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AN ANNUAL CHAIN INDEX OF ITALY'S "REAL" PRODUCT, 1861–1989

BY GIORGIO FUÀ

Facoltà di economia, Università di Ancona, Italy

AND

MAURO GALLEGATI

Facoltà di Giurisprudenza, Università di Teramo, Italy

The construction and application of a Paasche annual chain index to the Italian GDP figures between 1861 and 1989 is the subject of this article. Comparison with product at constant prices shows, except for war periods, a revaluation of GDP in favour of the annual chain index of around 0.10 percent. The most striking differences are concentrated into periods of marked changes in relative prices, and the new index modifies the temporal pattern of the growth trend by accentuating, without altering the periodization, cyclical fluctuations.

I. PRESENTATION OF THE FORMULA

1. *Introduction*

In 1959, Istat, the Italian statistical bureau, began publication (Istat, 1959) of a set of historical series concerning the annual values of certain aggregates of Italian national accounting since 1861 (a praiseworthy, but presumably risky undertaking). This set consisted of retrospective estimates of national product, measured at *current* prices and disaggregated with respect to both the formation of resources and their use; and also estimates of national product measured at 1938 *constant* prices and disaggregated with respect to use alone.

A few years later, as head of the Italian section of an international research project coordinated by Kuznets and Abramovitz, G. Fuà decided that in order to complete the Istat data, his research group should draw up historical series at constant prices of national product disaggregated with regard to the formation of resources as well. These estimates were carried out, under his direction, by Vitali, who although still very young at the time, combined great skill with direct knowledge of Istat's methods which he had acquired by working as an employee in that bureau. The methodology he used is explained in Vitali (1969), and the results are set out as a component part of a coordinated set of statistics on the long-period development of the Italian economy drawn from various sources, in Ercolani (1969).

The estimates at constant prices obtained by the Fuà group for the formation of resources were almost invariably in line with those previously obtained by Istat

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for their uses. Major differences—amounting to around 10 percent when measured on net national income at market prices—emerged only in the periods 1918-22 and 1949-52 (Vitali, 1969). As the Fuà group's work on Italian economic development continued, it used the constant price series of total product estimated by Vitali, presuming this to be more reliable than the pre-existing Istat estimate.

As the study was nearing completion, however, Fuà discovered some minor inconsistencies between Vitali's estimates and reliable evidence furnished by other sources; inconsistencies which he sought to remedy by revising a few points in Vitali's estimates. These corrections, however, were not included in the complete annual series; they appeared only in the "abridged" series (i.e. restricted to certain key-years: i.e. 1921 and 1949) set out in Fuà (1972) and (1981); and they were explained in those works.

The long-period income series at constant prices published in *Lo sviluppo economico in Italia*, and subsequently widely used, were therefore designed by their authors to supplement the Istat estimates already available. Accordingly, they were constructed by adhering as closely as possible to the sources, methods and techniques of the previous Istat work. It appeared entirely natural to adopt as Istat had done, without discussion and for the entire time-span, a measure at 1938 constant prices. However, care was taken to advise the reader that this choice heavily influenced the results, and example calculatioins were provided which illustrated the direction and the degree of this influence (Fuà, 1981: 368 ff).

With the passage of time, however, increasing dissatisfaction arose over the early exploratory work conducted in the 1950s and 1960s. It was felt that the old Istat series supplemented and adjusted by the Fuà group should be repalced by new estimates based on more careful analysis and incorporating new information. In particular, adequate account should be taken of:

(a) the increased knowledge about the physical indicators of quantities produced and about other base data of the national accounts that had, in the meantime, been accumulated by Fenoaltea and other authors;

(b) the advantages to be gained by using formulae for the calculation of real income other than those employing a single system of constant prices for the entire time-span.

Some studies along these lines have already been published (Maddison, 1991; Rossi, Toniolo and Sorgato, 1993), and various research projects are in progress, including one of very broad scope undertaken by the Banca d'Italia.

This article seeks to make a small contribution to the operation outlined in (b) above. It presents a possible alternative formula for constructing an index of real product, which it applies to the data already used in the old Vitali variant of the Istat estimate (while we await creation of a better data base).

2. *Chain Indices*

A long-period series of income expressed entirely at constant prices of one single year has notable advantages but also considerable drawbacks. Its principal advantage is the immediate evidence and concreteness of its significance, especially when we use as constant prices the prices prevailing in the situation closest to us, that is, today's prices. In a series expressed thus, the constant lire value of any

year t in the past tells us how we would value, on the basis of our present system of values, the product of that year t if we had it today. The ratio between the value of the product realized today and that realized in year t tells us to what extent—always on the basis of our present system of values—today's quantity of product has multiplied compared with that of year t.

If the series is expressed in the constant prices of a year in the distant past, its significance somewhat diminishes, but it preserves the advantage of simplicity. For example, in a series expressed at the constant prices of a remote year b, the ratio between today's product and that of year b tells us how much the quantity of annual product has increased, between then and now, if we measure it using the values system of people who lived in the long-past year b.

The principal disadvantage of a series of this kind emerges when it is used to compare the products of two years that are relatively close to each other but distant from the year whose prices are chosen as constants. There is, for instance, little justification for taking the system of values prevailing in 1990 to measure the progress achieved between 1860 and 1870; or the values system of 1913 to measure progress between 1960 and 1970. The measurement is more significant if it is carried out using a system of values prevailing at times closer to the dates which one wishes to compare.

This problem is usually avoided by dividing the long-period series into two or more sub-periods and assuming as constant prices for each sub-period those of a year chosen from within that sub-period.[1] However, although this procedure makes comparison between two periods belonging to the same sub-period less contrived, it raises a further difficulty when the intention is to compare two years belonging to different periods.

The discussion that follows will become clearer if the reader refers simultaneously to the next section, which provides a formalized description of the two types of index described verbally here.

Let us assume, for example, that we have a long-period series constructed by concatenating a series from year a to year m measured at constant prices of year l with a series from year m to year z measured at constant prices of year s. We may enquire of it to what extent product grew in year z compared with year f. The series replies that, according to the system of values of a hypothetical observer in year l, product multiplied a certain number of times (say, x times) between year f and year m; while, according to the system of values of a hypothetical observer in year s, it multiplied another y times between the year m and the year z. Hence if we wish to combine these two distinct valuations we may say that, in a certain sense, the product of year z is equal to $x \cdot y$ times that of year f.

