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DEFLATION OF INPUT–OUTPUT TABLES FROM THE USER'S POINT OF VIEW: A HEURISTIC APPROACH

BY ERIK DIETZENBACHER AND ALEX R. HOEN

University of Groningen

This paper considers the problem of deflating an input–output table from the viewpoint of the user. In many practical cases certain margins of this table are readily available in constant prices, whereas the entire table is not. This reduces the problem to estimating the matrix of sectoral intermediate deliveries in constant prices. The traditional approach for this purpose is based on the double deflation method. Since double deflation is sensitive to aggregation, however, it typically does not provide correct answers. Therefore, a heuristic approach is proposed as an alternative. It is based on the biproportional projection method. An empirical evaluation indicates that the heuristic approach clearly performs better.

1. INTRODUCTION

Input–output tables have been widely used for studying economic developments in a context where the interactions between sectors and/or regions are explicitly taken into account. Typical examples include analyses of multipliers and multiplier decompositions, of interindustry linkages and interregional feedbacks, of structural changes, and of a variety of impacts.[1] When carried out for a point in time, empirical studies of these types often aim at describing certain aspects of the production structure. Comparisons over time focus on changes in the production structure, which are frequently related to technological changes.[2]

Most input–output studies use technical (or input) coefficients which are interpreted in physical terms (or quantities). The input–output tables from which the technical coefficients are obtained, are prepared in money terms (or values), however. This leads to problems, in particular when developments over time are examined. Changes in the technical coefficients reflect changes in the cost structure rather than in the production structure. These changes comprise two effects; a quantity effect and a price effect. For the analysis of the production structure it is of major importance to separate these two effects. In other words, for a proper understanding of the changes in the production structure over time, it is necessary to use input–output tables in constant prices.

This paper considers the problem of deflating an input–output table. We explicitly adopt the viewpoint of the user who is interested in, for example, analyzing such changes in the production structure. For this purpose, the intermediate deliveries (i.e. from sector i to sector j) in constant prices are of particular interest.

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[1]See Rose and Miernyk (1989) for a detailed overview.

[2]See e.g. Carter (1970) for a seminal contribution to this type of research.

It is our experience that, when an entire input–output table in constant prices is not available, most of its margins are often known. Aggregate data, such as total output, final demand, imports, and value added (all per sector), frequently are published in constant prices. For the user this leaves the problem of estimating the matrix of intermediate deliveries in constant prices.

The method that has been used predominantly for the estimation of input–output tables in constant prices, is double deflation. Section 2 discusses the method of double deflation and some of its drawbacks. It explains why double deflation will most likely produce incorrect results. In Section 3 we propose an alternative method for the deflation of the matrix of intermediate deliveries. The intermediate deliveries in constant prices are estimated on the basis of the intermediate deliveries in current prices, and the row and column sums in constant prices. This estimation problem exactly satisfies the requirements for applying the biproportional projection (or RAS) method. We discuss four cases which differ with respect to the amount of available aggregate information in constant prices.

In comparing the two methods, it should be emphasized that the RAS-method needs more exogenous information than double deflation. In particular, the total value added (or GDP) is required to be known in constant prices. It should be noted that the double deflation method has been advocated, precisely for the purpose of estimating the total value added. Recall, however, that we have limited our scope to the user's point of view. In that case, figures in constant prices are often widely available for aggregates, including the value added per sector.

Section 4 presents the results for an empirical application to the Netherlands. It is indicated that the RAS-method clearly performs better than double deflation, for the practical purposes described above.

2. The Double Deflation Method

In most input–output studies on production structures the technical coefficients are interpreted in terms of physical quantities. In practice, however, input–output tables are prepared in money values. Deducing tables in quantities is a difficult (if not impossible) task. In certain sectors the commodities are too heterogeneous to use a physical measure while in other sectors (especially the service sectors) the production cannot be measured physically.

These problems can be overcome to some extent by choosing appropriate units of measurement. Defining these units as the quantity bought for one dollar allows for a physical interpretation of the coefficients in value terms. This is common practice when the analysis is with respect to only one year. For a comparison over time, however, it becomes necessary to value tables in constant prices.

