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POLLUTION AND POLLUTION ABATEMENT IN THE NATIONAL ACCOUNTS

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Building on the approach of Weitzman, as extended by Hartwick and Mäler, five models of national accounts in a dynamic competitive economy with pollution externalities are constructed: flow pollutants, stock pollutants, fossil fuels and CO_2, living resources and acid rain, and household defensive expenditures. The results measure welfare rather than national product *per se*. The general conclusions are that abatement expenditures should be treated as intermediate consumption, that adjustments need to be made for both pollution emissions and natural pollution dissipation processes, that marginal social costs should be used to value emissions, and that the level of environmental services must be valued in measuring welfare. Not only should household defensive expenditures not be subtracted from the welfare measure, under plausible assumptions the adjustment to welfare (as opposed to NNP) includes a value greater than thelevel of household defensive expenditure.

INTRODUCTION

There is a pervasive sense that the conventional national accounts overstate the measurement of "true" income and product because they do not account for the damage to the environment from pollution emissions. The basic notions are that the value of environmental damage should be deducted from domestic product and that at least some final expenditures on environmental protection, "defensive expenditures," should not be considered to be final demand. This paper develops a series of models to examine these claims and to suggest extensions to the standard accounts to account for environmental change.

By asking a simple question, why we measure both consumption and investment in national product when the economic goal is to maximize consumption, Weitzmann (1976) provided the theoretical framework for a fruitful line of inquiry into the relationship between resources, the environment and national product, the prime examples being Solow (1986), Hartwick (1990, 1992, 1993), and Mäler (1991). Hamilton (1993) looks more closely at the treatment of resource depletion and discoveries in the national accounts using this same framework.

Weitzman's answer to the question was that, if we assume we are on the optimal path of a dynamic competitive economy, then national product measured as the sum of consumption and investment in the current period is, if held constant and the present value taken, just equal to the present value of consumption along the optimal path—he calls it the *stationary equivalent* of future consumption. In an equally appealing interpretation of this framework, Solow (1986) showed that increases in national product from some assumed initial value are equal to the discount rate times the accumulation of capital from the initial period to the

Note: The comments of David Ulph, David Pearce, Giles Atkinson, Bart de Boer, John Hartwick and an anonymous reviewer are much appreciated. Any errors are those of the author.

present—national product can thus be conceived as the interest on total accumulated wealth.

There is perhaps a simpler welfare interpretation of national product: as Weitman (1976) noted, the current value Hamiltonian of an optimal control representation of the economy,

$$\max \int_0^\infty C e^{-rt}\, dt \quad \text{subject to}$$

$$F = C + p\dot{K},$$

is just $H = C + p\dot{K}$, i.e., it is equal to national product (in this formulation, F is production, C consumption, K capital and p the relative price of capital and consumption goods; r is the constant discount rate). From Pontryagin's Maximum Principle we know that the Hamiltonian is maximized at every point in time along the optimal path. Therefore, national product is simply that quantity that a planner would choose to maximize in each period in order to maximize the present value of consumption.

The present value of future consumption is a wealth measure, and Usher (1994) shows that the Hamiltonian is the return to this wealth under assumptions of fixed technology and endogenous consumption and capital formation. Usher demonstrates that the Hamiltonian is not equal to the return on wealth so defined if: (i) consumption can increase autonomously; (ii) there is autonomous technological change; or (iii) there are tax distortions in the economy. Hartwick (1990, 1993) and Mäler (1991) both explicitly extend Weitzman's approach to look at maximizing the present value of utility under different presumptions about the depletion of natural resources and damage to the environment from pollution. Mäler constructs one large model that contains, in addition to consumption and investment goods, a flow resource that is damaged by pollution emissions, a living resource that is harvested and whose growth is affected by inputs of goods and labour, and a household production function through which, by inputting goods and labour, households can increase their benefits from the environment (i.e., the flow resource). The key result in Mäler (1991) is that deductions for defensive expenditures should not be made in the measure of national welfare derived from the model.

Hartwick (1990) presents two pollution-related models, one in which there is a stock pollutant that accumulates emissions and is subject to a natural dissipation process—this pollutant appears (negatively) in the production function—and a second in which the rate of change of the stock pollutant appears in the utility function as well. In these models pollution is mitigated by expenditures that affect the rate of the natural dissipation process, an unlikely form of mitigation. Hartwick (1993) offers a more intuitive model in which utility is related to the accumulated stock of pollutant and abatement expenditures limit the quantity of pollution emissions.

This study builds on and extends the Hartwick and Mäler models in several directions: (i) an explicit approach is taken to pollution abatement expenditures, and these are related to optimal emission taxes; (ii) a series of models are constructed to examine individually the effects of flow pollutants, stock pollutants, stock pollutants linked directly to exhaustible resources (the CO_2 problem), and flow pollutants that damage living resources (the acid rain problem); and (iii) the treatment of household defensive expenditures is re-examined to yield a variation on Mäler's interpretation.

We begin by evaluating how the traditional "green national accounting" literature has approached pollution issues.