The chain series therefore answers our question, but it does so in a highly contrived and convential form. By accepting the chain comparison, therefore, we forgo the advantage of comparisons based on a single system of constant prices: namely, that this system provides answers which have immediate evidence and concreteness.

However, once we have decided to make this sacrifice, we are advised to be even more radical. There are no serious grounds for constructing the long-period

[1]This is the procedure adopted, for example, by Maddison, 1991, in his revision of the Italian series 1861–1989. The "abridged" series 1861–1967 presented in Fuà, 1972, also uses this procedure.

series as a chain of several-year-long segments, since this entails that the velocity of growth of product in all the individual years of a segment is measured with the same system of constant prices. It is true that this system of constant prices is usually that pertaining to one of the years in the segment. This, however, is not enough to allay our fears, since the choice of year inevitably brings with it an element of arbitrariness, and it is a choice which ends up by influencing the behaviour of the index. It therefore seems preferable to construct a chain composed of links so small that they enable us to measure the velocity of growth of product in each short period (for example, from year t to year $t+1$) with a system of prices prevailing at some time during the period. Hence a practical solution is an annual chain index, which is the subject of this article.[2]

Once the decision has been taken to adopt an annual chain index, one must then choose between a Laspeyres index or a Paasche index. When the Laspeyres index is adopted, the change in product between year t and year $t+1$ is measured according to the t year's value system. That is, one asks what valuation the population of year t would have made of the progress achieved in the following year (assuming that it was able to predict this progress and interested in measuring it). If the Paasche index is adopted, the variation between year t and year $t+1$ is measured according to the value system of year $t+1$; that is, one asks how far the population of year t would judge its product to have grown, according to its then prevailing system of values, compared with that of the previous year. This latter approach strikes us as less contrived and more interesting than the former. We therefore give priority to a Paasche formula in what follows, although we aknowledge that this choice is a debatable one.

3. *The Formulae*

As is customary, we refer to an ultrasimplified model of the economy which assumes that total product can be viewed as a set of goods distinguished by category. The goods in each category are all measurable by an appropriate physical unit (for example, by piece, kg, m) and they all have the same unit price. We use the symbol q_{it} to denote the "physical" quantity of goods in category i produced in year t; and the symbol p_{it} to denote the corresponding unit price.

The product of year t at current prices can thus be written:

$$\sum_i q_{it} \cdot p_{it}. \tag{1}$$

[2]The idea is an old one and has already appeared, in outline form, in a paper by Alfred Marshall, 1887: 208. It was given systematic development by Divisia, 1925, as part of an ambitious project of monetary theory, and then important theoretical elaboration appeared in works by various authors, as well as some limited practical application in the production of official statistical series. A useful historical-critical analysis of the annual chain index is to be found in Forsyth and Fowler, 1981. An application of different index formulae and weighting procedures, among which an annual chain index, for the inter-war period in the Netherlands can be found in den Bakker, 1991. Young, 1992 and 1993, compares the official U.S. GDP index, with fixed 1987 weights for a 65 year period, with the results of a chain index, and an alternative index with 5 year moving weights. Differently from Young's assertion, 1992: 35, we show that the chained-index of the real GDP *can* be obtained by "adding up" its components; moreover, we show that the contribution of each component to a given change in real GDP *can be made* readily apparent.

Henceforth we shall omit the index i. The formula for the product of year t valued at constant prices of year z is accordingly:

$$\sum q_t \cdot p_z \tag{2}$$

If we now wish to pass from the value of product expressed in money at constant prices given by (2) to the corresponding index number based on year $f=1$, we have:

$$\frac{\sum q_t \cdot p_z}{\sum q_f \cdot p_z} \tag{3}$$

We now consider a chain series consisting of two multi-year links. As in the example given in section 2 above, we assume that the first link comprises the years from a to m valued at the constant prices of year l, while the second comprises the years from m to z valued at constant prices s. The index for year z on base year $f=1$ is therefore:

$$\frac{\sum q_m \cdot p_l}{\sum q_f \cdot p_l} \cdot \frac{\sum q_z \cdot p_s}{\sum q_m \cdot p_s}. \tag{4}$$

Formula (4) can be rewritten in two other forms:

$$\frac{\sum q_z \cdot p_s}{\sum q_f \cdot p_l} \Big/ \frac{\sum p_s \cdot q_m}{\sum p_l \cdot q_m} \tag{5}$$

and

$$\frac{\sum q_z \cdot p_z}{\sum q_f \cdot p_f} \Big/ \left[\frac{\sum p_z \cdot q_z}{\sum p_s \cdot q_s} \cdot \frac{\sum p_f \cdot q_l}{\sum p_l \cdot q_l} \cdot \frac{\sum p_s \cdot q_m}{\sum p_l \cdot q_m}\right]. \tag{6}$$

Formula (5) shows that the index can also be obtained by taking the ratio between z and f, each valued with its own system of constant prices, and dividing it by the price index of s with respect to l, calculated with the weights (quantities) of the year conjoining the two links.

Formula (6) shows that the index can also be obtained by beginning with the ratio between z and f valued at current prices and dividing it by the product of three price indices (price index z on base s with s weights; price index f on base l with l weights; price index s on base l with m weights).

