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ATKINSON AND BOURGUIGNON'S DOMINANCE CRITERIA: EXTENDED AND APPLIED TO THE MEASUREMENT OF POVERTY IN FRANCE

BY CHRISTINE CHAMBAZ AND ERIC MAURIN
Institut National de la Statistique et des Etudes Economiques, France

We demonstrate that the criteria put forward by Atkinson and Bourguignon (1987) can be satisfactorily applied to the analysis of poverty even when the demographic composition of society and the definition of poverty change over time. We show that they provide both sufficient and necessary conditions for any robust conclusions concerning the changes in welfare and poverty. We apply these criteria to a set of data tracking annual income distribution movements in France from 1977 to 1994. The Atkinson and Bourguignon criteria posit that recent changes in poverty are a function of macroeconomic fluctuations in activity.

The needs of households differ according to their size and composition. These differences in needs must be taken into account when measuring and comparing living standards. Standard practice is to use equivalence scales, where each type of household is attributed a number of *consumption units* representing their specific needs. A distribution of income by consumption units is obtained by comparing the income of the different households with their number of consumption units. This distribution can be interpreted as a distribution of living standards. In this case, we can compare the welfare associated with two income distributions simply by comparing the generalized Lorenz curves of the two corresponding distributions of income by consumption units. The practical appeal of this method is considerable, but poses serious problems due to a lack of agreement about the supporting assumptions;

—assumptions regarding the types of equivalent households from the point of view of needs (identification problem);
—assumptions regarding the ranking of the different household types in terms of their relative needs (classification problem);
—assumptions regarding how much more needy one household type is compared to another (quantification problem).

This latter point is the aspect about which there is the least agreement.[1]

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[1]Blundell and Lewbel (1991) show, for instance, that it is impossible to estimate equivalence scales using household demand observed at a particular point in time without making *ad hoc* identification assumptions. So there are no more grounds for the common practice of estimating an equivalence scale than for those which simply adopt scales *ad hoc*. Lewbel (1989) maintains that the identification assumption usually adopted (where the equivalence scale is independent of the reference welfare level) is based on highly specific cost functions which, as empirical tests have shown, are not borne out by observed data.

One possible strategy for overcoming this problem is to choose a representative range of equivalence scales and to explore sensitivity of results to change in them. For instance, Bradbury (1997) makes some plausible assumptions about the needs differences between the different family types and provides an upper and lower bound for poverty increases in Australia between 1981 and 1990, using the Atkinson's dominance criteria (Atkinson, 1987).

An alternative method is the sequential dominance approach introduced by Atkinson and Bourguignon (1987). It does not require any *a priori* assumptions about how much different families differ in their needs. Let us assume that there are only two categories of households differentiated by their needs. The Atkinson and Bourguignon criteria involve comparing income distributions within the neediest subgroup of households and *then* across all the households. If one distribution dominates the other on both accounts (as shown by a generalized Lorenz curve), then social welfare is considered to be higher, *regardless of the method used to find how much more needy one household type is compared with the other.* The method put forward by Atkinson and Bourguignon (1987) does not quantify the welfare differentials and, in this respect, is somewhat less informative than sensitivity analyses. However the sequential criteria yield a much more precise picture of the set of diagnoses that can be considered independent of equivalence scale.

Sequential procedures were initially put forward for the case in which the demographic composition of households is assumed to be constant across all the income distributions. Atkinson (1992) extended the scope of these procedures to include poverty analysis. Jenkins and Lambert (1993) studied the validity of these procedures in the case where the distribution of needs and the composition of households differ from one distribution to the next.

Although the theoretical and methodological appeal of the sequential dominance approach is undeniable, its use is still not widespread. We suggest that there are at least three reasons for this. First of all, increasing the number of comparisons to be made is probably regarded as creating more work while reducing the chances of obtaining clear-cut results. Secondly, it has not yet been clearly established whether the method can satisfactorily compare societies with different demographic compositions. Thirdly, in poverty analysis, the method is normally based on a concept of *absolute* poverty (poverty line fixed over time or between countries), when it is usually taken to be *relative*, at least in Europe (poverty line varies over time or between countries).

In this article, we argue that these criticisms are not entirely justified. In particular, we set out to demonstrate that the Atkinson and Bourguignon criteria can be adapted to situations where a society's demographic composition and the definition of poverty vary over time, or from one country to the other. We would also suggest that, despite their elaborate appearance, the Atkinson and Bourguignon criteria can definitely be used for operational purposes.

The paper is organized as follows. In the first section, we look over the sequential dominance principles and identify the conditions required for their application to situations where the society's demographic composition changes. We demonstrate that the Atkinson and Bourguignon criteria are not only *sufficient* for the analysis of poverty, but are actually *necessary* conditions for robust poverty rankings, a fact we feel had not previously been established. In the second section, we put forward a simple theoretical framework to extend the scope of the sequential principles to the case in which the definition of poverty changes over time. The third section presents the data base used for the empirical analysis. This data base tracks the changes in annual income distributions in France over the 1977–94 period. In the fourth section, we apply the different generalized sequential comparison procedures to the data base.

1. Procedures for the Sequential Comparison of Income Distributions

Let us assume that there are n categories of households differentiated by their needs. On each date t, $q_{i,t}$ represents the share of subgroup i in the population of households. The income distribution function (resp. density) within the population is denoted by $F_t(y)$ (resp. $f_t(y)$), while $F_{i,t}(y)$ (resp. $f_{it}(y)$) represents the distribution function (resp. density) within subgroup i. In accordance with utilitarian tradition, the social welfare associated with the income distribution $F_t(y)$ is written as a linear function of the individual utilities, that is

$$W_t(v_1, \ldots v_n) = \sum_{i=1}^{n} q_{i,t} \int_0^{y_{\max}} v_i(y) f_{i,t}(y)\, dy \tag{1}$$

where $y_{\max}$ represents the maximum income level and $v_i(y)$ derivable, non-decreasing functions of income.