Green National Accounting

The interest in adjusting the national accounts to reflect environmental concerns has several broad motivations as summarized in Hamilton (1991): the accounts measure the goods but not the "bads" resulting from economic activity; depreciation of natural assets is not measured; no measure of the sustainability of economic development is possible with traditional national accounts; and the treatment of final expenditures by households on environmental protection is questionable. The approaches to altering the national accounts fall into roughly two categories, adjustments to reflect the exploitation of commercial natural resources, and adjustments to reflect the degradation of non-market environmental resources. The key papers on commercial natural resources include Repetto *et al.* (1989), El Serafy (1989), Hartwick (1990, 1992, 1993) and Hamilton (1993). However, the concern in this paper is with the treatment of pollution emissions and their effects, and it is the literature primarily dealing with these issues that will be discussed here. Peskin (1989) advocates two types of adjustments to standard gross national product to allow for the effects of environmental change: (i) the addition of some measure of environmental services, conceived primarily as waste disposal services provided free by the environment; and (ii) the deduction of damages to the environment, both directly through impacts on human health, for instance, and indirectly through the loss of use of non-market assets, such as a lake. Peskin notes that to the extent that environmental services are provided free to producers, then they are already measured to some degree in profits, which would otherwise be lower if the waste disposal services of the environment had to be bought. How to value environmental damage is not discussed.

Where Peskin would deduct damage to the environment, Harrison (1989) would add this to gross national product. The whole point about *gross* product, according to Harrison, is that it includes the value of depreciation of assets. Conceiving the effects of pollution emissions as depreciating environmental assets, therefore, suggests that GNP should increase to allow for environmental effects. Harrison then notes that in this scheme *net* product will fall or remain constant relative to its conventional measure depending on whether countries are making sufficient environmental expenditures to maintain environmental quality. Expenditures sufficient to maintain current environmental quality implicitly define the value of environmental degradation in Harrison's analysis, therefore. This fits with the example given in Harrison (1989) of the depreciation of major infrastructure assets, such as roads or dams, where is it assumed that current repair and maintenance expenditures are just enough to offset depreciation. Harrison's approach bears some resemblance to that of Bartelmus *et al.* (1989), which in turn led to the recommendations for an integrated System of Environmental and

Economic Accounting (SEEA) in United Nations (1993). The suggestion is to value environmental deterioration as the cost of returning the quality of the environment to its state at the beginning of the accounting period (or as the cost of maintaining this state, whence the term "maintenance costs"). Current environmental quality becomes a rather arbitrary yarkstick against which to value deterioration in this approach, therefore, but this could be generalized to the specification of some absolute level of quality, as determined by policy, as the reference point as well.

Hueting and Bosch (1990) extend these ideas further in the direction of the measurement of sustainability. Rather than valuing environmental deterioration as the cost of returning to some preceding environmental state, they propose to measure how much it would cost to achieve sustainable use of the environment. So instead of calculating a "green GNP" *per se* they attempt to arrive at a thorough estimate of the cost of, for instance, limiting pollutant emissions to the rate of assimilation by the environment, using renewable resources at a sustainable rate, and so on.

Turning to a different class of problem, Juster (1973) questions whether final expenditures on environmental protection ("defensive expenditures") should properly constitute a portion of GNP. The contention is that many such final expenditures do not increase welfare but simply preserve its level, e.g., when households spend on domestic water filters in order to avoid the effects of declining water quality. From this viewpoint environmentally adjusted GNP should be smaller than its conventionally measured counterpart. This is a view to which Leipert (1989) also subscribes. As noted above, Mäler (1991) explicitly concludes that such a deduction should *not* apply when calculating environmental adjustments to GNP.

As a brief summary of this literature, the general contention is that some measure of the cost of environmental protection should be deducted from GNP (or net product) to reflect damage to the environment, and that defensive expenditures should be deducted as well. In this view conventional national product is an overstatement of "true" product. The next section develops a series of models to test the extent to which theory supports the conclusions of this literature.

Models of Environmental National Accounts

Each of the models presented below is designed to examine a particular facet of the treatment of pollution in the national accounts. A number of simplifying assumptions are made: (i) technology is assumed to be unchanging; (ii) the production function F exhibits declining returns to factors; (iii) there is a single product that may be consumed, invested or used in abating pollution; (iv) labour markets are assumed be in equilibrium, so that the welfare effects of labour do not figure in what follows [as was derived in Mäler (1991)]; and (v) the discount rate r is constant. U is the utility function, and C consumption; in most of the models utility is assumed to be an increasing function of both consumption and the flow of environmental services B, measured in appropriate (but not necessarily monetary) units. B is assumed to measure pure *non-market* environmental services, so that there is no duplication with the indirect effects of environmental quality on production or asset values. With the exception of r, all variables are functions of time. Additional assumptions will be added as required.

The general ideas developed in the following models are that the natural environment provides a flow of non-market services that can be diminished by pollution emissions, that this flow of services yields utility, and that produced goods can be employed to abate pollution emissions.

Model 1: Flow Pollutant Related to Production

A flow pollutant is a pollutant whose current level of *emissions* can be assumed to affect the level of services derived from the environment. Any pollutant with noxious effects that are not cumulative, such as a toxin, could serve as an example. The simple economy for the model of green national accounts is therefore one where emissions are assumed to be related to the level of production, $e = e(F)$, production is a function of produced capital and labour, $F = F(K, L)$, and output of the composite good can either be consumed or invested, so that,

$$F = C + \dot{K}.$$

The objective of the social planner for this economy is to maximize the present value of utility over an infinite time horizon, where utility U is a function of both consumption C and the level of environmental services B. Utility is assumed to be discounted at a fixed rate r.

Environmental services are negatively related to pollution emissions as,

$$B(e) = B_0 - \alpha e.$$

Here B_0 is the level of environmental services that flow from a pristine environment, while α is the amount by which services decline when a unit of pollution is emitted. While it is not essential to specify a linear relationship between emissions and environmental services, it simplifies the exposition. The problem therefore is,

$$\max \int_0^{\infty} U(C, B)\, e^{-rt}\, dt \quad \text{subject to:}$$

$$\dot{K} = F - C$$

For γ_1 as the shadow price of capital, the current value Hamiltonian function for this programme is,

$$H = U + \gamma_1 \dot{K} = U + \gamma_1 (F - C).$$

The only control variable is consumption C, and therefore the first order condition for a maximum is,

$$\frac{\partial H}{\partial C} = 0 = U_C - \gamma_1 \Rightarrow \gamma_1 = U_C.$$

The second order condition for the Hamiltonian to be maximized is $U_{CC}<0$ (i.e., declining marginal utility of consumption).

