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AN ALTERNATIVE TO DOUBLE DEFLATION FOR MEASURING REAL INDUSTRY VALUE-ADDED

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This article proposes a new method to compute the real value-added of industries which would substitute for the traditional double deflation method. The new method consists of deflating industries' direct and indirect contributions to final demand deliveries (their value-added by commodity) by the respective final demand commodity prices. The article shows that the new industry real value-added measures have better statistical and analytical properties than those obtained by the double deflation method.

1. INTRODUCTION

The double deflation method is at the centre of the deflation of the gross domestic product of industries in Canada.[1] This method is also widely used in other countries as its application follows a recommendation from the UN.[2] Yet, the method has been strongly criticized in the economic literature as it provides a meaningful measure of the "net" output of industries only under extremely restrictive assumptions which are not likely to be realized in the real world (see for instance Bruno, 1978 and Denny and May, 1978). The method also presents major statistical difficulties of application at the disaggregated industry level and the more so the further one moves away from the base year.

This article proposes an alternative measure of real value-added which has better statistical and analytical properties than the measure obtained from the double deflation method. The alternative measure preserves and translates in real terms the usual economic notion of nominal value-added of an industry in national accounting. This notion relies on the basic idea that production units belong to a set of interdependent industries and carry individually only part of the transformation processes of goods and services (i.e. add value) which are necessary to produce and deliver them to final demand. The total value of these goods and services is therefore given by the sum of the values added to them by the industries which have participated in their production.

This idea, which is quite old, is simply extended here in real terms: The real value-added of commodities delivered to final demand must be identically equal to the sum of the real values added to them by the industries. This can be obtained by dividing their nominal value-added by their price both in the aggregate and at

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[1]The method, as is well known to national accountants, consists in deflating the outputs and intermediate inputs of industries in some base year prices, and in computing their real value-added by subtracting the deflated intermediate inputs from the deflated outputs.

[2]United Nations, 1968.

the industry level. In other words, industries' contributions to the nominal value-added of the commodities may be deflated by the corresponding commodity prices.

The plan of the article is as follows. Section 2 below compares, with the help of a simple numerical example, the alternative deflation method proposed in this article with the double deflation method. Section 3 presents the general mathematical derivations of the real value-added equation. Section 4 analyses the statistical and analytical properties of the new measure. The comparison with the double deflation method is sustained throughout showing the strong advantages of the new method. Some further interpretations and insight are provided in the conclusion.

2. A Numerical Comparison of Double Deflation with the New Method

Before presenting the formal derivation of the new real value-added measure, it may be preferable to start with a simple example as depicted on Figure 1. Figure 1 shows the gross output vector of the economy which is assumed, for simplicity, to have only two industries A and B. Real magnitudes are at the denominators of the fractions shown on the figure and nominal values are at the

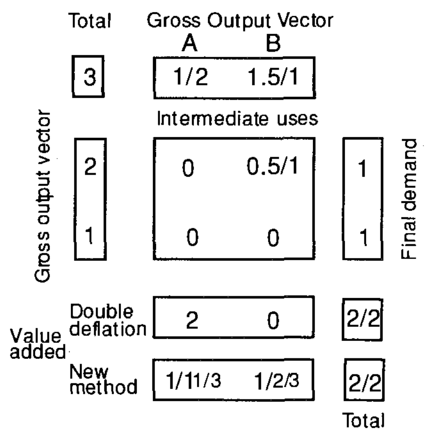

Figure 1

numerators so that the fractions give the deflators. The value of output of industry A is 2 and the one of industry B is 1. Industry A does not make use of commodity B while industry B uses one unit of commodity A as indicated in the intermediate uses matrix.

Real value-added by industry is given, according to the double deflation rule, by subtracting real intermediate inputs from real gross output. It has a value of 2 in industry A and a value of 0 in industry B. Similarly, real final demand deliveries can be computed by commodity by subtracting the quantity of each commodity used as intermediate inputs from the quantity produced which is reproduced for convenience in the vertical vector at the left of the figure. Final deliveries are 1 for commodity A and 1 for commodity B. Total real value-added is obtained either by summing value-added over industries or by summing over the commodities delivered to final demand. It is equal to 2.