We finally examine a chain series constituted by a set of annual links. Adopting a Laspeyres formula, the product index for years s on base year zero $=1$ assumes the following value:

$$\frac{\sum q_l \cdot p_0}{\sum q_0 \cdot p_0} \cdot \frac{\sum q_2 \cdot p_1}{\sum q_1 \cdot p_1} \cdot \frac{\sum q_3 \cdot p_2}{\sum q_2 \cdot p_2} \cdot \ldots \cdot \frac{\sum q_s-1 \cdot p_s-2}{\sum q_s-2 \cdot p_s-2} \cdot \frac{\sum q_s \cdot p_s-1}{\sum q_s-1 \cdot p_s-1}. \tag{7}$$

If instead we adopt a Paasche formula, the corresponding index is:

$$\frac{\sum q_1 \cdot p_1}{\sum q_0 \cdot p_1} \cdot \frac{\sum q_2 \cdot p_2}{\sum q_1 \cdot p_2} \cdot \frac{\sum q_3 \cdot p_3}{\sum q_2 \cdot p_3} \cdot \ldots \cdot \frac{\sum q_s-1 \cdot p_s-1}{\sum q_s-2 \cdot p_s-1} \cdot \frac{\sum q_s \cdot p_s}{\sum q_s-1 \cdot p_s}. \tag{8}$$

As in the case of the multi-year chain index, the above indices can be rewritten in other forms. For the sake of brevity, we restrict ourselves to formula (8), which can be rewritten as:

(9)
$$\frac{\sum q_s \cdot p_s}{\sum q_0 \cdot p_0} \Big/ \left[\frac{\sum p_1 \cdot q_0}{\sum p_0 \cdot q_0} \cdot \frac{\sum p_2 \cdot q_1}{\sum p_1 \cdot q_1} \cdot \frac{\sum p_3 \cdot q_2}{\sum p_2 \cdot q_2} \cdot \ldots \cdot \frac{\sum p_s - 1 \cdot q_s - 2}{\sum p_s - 2 \cdot q_s - 2} \cdot \frac{\sum p_s \cdot q_s - 1}{\sum p_s - 1 \cdot q_s - 1} \right].$$

Formula (9) shows that the annual chain Paasche index for year s on base year zero can be likewise obtained if one begins with the ratio between year s product and zero year product, both valued at current prices, and divides them by the product of the annual price indices (Laspeyres type) for all years from 1 to s.

4. *Construction of the Index on the Basis of Limited Information*

In actual fact, we do not possess information about the physical quantities and the unit prices of each category i of perfectly homogeneous products. However, we do possess information about broad sectors I (for example, "agriculture", "manufacturing", etc.) each of which groups together a number of categories and whose content is therefore heterogenous.

For each of these sectors we have:

—an indicator of the quantities produced (Q_I). This is an index constructed on the basis of various items of information with the intention of providing an approximate representation of the presumed average relative change, in physical terms, in the set of all the different productions included in the sector;

—the share at current prices of the sector's product in overall product (Y_I/Y_I).

Note that the original formula for the Laspeyres index given by (7) can also be rewritten as the sum of the quantity indices of each category of product, weighted with the share of the category in total product. For example, referring to the index of year 1 with respect to year zero, we have:

(10)
$$\frac{\sum_i q_{i1} \cdot p_{i0}}{\sum_i q_{i0} \cdot p_{i0}} = \sum_i \frac{q_{i1}}{q_{i0}} \cdot \frac{q_{i0} \cdot p_{i0}}{\sum_i q_{i0} \cdot p_{i0}}.$$

Assuming that the product of each sector is identical to the sum of all the categories of homogeneous products included within it, and assuming that the quantity index Q_1/Q_0 of one sector is sufficiently representive of the quantity variations in all the individual categories included within it, we may conclude that the Laspeyres index can be straightforwardly calculated on the basis of the sectoral data available by applying the formula:

(11)
$$\sum_I \frac{Q_{I1}}{Q_{I0}} \cdot \frac{Y_{I0}}{\sum_I Y_{I0}}.$$

The Paasche index cannot be calculated immediately in the same way, because:

$$(12) \qquad \frac{\sum_i q_{i1} \cdot p_{i1}}{\sum_i q_{i0} \cdot p_{i1}} \neq \sum_i \frac{q_{i1}}{q_{i0}} \cdot \frac{q_{i1} \cdot p_{il}}{\sum_i q^i \cdot p_{i1}}.$$

However, the problem can be solved by adopting an indirect approach. On the basis of the available data one may calculate the Laspeyres index for year zero based on year 1 and then take its reciprocal, which is identical to the Paasche index of year 1 based on year zero.

We have, in fact:

$$(13) \qquad \left(\sum_I \frac{Q_{I0}}{Q_{I1}} \cdot \frac{Y_1}{\sum_I Y_0}\right)^{-1} = \left(\sum_i \frac{q_{i0} \cdot p_{i1}}{q_{i1} \cdot p_{i1}}\right)^{-1} = \sum_i \frac{q_{i1} \cdot p_{i1}}{q_{i0} \cdot p_{i1}}.$$

The data can also be used to yield—as will be shown in Part II of this article—the annual rate of change in the index decomposed into the parts of it imputable to the change of each sector.

The operation is straightforward for the Laspeyres index. The rate of change of the index for year 1 on base year zero can, in fact, be written in the form:

$$(14) \qquad \left(\sum_I \frac{Y_{I0}}{\sum_I Y_{I0}} \cdot \frac{Q_{I1}}{Q_{I0}}\right) - 1 = \sum_I \frac{Y_{I0}}{\sum_I Y_{I0}} \cdot \frac{Q_{I1}}{Q_{I0}} - \sum_I \frac{Y_{I0}}{\sum_I Y_{I0}} \cdot \sum_I \frac{Y_{I1}}{Y_{I0}} \cdot \left(\frac{Q_{I1}}{Q_{I0}} - 1\right).$$

For the Paasche index the operation is somewhat more laborious. One must begin with the consideration that the Paasche index for year 1 based on year zero is identical to the reciprocal of the Laspeyres index for year zero based on year 1. The relative difference between this Paasche index and its base year therefore stands in a known relation to the difference between this Laspeyres index and its base-year. The former can be obtained by taking the latter, changing its sign and dividing it by the Laspeyres index for year zero. Since the data available allow calculation of the Laspeyres index, we shall perform this calculation to determine the rate of change in the Paasche index broken down into the parts of it imputable to changes in each sector, as follows:

$$(15) \qquad \left(\sum_i \frac{q_{i1} \cdot p_{i1}}{q_{i0} \cdot p_{i1}}\right) - 1 = \frac{1 - \left(\sum_I \frac{Y_{I1}}{\sum_I Y_{I1}} \cdot \frac{Q_{I0}}{Q_{I1}}\right)}{\sum_I \frac{Y_{I1}}{\sum_I Y_{I1}} \cdot \frac{Q_{I0}}{Q_{I1}}} = \sum_I \frac{\frac{Y_{I1}}{\sum_I Y_{I1}} \cdot \left(1 - \frac{Q_{I0}}{Q_{I1}}\right)}{\sum_I \frac{Y_{I1}}{\sum_I Y_{I1}} \cdot \frac{Q_{I0}}{Q_{I1}}}.$$

5. *Two Comments*

In conclusion to this presentation of the formula for the annual chain index, two comments are in order.