The compilation of input–output tables in constant prices is often based on the double deflation method.[3] Under the assumption that each sector produces one homogeneous good, each sector's gross output and intermediate and final deliveries are deflated by this sector's own price index. The United Nations (1973) advocates the use of the double deflation method for estimating the value-added

[3]For recent contributions on alternative methods, see De Boer and Broesterhuizen (1991), Durand (1994, 1996) or Folloni and Miglierina (1994), for example.

in constant prices. Each sector's value-added can be obtained as the difference between this sector's deflated gross output and the deflated intermediate inputs plus imports in constant prices.

The input–output table in current prices is given in Table 1a, the table in constant prices, using the double deflation method, in Table 1b.

TABLE 1a

I–O TABLE IN CURRENT PRICES

$\mathbf{Z}$	$\mathbf{f}$	$\mathbf{x}$
$\mathbf{M}$		
$\mathbf{v}'$		
$\mathbf{x}'$		

TABLE 1b

I–O TABLE IN CONSTANT PRICES

$\mathbf{Z}_d=\hat{\boldsymbol{\pi}}\mathbf{Z}$	$\mathbf{f}_d=\hat{\boldsymbol{\pi}}\mathbf{f}$	$\mathbf{x}_d=\hat{\boldsymbol{\pi}}\mathbf{x}$
$\mathbf{M}_d=\hat{\boldsymbol{\rho}}\mathbf{M}$		
$\mathbf{v}'_d$		
$\mathbf{x}'_d=\mathbf{x}'\hat{\boldsymbol{\pi}}$		

The $n\times n$ matrix $\mathbf{Z}$ denotes the intermediate deliveries, the vector $\mathbf{f}$ the final demands (private and government consumption and investment, and exports), $\mathbf{x}$ denotes the vector with sectoral outputs. The $k\times n$ matrix $\mathbf{M}$ gives the sectoral imports, where we have distinguished k different imported products.[4] $\mathbf{v}'$ is a row vector, the elements of which give the value added in each sector.[5] In Table 1b, the subscript d (for deflated) is used to indicate that the corresponding matrices and vectors are in constant prices. Let p_i denote the ratio of the current price and the base year price, for product i. Thus, $100p_i$ is the price index. The element π_i of the vector $\boldsymbol{\pi}$, denotes the *deflator* in sector i. It is defined as the reciprocal price ratio, that is, $\pi_i=1/p_i$. In the same way the import deflator ρ_j (with $j=1,\ldots,k$) is defined as $\rho_j=1/r_j$, where r_j denotes the price ratio between the current import price and the base year import price, for the imported product j.

In the double deflation method the deflators π_i and ρ_j are assumed to be given.[6] The value added vector $\mathbf{v}'_d$ is then obtained from the balancing equations. That is, the equality of the row sums and the column sums imply

$$\mathbf{v}'_d=\mathbf{x}'_d-\mathbf{e}'_{(n)}\mathbf{Z}_d-\mathbf{e}'_{(k)}\mathbf{M}_d \tag{1}$$

where $\mathbf{e}_{(n)}$ denotes the n-element summation vector consisting entirely of ones.

[4]If the input–output table records only a single row of total imports, k equals 1 and the matrix $\mathbf{M}$ becomes a vector.

[5]A prime (e.g. in $\mathbf{v}'$) is used to indicate transposition. As usual, vectors are column vectors. A hat (e.g. in $\hat{\boldsymbol{\pi}}$) is used to denote the diagonal matrix with the elements of the vector $\boldsymbol{\pi}$ on its main diagonal and all other entries zero.

[6]Alternatively, it may be assumed that the matrix $\mathbf{M}_d$ is given.

Although the double deflation method is generally accepted, it involves certain problems. Some of these may have serious effects in empirical studies (see e.g. Sevaldson, 1976). For example, using the price index of the gross output for deflating an entire row, can only be justified if this sector produces one good. Most sectors, however, produce more than one good. Since every sector requires a different mix of these goods as an input, the price indices are likely to be different within a row of intermediate deliveries.

Also when the double deflation method is used for estimating the value-added, certain difficulties may be encountered (see e.g. Wolff, 1994). For instance, since the value-added is obtained as the difference of variables, its measurement error equals the sum of the measurement errors of these variables.