Nominal value-added in both industry A and B is 1 dollar. With the alternative method, industry A is using half of a dollar of its value-added to contribute to the final delivery of commodity A. We inflate this component by the same factor that yields the real price of commodity A (two) and thus obtain one dollar of real value-added. It is also using half of a dollar of value-added to contribute to the final deliveries of commodity B. The inflation factor to obtain the real price is now $\frac{2}{3}$, yielding \$0.33 of real value-added. The total real value-added of industry A is given by the sum of its real contributions to final demand deliveries which amounts to $1\frac{1}{3}$ dollar. Industry B is contributing 1 dollar on commodity B at a price of 1.5 or $\frac{2}{3}$ of a dollar in constant prices.

Total real value-added, therefore, remains unchanged at 2 dollars. This is a property of the alternative method which it shares with the double deflation method as will be shown more formally later. This property is necessary to insure that the total real value-added of industries be equal to the total real value of commodities delivered to final demand. Hence the difference between the double deflation and the alternative method relies in the distribution of total real value-added between industries. The double deflation technique distributes total real value-added to industries according to their base year relative prices while the new method distributes real value-added according to nominal value-added and the prevailing relative prices. As relative prices shift from the base year in favor of commodity B, more real (and nominal) value-added is attributed to industry B relatively to industry A.

3. Mathematical Formulation

Let $\mathbf{V}$ be the real output matrix of industries (rows) by commodities (columns), $\mathbf{U}$, the matrix of intermediate uses by commodities (rows) and industries (columns). Let $\mathbf{p}$ be the price vector of the commodities (we assume, without loss of generality, a unique price for each commodity in all uses).[3] Then we have the following definitions:[4]

$$\mathbf{g} = \mathbf{V}\mathbf{p} \tag{1}$$

$$\hat{\mathbf{p}}\mathbf{v} = \hat{\mathbf{p}}\mathbf{V}^T\mathbf{i} \tag{2}$$

$$\hat{\mathbf{p}}\mathbf{u} = \hat{\mathbf{p}}\mathbf{U}\mathbf{i}. \tag{3}$$

The vector $\mathbf{g}$ is the vector of gross output by industry in current prices. By definition, the vector of real final demand deliveries of the business sector, $\mathbf{e}$, is identical to the vector of gross output by commodity, $\mathbf{v}$, minus, the vector of

[3]Note that as usual in the input–output literature, the dimensions of $\mathbf{V}$ and $\mathbf{U}$ are transposed.

[4]We use the standard Canadian input–output framework throughout. Final demand includes, therefore, gross fixed capital formation. Using alternative accounting frameworks with final demand including only net investment or excluding completely investment would not change the substance of the article.

intermediate inputs by commodity, $\mathbf{u}$.[5] Premultiplying by commodity prices, this gives:

(4) $$\hat{\mathbf{p}}\mathbf{e} = \hat{\mathbf{p}}\mathbf{v} - \hat{\mathbf{p}}\mathbf{u}.$$

If we define the technical parameter matrix of intermediate input consumption per unit of industry output, $\mathbf{B}$, by

(5) $$\mathbf{B} = \hat{\mathbf{p}}\mathbf{U}\hat{\mathbf{g}}^{-1}$$

and finally, the industry market share matrix $\mathbf{D}$ by

(6) $$\mathbf{D} = \mathbf{V}\hat{\mathbf{p}}(\hat{\mathbf{p}}\hat{\mathbf{v}})^{-1} = \mathbf{V}\hat{\mathbf{v}}^{-1}$$

then $\mathbf{g}$ is given by the usual "impact" equation as follows. From the identity (4), one has:

(7) $$\mathbf{D}\hat{\mathbf{p}}\mathbf{e} = \mathbf{D}\hat{\mathbf{p}}\mathbf{v} - \mathbf{D}\hat{\mathbf{p}}\mathbf{u}$$

which gives

(8) $$\mathbf{D}\hat{\mathbf{p}}\mathbf{e} = \mathbf{g} - \mathbf{D}\mathbf{B}\mathbf{g}.$$

The latter equation solves for $\mathbf{g}$ as:

(9) $$\mathbf{g} = [\mathbf{I} - \mathbf{D}\mathbf{B}]^{-1}\mathbf{D}\hat{\mathbf{p}}\mathbf{e}.$$

From equation (7) and as may also be seen from Figure 1 above, gross output by industry is, by definition, equal to intermediate input and final demand requirements. Since intermediate requirements are themselves proportional to output, gross output can be expressed only in terms of final demand requirements as in equation (9).