First, one may ask what differences are to be expected in the behaviour of the income index for a certain country in a certain period depending on whether we calculate this index using the annual chain technique or whether we use a single system of constant prices.

If we consider any particular year, we can establish what differences are to be expected between the *rates of change* of the two indices in that year. Differences may appear only on account of the fact that the two income indices use different price systems to weigh the sectoral indices of quantity, given that the quantity indices used are instead the same in both cases. This difference in weighting, moreover, is not enough to produce divergences between the two income indices unless there are growth rate differences among the various sectoral indices of quantity. The greater these differences, the more widely the rate of change of the two income indices may diverge. If certain sectoral indices of quantity should increas and others diminish, it may even happen that the variations in the two income indices assume contrasting signs.

Let us now see what differences can be expected between the levels assumed by the two income indices in any particular year, starting from the hypothesis, of course, that both indices are based on the same year = 1. There is a basic difference between the two indices due to the fact that the level of the index based on a single system of constant prices can be calculated for any year solely on the basis of the values of the sectoral quantity indices in that year and, obviously, of the constant price system. By contrast, in order to calculate the level of the annual chain index one must know the changes in the indices for each of the previous years, beginning with the year taken as base = 1; variations which, as we know, depend on the combinations among the weightings, which vary from year to year, and the changes in the indices of sectoral quantities. As we have seen, in each previous year the rate of change in the annual chain index may diverge from that of the index with fixed weighting, and these differences accumulate over time. To find the level reached by the index in a particular year, therefore, one must retrace its entire trajectory from the beginning.

There exists the rather widespread opinion that, in actual fact, there is an inverse correlation between the trend of the relative prices of the individual goods constituting the national product and that of relative quanities. In short, there is supposedly a more or less distinct tendency for those goods whose production increases more rapidly (or declines less slowly) than others to exhibit slighter price increases (or more marked price reductions) than others. Although this presumed statistical regularity has yet to receive more satisfactory verification, we may for the moment consider what expectations those who provisionally accept it as valid must form regarding the behaviour of the income indices being compared here. We find that they must expect *rates of change* generally higher for the annual chain index than for the index with fixed weightings, as long as years previous to the year from which the fixed weighting index takes its system of constant prices are considered. They would expect the difference between the rates of change of the two indices to diminish as one approaches the year from which the single constant price system is taken; and to change sign as one passes beyond that year. In accordance with our argument so far, they would expect the *level* of the annual chain index increasingly to diverge upwards from that of the constant-price index until the year from which the single constant-price system is taken.

Our discussion so far regarding the annual chain index applies to both the Paasche formula and to the Laspeyres formula. We would add, however, that the presumed inverse correlation between prices and quantities induces us to expect

the Laspeyres index to increase more rapidly than the Paasche index, due to the fact that the former weights the sectoral changes in quantity for any given year with a system of prices preceding by one year that of the latter.

A second comment concerns the presentation of product series in the form of indices or of values expressed in monetary units.

The technique of the single cosntant-price system yields as its most immediate result (at least logically) a series expressed in monetary units, which can then be straightforwardly "translated" in index numbers. Use of the chain index technique only yields, as the direct result of the operation, a series of index numbers. Those who then wish to translate this series into values expressed in monetary units are obliged to resort to a logical contortion that we do not think advisable.

This, however, is the expedient they may adopt. Let us assume that the intention is to express the Italian product of past years in monetary units in some sense "familiar" to today's public. One finds that the value of last year's Italian product, at current prices, was around 1,500,000 billion lire. Let us suppose that its annual chain series shows that the index of 1870 was equal to, say, 5 percent of that for last year, and that the corresponding ratios were 10 percent in 1913 and 15 percent in 1938. One may therefore construct an estimate in "1992 conventional lire" calculating the 1870 product as $1{,}500{,}000 \cdot 0.05 = 75{,}000$ billion, that of 1913 as $1{,}500{,}000 \cdot 0.10 = 150{,}000$ billion, that of 1938 as $1{,}500{,}000 \cdot 0.15 = 225{,}000$ billion, and so on for each year in the past.

This, however, is an entirely conventional estimate. It certainly does not mean that 1870 product valued at last year's prices would have been worth 75,000 billion lire. It has a much more contorted sense, as the explanations of previous sections should have made clear. Hence these measurements in vague monetary units are to be avoided whenever possible. Note that these considerations apply not only to annual chain indices but also to those (widely used) which are constructed on a chain consisting of a few long multi-year links.

II. Application of the Formula

6. *General Framework*

For the construction of the Paasche and Laspeyres annual chain indices, we used the series of value added at factor cost for the periods 1861–1950, 1951–70 (Ercolani, 1978) and 1971–85 (ISCO, 1990) at 1938, 1963 and 1980 constant prices, respectively. The weights of individual sectors (10 for the period 1861–1971: agriculture, mining, manufacturing, construction, electricity, transport, commerce and miscellaneous services, credit, buildings, and public services) were obtained from the value added at factor cost at current prices and at present boundaries (Annali di Statistica, 1957 and 1975). For the period 1970–85 we used value added at factor cost (ISCO, 1990) disaggregated into 17 sectors (agriculture, construction, energy, minerals and metals, non-metalliferous minerals, chemicals, mechanical engineering, vehicles, food, textiles, wood and paper etc., transport, commerce, miscellaneous services, credit, buildings, public services and other non-marketable goods). The population series were taken from Ercolani (1978) for

the period 1861–1960, and from ISCO (1986 and 1990) for the periods 1960–70 and 1970–89.