Having established these notations, let us suppose that the different household categories can be *ranked* in order of needs. We could then identify which one of two different types of households with the same income would benefit the most from an increase in income. The principles established by Atkinson and Bourguignon (1987) are based on this sole identification assumption. To extend the scope of their method to the case in which the demographic structure of society changes, we shall assume that the differences in the needs of those households with the highest incomes can be disregarded. We shall explicitly focus on planners such that,

$$\begin{cases} v_1'(y) \geq v_2'(y) \geq \cdots \geq v_n'(y) \geq 0, \forall y \leq y_{\max} \\ v_1(y_{\max}) = v_2(y_{\max}) = \cdots = v_n(y_{\max}) = \bar{v} \end{cases} \tag{2}$$

Using this framework, the following fundamental proposition can be stated:

Proposition 1. $W_{t+1}(v) \geq W_t(v), \forall v = (v_1, \ldots, v_n)$ *satisfying* (2)
$$\Leftrightarrow \sum_{i=1}^{j} (q_{i,t+1} F_{i,t+1}(y) - q_{i,t} F_{i,t}(y)) \leq 0, \forall y \text{ and } \forall j.$$

In other words, for the level of social welfare to be unanimously regarded as higher at $t+1$ than at t, it is necessary and sufficient to find that, for any income threshold y and any division of the population into two subgroups ranked in terms of needs, the share of "needy" and "poor" households (i.e. those below the y threshold) decreased between the two dates.

The proof of Proposition 1 is given in Appendix A. The Atkinson and Bourguignon (1987) approach can simply be adapted to suit the case where there may be variation in the society's demographic composition.

Before going any further, bear in mind that the extension of the Atkinson-Bourguignon theorem is restricted to planners who believe that the best option for any household not among the wealthiest is to have as few needs as possible. Assumption (2) implies in particular $v_j(y) \leq v_{j+1}(y)$ for all j and y. If $v_i(y)$ is interpreted as the individual utility of type i households, then Proposition 1 is only valid in the restrictive case where families do not experience an increase in welfare when their size increases while resources remain constant (see Bourguignon, 1989, for a discussion on this point). As we demonstrate further on, this kind of assumption is typical of those generally employed in poverty analysis. So assumption (2) will not be particularly restrictive when we shall adapt it to the measurement of poverty. In this theoretical framework, where there may be variations in the population shares of the different household types, notice that Proposition 1 displays an interesting corollary:

Corollary 1: $\exists h \in [0, \ldots, n] / \sum_{i=1}^{h} (q_{i,t+1} - q_{i,t}) > 0$

$$\Rightarrow \exists (u_1, \ldots, u_n) \textit{ satisfying } (2) / W_{t+1}(u_1, \ldots, u_n) < W_t(u_1, \ldots u_n).$$

If the possible partitions of the household population into two subgroups ranked according to their needs contains even one division where the weight of the "needy" (in terms of that division) increases between the given dates, then there is no way that we can unambiguously conclude a rise in living standards between these dates, *regardless of the redistributions of wealth that may have occurred during the period.*

This result implies that—even with the restrictive constraints contained in assumption (2)—the conditions for a unequivocal conclusion about the change in welfare could be extremely hard to satisfy. The sequential approach detailed above could, nevertheless, prove to be a most useful tool in the analysis of poverty and changes in poverty.

1.1. *First-Degree Sequential Dominance and Changes in Poverty*

For all sets of poverty lines $Z_1 \geq \cdots \geq Z_n \geq 0$, we shall consider the class $\Pi(Z_1, \ldots, Z_n)$ of poverty measures P defined as follows:

$$P(F_t) = \sum_{i=1}^{n} q_{i,t} \int_0^{y_{\max}} p_i(y) f_{i,t}(y)\, dy, \tag{3}$$

where the $p_i(y)$ functions are derivable, non-increasing, non-negative over $[0, Z_i]$, zero for $Z_i < y \leq y_{\max}$ and satisfy the equivalent condition of (2), that is:

$$p_1'(y) \leq p_2'(y) \leq \cdots \leq p_n'(y) \leq 0, \forall y. \tag{4}$$

In other words, we restrict the analysis to the class of measures studied by Atkinson (1992) and Jenkins and Lambert (1993). These include, in particular, the non-normalized formulations of the measures proposed by Foster *et al.* (1984) ($p_i(y) = (Z_i - y)^a / a$ for all $a \geq 1$) as well as those put forward by Clark *et al.* (1981)

$(p_i(y) = (Z_i^c - y^c)/c$, for all positive $c)$.[2] They do not include the headcount ratio, which is not continuous in Z_i.[3] In this framework, it is relatively easy to state sequential dominance criteria similar to those covered in Proposition 1. In fact, for all sets of thresholds $(Z_1 \geq \cdots \geq Z_n)$, we have:

Proposition 2. $$\left\{\sum_{i=1}^{j} \Delta(q_{i,t}F_{i,t}(y)) \leq 0, \forall y < Z_j \text{ and } \forall j\right\}$$
$$\Leftrightarrow \{P(F_{t+1}) \leq P(F_t), \forall P \in \Pi(Z_1, \ldots Z_n)\}$$

where $\Delta(q_{i,t}F_{i,t})$ stands for $(q_{i,t+1}F_{i,t+1} - q_{i,t}F_{i,t})$. This proposition describes the *equivalence* between sequential dominance "restricted" over $(Z_1, \ldots, Z_n)$ and a decrease in poverty for all measures belonging to $\Pi(Z_1, \ldots, Z_n)$.

The *sufficiency* of restricted sequential dominance is shown by Atkinson (1992) in the case where needs structures are constant and Jenkins and Lambert (1993) in the general case. In Appendix B, we show the *necessity* of this condition in the general case.

It should be noted that for all v satisfying assumption (2), the poverty index defined by $(v_i(Z_i) - v_i(y))I_{[0,Z_i]}(y)$, $i = 1, \ldots, n)$—where I_A is the indicator function for a set A—belongs to $\Pi(Z_1, \ldots, Z_n)$ and that for all p in $\Pi(Z_1, \ldots, Z_n)$, the family $(-p_i(y)$, $i = 1, \ldots, n)$ satisfies assumption (2). In other words, a poverty index p belongs to $\Pi(Z_1, \ldots, Z_n)$ *if and only if* there is a v which satisfies (2) and such that p is equal to $((v_i(Z_i) - v_i(y))I_{[0,Z_i]}(y)$, $i = 1, \ldots, n)$.

Thus, if the principle of restricted sequential dominance is to be fully understood, we must examine the group of social decision-makers whose preferences satisfy assumption (2) and who evaluate poverty as the gap between the current welfare of the poor and the welfare they would have if they were to reach the poverty lines $Z_1 \geq \cdots \geq Z_n$. These planners would unanimously agree that a decrease in poverty had occurred if, and only if, first-degree sequential dominance over segments $[0, Z_1], \ldots, [0, Z_n]$ was observed.