It is assumed for simplicity and without loss of generality that the economy is closed and that all commodity supply comes from the business sector of the economy, i.e. that there is no leakages associated with imports, government supply of goods and services, inventory depletion, etc.[6] The nominal value-added of industries, $\mathbf{y}$, are given by their gross output minus their use of intermediate inputs, $(\mathbf{V}\mathbf{p} - \mathbf{U}^{\mathbf{T}}\mathbf{p})$. They are thus fractions, say λ, of their nominal gross output. Therefore, by construction, value-added may be expressed as:

(10) $$\mathbf{y} = \hat{\lambda}\mathbf{g}$$

By virtue of (9), this may also be written as:

(11) $$\mathbf{y} = \hat{\lambda}[\mathbf{I} - \mathbf{D}\mathbf{B}]^{-1}\mathbf{D}\hat{\mathbf{p}}\mathbf{e}.$$

The vector $\mathbf{y}$ gives value-added by industry in current prices. To get value-added decomposed also by commodity, it suffices to replace the vector $\mathbf{e}$ by its diagonal in equation (11). This amounts to applying the impact matrix to each commodity separately. The matrix of value-added $\mathbf{Y}$ by industries (rows) and

[5] Bold faces types are used throughout for vectors and matrices. The "hat" symbol is used to indicate a diagonal matrix formed from a vector.

[6] The results extended to the open economy case can be obtained from the author on request.

commodities (columns) is given by

$$(12) \qquad \mathbf{Y} = \hat{\lambda}[\mathbf{I} - \mathbf{DB}]^{-1}\mathbf{D}\hat{\mathbf{p}}\hat{\mathbf{e}}.$$

Thus, considering all industries and all commodities together, one can see that the value-added generated in the economy appears in the form of a two dimensional array showing value-added by industry and by commodity. Each industry contributes to the value-added of many commodities and conversely each commodity is receiving value-added from many industries. The array could be depicted as on Figure 2 below.

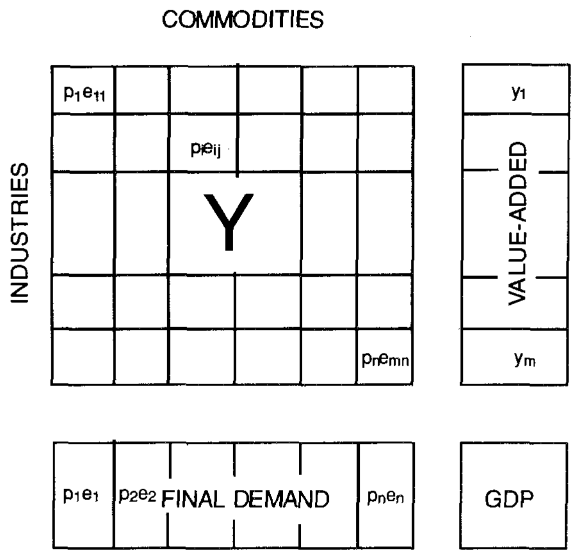

Figure 2. The Value-Added Matrix

The **n** commodities form the **n** columns of the matrix and the **m** industries form the **m** rows. Any given row contains the direct and indirect contributions (value-added) of an industry to all final demand commodities. The sum of an industry's contributions over all commodities, therefore, gives its total value-added. Similarly, any given column gives the direct and indirect contributions of all industries to the value of a commodity. The sum over all industries of their contributions to the value-added of a given commodity gives the total value of that commodity delivered to final demand.

Each cell of the value-added matrix **Y** represents the contribution of a specific industry to the value-added of a given final demand commodity. It then seems only natural to estimate the contribution of an industry to the real value-added of a commodity by deflating its nominal contribution by that particular commodity price. The nominal contributions of all other industries to the real value-added of that commodity may similarly be deflated by the same commodity price. In other words, the proposed deflation method consists in deflating each column of the **Y** matrix by the corresponding commodity price.