The Hamiltonian function is measured in utils, and so must be transformed into consumption units in order to yield an expression that conforms more closely to conventional national accounting aggregates. This is done in two steps: (i) each flow in the Hamiltonian—consumption, environmental services and investment—is valued at its shadow price in utils; and (ii) the resulting expression is divided by the marginal utility of consumption U_c to give a measure of economic welfare (MEW) in consumption units. Scaling by the marginal utility of consumption yields the correct relative prices between flows at each point in time. The resulting expression is,

$$\text{MEW} = C + \dot{K} + \frac{U_B}{U_C} B.$$

Here economic welfare is measured as the sum of GNP $(C+\dot{K})$ and the value of the flow of environmental services. Note that U_B/U_C is the price that utility-maximizing consumers would be willing to pay for a marginal unit of environmental service. Pollution flows can be brought explicitly into the picture by substituting the expression for B,

(1) $$\text{MEW} = C + \dot{K} - \alpha \frac{U_B}{U_C} e + \frac{U_B}{U_C} B_0.$$

Here $\alpha U_B/U_C$ is the marginal social cost of a unit of emissions, yielding the correct valuation of pollution in the aggregate welfare measure. This is also clearly the level of a Pigovian emissions tax sufficient to maximize welfare in each period. The last term in this expression is the (constant) environmental service flow from a pristine environment valued at (varying) current prices.

This model can be made more realistic and more general if we assume that the composite good can both be invested in pollution abatement capital K_a and spent on current abatement expenditures a in order to reduce pollution emissions to welfare-maximizing levels. The emission function therefore becomes,

$$e = e(F, K_a, a) \qquad \text{with } e_a < 0 \quad \text{and} \quad e_{K_a} < 0.$$

Introducing a new control variable m for investment in pollution abatement capital, the maximization problem becomes,

$$\max \int_0^\infty U(C, B)\, e^{-rt}\, dt \quad \text{subject to:}$$

$$\dot{K} = F - C - a - m$$

$$\dot{K}_a = m.$$

The current value Hamiltonian for this programme is,

$$H = U + \gamma_1 \dot{K} + \gamma_2 \dot{K}_a$$
$$= U + \gamma_1 (F - C - a - m) + \gamma_2 m$$

and the first order conditions for a maximum are:

$$\frac{\partial H}{\partial C} = 0 = U_C - \gamma_1 \Rightarrow \gamma_1 = U_C$$

$$\frac{\partial H}{\partial m} = 0 = -\gamma_1 + \gamma_2 \Rightarrow \gamma_2 = U_C$$

$$\frac{\partial H}{\partial a} = 0 = -\alpha U_B e_a - \gamma_1 \Rightarrow -\alpha U_B e_a = U_C.$$

The additional second order condition for a maximum is therefore $e_{aa} > 0$. Defining $b \equiv -1/e_a$ as the *marginal cost of pollution abatement*, this condition amounts to increasing marginal abatement costs. This marginal cost, from the first order condition on a, is equal to the marginal social cost of emissions,

$$b = \alpha \frac{U_B}{U_C}.$$

Transforming the Hamiltonian as in the model without abatement expenditures yields,

(2) $$\text{MEW} = C + \dot{K} + \dot{K}_a - be + \frac{U_B}{U_C} B_0.$$

Note first that all investment, whether in productive capital or in abatement capital, is counted in the aggregate welfare measure. Second, current abatement expenditure a is not measured in welfare—these expenditures are essentially intermediate in character. Third, current pollution emissions are represented as a deduction from welfare, valued either at marginal abatement costs or marginal social costs, both of which in turn are equal to the level of a Pigovian emissions tax. The equivalence of these marginal costs is, of course, a consequence of MEW being measured on the optimum path.

Model 2: A Cumulative Pollutant and a Stock Pollutant

We next wish to model a pollutant whose effects are cumulative.[1] The level of the flow of environmental services is therefore related negatively to the cumulative amount of pollution emitted, X, so that

$$\dot{B} = B_0 - \beta X.$$

[1]As a simplication in this and all subsequent models, investment in pollution abatement will be ignored, since its effects on the welfare measure have been explained in Model 1.

We first assume no abatement expenditures, so that $e=e(F)$. The model is therefore,

$$\max \int_0^\infty U(C, B)\, e^{-rt}\, dt \quad \text{subject to:}$$

$$\dot{K}=F-C$$

$$\dot{X}=e.$$

Here C is the only control variable. The current value Hamiltonian for this problem is,

$$H=U+\gamma_1(F-C)+\gamma_2 e,$$

for co-state variables γ_1 and γ_2, and the first order condition for a maximum (ignoring the dynamic conditions) is,

$$\frac{\partial H}{\partial C}=0=U_C-\gamma_1 \Rightarrow \gamma_1=U_C.$$

For the first order condition to yield a maximum, a necessary condition is that $U_{CC}<0$, i.e., that there be declining marginal utility of consumption.