Let us now consider a particular system of poverty lines $Z_1^+ \geq \cdots \geq Z_n^+$. If we denote $L(Z_1^+, \ldots, Z_n^+)$ as the entire set of thresholds $Z_1 \geq \cdots \geq Z_n$ such that $(Z_j \leq Z_j^+, \forall j)$, a corollary can be inferred from Proposition 2 which slightly broadens its scope:

Corollary 2. $$\sum_{i=1}^{j} \Delta(q_{i,t}F_{i,t}(y)) \leq 0, \forall y < Z_j^+ \text{ and } \forall j$$
$$\Leftrightarrow \forall (Z_1, \ldots, Z_n) \in L(Z_1^+, \ldots, Z_n^+)$$
$$\{P(F_{t+1}) \leq P(F_t), \forall P \in \Pi(Z_1, \ldots, Z_n)\}.$$

Corollary 2 shows the equivalence between the limited sequential dominance over $Z_1^+ \geq \cdots \geq Z_n^+$, and the decrease in poverty for *all systems of poverty lines* chosen below the thresholds defined by $Z_1^+ \geq \cdots \geq Z_n^+$. This is a direct result of sequential dominance over $(Z_1^+, \ldots, Z_n^+)$ being equivalent to sequential dominance over all the poverty line systems belonging to $L(Z_1^+, \ldots, Z_n^+)$.

[2]See Atkinson (1987) for a discussion of the problems posed by the normalization of these indices.
[3]For a seminal critique of the headcount ratio, see Sen (1973).

An interesting consequence of this corollary is that where a system of poverty lines exist such that F_t dominates F_{t+1} at one stage in the sequential procedure and F_{t+1} dominates F_t at another, there is no system of poverty lines such that the planners satisfying (2) unanimously agree about the change in the welfare deficit associated with poverty during this time period. In a recent contribution, Jenkins and Lambert (1997) provide a new graphical method for checking the robustness of poverty orderings to change in variation in deflators used to render nominal income comparable (i.e., to change in the poverty line in one distribution). However, their method requires the choice of one specific equivalence scale, which is not the case of the sequential approach.

1.2. *Second-Degree Sequential Dominance*

Let us now suppose that the impact of increases in income on the households' living standards weakens as their income rises—a fairly common assumption of concave individual welfare (where $v_i'(y)$ decreases with y). We shall also assume that in addition to the neediest households benefiting more from a rise in income, this is especially the case when low-income households are compared (i.e. $v_i'(y) - v_{i-1}'(y)$ grows with y for all $i > 1$). These assumptions can be written:

$$v_1''(y) \le v_2''(y) \le \cdots \le v_n''(y) \le 0, \forall y. \tag{5}$$

In other words, we are now looking above all at differences in subsistence needs. In this framework, we obtain the following fundamental result:

Proposition 3. $W_{t+1}(v) \ge W_t(v), \forall v$ *satisfying* (2) *and* (5)

$$\Leftrightarrow \sum_{i=1}^{j} \int_0^y \Delta(q_{i,t} F_{i,t}(x))\, dx < 0, \forall y \text{ and } \forall j.$$

Atkinson and Bourguignon (1987) put forward a proof of this theorem under the assumption that $q_{i,t}$ is constant. Jenkins and Lambert (1993) show the sufficiency of the second-degree sequential dominance in the more general case in which the structure of needs $q_{i,t}$ can vary. In Appendix C, we propose a proof of *necessity* in this same general case.

1.3 *Second-Degree Sequential Dominance and Changes in Poverty*

We shall now be concerned with poverty measures that satisfy both assumption (4) and:

$$p_1''(y) \ge p_2''(y) \ge \cdots \ge p_n''(y) \ge 0, \forall y. \tag{6}$$

Later, for all systems of poverty lines $Z_1 \ge \cdots \ge Z_n$, we use $\Pi^2(Z_1, \ldots, Z_n)$ to denote the set of measures belonging to $\Pi(Z_1, \ldots, Z_n)$ and satisfying (6). These poverty measures correspond to the welfare functions satisfying (2) and (5). *De facto*, for all v satisfying assumptions (2) and (5), the poverty index defined by $(v_i(Z_i) - v_i(y))I_{[0,Z_i]}(y)$ belongs to $\Pi^2(Z_1, \ldots, Z_n)$ and for all p in $\Pi^2(Z_1, \ldots, Z_n)$, the family $(-p_i(y), i = 1, \ldots, n)$ satisfies assumptions (2) and (5). On these bases, we can state sequential dominance criteria similar to those established in the previous sections. In fact, for any set of thresholds $(Z_1 \ge \cdots \ge Z_n)$, we can deduce:

Proposition 4. $\left\{\sum_{i=1}^{j}\int_0^{Z_i}\Delta(q_{i,t}F_{i,t}(y))\,dy\le 0, \forall y<Z_i \text{ and } \forall j\right\}$

$$\Leftrightarrow\{P(F_{t+1})\le P(F_t),\ \forall P\in\Pi^2(Z_1,\ldots,Z_n)\}.$$

This result expresses the *equivalence* between "restricted" sequential dominance over $(Z_1,\ldots,Z_n)$ and a decrease in poverty for all measures belonging to Π^2 $(Z_1,\ldots,Z_n)$, (i.e. planners sensitive to transfers are unanimous in concluding that the standard of living of the "poor" has approached the level it would be at if all the poor reached the poverty line). Proof of this is given in Appendix D.

Let us now consider a system of poverty lines $(Z_1^+\ge\cdots\ge Z_n^+)$. If we continue to use $L(Z_1^+,\ldots,Z_n^+)$ to denote the set of thresholds $Z_1\ge\cdots\ge Z_n$ such that $(Z_j\le Z_j^+, \forall j)$, a corollary can be derived from Proposition 4 which slightly broadens its scope.

Corollary 3. $\sum_{i=1}^{j}\int_0^{Z_i^+}\Delta(q_{i,t}F_{i,t}(y))\,dy\le 0, \forall y<Z_j^+ \text{ and } \forall j$

$$\Leftrightarrow \forall(Z_1,\ldots,Z_n)\in L(Z_1^+,\ldots,Z_n^+)$$

$$\{P(F_{t+1})\le P(F_t),\ \forall P\in\Pi^2(Z_1,\ldots,Z_n)\}.$$

Corollary 3 is symmetrical to Corollary 2 and bears the same useful practical implications. It states the equivalence between the restricted sequential dominance over $Z_1^+\ge\cdots\ge Z_n^+$ and the decline in poverty (in terms of (5)), *whatever the system of poverty lines* chosen below the thresholds defined by $Z_1^+\ge\cdots\ge Z_n^+$.