As one would expect from the evolution of relative prices and of changes in sectoral weights over time, the product series calculated using both Paasche and Laspeyres annual chain indices yields a higher annual growth rate than the corresponding series calculated at constant prices (Table 1; Figures 1 and 2). Perhaps it should be pointed out that the comparison with the constant-price index is only valid for those periods in which the base year of the deflation does not change: in our case this restricts the homogeneity of the comparison to the period 1861–1951. We should therefore expect low divergence between the indices in the years 1951–89, these too calculated "in chain" (i.e. with change in the constant prices index), and a more substantial revaluation for the preceding period.[3]

Comparison of the series (Table 2) confirms this expectation. One may argue, in fact, that the average revaluation of the GDP calculated with the Paasche criterion compared with the constant price series in the period 1897–1989 is around 0.15 percent per year (if we exclude the war years 1913–21 and 1938–50, this average revaluation drops to 0.10 percent). The revision when the Lapeyres annual chain index is used appears more substantial. This, again in the post 1897 years, reached 0.27 percent. It is well known that systematic divergences between the Paasche and Laspeyres indices appears if there are correlations between prices and quantities. In our case, the percentage divergence between the two criteria is practically zero, oscillating between 0.03 and 0.07 if war periods are excluded—when the Laspeyres annual chain index surpasses the corresponding Paasche index by 0.88 (between 1913 and 1921) and 1.34 (between 1938 and 1951) percentage points.

Since the quotient between the two chain indices remains constant during peace-time years (all differences concentrated into war periods), it is possible to

[3]In the very recent years there has been a flourishing of new estimates of the very long data of the Italian GDP. Besides the "old" Fuà-Istat series, we can refer to: (i) Maddison, 1991; (ii) Rossi, Toniolo and Sorgato, 1993; and, (iii) Fuà and Gallegati. Basically, Maddison's series utilize Fenoaltea's, 1982, 1987, 1988, estimation for mines, construction and public consumption in the years 1861–1913. For the period 1861–1913 (1913–38) he recalculated the added value with weight of the 1870 (of the 1938) instead of 1913. Finally for the years 1951–70 he used Golinelli and Monterastelli (1990). Rossi, Toniolo and Sorgato, 1993, utilize for the years 1951–90, Golinelli and Monterastelli 1990. As for the years 1890–1951, they get a new trend connecting the years 1911 (for which there exists a new Banca d'Italia, 1992, reconstruction) and 1951 (Golinelli and Monterastelli, 1990) to which the old cyclical component of the Fuà-Istat series is superimposed. As the following table shows (1951 = 100; the annual rate of growth in brackets), there exist some differences in term of growth rates between the above mentioned series. In particular, Maddison obtains an higher growth rate for the years 1861–1938 and 1951–70; while Rossi *et al.* show a sensitive higher (smaller) rate of growth between 1913–38 (1890–1913 and 1938–51).

	Old Series		*Fuà, Gallegati*		*Maddison*		*Rossi et al.*	
1861	27		24		21			
1890	34	[0.8]	30	[0.8]	30	[1.1]	30	
1913	55	[2.2]	50	[2.2]	54	[2.6]	48	[2.0]
1938	81	[1.6]	78	[1.8]	81	[1.7]	85	[2.3]
1951	100	[1.6]	100	[1.9]	100	[1.6]	100	[1.2]
1960	159	[5.3]	161	[5.4]	168	[5.9]	164	[5.6]
1970	272	[5.3]	274	[5.4]	294	[5.8]	282	[5.7]
1980	394	[3.8]	403	[3.9]	426	[3.8]	408	[3.8]
1990	482	[2.3]	495	[2.3]	522	[2.3]	498	[2.2]

Figure 1 - Index number of value added (gross of duplications) at factor cost, present boundaries (1861-1945)

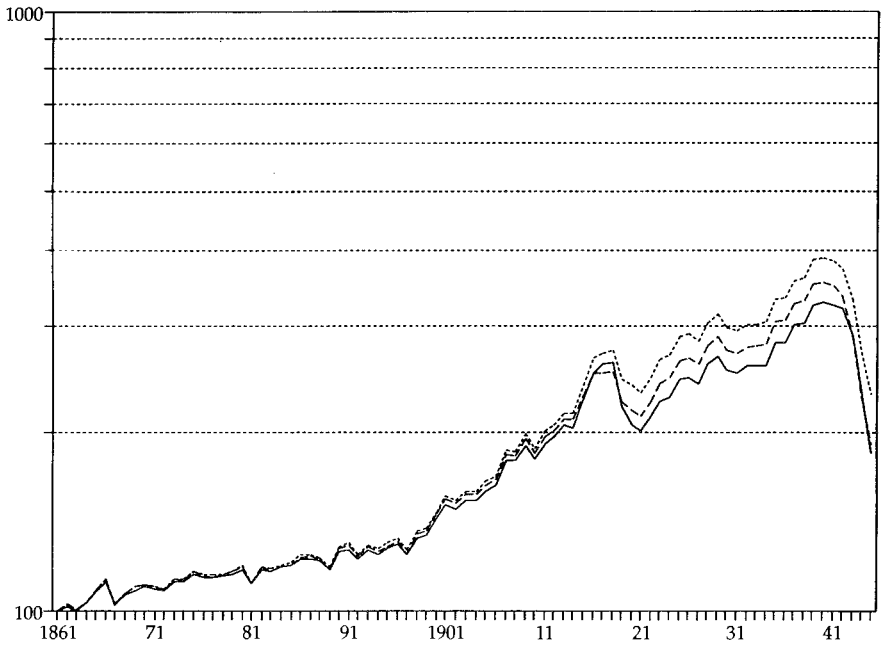

Figure 1. Index Number of Value Added (Gross of Duplications) at Factor Cost, Present Boundaries (1861–1945). —— Constant Prices Index; – – – Annual Chain Paasche Index; - - - - - Annual Chain Laspeyres Index

conclude that, whatever criterion for constructing the index is adopted, the average revaluation of the annual compound growth rate is 0.10 percent.[4] Given the questionable nature of the choice of criterion in the construction of the index, in the next section we shall concentrate on comparison between the constant price series and the annual chain index constructed on the Paasche criterion, still maintaining that the choice of criterion in the construction of the index is of a questionable nature.