Note that $\gamma_2<0$, since increases in the accumulation of the pollutant decrease welfare. The measure of economic welfare is obtained by transforming the Hamiltonian as in Model 1, to yield,

$$\text{MEW}=C+\dot{K}+\frac{\gamma_2}{U_C}e+\frac{U_B}{U_C}B.$$

There are several things to note about this expression, beginning with why it should be interpreted as a welfare measure rather than net national product. The term in environmental services B provides the answer: as a purely external phenomenon it reflects adjustments to utility rather than to market production. This expression should be interpreted as what a planner should maximize at each point in time in order to maximize the present value of utility, in keeping with our earlier interpretation of the Weitzman model. The expression U_B/U_C is the price that utility-maximizing consumers would be willing to pay for a unit of environmental service, and so a key component of welfare in this model, as with the flow pollutant, is the monetized value of the *level* of environmental services.

Since γ_2 is the shadow price of the accumulation of the pollutant measured in utils, it is natural to define $\sigma\equiv-\gamma_2/U_C$ as the marginal social cost of a unit of the pollutant, and, as in Model 1, this will equal the Pigovian tax required to maximize utility. If $p_B\equiv U_B/U_C$, then the expression for economic welfare becomes,

(3) $$\text{MEW}=C+\dot{K}-\sigma e+p_B B.$$

Economic welfare is therefore measured as consumption plus investment less the value of an optimal emissions tax plus the value of the level of environmental services.

Abatement expenditures, a, are introduced into this model as the use of current production to reduce the level of emissions, so that the emission function is re-defined as folllows:

$$e = e(F, a),\ e_F > 0,\ e_a < 0.$$

The maximization problem is now specified as:

$$\max \int_0^\infty U(C, B)\, e^{-rt}\, dt \quad \text{subject to:}$$

$$\dot{K} = F - C - a$$

$$\dot{X} = e.$$

The control variables are C and a and the current value Hamiltonian is as specified about. The first order condition for γ_1 is again that it should equal the marginal utility of consumption. For γ_2 we now have,

(4) $$\frac{\partial H}{\partial a} = 0 = -\gamma_1 + \gamma_2 e_a \Rightarrow \gamma_2 = \frac{U_C}{e_a}.$$

It will be useful in what follows to define $b \equiv -1/e_a$; this is just the *marginal cost of pollution abatement*. Transforming the Hamiltonian into consumption units, we therefore derive,

(5) $$\text{MEW} = C + \dot{K} - be + p_B B.$$

Expression (4) implies that $b = -\gamma_2 / U_C$. The marginal cost of abatement is identically equal to the marginal social costs of emissions and to the value of the optimal unit emissions tax. Given that $\gamma_1 > 0$ and $\gamma_2 < 0$, a necessary condition for the Hamiltonian to produce a maximum of utility is $e_{aa} > 0$, i.e., increasing marginal abatement costs.

Economic welfare, therefore, is measured as consumption plus investment, less the value of pollution, plus the value of environmental services. Note the valuation of pollution in expression (5). While this may appear similar to valuing environmental damage as the current cost of abatement, a moment's reflection shows that this is not so: valuation is based on the *marginal* cost of abatement, and emissions are implicitly held to their optimal value, because welfare is being maximized.

Expression (5) can also be written as,

(6) $$\text{MEW} = C + p_B B + \dot{K} + \frac{b}{\beta} \dot{B},$$

so that economic welfare consists of the proximate sources of utility, C and $p_B B$, plus the adjustments required to ensure utility maximization over time, $\dot{K}$ and $(b/\beta)\dot{B}$. Note that $\dot{B} < 0$ for any non-zero production level because pollution accumulates.

Expression (5) yields another interpretation. First, GNP $= F = C + \dot{K} + a$. This implies that

(7) $$\text{MEW} = \text{GNP} - a - be + p_B B.$$

So we conclude that, in order to arrive at a welfare measure, abatement expenditures should be subtracted from GNP—they become, in effect, intermediate consumption.[2] This is consistent with the notions of Juster (1973) and Leipert (1989). What goes beyong the conclusions in these studies is the subtraction of emissions valued at the marginal cost of abatement and the addition of the value of environmental services.

One unsatisfactory aspect of the previous model is that it treats the environment as purely exhaustible: the flow of environmental services can only decline for any non-zero level of output. In a variation on this model we therefore assume a simple representation of a pollutant that both accumulates and dissipates:

$$\dot{X} = e - d(X)$$
$$B = B_0 - \beta(X - X_0).$$

Here X_0 is the initial stock of the pollutant and $d(X)$ is the dissipation function for this stock, representing physical processes that reduce and render harmless some amount of the accumulated pollutant. Environmental services B are assumed to be negatively related to the stock of pollutant, with β being the fixed rate at which services decrease with accumulation of the stock. As a consequence, $\dot{B} = -\beta\dot{X}$. As in the preceding model, $e = e(F, a)$. The overall model therefore becomes[3], for control variables C and a,

$$\max \int_0^\infty U(C, B)\, e^{-rt}\, dt \quad \text{subject to:}$$

$$\dot{K} = F - C - a$$
$$\dot{X} = e - d.$$

The current value Hamiltonian for this problem is given by,

$$H = U + \gamma_1(F - C - a) + \gamma_2(e - d).$$

As in the previous model, the first order conditions for a maximum give $\gamma_1 = U_C$. For marginal abatement cost $b \equiv -1/e_a$, the first order conditions then imply that $\gamma_2 = -U_C b$. Transforming the Hamiltonian into consumption units, the measure of economic welfare therefore reduces to:

(8) $$\text{MEW} = C + \dot{K} - be + bd + p_B B.$$

In this expression the term bd represents the dissipation of the pollutant stock valued at the marginal cost of abatement. Emissions are a deduction from welfare in this model, while dissipation of the stock of pollutant represents an increase in

[2]This is true for all the following mdoels. The interpretation of household defensive expenditures will be derived in Model 5.