A third drawback of the method of double deflation is that it is subject to aggregation problems. In input–output analysis, the aggregation problem typically refers to the fact that an aggregated Leontief inverse may be obtained from an input–output table in two alternative ways. First, aggregation after inversion, which means that the original table is used to derive a large Leontief inverse, which is then aggregated. Second, inversion after aggregation, where the original table is aggregated into a smaller table, which is then used to derive a small Leontief inverse. Unless some very stringent conditions are satisfied, the two alternative procedures will yield different answers (see e.g. Kymn, 1990, for a detailed overview of the literature and results).

A similar argument holds when double deflation is applied to compute an aggregaged input–output table in constant prices. On the one hand, aggregation after deflation means that the original table is deflated first, resulting in a value added vector in constant prices, which then is aggregated. On the other hand, deflation after aggregation means that the original table is aggregated first, after which it is deflated. Both procedures yield an estimate of the aggregated value added vector in constant prices and of the matrix of intermediate deliveries in constant prices. It can be shown, however, that only under stringent conditions are the two answers for the value added vector equal to each other. The same result holds for the column sums of the two matrices of intermediate deliveries (see Dietzenbacher and Hoen, 1996).

Clearly, if detailed or even "ideal" information is available, aggregation after deflation leads to the correct answer. Unfortunately, however, in practical cases the published information available to the user already is largely aggregated. Consequently, the only possibility for calculating an input–output table in constant prices is via deflation after aggregation. It is likely, however, that this answer will differ from the correct (but unknown) answer.

3. A Heuristic Approach

In this section we propose an alternative approach for deflating input–output tables. As stated earlier, we adopt the viewpoint of the user who is interested in the intermediate deliveries in constant prices. The deflation therefore does not aim at providing an estimate for the value-added (or GDP). As a matter of fact, it is our experience that for most practical cases the National Accounts frequently provide a wealth of deflated data. For example, for each sector figures on the output, the value-added, the import and the final demand are usually recorded both in current and in constant prices. This implies that all margins of the input–output table are known in constant prices. As a consequence, we may use the RAS-procedure to estimate the table itself in constant prices.

The RAS-procedure is a biproportional projection method that was developed for "updating" a given matrix (say $\mathbf{A}_0$, not necessarily square), such that the updated matrix ($\tilde{\mathbf{A}}_1$) satisfies exogenously given row and column sums.[7] The RAS-method proceeds iteratively. In the first step the rows are adjusted. Each row i is multiplied by a scalar r_i such that the i-th row sum equals the prespecified row sum of $\mathbf{A}_1$. The resulting matrix after step 1 may be denoted as $\tilde{\mathbf{A}}_1(1)=\hat{\mathbf{r}}_1\mathbf{A}_0$. In the second step, the columns of $\tilde{\mathbf{A}}_1(1)$ are adjusted so as to satisfy the column sum requirements. This yield $\tilde{\mathbf{A}}_1(2)=\tilde{\mathbf{A}}_1(1)\hat{\mathbf{s}}_2=\hat{\mathbf{r}}_1\mathbf{A}_0\hat{\mathbf{s}}_2$. It is likely, however, that the row sum requirements are violated. Therefore the rows are adjusted again; $\tilde{\mathbf{A}}_1(3)=\hat{\mathbf{r}}_3\tilde{\mathbf{A}}_1(2)=\hat{\mathbf{r}}_3\hat{\mathbf{r}}_1\mathbf{A}_0\hat{\mathbf{s}}_2$. Next, the columns are adjusted again: $\tilde{\mathbf{A}}_1(4)=\tilde{\mathbf{A}}_1(3)\hat{\mathbf{s}}_4=\hat{\mathbf{r}}_3\hat{\mathbf{r}}_1\mathbf{A}_0\hat{\mathbf{s}}_2\hat{\mathbf{s}}_4$, and so forth. Starting with column adjustments in the first step yields $\tilde{\mathbf{A}}_1(4)=\hat{\mathbf{r}}_4\hat{\mathbf{r}}_2\mathbf{A}_0\hat{\mathbf{s}}_1\hat{\mathbf{s}}_3$ after the fourth step. It can be shown that under mild conditions the iterative procedure converges. The updated matrix can be written as $\tilde{\mathbf{A}}_1=\hat{\mathbf{r}}\mathbf{A}_0\hat{\mathbf{s}}$ and does not depend on whether the procedure is started with a row adjustment or with a column adjustment.[8]

Typically, the RAS-method has been applied to estimate next year's coefficients matrix ($\mathbf{A}_1$) on the basis of this year's matrix ($\mathbf{A}_0$), given next year's row and column sums. In this paper we apply the RAS-procedure to estimate the input–output table (or parts thereof) in constant prices, on the basis of the table in current prices, given the row and column totals in constant prices.