7. *Analysis by Periods*

Compared to its analogous constant-price chain index with three multi-year links, the Paasche annual chain index revalues the growth trend and accentuates the amplitude of cyclical fluctuations, whose standard deviation increases on average by almost 8 percent, although respecting the chronology of short cycles identified in Fuà (1981). If the value of the initial year is set at 100, the per capita figures for the constant-price chain series with three multi-year links and for the annual chain series indicate values for the final year of 820 and 953, respectively

[4]The quotient between the Lapeyres and Paasche annual chain indices remained around 1 between 1861 and 1931. During the First World War it steadily increased until, in 1921, it reached 1.1. The quotient remained at roughly this value until 1938. During the Second World War it increased considerably until, in 1950, it stood at 1.3; a level which it maintained until 1989.

Figure 2 - Index number of value added (gross of duplications) at factor cost, present boundaries (1861-1989)

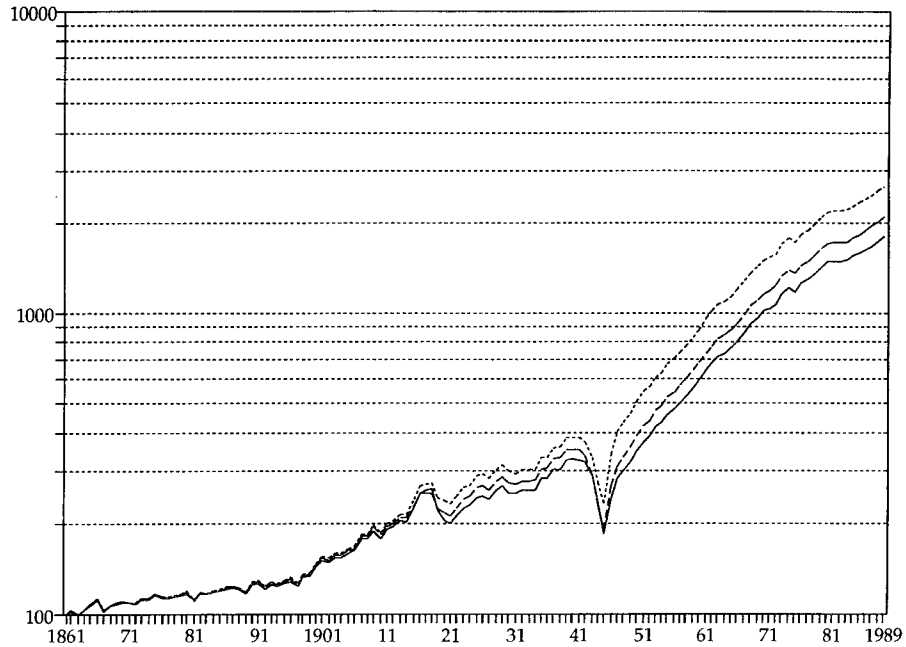

Figure 2. Index Number of Value Added (Gross of Duplications) at Factor Cost, Present Boundaries (1861-1989). —— Constant Prices Index; – – – Annual Chain Paasche Index; - - - - - Annual Chain Laspeyres Index

(Table 1). Whereas the per capita multi-year series over the entire period gives an annual compound rate of change of 1.64 percent, the corresponding series estimated using the annual chain index is revalued by around 0.12 percentage points. This revaluation becomes more marked (0.15 percent) if we restrict the comparison to the period subsequent to 1897, the year in which "modern economic development" in Kuznets' sense began.

There is therefore substantial uniformity in estimates at constant prices and by means of the annual chain index for the period 1861–97. In those years, in fact, whatever method of intertemporal comparison is used, the relation between prices and quantities is too slight to arouse notice, due to the fact that the quantities change very little. However, the period prior to capitalist development is not of major importance as far as our present interests are concerned; it is only after 1897, in concomitance with the period of modern economic development, that the results are affected by the method of estimation used. For the years since 1897, the annual chain index displays a revaluation which, especially during war years becomes sizeable as regards the amplitude of cyclical fluctuations as well.

Between 1913 and 1921, in fact, the two series differ considerably. Specifically, in 1920 the reduction in product using the Paasche annual chain index is half that of the constant price series. Between 1921 and 1938 the revaluation of the series

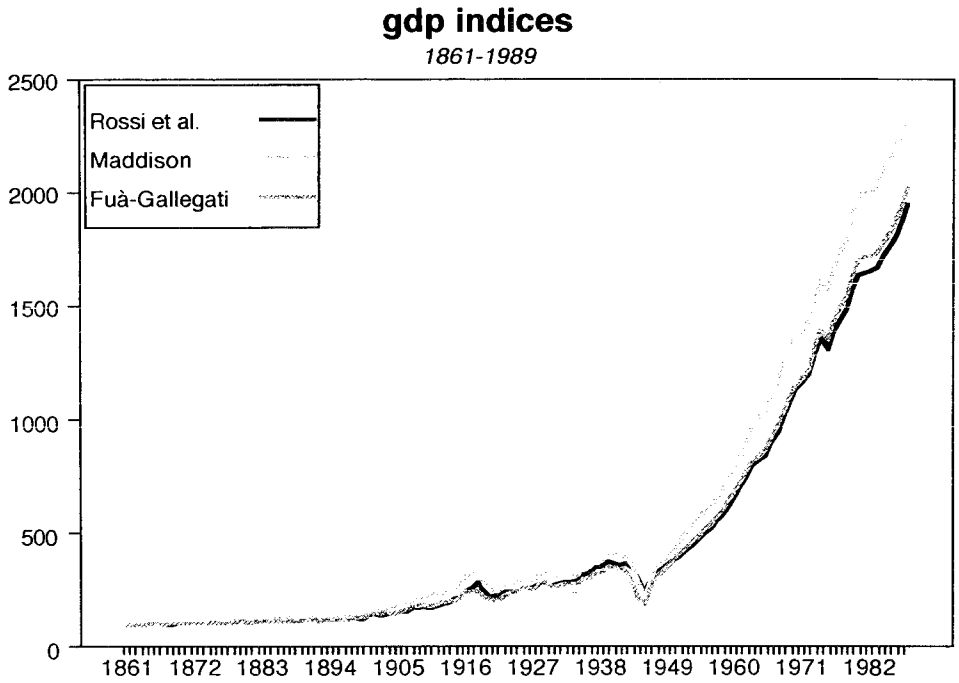

Figure 3.

assumes roughly the values of the preceding period (+0.14 percent). However, it is once again in war years, between 1942 and 1946, that we find a noticeable difference in the rates of change. Whereas during the First World War the annual chain index corrects mainly the value of the rates of change in recession, between 1942 and 1946 the revision involves both the decline and the subsequent revival in real product. After 1950, calculation of domestic product using the annual chain index again shows the revaluation of the series, although to a lesser extent than preceding periods because of the concatenation with the deflation base of the constant-price series.