[3]This model is formally similar to one in Hartwick (1993). It is here given a more careful interpretation and is used to set the stage for models that follow.

welfare. The value of the flow of environmental services is again included in economic welfare, owing to the fact that environmental services are a direct source of utility.

Model 3: Pollution is Linked to Exhaustible Resource Use

This can be dubbed "the CO_2 problem" because we view the level of pollution emissions as being linked directly to the quantity of resource use, much as carbon dioxide emissions are related stoichiometrically to the carbon content of fossil fuels.[4] We will assume in what follows that we are dealing with an exhaustible fossil fuel resource. As in Model 2, utility is a function of both consumption and environmental services, and the environment regenerates as a result of dissipation processes. The key differences are that $e=e(R, a)$, $e_R>0$, R measures the quantity of resource extracted and used, S is the resource stock, resources are inputs into production, so that $F=F(K, L, R)$, $F_R>0$, and resources are costly to produce, so that $f=f(R)$, $f_R>0$ specifies the cost of resource extraction. This treatment of exhaustible resources follows Hartwick (1990) and Hamilton (1993).

The following stock-flow relationships for pollution stock and environmental services hold,

$$\dot{X}=e(R, a)-d(X), \quad \text{and} \quad B=B_0-\beta(X-X_0).$$

The model is specified as:

$$\max \int_0^\infty U(C, B)\, e^{-rt}\, dt \quad \text{subject to:}$$

$$\dot{K}=F-C-a-f$$

$$\dot{X}=e-d$$

$$\dot{S}=-R$$

$$S \rightarrow 0 \quad \text{as} \quad t \rightarrow \infty.$$

The final part of this expression just says that resources must be exhausted over the program; this is an efficiency condition. In this model K, X, and S are the state variables and C, a and R are the control variables. This optimal control problem has the current value Hamiltonian function,

$$H=U+\gamma_1(F-C-a-f)+\gamma_2(e-d)-\gamma_3 R$$

where γ_1, γ_2 and γ_3 are the co-state variables corresponding to capital, carbon dioxide stocks, and resource stocks respectively. If we define $b\equiv -1/e_a$ to be the marginal cost of abatement as before, then derivation of the first-order conditions for this problem yields,

$$\gamma_1=U_C, \qquad \gamma_2=-U_C b, \qquad \gamma_3=U_c(F_R-f_R-be_R).$$

The Hamiltonian is measured in utils and is maximized at each point in time under the optimal program—it is therefore a current measure of welfare. The

[4]This model is similar to that developed in Hamilton and Ulph (1994).

conditions for a maximum include $F_{RR} < 0$, so resources have declining marginal product, and $f_{RR} \geq 0$, i.e., constant or increasing marginal extraction costs. As in previous models, we define the measure of economic welfare by transforming the Hamiltonian into consumption units. Substitution of the above expressions for the co-state variables into the Hamiltonian therefore gives,

$$\text{(9)} \qquad \text{MEW} = C + \dot{K} - b(e-d) - (F_R - f_R - be_R)R + p_B B.$$

This expression says that emissions decrease welfare while regeneration of the environment, through the dissipation of CO_2, increases it (i.e., the environment is productive); in both cases the appropriate unit of valuation is the marginal cost of abatement, b. Assuming profit-maximizing producers, F_R is the market price of the resource and f_R its marginal cost of extraction. The next-to-last term in this expression therefore relates to the value of resource depletion, being of the form "price minus marginal cost."[5] However, the unit resource rent $F_R - f_R$ is reduced by a Pigovian tax, at rate be_R. This is a *carbon tax on resource use*, a specific tax required to achieve both the maximization of the present value of utility and the efficient extraction of the resource when its use leads to CO_2 emissions. The net rental value of fossil fuels decreases when account is taken of their environmental externalities.

It might be argued that carbon emissions cannot be abated in any practical manner, calling the dependence of expression (9) on marginal abatement costs into question. One response to this is to argue that in any model of pollution emissions it is more general to assume some level of abatement effort. The second response is that, as in Models 1 and 2, the same results can be obtained by assuming no abatement effort—what results is an expression containing the level of an optimal carbon emissions tax, σ, in place of the marginal abatement cost b in expression (9) (and the preceding expressions as well).

As a final note, this model assumes a single country dealing with the welfare effects of its own carbon emissions. The situation is reality, of course, is much more complex, with multiple emitting countries facing different marginal abatement cost schedules, so that finding a global optimum would require, for example, some form of emission trading.

Model 4: Living Resources are Damaged by Pollution

This model examines the "acid rain" problem: it is assumed that living resources with economic value are damaged by emissions resulting from production. To keep the analysis to its essentials we will make a couple of simplifying assumptions. First, harvest of the living resource is assumed to be costless. And secondly, utility is derived only from consumption and not from the resource or the quality of the environment is general.

Production is characterized by the production function $F = F(K, L, R)$, where R is the quantity of resource harvested, and emissions (as in Models 1 and 2) are related to the level of production and abatement expenditures a, so that $e = e(F, a)$. The resource stock S is augmented by natural growth $g(S)$ and diminished by

[5]This is comparable to the value of depletion, based on resource rentals, that appears in Hartwick (1990).

harvesting and the amount of damage resulting from pollution emissions w, such that,

$$w = w(S, e),\ w_S > 0 \quad \text{and} \quad w_e > 0.$$

This formulation implies that acid emissions cause direct damage only, and have no cumulative effect. This is obviously another simplification.