Below we discuss four cases (see Table 2a–2d) which differ with respect to the information that is available. Without loss of generality it is assumed that the imports are given as a row vector ($\mathbf{m}'$ in current prices or $\mathbf{m}'_d$ in constant prices) and that no imports are used for final demand purposes. Similarly, all value-added is assumed to be recorded in the production sectors. Total imports are denoted as $m=\mathbf{m}'\mathbf{e}_{(n)}$ and $m_d=\mathbf{m}'_d\mathbf{e}_{(n)}$, total value-added as $v=\mathbf{v}'\mathbf{e}_{(n)}$ and $v_d=\mathbf{v}'_d\mathbf{e}_{(n)}$, and total final demand as $f=\mathbf{e}'_{(n)}\mathbf{f}$ and $f_d=\mathbf{e}'_{(n)}\mathbf{f}_d=m_d+v_d$.

In the first case (Table 2a), the $(n+2)\times(n+1)$ matrix in the upper-left corner (i.e. North-West of the double lines) may be estimated in constant prices by the RAS-method.[9] The sectoral outputs ($\mathbf{x}_d$), the total imports (m_d) and the total value-added (v_d) or, equivalently, the total final demand (f_d) are required to be known. The double deflation method, in contrast, only requires that $\mathbf{x}_d$ and m_d are known and additionally provides an estimate of v_d.

[7]Although the method was known in the field of demographics, its introduction to economics was due to Stone. Early applications are reported in Stone (1961) and Cambridge University (1963).

[8]The reader is referred to Miller and Blair (1985) for a detailed introduction, to Bacharach (1970) or Macgill (1977) for technical aspects. The RAS-procedure can be reformulated as an optimization problem with a specific objective function. Also alternative updating procedures based on other objective functions have been analyzed (for recent contributions, see Kaneko, 1988, or Golan *et al.*, 1994). Critical surveys and comparative evaluations of such updating procedures can be found in Allen and Gossling (1975) or Lynch (1986).

[9]A feature of the RAS-procedure for updating matrices is that additional information can be incorporated. For the present case, this applies to the zeros, which are not adjusted and thus remain zero.

TABLE 2a

$\mathbf{Z}_d$	$\mathbf{f}_d$	$\mathbf{x}_d$
$\mathbf{m}'_d$ $\mathbf{v}'_d$	0 0	m_d v_d
$\mathbf{x}'_d$	f_d	—

TABLE 2b

$\mathbf{Z}_d$	$\mathbf{x}_d - \mathbf{f}_d$
$\mathbf{m}'_d$ $\mathbf{v}'_d$	m_d v_d
$\mathbf{x}'_d$	—

TABLE 2c

$\mathbf{Z}_d$	$\mathbf{x}_d - \mathbf{f}_d$
$\mathbf{v}'_d$	v_d
$\mathbf{x}'_d - \mathbf{m}'_d$	—

TABLE 2d

$\mathbf{Z}_d$	$\mathbf{x}_d - \mathbf{f}_d$
$\mathbf{x}'_d - \mathbf{m}'_d - \mathbf{v}'_d$	—

In the second case (Table 2b), the sectoral outputs ($\mathbf{x}_d$), the final demands ($\mathbf{f}_d$) and the total imports (m_d) are required to be known. From this information the total value-added (v_d) can be deduced. Then, the $(n+2) \times n$ matrix in the upper-left corner may be estimated by the RAS-method. The double deflation method uses the same information and provides no additional results.

In the third case (Table 2c), the sectoral outputs ($\mathbf{x}_d$), the final demands ($\mathbf{f}_d$) and the imports ($\mathbf{m}'_d$) are required to be known, from which the total value-added (v_d) can be deduced. The $(n+1) \times n$ matrix in the upper-left corner may then be estimated by the RAS-method. The double deflation method uses the same information and provides no additional results.