2. Sequential Dominance and Variable Poverty Lines

The previous analyses assume that the thresholds below which households are considered to be poor remain constant from one period to the next. It is possible to extend the sequential comparison principles to societies in which the definition of poverty changes over time. Let us assume that poverty is defined at date t in relation to $Z_{1,t}\ge\cdots\ge Z_{n,t}$, and let us suppose that the planners agree that the definition of poverty changes at the same rate for all the households. In this framework, there exist $Z_1\ge\cdots\ge Z_n$ and M_t such that $Z_{j,t}=M_tZ_j$ for all j and t. The M_t factor is interpreted as a social standard used as a reference by the planners studied. A non-poor household whose income grows faster than M_t cannot become poor. Conversely, a poor household whose income grows more slowly than M_t cannot break out of poverty. In our empirical study, M_t is taken as the average of the total income.[4]

We assume that the poverty measures $(p_{j,t}(y))$ satisfy assumption (4) at each date. Moreover, we apply measures such that the poverty of the poorest households changes over time at the same rate d_t regardless of their needs category, that is:

$$p_{j,t+1}(0)=(1+d_t)p_{j,t}(0),\ \forall j,t. \tag{7}$$

[4]There are many possible modeling assumptions about poverty changes. In this paper, we study only one of them and there is scope for further research.

The relative poverty of households with different needs and no income is thus assumed to be constant over time. Finally, we assume that, compared to the situation of the poorest, that of positive income households ($y > 0$) only improves when their income approaches the poverty line, that is

$$(8) \qquad \frac{p_{j,t}(y)}{p_{j,t}(0)} = \varphi_j\left(\frac{y}{Z_{j,t}}\right), \; \forall j, t, \text{ with } \varphi_j(0) = 1, \varphi_j(1) = 0, \; \varphi_j'(z) < 0, \forall j, t.$$

It is easy to check that the non-normalized formulations of the Clark *et al.* (1981) measures and Foster *et al.* (1984) measures belong to the set of poverty measures satisfying (4), (7) and (8).

Having stated these assumptions, poverty at t may now be expressed as

$$(9) \qquad P_t = \sum_{i=1}^{n} q_{i,t} \int_0^{Z_i} p_{i,t}(0)\varphi_i\left(\frac{r}{Z_i}\right)\tilde{f}_{i,t}(r)\, dr,$$

where r represents the relative income y/M_t and $\tilde{f}_{i,t}$ the density function of relative income in subgroup i. From (9) we derive

$$(10) \qquad \frac{P_{t+1}}{1+d_t} - P_t = -\sum_{i=1}^{n} \int_0^{Z_i} p_{i,t}(0)\varphi_i'\left(\frac{r}{Z_i}\right)\Delta(q_{i,t}\tilde{F}_{i,t}(r))\, dr = \tilde{\Delta}P_t,$$

where $\tilde{F}_{j,t}$ represents the distribution function of relative incomes in subgroup j.

$\tilde{\Delta}P_t$ represents the change in "adjusted" poverty. If we assume by convention that the poverty of the poorest households increases at the same rate as their distance from the average income (i.e. if d_t represents the total income growth rate), then $\tilde{\Delta}P_t$ is interpreted as the growth in the *share* of poverty in the national wealth. The sign of $\tilde{\Delta}P_t$ can obviously be studied using strictly the same sequential principles as those presented in the previous sections:

Proposition 5. $\tilde{\Delta}P_t \geq 0 \Leftrightarrow \sum_{i=1}^{j} \Delta q_{i,t}\tilde{F}_{i,t}(r) \geq 0, \forall r \in [0, Z_j]$ *and* $\forall j \leq n$.

Therefore the conclusions regarding the $\tilde{\Delta}P_t$ changes in adjusted poverty (and the identification of the structure of the poverty lines *below which these findings are possible*) can easily be drawn when sequential comparison procedures are applied to distribution of *relative incomes.*

3. The Data Sources and Conventions

The French National Institute of Statistics and Economic Studies (INSEE) has been conducting Household Living Standards (ECVM) surveys since 1977. Three surveys are made every year, with a sample of approximately 8,000 households for each survey. For the purposes of this study, we have used data from the October surveys. Interviewers asked each household to place itself in an annual income bracket. From 1977 to 1986, there were nine brackets to choose from, and eleven from 1987 to 1994. Our bracket system was set up to produce a discrete variable. To convert this into a continuous distribution, we used a fairly standard simulation method as described in Appendix E. We expressed all income distributions in 1994 constant French francs. These income distributions are consistent with those derived from surveys in which income is precisely stated, such as the Family Budget and Tax Revenue surveys (see Accardo and Fall, 1996). The income data were rounded out with information on the age and socio-economic group of the household's reference person, the household's composition, the size of the town of residence, and its status in the place of residence (owner, tenant, etc.). We assumed that four broad household types could be defined according to their needs: couples with two or more children, couples with one child, childless couples and lone-parent families, and single adults.

On the basis of these conventions, we made a sequential comparison of each income distribution (and *relative* income distribution) observed from 1977 to 1994 in relation to all the others (i.e. $18 \times 17/2 = 153$ comparisons). We used Kolmogorov tests to measure the significance of the deviations between the observed distribution functions.

4. Results

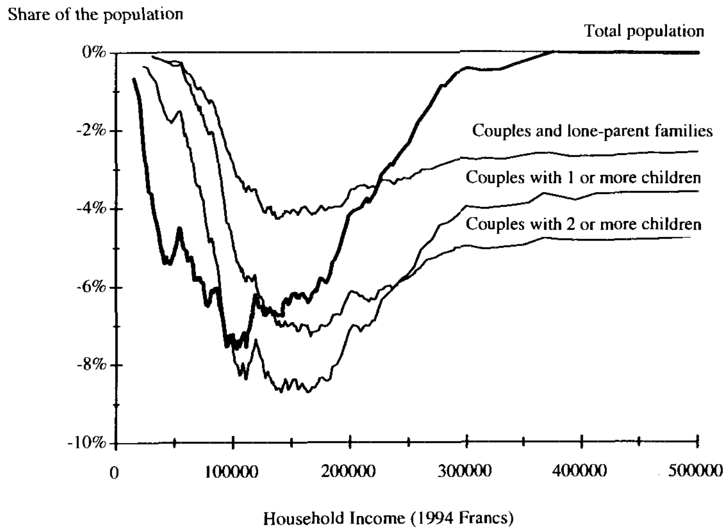
When the two income distributions at the extremes of our database (i.e., 1977 and 1994) are sequentially compared, we find first-degree sequential dominance of the 1994 distribution over the 1977 distribution (Graph 1a). Consequently, the set of planners satisfying assumption (2) unanimously conclude that there was a rise in social welfare from 1977 to 1994. Those planners who moreover agree about measuring poverty as the welfare deficit of the poor in relation to a system of *fixed* poverty lines (i.e. planners satisfying (4)), will also unanimously conclude that poverty declined, *regardless of the poverty line system chosen* to compare 1977 with 1994.