Thus, the matrix **Y** is such that summing over its rows gives the value of the final demand vector $\hat{\mathbf{p}}\mathbf{e}$ and summing over its columns, gives the value-added by industry vector, $\mathbf{y}$:

(13) $$\mathbf{i}^T\mathbf{Y} = \mathbf{p}^T\hat{\mathbf{e}}$$

(14) $$\mathbf{Yi} = \mathbf{y}$$

where $\mathbf{i}$ is used as a summation (unit) vector of appropriate size. Property (13) follows from the fact that the coefficients of the impact matrix yielding primary inputs add up to one. Indeed:

(15) $$i^T\hat{\lambda} + i^T B = i^T(\hat{\lambda} + DB) = i^T$$

since input shares sum to one and since, in the second equality, market shares sum to one. The result follows by solving the latter equation for λ and substituting in the impact matrix. Property (14) follows by construction from (11). The constant prices matrix $\mathbf{Y}^k$ is obtained by post multiplying both sides of equation (12) by the inverse of the commodity prices $\hat{\mathbf{p}}^{-1}$:

(16) $$\mathbf{Y}^K = \mathbf{Y}\hat{\mathbf{p}}^{-1}$$

(17) $$\mathbf{Y}^k = \hat{\lambda}[\mathbf{I} - \mathbf{DB}]^{-1}\mathbf{D}\hat{\mathbf{e}}.$$

The Laspeyres aggregate of real value-added by industry, $\mathbf{y}^k$, is obtained from (16) or (17) by summing over commodities, i.e. from:

(18) $$\mathbf{y}^k = \mathbf{Y}^k\mathbf{i}$$

(19) $$\mathbf{y}^k = \hat{\lambda}[\mathbf{I} - \mathbf{DB}]^{-1}\mathbf{De}.$$

In equation (19), the impact matrix is in current prices. It would be equal to the constant price impact matrix if the matrices $\mathbf{D}$, $\mathbf{B}$ and $\hat{\lambda}$ were identical to their constant prices counterparts. In such a case, equation (19) would yield, by the very definition of λ, industries' constant price estimates of value-added identical to those obtained from the usual application of the double deflation method. Hence the double deflation method can also be viewed as a method of distributing the same final demand real output by industry.

The market share matrix, $\mathbf{D}$, in our simplified framework, in which commodity prices are identical in all uses, is identical to its constant price counterpart. The matrices $\mathbf{B}$ and λ would be identical to their constant prices if, and only if, all relative prices of inputs to outputs in any year $\mathbf{t}$ would be the same as those of the arbitrarily chosen base year. However, only in that case would double deflation give results identical to the alternative method proposed here. In addition, only in that case would the separability conditions necessary to validate the double deflation method be satisfied. In all other cases, the distribution of real value-added estimates by industry obtained from the two alternative methods differ even though their total real value-added summed over all industries are equal.

Further inspection of equation (19) leads to an interesting interpretation. This equation applies current prices and current year weights to real final demand expenditure by commodities. This means that each industry's share in real final output is directly associated with its nominal value share of that output (according to the relative prices prevailing in any period). In contrast, with the double deflation method, each industry's share in real final output is given by what it would have been with the relative prices prevailing in the base year. This seems to be a somewhat irrelevant measure for economic analysis as it answers the question "what would have been the real value-added of a given industry had relative prices been the same as in the base year?".

4. Properties of the Alternative Measure

4.1. *Statistical Properties*

The new real value-added measure has some interesting statistical properties which makes it more desirable than its alternative obtained from the double deflation method. In particular, the corresponding price index (using a fixed base year Paasche implicit price approximation, i.e. dividing nominal value-added by fixed base year constant price value-added) is a weighted average of the prices of the commodities to which each industry has contributed directly and indirectly. The value-added implicit price index is therefore well bounded from below by the smallest commodity price index and, bounded from above, by the highest commodity price index.

Sensible bounds cannot so easily be established in the case of the implicit value-added price index derived from the double deflation method. This price index tends to behave erratically and sometimes turns out to be negative (which, obviously, has no economic meaning) as relative input to output prices change through time.

The sensitivity of the implicit value-added price index derived from the double deflation methods tends to be greater, the smaller is the share of nominal value-added in the gross output of industries (see Lal, 1988). National accountants recommend, in such a case, abandoning the double deflation method for some alternative method. Such alternative methods could include using the gross output deflator to deflate value-added or aggregating the problematic industries before deflating. The further away the current year is from the base year, the more likely such situations tend to occur. Problem cases also tend to increase with the disaggregation of real value-added by industry. This is one of the major reasons why national accountants recommend updating the base year periodically.