In the last case (Table 2d), the sectoral outputs ($\mathbf{x}_d$), the final demands ($\mathbf{f}_d$), the imports ($\mathbf{m}'_d$) and the value-added vector ($\mathbf{v}'_d$) are required to be known. The matrix of intermediate deliveries ($\mathbf{Z}_d$) may be estimated by the RAS-method. When compared to the previous case, the additional information concerns the sectoral value-added figures in constant prices. Since the double deflation method typically does not use this sort of information, the results for this method would be obtained as in the third case.

A comparison of the double deflation method and the RAS-procedure for estimating an input–output table in constant prices may be summarized as follows. The advantage of the double deflation method is that it provides an estimate of the value-added in the first case (Table 2a). For that case, the total value-added in constant prices is required to be specified exogenously for the RAS-method. Recall, however, that for many practical purposes, figures in constant prices are often widely available for aggregates (including the value-added per sector). The advantage of the RAS-procedure is that every element of the deflated input–output table has its own, cell-specific price index. In particular for aggregated input–output tables, price indexes that apply uniformly within a row are implausible. Furthermore, the estimation errors with the RAS-method are smoothed out over the entire table, instead of being cumulated in the value-added vector as is the case for the double deflation method.

In general, the input–output table in constant prices obtained by double deflation will be different from the one obtained by the RAS-method. Note, however, that the first step of the RAS-procedure yields the same estimate for the intermediate deliveries ($\mathbf{Z}_d$) as the double deflation method, provided that the RAS-procedure is started with row adjustments. Also, when the correct table in constant prices satisfies the double deflation method and when the value-added vector in constant prices is available, the RAS-procedure yields the correct result. The first step of the RAS-method gives the same result as double deflation while the sectoral values-added are given. Hence, the result after the first step also satisfies the prescribed column sums, implying that the RAS-procedure terminates.

4. An Empirical Evaluation

In this section we present the results of an empirical application of the two methods. The intermediate deliveries are estimated by the RAS-procedure for the case described by Table 2d. That is, given the vectors $\mathbf{x}_d-\mathbf{f}_d$ and $\mathbf{x}_d'-\mathbf{m}_d'-\mathbf{v}_d'$, the matrix $\mathbf{Z}_d$ is estimated. If we denote this estimate as $\mathbf{Z}_d^{\mathrm{RAS}}$, then $\mathbf{Z}_d^{\mathrm{RAS}}=\hat{\mathbf{r}}\mathbf{Z}\hat{\mathbf{s}}$ where $\mathbf{Z}_d^{\mathrm{RAS}}$ satisfies $\mathbf{Z}_d^{\mathrm{RAS}}\mathbf{e}_{(n)}=\mathbf{x}_d-\mathbf{f}_d$ and $\mathbf{e}_{(n)}'\mathbf{Z}_d^{\mathrm{RAS}}=\mathbf{x}_d'-\mathbf{m}_d'-\mathbf{v}_d'$. Applying the double deflation method yields the estimate $\mathbf{Z}_d^{\mathrm{DD}}=\hat{\boldsymbol{\pi}}\mathbf{Z}$ with $\boldsymbol{\pi}=(\widehat{\mathbf{Z}\mathbf{e}_{(n)}})^{-1}(\mathbf{x}_d-\mathbf{f}_d)$. Note that the calculation of $\mathbf{Z}_d^{\mathrm{DD}}$ requires only information with respect to $\mathbf{x}_d-\mathbf{f}_d$. If in addition also $\mathbf{x}_d'-\mathbf{m}_d'$ is known, the value-added vector in constant prices can be estimated as $\mathbf{x}_d'-\mathbf{m}_d'-\mathbf{e}_{(n)}'\mathbf{Z}_d^{\mathrm{DD}}$.

Our calculations are based on two input–output tables for the Netherlands for 1988 (see Central Bureau of Statistics, 1990, 1994). One is in current prices, the other in 1987 prices. The latter table provides the row and column sums which are required for the estimation. Using these margins in constant prices, the intermediate deliveries in current prices are then "updated" by the RAS-procedure and the double deflation method, so as to yield estimates for the intermediate deliveries in 1987 prices. These estimates, $\mathbf{Z}_d^{\mathrm{RAS}}$ and $\mathbf{Z}_d^{\mathrm{DD}}$, are then compared with the published $\mathbf{Z}_d$.