The results of a comparison of the 1989 and 1994 distributions are not so clear-cut (graph 1b). There is indeed no clear sequential dominance of one distribution over another in terms of Proposition 1 (or Proposition 3). There is no one welfare change conclusion likely to be upheld by all the utilitarian planners satisfying (2) [or (5)]. As regards poverty, however, the first-degree sequential dominance analysis does show that, for a system of poverty lines $Z_1 \geq Z_2 \geq Z_3 \geq Z_4$ such that ($Z_1 \leq 95{,}000\,F = 0.66\,med_1$, $Z_2 \leq 93{,}000\,F = 0.65\,med_2$, $Z_3 \leq 90{,}000\,F = 0.83\,med_3$, $Z_4 \leq 76{,}000\,F = 1.07\,med_4$), where med_i is the median of incomes for subgroup i in 1994, the planners satisfying (4) would agree that poverty had *risen*.

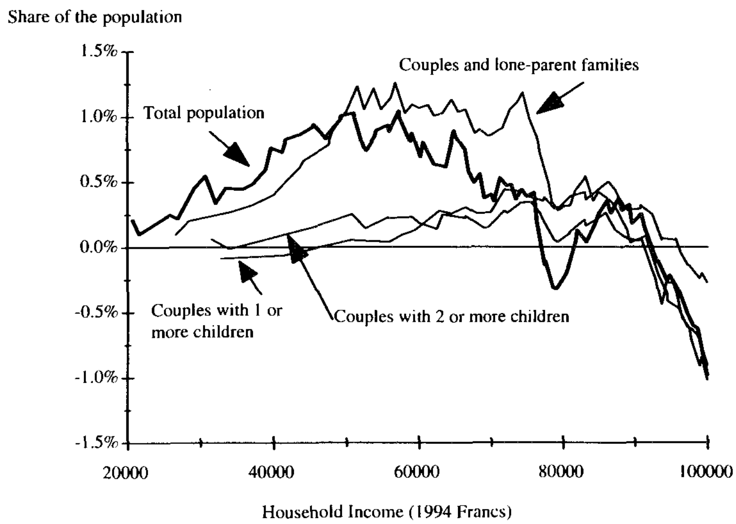
Corollary 2 provides an interesting additional result: it would be impossible to reach unanimous agreement [among the planners satisfying (4)] about the change in poverty if a more rigid system over poverty lines than the one adopted by the analysis were imposed (i.e. such that one of the thresholds would exceed the upper threshold associated with the analysis).

It would be an elaborate process to describe all the comparisons made in detail, so they are summarized in Graph 2. As regards poverty, these comparisons lead to a partial ordering of the set of eighteen, systematically compared income distributions. Of the 153 possible pairwise comparisons, the dominance criteria order 137 or 89.5 percent.

1.a - 1977 and 1994 Income distributions

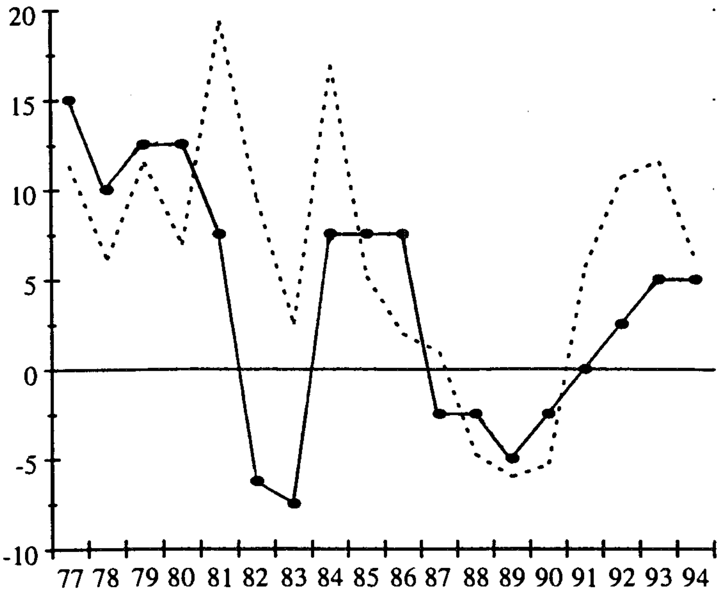


1.b: 1989 and 1994 Income Distributions



Reading: Each figure correponds to a given pair (t, t') of income distributions, with $t < t'$. Each figure represents the four curves $(\sum_{i=1}^{j}(q_{i,t}F_{i,t'}(y) - q_{i,t}F_{i,t}(y))$ for $j = 1, \ldots, 4$. For instance, figure 1b shows that in the French households population, the share of couples with two or more children with an annual income less than or equal to 70,000 French francs (1994 constant French francs) rose by nearly 0.3 points from 1989 to 1994.

Figure 1. Sequential comparison of French Income Distributions



Sources INSEE, Household Living Standards surveys, October 1977 to 1994.
INSEE, National Accounts.

Reading: The dotted lines show the series of annual variations in the unemployment rate (in %) in France (CVS data). The solid line represents the order in which the series of sequential comparisons of income distributions places the different distributions, in terms of poverty. To make the ranking consistent with all possible pair-comparisons, we laid down as a rule that:

- a distribution F is placed above a distribution G if (i) there does not exist a distribution which does not dominate G and dominate F; and (ii) there is a distribution which does not dominate F but dominates G;
- two distributions are placed on the same level only if they cannot be ordered and they do not verify one of the two conditions (i) and (ii).

Figure 2. Poverty in Terms of Sequential Dominance and Variations in the Unemployment Rate.

Following the reduction in poverty during the economic boom period in France in the late 1980s, poverty would seem to have risen again with the recession and economic slowdown in the early 1990s. This said, it has not regained its level of twenty years ago.

However elaborate the sequential method may seem, it would appear to be capable of identifying the relationship between poverty and the business cycle and has the added benefit of specifically isolating the conditions under which fluctuations would be unanimously recognized by all the utilitarian planners.

