income is in the two lowest deciles and in the highest decile of intertemporal income. For this panel data set and for this boom period it seems more transitory income are found in the lowest brackets of income (probably young people), as Blundell and Preston (1994) suggest and in the very rich (probably due to transitory of capital incomes). Inclusion of the cycle decline period will probably lead to a reassessment of these results.

In Table 3.1 we present overall, vertical intertemporal redistribution and income smoothing defined as the difference between corresponding indices evaluated over the pre- and post-tax income levels. These results show, for the whole

TABLE 3

INTERTEMPORAL INCOME SMOOTHING AND REDISTRIBUTION

Table 3.1: Overall, Income Smoothing and Vertical Redistribution Discounted Indices

Index	Overall	W-Households Smoothing	B-Periods Smoothing	W-Periods Redistribution	B-Households Redistribution
Atkinson	0.04061	0.00379	−0.000066	0.04068	0.03674
Theil	0.06458	0.00682	−0.000066	0.06700	0.05858
Gini	0.04250	0.00981	−0.00947	0.048214	0.04168

Table 3.2: Cross Section Within-period Vertical Redistribution Indices

Index	1985	1986	1987	1988
Atkinson	0.03514	0.03563	0.03883	0.04032
Theil	0.05203	0.05440	0.06108	0.06680
Gini	0.03805	0.03827	0.04091	0.04353
Average Discount Tax	209,163	228,312	251,425	235,573
Index	1989	1990	1991	1985–91
Atkinson	0.04324	0.04438	0.04460	0.04068
Theil	0.07189	0.07348	0.07147	0.06700
Gini	0.04522	0.04542	0.04588	0.04821
Average Discount Tax	253,951	272,068	271,809	246,043

income tax policy over the period, little change in either estimates of vertical redistribution. However the sign of income smoothing is opposite (showing positive or negative) as measured by either index. Aggregate between-period income smoothing seems to be biased due to the use of inadequate aggregates in this small number of years data set.

Finally, in Table 3.2 we present the evolution of all the cross-section within-period vertical redistribution indices, traditionally used to evaluate intertemporal changes in vertical redistribution, where within-households life-cycle income effects are not taken into account.

7. CONCLUSIONS

This paper has addressed the issue of measurement of intertemporal inequality and public sector redistribution and income smoothing within a social welfare framework. We have established a microeconomic social welfare framework to consider the social objectives in terms of efficiency, intertemporal vertical redistribution and income smoothing (i.e. cyclical stabilization).

We have defined the concepts of intertemporal vertical redistribution and income smoothing as the relative effect of the policy on social welfare, by means of the relative increase in intertemporal vertical and lifetime equity, respectively. Both elements constitute the overall intertemporal equity. We propose the "between-households vertical intertemporal" and the "within-households cyclical" inequality indices as the adequate measures of vertical and lifetime (i.e. cyclical) intertemporal inequality components. Another decomposition of the overall inequality is also possible into a between-periods cyclical and within-periods vertical static inequality indices. However we find several drawbacks with this approach which are important when used in empirical work.

We have found a set of theoretical results which relates these indices and gives some ideas for their appropriate empirical use:

(1) Any overall convex inequality index is not lower than the sum of its between-periods cyclical inequality index and its between-households vertical intertemporal inequality index.

As a corollary, we find a general result that justifies the empirical results of Shorrocks (1978) and Slemrod (1992). The global inequality index is not lower than the classical overall within-periods vertical static inequality index, and the latter is not lower (equal, at the most) than the proposed between-households intertemporal vertical inequality index. As a result the use of the classical static-based index tends to overestimate real vertical intertemporal inequality. The reason is the loss of income variability by using permanent income-based estimates to evaluate vertical inequality.

As far as intertemporal redistribution is concerned, the traditional view of observing the evolution of vertical redistribution by using a sequence of cross-sectional vertical inequality reducing indices is very likely to be biased. This approach does not capture the real redistribution effects between rich and poor on the basis of the lifetime income.

However, the redistribution between rich and poor is not the only objective of public policy. Much of what public policy is aimed at is to redistribute across the life cycle, that is between higher income to lower income level periods. In general, this cyclical component is not addressed in the literature.

We have proved that the global inequality index is not lower than the overall within-households cyclical inequality index, and the latter is not lower (equal, at the most) than the between-periods cyclical inequality index. As a result the use of this latter period-aggregate-based index tends to infraestimate the real individually-based cyclical inequality index. The reason is the loss of income variability due to use of aggregate-based estimates to evaluate cyclical inequality.

(2) In general we have observed that all the results stated theoretically are verified empirically. We obtain that the vertical intertemporal redistribution shows, for the whole income tax policy over the period, little change in either estimate of vertical redistribution. However, the sign of income smoothing is clearly biased by using aggregate between-periods estimates, showing a negative effect of progressive income tax on stabilization. This suggests that the need for more adequate within-households estimates, which requires the availability of individual panel data sets.

(3) We have appropriately decomposed the overall within-households cyclical inequality index into the subgroup within-households cyclical inequality indices. We have also observed the higher volatility of income in the two lowest deciles and in the highest decile of intertemporal income. For this panel data set and for this expansion period it seems that more transitory income are found in lowest brackets of income (probably young people), as Blundell and Preston (1994) suggest and in the very rich (probably due to the transitory nature of capital

incomes). Inclusion of the cycle decline period may cause reassessment of these results.

(4) Under discounted income inequality indices, all previous propositions remain basically unchanged. The main changes affect the benchmark from which inequality is measured. Extensions can be proposed to incorporate alternative concepts of discounted income measures which affect the concept of permanent income. This will be analyzed in future research, but the main results can be extrapolated.