Both published tables record 58 sectors. Recall that the RAS-procedure was proposed as an alternative for the double deflation method because the latter was shown to suffer from, among other things, aggregation problems. In order to be able to indicate the effects of aggregation, we carry out the calculations at an aggregate level of 12 sectors.

Table 3 summarizes the estimation errors. The results are based on $\mathbf{Z}_d^{\mathrm{RAS}}-\mathbf{Z}_d$ and $\mathbf{Z}_d^{\mathrm{DD}}-\mathbf{Z}_d$. The second and third column consider the errors within the rows, the fourth and fifth column in Table 3 focus on the errors within the columns. As summary statistics, the table reports for each sector the weighted mean absolute percentage error (*WMAPE*). The magnitudes of the transactions within a row or column are used as weights.[10]

Looking at the errors in the rows of the intermediate deliveries matrix $\mathbf{Z}_d$, Table 3 shows that RAS and DD perform more or less equally. According to the

TABLE 3

ERRORS OF THE DEFLATION PROCEDURES, MEASURED BY THE *WMAPE*

	Rows		Columns	
Sector	RAS	DD	RAS	DD
1	0.36	0.50	1.99	2.42
2	0.34	1.64	0.44	0.78
3	1.80	1.63	2.70	3.54
4	5.26	15.06	1.15	4.82
5	1.07	0.49	0.70	1.24
6	0.57	0.19	1.07	1.26
7	0.77	0.76	0.82	1.51
8	7.63	4.27	11.68	18.00
9	0.16	0.16	1.48	1.43
10	1.65	1.06	1.90	2.76
11	1.77	1.96	1.69	1.66
12	1.02	0.52	0.71	0.81
Total	1.43	2.13	1.43	2.13

Note: 1 = Agriculture; 2 = Food; 3 = Textiles, leather and rubber; 4 = Mineral and chemical products; 5 = Metals, electrical goods and transport equipment; 6 = Building and construction; 7 = Wood, paper and glass; 8 = Public services; 9 = Wholesale and retail trade; 10 = Transport and communication; 11 = Credit and insurance; 12 = Other services.

WMAPE, RAS is better than DD in 5 sectors and worse in 7 sectors. The detailed results, which have been used for obtaining the summary statistics, indicate that RAS yields—in comparison with DD—larger percentage errors in small elements and smaller percentage errors in large (and therefore more relevant) elements.

The picture of both methods performing equally well, changes drastically when the errors are considered over the columns. RAS has a better *WMAPE* than DD in no less than 10 sectors. Intuitively speaking, this result is not so surprising. The extra information used by the RAS procedure are mainly value added data. Adding these data does not change the outcomes of DD, whereas the RAS procedure is able to exploit this information. Since value added data are used in the estimation of the columns, the better performance of RAS with respect to the columns is not surprising. Another reason stems from the procedure itself. DD adapts the rows of the intermediate deliveries matrix, so does RAS in its first

[10]An alternative measure for errors that is also used frequently, is the mean absolute error (*MAE*). It should be noted, however, that the *WMAPE* is related to the *MAE*, in the sense that correcting the *MAE* for the total intermediate deliveries of a sector yields the *WMAPE*. As a consequence, the ratio between these two measures is the same. That is, $WMAPE^{\mathrm{RAS}}/WMAPE^{\mathrm{DD}} = MAE^{\mathrm{RAS}}/MAE^{\mathrm{DD}}$, which holds for each sector.

step. In the subsequent steps, the RAS-method also adapts the columns so as to meet certain requirements. Therefore, we may expect that the performance within the columns improves. The performance within the rows is expected to remain more or less the same, i.e. some rows will gain from the iterative adaptation so that RAS becomes better than DD, other rows will lose.

The overall results are given in the last row (*Total*) of Table 3. The sectoral results are weighted with the sectoral intermediate deliveries. The total *WMAPE* is clearly smaller (more than 30 percent) for RAS than for DD. It should be mentioned, however, that in part this is caused by the very bad result for the deliveries from sector 4 to sector 8, which is one of the larger cells of $\mathbf{Z}_d$. Although the difference between the total *WMAPE*s may seem small, it should be borne in mind that we are considering the effects of price changes in only one year.