The model therefore becomes,

$$\max \int_0^\infty U(C)\, e^{-rt}\, dt \quad \text{subject to:}$$

$$\dot{K} = F - C - a$$

$$\dot{S} = -R - w + g.$$

Here C, a, and R are the control variables. The current value Hamiltonian for this problem is,

$$H = U + \gamma_1 (F - C - a) + \gamma_2 (-R - w + g).$$

As in previous models, the efficiency condition on consumption implies that $\gamma_1 = U_C$. Optimality also requires, assuming marginal abatement costs $b \equiv -1/e_a$,

$$\frac{\partial H}{\partial a} = 0 = -\gamma_1 - \gamma_2 w_e e_a \Rightarrow \gamma_2 = \frac{U_C b}{w_e} \quad \text{and} \quad \frac{\partial H}{\partial R} = 0 = \gamma_1 F_R - \gamma_2 \Rightarrow \gamma_2 = U_C F_R.$$

These expressions imply the interdependence of abatement costs, marginal emission damages and resource rents, so that $F_R = b/w_e$. This interdependence arises from having three control variables but only two state variables, K and S, in the model. The measure of economic welfare is,

$$\text{(10)} \qquad \text{MEW} = C + \dot{K} - F_R (R + w - g)$$

$$= C + \dot{K} - \frac{b}{w_e} (R + w - g).$$

Note that these expressions can be considered to be a measure of net national product, since there are no terms representing household welfare. The first is similar to that of Hartwick (1990)—when living resources are exploited, net national product is adjusted by deducting resource harvest and dieback and adding resource growth, all valued at the resource rental rate. The second expression yields a mildly counter-intuitive result: other things being equal, the greater the marginal damage associated with emissions, the smaller is the adjustment (associated in this case with resource harvest, dieback and growth) to national product; this interpretation cannot be carried too far, however, because resource rents, marginal abatement costs and marginal damages from emissions are all interrelated.

Model 5: Household Defensive Expenditures

This model explores the situation where households make expenditures that directly affect the benefits obtained from the environment. We assume, therefore, that rather than deriving utility from environmental services B directly, utility is obtained via some *benefit* function Φ, that is in turn a function of environmental services and household expenditures that enhance benefits from the environment (or, equivalently, that can compensate for decreases in flows of environmental services—e.g., as suggested earlier, purchasing a water filter to purify drinking water that is declining in quality). The model is otherwise very similar to Model 2 for a pollutant with cumulative effects.

We therefore have $U = U(C, \Phi)$, $U_{\Phi} > 0$, $\Phi = \Phi(B, h)$, $\Phi_B > 0$, and $\Phi_h > 0$ for household defensive expenditures h. As in the first models, emissions are given by $e = e(F, a)$. For state variables K and X (the stock of pollutant) and control variables C, a, and h, therefore, the optimal control program is:

$$\max \int_0^{\infty} U(C, \Phi)\, e^{-rt}\, dt \quad \text{subject to:}$$

$$\dot{K} = F - C - a - h$$

$$\dot{X} = e.$$

The current value Hamiltonian for the optimal control program is:

$$H = U + \gamma_1 (F - C - a - h) + \gamma_2 e.$$

The first order conditions for the optimum yield $\gamma_1 = U_C = U_{\Phi} \Phi_h$. At the optimum we therefore have the following constraint,

$$\frac{1}{\Phi_h} = \frac{U_{\Phi}}{U_C},$$

where $1/\Phi_h$ is the *marginal defensive cost*. Note that this means that the price of environmental benefits just equals the marginal defensive cost, which is to be expected for a utility-maximizing consumer.[6] Sufficient conditions for a maximum are $U_{\Phi\Phi} < 0$, so there is declining marginal utility of environmental benefits, and $\Phi_{hh} < 0$, which implies increasing marginal defensive costs. As in Model 2, $\gamma_2 = -U_C b$. As a result, the measure of economic welfare is:

(11) $$\text{MEW} = C + \dot{K} - be + \frac{\Phi}{\Phi_h}.$$

Where households can compensate for changing environmental service flows, therefore, economic welfare is measured as consumption plus investment, less the value of pollution emissions (where pollution is priced at the marginal abatement cost), plus the value of environmental benefits to households (where benefits are priced at the marginal defensive cost). The term Φ/Φ_h is conceptually similar to $p_B B$, the value of environmental services, that appears in the other models.

[6]This is obviously related to the notion of using avertive expenditures to value environmental benefits as described in Smith (1991).

This result requires careful interpretation. The measure of economic welfare can also be written as,

$$\text{MEW} = \text{GNP} - a - h - be + \frac{\Phi}{\Phi_h}. \tag{12}$$

Because $\Phi_{hh} < 0$, i.e., increasing marginal defensive costs, is part of the sufficient condition for a maximum, we can conclude that the welfare measure includes a value larger than household defensive expenditures h.

This should be compared with Mäler's (1991) interpretation of household defensive expenditures, which is basically that they should not be deducted in arriving at a "green" welfare measure. Rather than deducting defensive expenditures, this model suggests that the welfare measure should include some value greater than defensive expenditures. In addition, the model says that abatement expenditures and emissions valued at their marginal cost of abatement (or, equivalently, the value of an optimal emissions tax) should be deducted from GNP in arriving at welfare.

National Income and Genuine Saving

One of the key characteristics of the foregoing models is that they measure welfare rather than an explicit "green NNP." This is a natural consequence of the optimization problem, to maximize the present value of utility, that underlies the models. An important issue is the relationship between MEW and a measure of income.