In general, if measured in levels, the trend of per capita product using the annual chain index is always higher than the corresponding 1938 constant-price index, with four exceptions: 1867, 1881, 1917–18, 1943–44. In the first two cases, we find exceptional falls in output (−9.81 percent and −7.25 percent) which are wholly specific and therefore marginal. Entirely different, though, is interpretation of the divergences during the war years. These require more careful analysis.

As we have seen, estimates of the series until 1913 diverge little. Whereas between 1915 and 1918 the constant-price series grows more rapidly than the chain series (30.3 percent as opposed to 21.8 percent), in 1919–20 the decrease in the former is more marked (−24.7 percent and −17.6 percent respectively). Construction of the Paasche annual chain index enables us to ascertain the effective contribution of each production sector to the overall change in product (section 5). By applying this methodology, it is possible to attribute the overall

TABLE 1

INDEX NUMBER OF VALUE ADDED (GROSS OF DUPLICATIONS) AT FACTOR COST, PRESENT BOUNDARIES

Years	Constant Prices Index	Annual Chain Index	
		Paasche	Laspeyres
	1938 Prices		
1861	100.00	100.00	100.00
1862	102.01	102.54	102.52
1863	100.40	100.42	100.35
1864	103.02	103.42	103.39
1865	107.65	108.47	108.46
1866	112.27	113.27	113.46
1867	102.82	102.42	102.72
1868	106.44	107.04	107.21
1869	108.65	109.62	109.76
1870	109.86	110.67	110.79
1871	109.05	109.97	110.11
1872	108.05	108.47	108.72
1873	112.27	113.20	113.43
1874	112.27	112.79	113.06
1875	115.29	116.24	116.70
1876	114.08	114.48	114.95
1877	113.88	114.91	114.74
1878	114.29	114.91	115.44
1879	115.49	116.37	116.91
1880	117.30	118.88	119.38
1881	111.67	111.22	111.72
1882	117.30	118.26	118.96
1883	116.70	117.09	117.75
1884	118.31	118.88	119.54
1885	119.52	119.59	120.31
1886	121.73	122.84	123.72
1887	122.54	123.28	124.02
1888	121.13	122.34	123.09
1889	117.30	117.54	118.54
1890	125.55	126.89	128.04
1891	126.76	128.89	129.92
1892	121.93	123.28	123.99
1893	125.96	127.93	128.81
1894	124.55	125.72	126.87
1895	127.16	128.36	129.67
1896	129.38	131.02	132.55
1897	124.14	124.62	126.39
1898	132.19	134.43	136.03
1899	134.00	135.93	137.46
1900	143.26	145.53	147.24
1901	150.70	154.37	156.74
1902	148.49	151.39	153.52
1903	153.52	157.05	159.12
1904	153.92	156.83	158.93
1905	158.95	162.75	164.89
1906	162.37	165.78	168.17
1907	178.67	183.51	186.36
1908	178.47	182.80	185.39
1909	189.54	194.84	198.11
1910	180.08	183.95	187.72
1911	191.15	196.78	200.85
1912	195.57	200.45	204.78
1913	205.23	210.66	215.64
1914	203.22	210.11	215.25
1915	227.36	230.68	240.43

TABLE 1—Continued

Years	Constant Prices Index	Annual Chain Index	
		Paasche	Laspeyres
1916	251.51	251.52	267.11
1917	259.56	251.10	271.53
1918	261.57	252.41	273.07
1919	221.33	224.39	245.44
1920	205.23	217.38	238.88
1921	201.21	212.48	232.88
1922	213.28	226.04	247.83
1923	225.35	241.38	264.77
1924	229.38	245.82	269.01
1925	245.47	263.55	288.67
1926	247.48	265.79	291.15
1927	241.45	259.71	283.65
1928	259.56	278.20	303.92
1929	267.61	287.34	313.79
1930	253.52	273.63	297.87
1931	251.51	270.23	294.40
1932	257.55	277.18	301.90
1933	257.55	277.80	301.88
1934	257.55	279.83	304.91
1935	281.69	304.89	332.67
1936	281.69	305.82	334.14
1937	301.81	327.45	357.10
1938	303.82	329.78	359.64
1939	325.96	352.74	384.96
1940	327.97	354.17	387.35
1941	323.94	349.23	383.14
1942	319.92	336.09	374.27
1943	289.74	288.17	332.92
1944	235.41	230.36	272.68
1945	184.31	190.79	232.49
1946	241.45	268.04	340.79
1947	283.70	312.76	404.06
1948	299.80	332.81	429.98
1949	321.93	359.35	464.03
1950	348.09	390.49	505.45
1951	374.25	423.41	547.74
	1963 Prices		
1952	388.79	439.47	568.56
1953	415.99	472.40	611.52
1954	429.86	488.50	633.13
1955	458.39	521.90	676.76
1956	478.26	545.11	707.10
1957	502.85	573.36	743.67
1958	527.39	601.49	780.55
1959	562.38	642.26	833.53
1960	596.20	682.37	885.73
1961	642.97	736.51	956.28
1962	681.10	780.09	1013.93
1963	716.37	820.50	1066.36
1964	737.15	844.09	1097.29
1965	763.17	873.04	1135.38
1966	807.32	922.69	1200.09
1967	862.47	984.45	1280.74
1968	917.11	1047.18	1362.17
1969	968.59	1105.34	1438.18
1970	1017.37	1159.23	1508.33
1971	1037.85	1185.13	1536.27