In conclusion, RAS clearly performs better than DD in estimating the columns in constant prices. Both methods perform more or less the same when the estimation is considered rowwise. As an overall measure, the weighted mean absolute percentage error for RAS is more than 30 percent smaller than for DD.

The comparison above, between RAS and DD, addressed the user's problem of estimating the intermediate deliveries in constant prices, in the case where deflated data for the aggregates are available. It is our experience that this is usually the case, whenever a full input–output table in constant prices is not readily published. Therefore the RAS-method was applied to the situation as

TABLE 4

SUMMARY OF ERRORS FOR THE CASES IN TABLES 2a–2d

Table	RAS	DD
2a	2.86	2.25
2b	2.71	2.13
2c	2.76	2.13
2d	1.43	2.13

described in Table 2d. Table 4 summarizes the results (total *WMAPE*s) for all four cases, see Tables 2a–2d. Given the user's focus, the errors are presented only for the intermediate deliveries and are obtained in the same way as in Table 3. Note that for the double deflation method the results are the same for the cases 2b–2d, since only the errors in the intermediate deliveries are taken into consideration. The results in Table 4 indicate that knowledge of the sectoral value-added in constant prices allows for a substantial reduction in the errors by using the RAS-method. In the other three cases (Tables 2a–2c), in which the value-added in constant prices is not known by sector, the double deflation method performs better than the RAS-method.

Recall that the RAS-procedure was proposed as a heuristic alternative for double deflation since it was shown that the latter method is subject to aggregation problems. As a second exercise, we now focus explicitly on the effects of aggregation. To this end we have carried out the same calculations as above, but on the original 58-sector level instead. The detailed results are given in the Appendix.

The results in the Appendix seem to suggest that DD suffers more from aggregation than RAS. That is, at the 58-sector level DD is better (i.e. has a lower

WMAPE) than RAS in 37 rows and in 22 columns. Recall that for the 12-sector classification DD was better in 7 rows and 2 columns. The total *WMAPE*, however, is still significantly better for RAS than for DD. Also with respect to the total *WMAPE*, the results for DD have improved; at the 58-sector level the total *WMAPE* is 20 percent lower for RAS than for DD, while it was more than 30 percent lower at the 12-sector level.

The aggregation effects can be studied when the deflated 58×58 matrices $\mathbf{Z}_d^{\mathrm{RAS}}$ and $\mathbf{Z}_d^{\mathrm{DD}}$ are aggregated into 12×12 matrices. The results are summarized in Table 5. In contrast to the results of Table 3, which are obtained by deflation after aggregation, the results in Table 5 are thus obtained by aggregation after deflation. It is well-known from the literature that calculations can best be carried out at the most detailed level after which the results are aggregated. The empirical results indicate that this holds also true for deflation. For the columns, all *WMAPE*s in Table 5 are lower than in Table 3. For the rows we find that almost all *WMAPE*s are lower in Table 5 than in Table 3. Due to aggregation, positive and negative errors cancel out each other. This also applies to the total *WMAPE*s which are reduced by more than 50 percent. So, aggregation after deflation yields substantially better results than deflation after aggregation.

TABLE 5

ERRORS FOR AGGREGATION AFTER DEFLATION

	Rows		Columns	
Sector	RAS	DD	RAS	DD
1	0.37	0.41	1.23	1.21
2	0.61	0.66	0.41	0.51
3	1.47	1.52	1.54	1.73
4	1.30	3.60	1.08	2.23
5	0.48	0.41	0.35	0.30
6	0.22	0.19	0.20	0.39
7	0.31	0.38	0.60	0.85
8	2.65	2.41	2.08	4.62
9	0.14	0.16	0.98	0.92
10	0.74	0.62	0.72	0.78
11	1.67	1.96	1.57	1.48
12	0.48	0.38	0.29	0.29
Total	0.64	0.84	0.64	0.84

A somewhat surprising outcome in this empirical example is that the gains of DD over RAS are lost again by aggregation. Comparing the results in the Appendix with those in Table 3 indicates that deflation at the detailed level was beneficial for DD in particular. That is, the number of cases in which DD has a lower *WMAPE* than RAS improved from 9 out of 24 (Table 3) to 59 out of 116 (the Appendix). Apparently, this improvement in the performance of DD is offset again when the results in the Appendix are aggregated. As was the case in Table 3, DD has a lower *WMAPE* than RAS in 9 out of 24 instances in Table 5. This effect is also reflected by the total measures. RAS had a total *WMAPE* that was 33 percent lower than the one for DD in Table 3, this difference reduced to 20 percent in the Appendix, but increases again to 24 percent in Table 5.