4.1. *Variable Poverty Lines and Comparison with the Sen index*

To round off our analysis, we made a sequential comparison of the distributions of *relative* income. This involves analyzing the change in poverty given the assumption that the poverty thresholds change over time at the same rate as mean income. Within this framework, the application of the principles contained in proposition 5 produces quite an interesting result: whether based on the assumption of relative or absolute poverty lines, the diagnosis remains almost unchanged. In other words, although the measure of poverty at any given date may be significantly altered by switching from an absolute notion (fixed poverty lines) to a relative notion (variable poverty lines), this adjustment barely affects the course of the fluctuations of the measure over time.

How should this convergence of findings be interpreted? When a fixed definition of poverty is adopted, the process is fueled not only by the changes in inequalities between rich and poor, but also by the rate at which the average income of the population grows. When a relative definition is applied, changes in inequalities form the main motive force. The similarity betwen the curves obtained when either definition is adopted could therefore be explained by the fact that income inequalities increase at the expense of the poor more readily when overall income growth is slow. In short, the less surplus there is to be shared, the more unequal shares will be.

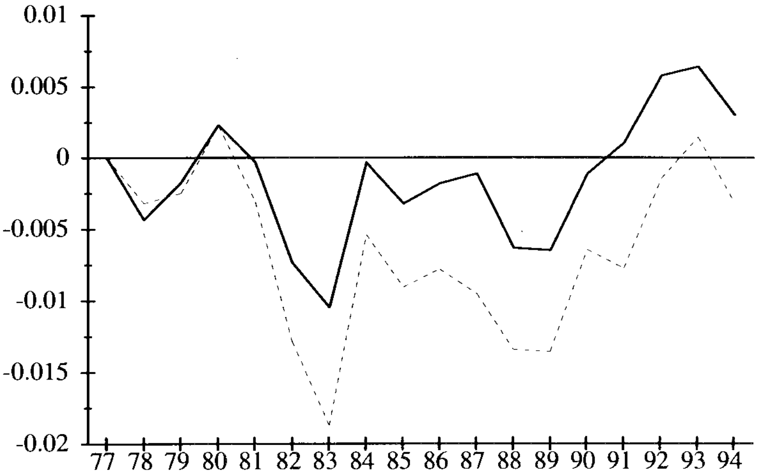
Finally, we felt it useful to compare the curve obtained from adopting sequential procedures with the curve produced by the poverty indicator proposed by A. Sen (1976) (Graph 3) for the same period. This indicator does not belong to the class of poverty measures studied in this paper. It implicitly attributes importance to rank in the income hierarchy and assumes discontinuity between the state of "being poor' and being "non-poor."[5] Regardless of the type of poverty lines chosen *a priori* (half-median, half-mean) for the 1977–94 period, the Sen index confirms the general course of fluctuations found earlier using the sequential procedure. Similarly, in a detailed analysis, this type of calculation confirms how useful it would be to produce for the Sen indictors the same kind of normative results as those produced for utilitarian indicators. For example, the Sen index leads to a different ranking of income distributions for 1977 and 1994 depending on whether the half-median or half-mean is chosen to define poverty. As we have seen, when utilitarian poverty measures are used, the application of sequential procedures makes it possible to reach more certain conclusions.

Conclusion

In this paper, we show that Atkinson and Bourguignon's criteria provide both sufficient and necessary conditions for any robust conclusions concerning the changes in welfare and poverty. We also demonstrate that their usefulness is not limited to studies of societies whose demographic composition remains constant. By the same token, we believe it is possible to satisfactorily extend the scope of these criteria to situations where the definition of poverty changes over time.

Moreover, sequential dominance principles would appear to be useful from more than just a methodological and theoretical point of view. A study of recent French data shows that they can also be used for operational purposes. They highlight the factors in the recent income distribution changes in France that can be considered to be independent of the choice of equivalence scales and poverty

[5]See Shorrocks (1995) for a recent discussion of the Sen index principle.



Source: INSEE, Household Living Standards surveys, October 1977 to 1994.
Reading: The Sen indices are shown in terms of their deviation from their 1977 value. The solid line represents the series corresponding to a poverty threshold defined as the half-median of incomes by consumption units (C.U.), which are measured using the Oxford scale. The dotted line shows the series corresponding to a poverty threshold defined as the half-mean of incomes by consumption units (C.U.), which are measured using the Oxford scale.

Figure 3. The Sen Indices for the 1977–94 Period

lines: poverty has not really risen or fallen since the mid-1980s, but has rather fluctuated with the business cycle. Regardless of the method used to quantify household needs, a set of poverty lines exists that is such that all utilitarian planners would be in agreement that poverty is greater today than in 1989 or 1983.

If the irrefutability of these findings is to be further assured, we must aim to bridge the gap between the purely utilitarian school and rival schools such as the one initiated by A. Sen (1976) in economics or the *Class Analysis* school in sociology. These schools of thought consider that the situation of the poor differs *qualitatively* from that of the non-poor and, more generally, believe that the social environment cannot be measured solely by the distribution of *continuous* variables such as income.

Appendix

A. Proof of Proposition 1

The proof of Proposition 1 proceeds in exactly the same way as that proposed by Atkinson and Bourguignon (1987). Only one of the lemmas taken up differs slightly. First of all, integrating ΔW_t by parts, we obtain

$$\Delta W_t(v_1, \ldots, v_n) = -\int_0^{y_{\max}} \sum_{i=1}^{n} v_i'(y)\Delta(q_{i,t}F_{i,t}(y))\, dy. \tag{A1}$$

Let us consider $\varepsilon_i(y) = v_i'(y) - v_{i+1}'(y)$ for $1 \leq i < n$ and $\varepsilon_n(y) = v_n'(y)$. By assumption, we have $(\varepsilon_i(y) \geq 0, \forall i)$ and we can write:

$$\text{(A2)} \quad \Delta W_t(v_1, \ldots, v_n) = -\int_0^{y_{\max}} \sum_{i=1}^{n} \varepsilon_i(y)\Delta_i(y)\, dy \text{ with } \Delta_k(y) = \sum_{i=1}^{k} \Delta(q_{i,t}F_{i,t}(y)).$$

From (A2), it is clear that $(\Delta_i(y) \leq 0, \forall i)$ is a *sufficient* condition for $\Delta W_t \geq 0$. To show that it is a *necessary* condition, we use the two following lemmas:

Lemma 1. Let $I = [0, a]$ be an interval, V the set of continuous functions over I, and V^+ the set of non-negative continuous functions over I. Now let $(w_1(y), \ldots, w_n(y))$ be a set of continuous functions over I, (i.e. belonging to V^n).

$$\text{(A3)} \qquad \sum_{i=1}^{n} w_i(y)v_i(y) \in V^+, \forall v_1, \ldots, v_n \in V^+ \Leftrightarrow w_i \in V^+, \forall i$$

[Proof: Sufficiency of $(w_i \in V^+, \forall i)$ is direct. We prove necessity by reducing it to the absurd. Assume there exists a j such that $w_j \notin V^+$. Then, there exists $]\alpha, \beta[\subset I$, such that $(w_j(y) < 0, \forall y \in]\alpha, \beta[)$. By choosing $(v_i = 0, \forall i \neq j)$ and making v_j zero over $]\alpha, \beta[$ and strictly positive at least one point of $]\alpha, \beta[$, we build a set $v_1, \ldots, v_n \in V^+$ such that $\sum_{i=1}^{n} w_i(y)v_i(y) \notin V^+$].

Atkinson and Bourguignon (1987) use a lemma that does not explicitly take the continuity conditions into account, which we feel leaves the rest of their proof slightly wanting. We now move on to Lemma 2, which is analogous to that used by Atkinson and Bourguignon (1987).

Lemma 2. Using the same notations as for all f in V we have,

$$\text{(A4)} \qquad \int_0^{a} f(y)u(y)\, dy \geq 0, \forall u \in V^+ \Leftrightarrow f \in V^+.$$

[The proof is direct as the proposition displays a straightforward extension of Lemma 1.]

Let us now assume that there is an integer k such that $\Delta_k(y)$ is somewhere positive in $[0, y_{\max}]$. Lemma 1 shows that there exists an interval $J \subset \mathrm{I} = [0, y_{\max}]$ and a set of non-negative continuous $(\hat{\varepsilon}_1(y), \ldots, \hat{\varepsilon}_n(y))$ functions over $[0, y_{\max}]$ such that $(\sum_{i=1}^{n} \hat{\varepsilon}_i(y)\Delta_i(y) > 0, \forall y \in J)$. So according to Lemma 2, there exists a $u \in V^+$ such that,

$$\text{(A5)} \qquad \int_0^{y_{\max}} u(y)\left(\sum_{i=1}^{n} \hat{\varepsilon}_i(y)\Delta_i(y)\right) dy > 0.$$

Let us consider $v_i(x) = -\sum_{j=i}^{n} \int_x^{y_{\max}} \hat{\varepsilon}_j(y)u(y)\, dy$. We have $v_i' - v_{i+1}' = u(x)\hat{\varepsilon}_i(x) \geq 0$ and $v_i(y_{\max}) = 0$ for all i. The v_i functions thus verify conditions (2) and condition (A5) yields $\Delta W_t(v_1, \ldots, v_n) < 0$. Thus $\Delta_k(y) > 0$ for some k is incompatible with $\Delta W_t \geq 0$ for all v satisfying (2).

B. *Proof of Proposition 2*

If we adapt the notations in Appendix A, integrating by parts we obtain:

$$\text{(B1)} \qquad P(F_{t+1}) - P(F_t) = -\sum_{i=1}^{n} \int_0^{Z_i} \varepsilon_i(y)\Delta_i(y)\, dy, \ \forall Z_1 \geq \cdots \geq Z_n$$

$$\text{and } \forall P \in (Z_1, \ldots, Z_n)$$

with $\varepsilon_i(y)=p_i'(y)-p_{i+1}'(y)$ for $i<n$ and $\varepsilon_n(y)=p_n'(y)$, and $\Delta_i(y)=\sum_{k=1}^{i}\Delta(q_{k,t}F_{k,t}(y))$. Each $\varepsilon_k(y)$ is a non-positive continuous function over $[0, Z_k]$ and zero over the complementary interval. The relationship (B1) clearly shows that sequential dominance ($\Delta_j(y)\le 0$, $\forall j$ *and* $\forall y\le Z_j$) is a sufficient condition for a decline in poverty. We again prove necessity by reducing it to the absurd.

Let us take the case where poverty decreases for all $\Pi(Z_1,\ldots,Z_n)$ measures and assume that an integer k exists such that $\Delta_k(y)$ is not constantly negative or zero over $[0, Z_k]$. Using Lemma 1 in Appendix A guarantees the existence of a set of non-positive continuous functions $(\hat{\varepsilon}_1,\ldots,\hat{\varepsilon}_k)$ over an interval $I=[0, X]$ strictly included in $[0, Z_k]$ (i.e. $X<Z_k$), and an open interval J in I such that

$$\text{(B2)}\qquad \sum_{s=1}^{k}\hat{\varepsilon}_s(y)\Delta_s(y)<0, \forall y\in J.$$

For all $l\le k$, it is possible to continue $\hat{\varepsilon}_l$ over $[0, y_{\max}]$ so that it remains non-positive over $[0, Z_l]$ and zero over the complementary interval of $[0, Z_l]$ in $[0, y_{\max}]$. We now decide that $\hat{\varepsilon}_l=0$ for all $l>k$.

For all u non-negative continuous functions over $[0, Z_k]$, which are zero elsewhere, the functions $(p_j(x)= -\sum_{l=j}^{n}\int_x^{Z_l}u(y)\hat{\varepsilon}_l(y)\,dy)$ define a measure of poverty over $[0, y_{\max}]$ belonging to $\Pi(Z_1,\ldots,Z_n)$. The assumption of a decrease in poverty for all measures of $\Pi(Z_1,\ldots,Z_n)$ implies that:

$$\int_0^{y_{\max}}(\sum_{s=1}^{k}\hat{\varepsilon}_s(y)\Delta_s(y))u(y)\,dy\ge 0,$$

for all u non-negative continuous functions over $[0, Z_k]$, which, according to Lemma 2, is incompatible with (B2).

C. *Proof of Proposition 3*

Again using the notation of Appendix A, after integrating again by parts, the welfare variation can be written:

$$\text{(C1)}\qquad \Delta W_t(v_1,\ldots,v_n)=\int_0^{y_{\max}}\sum_{i=1}^{n}\varepsilon_i'(y)D_i((y)\,dy-\sum_{i=1}^{n}\varepsilon_i(y_{\max})D_i(y_{\max})$$

where

$$D_i(y)=\int_0^y\Delta_i(x)\,dx=\Delta\left[\sum_{j=1}^{i}\int_0^y q_{j,t}F_{j,t}(x)\,dx\right].$$

($D_i(y)\le 0, \forall j$ and $\forall y$) is thus clearly a sufficient condition for ($\Delta W_t\ge 0$, $\forall(v_1,\ldots,v_n)$ satisfying (2) and (5)). We again prove necessity by reducing it to the absurd.