Model 3, the CO_2 problem, provides a general framework for examining this question since it concerns both an exhaustible resource and a stock pollutant. It will be convenient to define the net resource rent as $n \equiv F_R - f_R - be_R$. MEW is therefore given by,

$$\text{MEW} = C + \dot{K} - b(e - d) - nR + p_B B. \tag{13}$$

Considering natural resources to be assets in the national balance sheet, and pollution stocks to be liabilities, the definition of genuine saving is,

$$GS \equiv \dot{K} - b\dot{X} + n\dot{S} = \dot{K} - b(e - d) - nR.$$

As in Hamilton (1994), this is called "genuine" saving to distinguish it from traditional net saving measures in the national accounts, which deduct only the depreciation of produced assets. Genuine saving equals the change in the real value of all tangible assets. A green measure of national income, or NNP, should therefore be defined as the sum of any production that is not invested or used to abate pollution and the change in the real value of assets,

$$\begin{aligned} \text{NNP} &= F - \dot{K} - a + GS \\ &= C + \dot{K} - b(e - d) - nR. \end{aligned}$$

As shown in Hamilton (1995), an important property of NNP so defined is that it represents the maximum amount that could be consumed while leaving utility *instantaneously* constant, under appropriate assumptions about the rate of change of investment [under weaker assumptions Pemberton *et al.* (1995) show that this is the maximum amount that could be consumed while leaving the present value of utility instantaneously constant]. While this is a valid conception of income, it does not measure sustainable income, the maximum amount that could be consumed along a constant-utility path.[7]

Hamilton (1995) also shows that persistently negative genuine savings are not sustainable—eventually welfare must decline. As a guide to policies for sustainable development, therefore, genuine savings rates are a key indicator.

If environmental services are an input to production in this model, so that $F=F(K, L, R, B)$, it is straightforward to show that the formula for MEW that results is precisely that of expression (13). No explicit adjustment to the welfare measure to represent the effect of environmental services on production is required, therefore. This effect is entirely reflected in the level of production.

This helps to clarify the role of environmental services in green national accounting: they have both an implicit effect on the level of production and contribute directly to the welfare of households. So while Peskin (1989) is correct to highlight the role that environmental services may play, the explicit adjustment to national accounting aggregates should be restricted to households' valuation of environmental services. The result of this adjustment should be considered to be a welfare measure rather than NNP *per se*, since NNP is the amount of *produced* output that can be consumed while maintaining zero change in the real value of assets.

MEASUREMENT

Since national accounting is a practical exercise, built upon measurement and estimation as well as a core of theory, it is fair to ask what practical consequences these results might have for "green" national accounting.

Because these results are based upon optimizing models, it might be tempting to agree with Usher (1994) that the results are valid only if all of the underlying assumptions of the optimal control problem hold. There are two responses to this criticism. First, the model results correspond to those of a competitive economy with a Pigovian tax—factors are valued at their marginal products and households' valuation of environmental services is the ratio of the corresponding marginal utilities. The model results are not different in kind, therefore, from the other models used in standard economic analysis. Second, the models provide a consistent framework within which to analyze the adjustments that should be made to "green" the national accounts. The formal aspects of the model results are therefore important, in the sense of identifying explicitly which flows should be added or subtracted to standard GNP, and in the sense of indicating what are the "ideal" unit values to employ.

[7]This is shown in Pezzey and Withagen (1994) for an economy with a single exhaustible resource.

One set of practical questions concerns the measurement of the flow of environmental services. The appropriate quantitative measure of the services provided by clean air, for instance, is not obvious. However, one proxy might be provided by air quality indices; the task is then to measure the willingness of consumers to pay for marginal changes in these indices. Where such indices have been reported for a long period of time and consumers have developed a sense of what subjective environmental quality they associate with given index levels, this might be a practical approach.

Alternatively, there is by now a substantial literature on the measurement of willingness to pay for environmental amenities [see, for instance, Cropper and Oates (1992) for a review], including travel cost methods, hedonic pricing and contingent valuation. Since an important issue identified in the green national accounts models is the valuation of pure non-market environmental services [B in expression (13)], contingent valuation is a key technique for deriving the measure of economic welfare (MEW). Much of the valuation literature is concerned with valuing individual sites or environmental assets, however. The question of how to sum across the myriad environmental assets within a country in a consistent, non-duplicating manner is an unanswered question.

The models point to the need to value pollution emissions and the regeneration of the environment (through dissipation of pollution stocks) at the marginal abatement cost or the marginal social cost. Measuring pollution emissions is in principle a straightforward matter. Measuring regeneration is more problematic—it might be necessary to model the physical processes involved rather than measuring them directly. It will also be difficult to measure *marginal*, as opposed to average, abatement costs in the current period. Measuring marginal social costs is increasingly viable, however, as evidenced by the work on the social costs of fuel cycles (CEC/US 1993).

The question of measurement away from the optimum is an interesting one, given that most real economies would not be expected to be operating at the environmental optimum. Figure 1 provides a way to think about this issue.

This is the textbook figure used to derive the notion of optimal pollution. In this figure the horizontal axis refers to reductions in pollution emissions. "MCA"

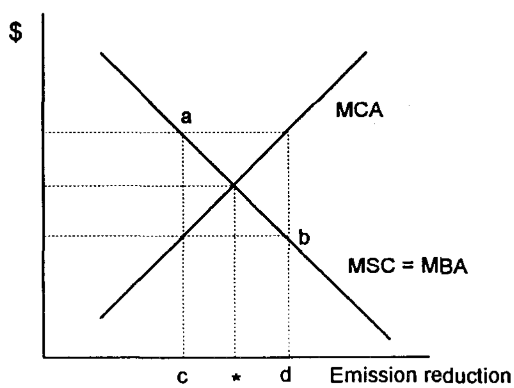

Figure 1. Marginal Costs and Benefits of Abatement

is the curve for marginal cost of abatement. "MSC" is the marginal social cost curve, which is equal to the marginal benefit of abatement (MBA). The optimal emission reduction occurs at level, "*" while level "c" represents over-polluting and "d" under-polluting.