5. Conclusions

This paper considered the problem of deflating an input–output table. To solve this problem, it took the view of the user as a point of departure. Therefore, data which are generally available for deflation purposes, were assumed to be known. Given these data, it was argued that the method of double deflation—which is the method that is used predominantly—will most likely produce incorrect results, since it suffers from aggregation problems. As an alternative, a heuristic method based on the RAS procedure was proposed for the purpose of deflating.

An empirical analysis showed that the results of the RAS procedure were indeed better than the results of the double deflation method, if the value added is known in constant prices. The empirical results also indicated that double deflation suffers severely from aggregation problems. The more the tables were aggregated, the better the performance of the RAS method became in comparison to the performance of the double deflation method. Since the first step of the RAS procedure will often be equal to the method of double deflation, the errors within the columns exhibited a substantial improvement. The performance within the rows was found to be more or less similar for both methods.

APPENDIX

Deflation Errors, 58 Sectors

	Rows		Columns			Rows		Columns	
Sector	RAS	DD	RAS	DD	Sector	RAS	DD	RAS	DD
1	1.27	4.51	1.60	1.59	30	1.84	1.61	2.64	2.84
2	7.21	8.33	5.19	5.02	31	1.62	1.34	1.18	1.30
3	3.28	9.76	6.86	6.63	32	2.15	1.83	1.01	1.00
4	4.22	4.16	3.29	4.23	33	2.01	1.85	1.50	1.50
5	12.27	12.25	2.89	4.97	34	1.71	1.55	1.91	2.10
6	4.58	6.30	1.28	4.91	35	2.37	2.04	1.30	1.39
7	4.29	4.85	6.84	6.41	36	4.29	4.04	3.31	3.43
8	1.38	1.91	5.43	6.74	37	2.42	2.40	2.45	6.95
9	4.92	5.03	1.31	2.51	38	5.88	5.24	2.92	8.11
10	9.80	10.61	4.29	5.49	39	2.36	2.13	3.82	3.85
11	7.89	9.32	5.09	4.60	40	0.57	0.43	0.41	0.56
12	13.65	13.59	12.11	11.27	41	0.22	0.23	1.67	1.79
13	1.48	1.57	2.71	3.12	42	1.08	0.89	2.74	3.02
14	4.60	3.32	2.81	2.83	43	1.67	1.76	2.06	2.10
15	7.94	7.59	5.96	5.88	44	3.91	3.60	4.46	4.41
16	6.36	6.62	5.76	5.57	45	0.85	0.74	1.82	1.98
17	5.12	5.14	3.69	4.06	46	1.81	1.26	3.10	3.01
18	4.97	4.95	4.83	5.05	47	1.70	1.98	1.68	1.64
19	2.62	2.42	3.41	3.60	48	3.21	3.18	1.25	1.37
20	6.56	5.00	9.90	8.75	49	1.76	1.24	1.13	1.20
21	1.89	1.59	1.68	1.63	50	2.58	2.68	2.52	2.64
22	2.59	3.38	7.01	6.94	51	9.51	9.09	3.20	3.05
23	2.76	2.49	1.84	1.99	52	4.38	4.06	1.41	1.48
24	1.16	1.12	1.01	1.26	53	1.50	0.72	2.66	2.67
25	4.95	5.47	15.44	20.35	54	1.94	1.22	2.17	2.24
26	3.90	3.54	2.18	1.39	55	0.96	0.63	2.34	2.18
27	2.89	3.12	1.68	1.53	56	1.13	0.74	2.67	2.66
28	3.02	2.81	2.84	3.30	57	1.22	0.83	6.42	5.44
29	1.09	1.19	2.53	3.01	58	1.40	1.50	0.17	0.22
					Total	1.80	2.26	1.80	2.26

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