TABLE 1—Continued

Years	Constant Prices Index	Annual Chain Index	
		Paasche	Laspeyres
	1985 Prices		
1972	1071.27	1228.51	1586.39
1973	1149.44	1327.26	1699.48
1974	1207.61	1394.21	1783.40
1975	1174.97	1350.71	1733.56
1976	1250.15	1444.17	1843.00
1977	1290.77	1493.66	1903.99
1978	1339.81	1550.09	1975.75
1979	1418.39	1643.67	2091.76
1980	1473.43	1706.76	2174.88
1981	1485.34	1724.03	2190.36
1982	1489.06	1720.74	2194.82
1983	1505.72	1736.04	2215.34
1984	1549.37	1782.31	2275.91
1985	1591.39	1832.09	2335.95
1986	1631.07	1884.32	2393.51
1987	1680.46	1941.90	2463.45
1988	1749.68	2026.77	2559.35
1989	1804.05	2094.97	2637.16

change in product between 1915 and 1918 almost entirely to the increase in public administration (Table 2). Similarly, in 1919 more than three-quarters of the fall in output is due to the contraction of the public sector (−8.58 percent) while agriculture (−2.53 percent) and transport (−1.68 percent) are responsible for the rest. The negative contribution (−6.56 percent) by the public sector continues in 1920, in contrast to the positive contributions made in that year by agriculture (2.66 percent) and services (1.18 percent). The dissimilarity in the trends of the growth rates of the two series, which is almost entirely due to their different estimates of the contribution by the public sector in 1917 and in the two-year period 1919–20 (see Table 3), induces a first substantial divergence in the levels of the indices. A divergence which, from 1921 onwards, extends to the annual compound rate, which was 0.14 percent until 1938. Then in the immediate postwar years there is once again a "leap" in the series.

In contrast to the situation during the First World War, the fall in per capita product between 1938 and 1945 is roughly the same whichever deflation method is used (−41.92 percent for the constant-price series, and −44.61 percent with the annual chain index). Moreover, the chief contributor to the decrease in output is not public administration. The largest difference between the series emerges during the years of postwar revival (between 1945 and 1949 the annual chain index shows an increase of 82.73 percent, whereas at constant prices the overall increase is 69.46 percent). More than 70 percent of this difference is concentrated in 1946, when the contributions (very much undervalued in the 1938 price series: see Table 3) by agriculture (11.14 percent), manufacturing (21.3 percent) and, in part, commerce (5.09 percent) induce a growth in output of more than 40 percent.

After 1949, the year in which per capita product exceeds the level of 1938, the series continue to diverge until, in 1989, their ratio reached the value of 1.16.

TABLE 2

COMPOUND RATE OF CHANGE (%) OF REAL INCOME PER CAPITA
(Different index formulae compared)

	Pluri-Annual Chain index	Annual Chain Paasche Index	Annual Chain Laspeyres Index
	1938 constant prices		
1861–1989	1.64	1.76	1.96
1861–1951	0.78	0.91	1.19
1861–1897	0.01	0.03	0.06
1897–1913	2.14	2.24	2.31
1913–1921	0.05	0.41	1.29
1921–1938	1.55	1.69	1.66
1989–1951	0.28	0.58	2.02
	1963 constant prices		
1951–1971	4.68	4.71	4.77
	1985 constant prices		
1971–1989	2.65	2.77	2.71

TABLE 3

ANNUAL PERCENTAGE RATE OF CHANGE OF REAL INCOME, BROKEN DOWN INTO 4 SECTORAL COMPONENTS. ANNUAL CHAIN PAASCHE INDEX (A) AND CONSTANT 1938 PRICES INDEX (B) COMPARED

		Agriculture	Industry	Private Services	Government	Total Value Added
1915	A	−3.69	4.36	0.96	8.16	9.79
	B	−3.09	5.70	1.19	8.09	11.88
1916	A	2.70	−0.01	−0.88	7.23	9.03
	B	2.12	0.21	−0.74	9.03	10.62
1917	A	−0.11	−3.45	−1.17	4.56	−0.17
	B	−0.10	−2.82	−1.22	7.30	3.20
1918	A	0.90	−1.26	−0.03	0.92	0.52
	B	0.62	−0.94	−0.39	1.48	0.78
1919	A	−2.53	0.23	−0.21	−8.58	−11.1
	B	−1.85	−0.31	—	−13.38	−15.38
1920	A	2.66	−0.40	1.18	−6.56	−3.12
	B	1.89	−0.36	1.45	−10.25	−7.27
1942	A	−4.46	−2.86	0.99	2.57	−3.76
	B	−2.60	−3.47	1.43	3.41	−1.24
1943	A	−5.52	−4.98	−7.55	3.77	−14.26
	B	−3.21	−6.13	−7.73	7.45	−9.43
1944	A	−1.53	−9.02	−9.71	0.20	−20.06
	B	−0.75	−8.81	−9.75	0.56	−21.71
1945	A	−2.57	−6.46	−2.33	−5.82	−17.18
	B	−1.30	−4.34	−2.17	−13.89	−21.71
1946	A	11.14	25.04	8.26	−3.96	40.49
	B	6.51	20.77	9.92	−6.20	31.00

8. *Conclusions*

The literature on chain indices has evidenced two major advantages of the chain method with respect to constant-price reduction: (i) the choice of the deflation formula becomes less crucial; (ii) an annual chain index automatically incorporates those effects deriving from the change in sectoral weights which inevitably arise over long periods of time.

In section 2 we argued in favour of a Paasche annual chain index, and its construction and application to the Italian GDP figures between 1861 and 1989 has accordingly been the subject of this article. Comparison with product at 1938 constant prices has yielded homogeneous results which can be summarized as follows: (i) there is no appreciable difference between Laspeyres and Paasche annual chain estimates for the period of pre-capitalist development (1861–97), (ii) except for war periods (1913–21 and 1938–49, when the divergence between the constant price series and the annual chain indices is on average 0.4 percent for the Paasche index and 1.5 percent for the Laspeyres index), there is a revaluation of GDP in favour of the annual chain index, however calculated, of around 0.10 percent, (iii) the divergence between the Paasche and the Laspeyres annual chain indices is entirely confined to war years.

The most striking differences are therefore concentrated into period of marked changes in relative prices, and the new index modifies the temporal pattern of the growth trend by accentuating, without altering the periodization, cyclical fluctuations.

It goes without saying that this work represents only a first, modest step towards reconsideration of the problem of intertemporal comparisons of income. In this sense its purpose has been to illustrate a possible reasonable alternative to the methods now widely used.

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