If it is assumed that the current state of the economy is one of over-polluting, then marginal social costs at level "a" will provide an upper bound on the value of optimal pollution emissions; if the current state is one of under-polluting then marginal social costs at level "b" will provide a lower bound on the optimal emission value. As long as it is reasonably certain that the economy is over-polluting, therefore, using marginal social costs to value emissions should provide an upper bound estimate. This implies in addition that the deduction for pollution emissions in the green national accounting aggregate will decrease as the optimum is approached.

Figure 1 also makes it clear that using marginal abatement costs to value emissions will not lead to an unequivocal direction of bias in the estimates of the value of pollution. If the economy is over-polluting then marginal abatement costs will be below the optimum, while emissions are above their optimal level; the opposite applies to an economy that is under-polluting.

Conclusions

The analysis in this paper has considered models where explicit current and capital expenditures are made to abate pollution. In the real world, of course, many capital investments jointly increase productivity and reduce the uncontrolled level of pollution emissions. Under these circumstances the notion of "marginal cost of abatement" is not well defined, but it is still possible to measure the marginal social costs of pollution emissions.

Introducing external trade in the composite good into the models has no major effect on the results. Gomez-Lobo in Gomez-Lobo *et al.* (1993) shows that when there are exports and imports of the produced good, a term of the form iA must be added to the derived welfare measure, where i is a fixed international interest rate and A the net accumulation of foreign assets resulting from external trade. Of course this assumes a small open economy, so that the interest rate can be taken as given, and it does not deal with transboundary pollution. In the latter case Hamilton and Atkinson (forthcoming) argue that, by analogy with the "polluter pays principle," the total social costs of pollution, both domestic and foreign, should be charged against the income or welfare measure of the polluting country.

It should be obvious that these separate models could be combined to represent the more realistic assumption of multiple pollutants and multiple control efforts. Each pollutant would have its own emissions function (including abatement, where appropriate), separate accumulation and dissipation of pollution stock, and distinct environmental service flow that is associated with the level of emission (in the case of flow pollutants) or the level of the pollutant stock. Alternatively, a single environmental service flow could be the result of taking a weighted combination of the separate emissions and dissipation of the various pollutants. Living resources, fossil fuels, and other exhaustible resources could be added as well, including the effects of acid rain and CO_2, as long as the cross-effects are accounted for in the analysis (e.g., the reduction in the level of natural resource rents resulting from pollution emissions associated with production, as in Model 3).

The following is a brief summary of the model results, concentrating on the versions of the models with abatement expenditures. In each model the starting point in measuring welfare is GNP less abatement expenditures.

Model 1. For flow pollutants, deduct emissions valued at the marginal cost of abatement, and add the level of environmental services that would flow from a pristine environment, valued at consumers' marginal willingness to pay.

Model 2. For the cumulative pollutant, rather than adding the value of the service flow from a pristine environment, add instead the value of the current level of environmental services. For a stock pollutant that dissipates, in addition to the preceding adjustments, add the dissipation of the pollutant stock, valued at the marginal cost of abatement.

Model 3. For the CO_2 problem, in addition to the adjustments in Model 2, subtract the value of net fossil fuel rents—the price minus the marginal cost of extraction less the value of an optimal carbon tax.

Model 4. The acid rain problem yields the following adjustment for living resources: from GNP less abatement expenditures subtract net resource depletion (harvest plus dieback minus growth) valued at the resource rental rate. The resource rental rate must be equal to the marginal cost of abatement divided by the marginal dieback rate for the living resource.

Model 5. for household defensive expenditures, the result is formally the same as for the cumulative pollutant of Model 2, except that the flow of environmental services valued at marginal willingness to pay is replaced by the flow of environmental benefits (i.e., service levels as affected by defensive expenditures) valued at the marginal defensive cost. Assuming increasing marginal defensive costs, this term is in fact greater than the level of household defensive expenditures. However, these expenditures would not be included in the adjusted measure of NNP.

There are several general conclusions from this analysis. Abatement expenditures are essentially intermediate in character—the practical consequence of this is that any abatement expenditures in final demand in the standard national accounts should be deducted in order to arrive at the measure of economic welfare. Secondly, the natural dissipation of pollutants should be added to traditional GNP in order to measure economic welfare. Third, while the analytical framework leads in a natural way to a measure of economic welfare, dropping the term for households' valuation of the flow of environmental services produces a green NNP measure. Although both marginal social costs and marginal abatement costs may be used to value pollution emissions and dissipation, marginal social costs may be preferable because there are practical methods to measure them, because the direction of bias in the valuation of emissions is known, and because the direction of movement of these values as emissions levels decrease is correct. Valuation at marginal social costs is distinctly at variance with valuation using maintenance costs, the preferred option in the UN SEEA (United Nations 1993).

A natural outcome of expanding the asset base for national accounting to include pollutant stocks and flows is to shift the focus from income to welfare. This partly explains the contrast between the results presented here and the traditional literature. However, a consistent measure of national income can be derived from the models presented in this paper by dropping the pure welfare term, households' valuation of the level of environmental services.

With respect to the policy relevance of green national accounting, there are two conclusions. First, the measure of economic welfare (MEW) can be viewed as an indicator to guide policies for optimal growth. Secondly, measures of genuine saving can serve as indicators of sustainability, since persistent negative genuine savings must lead to declines in welfare.

Measurement problems abound, but the models presented suggest the way to think clearly and consistently about how conventional national accounts can be extended to account for the effects of environmental change.

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