APPENDIX 1

If we assume that $W(\cdot)$ is additively separable into identical utility of households income levels:[12]

$$W=\sum_{h=1}^{H} U(Y_{h1},\ldots,Y_{hT}). \tag{40}$$

Homotheticity implies that the intertemporal vertical equity index may be expressed as follows:

$$1-I(A_\varepsilon)_{h\cdot}^{B}=\left[\frac{1}{H}\sum_{h=1}^{H}\left[\frac{Y_{h\cdot}^{M}}{Y^{M}}\right]^{1-\varepsilon}\right]^{1/(1-\varepsilon)},\qquad \varepsilon\neq 1 \tag{41}$$

$$1-I(A_\varepsilon)_{h\cdot}^{B}=\exp\left[\frac{1}{H}\sum_{h=1}^{H}\mathrm{Ln}\left[\frac{Y_{h\cdot}^{M}}{Y_{M}}\right]\right],\qquad \varepsilon=1 \tag{42}$$

where ε is the degree of aversion to intertemporal vertical inequality, which is greater than zero if we require concavity. In this case, the cyclical equity index can be expressed as follows:

$$1-I(A_\varepsilon)_{h\cdot}^{W}=\left[\sum_{h=1}^{H}\alpha_{h\cdot}[1-I(A_\varepsilon)_{h\cdot}]^{1-\varepsilon}\right]^{1/(1-\varepsilon)},\qquad \varepsilon\neq 1 \tag{43}$$

being $\alpha_{h\cdot}=(Y_{h\cdot}^{M})^{1-\varepsilon}/\sum(Y_{h\cdot}^{M})^{1-\varepsilon}$ and where:

$$1-I(A_\varepsilon)_{h\cdot}=\left[\frac{1}{T}\sum_{t=1}^{T}\left[\frac{Y_{ht}}{Y_{h\cdot}^{M}}\right]^{1-\varepsilon}\right]^{1/(1-\varepsilon)},\qquad \varepsilon\neq 1 \tag{44}$$

and

$$1-I(A_\varepsilon)_{h\cdot}^{W}=\exp\left[\sum_{h=1}^{H}\frac{1}{H}\mathrm{Ln}\,[1-I(A_\varepsilon)_{h\cdot}]\right],\qquad \varepsilon=1 \tag{45}$$

where:

$$1-I(A_\varepsilon)_{h\cdot}=\exp\left[\frac{1}{T}\sum_{t=1}^{T}\mathrm{Ln}\left[\frac{Y_{ht}}{Y_{h\cdot}^{M}}\right]\right],\qquad \varepsilon=1 \tag{46}$$

[12]Separability implies that relative social valuation of the income or tax rates between two households does not depend on the other households.

Analogous decomposition can be found in von Weizsäcker (1978) and in Blackorby *et al.* (1981) in a context where between-groups inequality is differently defined, from a distribution where each household receives its subgroup's equally-distributed equivalent income, instead of its subgroup mean income.

APPENDIX 2

Proof of Theorem 1: For any convex inequality index:

(47) $$I(Y^e_{11}, Y^e_{12}, \ldots, Y^e_{1T}; \ldots; Y^e_{H1}, Y^e_{H2}, \ldots, Y^e_{HT}).$$

Define the between-periods cyclical inequality $I^B_{\cdot t}$ and the between-households vertical intertemporal inequality $I^B_{h\cdot}$ as the inequality index of all households having the same period mean income and the same personal income across time, respectively. If H, $T \geq 2$, then:

(48) $$I \geq I^B_{h\cdot} + I^B_{\cdot t}.$$

Let us define the following expressions:

(49) $$Y^M_{h\cdot} = \frac{1}{T} \sum_{t=1}^{T} Y_{ht}$$

(50) $$Y^M_{\cdot t} = \frac{1}{H} \sum_{h=1}^{H} Y_{ht}.$$

If Ψ is a convex function then

(51) $$\psi(Y^M_{h\cdot}) = \psi\left(\frac{1}{T} \sum_{t=1}^{T} Y_{ht}\right) \leq \frac{1}{T} \sum_{t=1}^{T} \psi(Y_{ht})$$

(52) $$\psi(Y^M_{\cdot t}) = \psi\left(\frac{1}{H} \sum_{h=1}^{H} Y_{ht}\right) \leq \frac{1}{H} \sum_{h=1}^{H} \psi(Y_{ht}).$$

It follows that:

(53) $$\sum_{h=1}^{H} \psi(Y^M_{h\cdot}) + \sum_{t=1}^{T} \psi(Y^M_{\cdot t}) \leq \sum_{h=1}^{H} \frac{1}{T} \sum_{t=1}^{T} \psi(Y_{ht}) + \sum_{t=1}^{T} \frac{1}{H} \sum_{h=1}^{H} \psi(Y_{ht})$$
$$= \left(\frac{1}{T} + \frac{1}{H}\right) \sum_{h=1}^{H} \sum_{t=1}^{T} \psi(Y_{ht})$$

if

(54) $$\frac{1}{T} + \frac{1}{H} \leq 1, \qquad \text{if } T, H \geq 2.$$

We can finally conclude:

$$\sum_{h=1}^{H} \psi(Y_{h\cdot}^{M}) + \sum_{t=1}^{T} \psi(Y_{\cdot t}^{M}) \leq \sum_{h=1}^{H} \sum_{t=1}^{T} \psi(Y_{ht}) \tag{55}$$

so Theorem 1 is proved.

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