Suppose there is an integer k such that $D_k(y)$ is somewhere positive in $[0, y_{\max}]$. From Lemma 1 of Appendix 1, we know that there is an interval $I\subset[0, y_{\max}]$ and a set $(\eta_1,\ldots,\eta_n)$ of non-positive continuous functions over $[0, y_{\max}]$ such that $(\sum_{i=1}^{n}\eta_i(y)D_i(y)<0, \forall y\in I)$, which can also be written: $(-\sum_{i=1}^{n}\eta_i(y)D_i(y)>0, \forall y\in I)$. Lemma 2 of Appendix A informs us of the necessary existence of a non-negative continuous function u over $[0, y_{\max}]$ such that $(-\int_0^{y_{\max}}u(y)\sum_{i=1}^{n}\eta_i(y)D_i(y)\,dy>0)$.

If we take $\varepsilon_k(y) = -\int_y^{y_{\max}} u(x)\eta_k(x)\,dx$, ε_k satisfies $\varepsilon_k' = u\eta_k$ and therefore $\varepsilon_k' \leq 0$. As ε_k is decreasing and $\varepsilon_k(y_{\max}) = 0$, we can also be sure that $\varepsilon_k \geq 0$. Now consider a planner whose preferences $(v_1, \ldots, v_n)$ can be inferred from $(\varepsilon_1, \ldots. \varepsilon_n)$. By definition, these preferences satisfy (2) and (5) and we have:

$$\Delta W_t(v_1, \ldots, v_n) = \int_0^{y_{\max}} u(y) \sum_{i=1}^{n} \eta_i(y) D_i(y)\,dy < 0.$$

D. *Proof of Proposition 4*

Using the notations in appendices B, C and D, and integrating by parts again, we obtain:

$$P(F_{t+1}) - P(F_t) = \sum_{i=1}^{n} \int_0^{Z_i} \varepsilon_i'(y) D_i(y)\,dy - \sum_{i=1}^{n} \varepsilon_i(Z_i) D_i(Z_i).$$

$(D_j(y) < 0, \forall j$ and $\forall y \leq Z_j^+)$ is thus clearly a sufficient condition for a decline in poverty. We again prove necessity by reducing it to the absurd.

Let us assume that there is a decrease in poverty and that there is also an integer k such that $D_k(y)$ is not constantly negative or zero over $[0, Z_k]$. From Lemma 1 of Appendix A, we know that there is a set $(\eta_1, \ldots, \eta_k)$ of non-positive continuous functions over $[0, Z_k]$ and an interval $I \subset [0, Z_k]$ such that $(\sum_{i=1}^{k} \eta_i(y) D_i(y) < 0, \forall y \in I)$.

For all $l \leq k$ we continue η_l over $[0, y_{\max}]$ such that it remains non-positive over $[0, Z_k]$ and zero over $[Z_k, y_{\max}]$. We also let $\eta_l = 0$ for all $l > k$.

Lemma 2 of Appendix A confirms the existence of a non-negative continuous function u over $[0, y_{\max}]$, such that

$$\text{(D1)} \qquad \left(\int_0^{y_{\max}} \sum_{i=1}^{n} u(y)\eta_i(y) D_i(y)\,dy < 0\right).$$

Given $\hat{\varepsilon}_j(y) = \int_y^{Z_j} -u(x)\eta_j(x)\,dx$, $\forall y \leq Z_j$. We have $\varepsilon_j' = u\eta_j$.

Since $\hat{\varepsilon}_j \leq 0$ and $\hat{\varepsilon}_j' \geq 0$ over $[0, Z_j]$, all poverty measures P based on ε belong to $\Pi^2(Z_1, \ldots, Z_n)$.

Elsewhere,

$$P(F_{t+1}) - P(F_t) = \int_0^{y_{\max}} -\sum_{i=1}^{n} \hat{\varepsilon}_i'(y) D_i(y)\,dy + \sum_{i=1}^{n} \hat{\varepsilon}_i(Z_i) D_i(Z_i).$$

The first term corresponds to (D1) and is therefore strictly positive. The second term $(+\sum_{i=1}^{n} \hat{\varepsilon}_i(Z_i) D_i(Z_i))$ is by definition zero. So all measures P based on $-\hat{\varepsilon}$ generate $P(F_{t+1}) - P(F_t) > 0$, which refutes the initial assumption.

E. *The Simulated Residuals Approach*

If Y_i denotes the income of household i and z_i the income bracket stated for the survey, we have the equivalence $(z_i = k) \Leftrightarrow (R_k \leq Y_i \leq R_{k+1})$, where R_k and R_{k+1} are the bounds of bracket $k = 0, \ldots, K$. At the extremes, let $R_0 = -\infty$ and $R_{k+1} = +\infty$. We postulate that variable Y_i behaves according to: Log $Y_i = y_i = x_i b + u_i$ where x_i represents the characterisitcs of hosehold i and u_i a random disturbance such that: $Eu_i = 0$, $Eu_i^2 = s^2$ and u_i performs like a normal function for all i. For each k we have $P(z_i = k) = F((r_{k+1} - x_i b)/s) - F((r_k - x_i b)/s)$ where F is the normal distribution function and $r_k = \mathrm{Log}\, R_k$.

The maximum likelihood method is used to estimate bn and b/s, which provides estimators $x_i\hat{b}$ and $\hat{s}$ of $x_i b$ and s. The income attributed to household i is then $x_i\hat{b} + \hat{s}\tilde{w}_i$, where $\tilde{w}_i$ is the first draw of a $N(0, 1)$ random variable such that $r_{Z_i} \leq x_i\hat{b} + \hat{s}\tilde{v}_i < r_{Z_i+1}$ (see Gouriéroux *et al.*, 1985.

When the value z_i for a household is not available, we reconstruct its income by adding an unconstrained $N(0, 1)$ residual to $x_i\hat{b}$. This way, there are no values missing. This method has been applied to the data from the Households Living Standards surveys. The following explanatory income variables x_i were chosen: household type, the reference person's age, the reference person's socio-economic group, status in the usual residence (owner, tenant, etc.) and the size of the urban unit of residence.

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