Turning back to our earlier example might still be more convincing. Indeed, dividing the nominal value added of industries reported on Figure 1 (which are 1 dollar for both industry A and B) by their real value added obtained through double deflation (respectively of 2 and 0 dollars), one obtains an implicit price deflator of 0.5 for industry A and an infinite price deflator for industry B! This extreme result was achieved with plausible values of the commodity price deflators and well illustrates the practical problems in the application of the method.

The alternative value-added price index proposed here, being a weighted average of final demand commodity prices, is, by construction, always positive and it is completely insensitive to the share of nominal industry value-added into gross output. Therefore, a selected fixed base year can be maintained for longer time spans, although it may still be revised for other reasons.[7] Turning back again

[7]Many economists would indeed prefer chained indices of output on the ground that they yield "quantities" and corresponding prices which are actually those economic units have in mind when making buying or selling decisions.

to our numerical example, the alternative price indices for industry A and B are respectively of 0.75 ($1/1\frac{1}{3}$) and 1.5 ($1/(\frac{2}{3})$). This is much more in line with the commodity prices and, as stated above, in between the highest and lowest commodity prices.

A last, but no less important, statistical property of the alternative deflation method is that it requires only final demand commodity prices.[8] Intermediate input prices, particularly prices of service inputs do not need to be known. This property is important in light of the difficulties encountered in deflating many services and, in particular, business services. Indeed, business services and many other services are deflated using input prices with the effect of eliminating the possibility of measuring the productivity gains of these industries.

4.2. *Analytical Properties of the New Real Value-added Measure*

The main focus of this article is to define a meaningful concept and the associated measure of real value-added for industries which could be used for economic analysis and, in particular, for production analysis. We have already seen that real value-added obtained through double deflation cannot meet that objective, in addition to its statistical drawbacks. We have introduced an alternative measure of real-value-added which has much better statistical properties and, at face value, a better interpretation that parallels closely the interpretation given to nominal value-added in national accounting. It may also be interpreted, as we have already seen, as a measure of the real income of primary inputs when the production process is specified on final output with the associated direct and indirect inputs located in all upstream industries. Still we need to go much deeper into the analysis of the concept before drawing conclusions on its usefulness for economic analysis. This is the object of the present section.

As already mentioned, the real value-added of an industry can be interpreted as its direct and indirect contribution to the business sector real output. It is generally recognized that the latter consists of deliveries of commodities to final demand only. Deliveries of commodities to other industries are excluded from the business sector definition of output. These intermediate deliveries are also excluded from the input set of the business sector. The input set comprises only primary inputs, namely inputs of capital and labour which are not supplied by the business sector.

These notions of aggregate outputs and inputs are best understood using the concept of vertical integration as follows. At the industry level, the sales of establishments are usually aggregated to give a measure of output called the gross output of the industry. This gross output comprises the sales (and purchases) of establishments to (from) other establishments of the same industry in addition to sales to and purchases from other industries. If these establishments were all integrated together, their gross output would exclude these intra-industry sales. Similarly, inputs would exclude these purchases. The focus of production analysis would shift from the establishment to the industry level. That is, inputs and

[8] This important property was noticed and communicated to the author by T. Gigantes.

outputs would be computed by accounting only for the inputs coming from outside the industry and the outputs delivered outside the industry.

Aggregating industries together into larger industry groups and shifting the focus of analysis from the single industry to the industry group, that is by vertically integrating industries together, would lead to an additional reduction of intermediate outputs and inputs. Considering complete aggregation and integration of establishments at the business sector level, therefore, leads to the notion of business sector's net *real* value-added output concept with associated primary inputs. The concept of output at the aggregate level is essentially an *integrated* concept of output as if that output was produced by a single economy-wide establishment.

The alternative measure of real value-added proposed in this note simply extends this idea by disaggregating business sector's output by commodity. Each commodity is assumed to be produced by an integrated process which goes across all industries: Industries join their primary resources of labour and capital to produce commodities delivered to final demand. In other words, they have a joint output which they share according to the value of the resources they have spent on their production. This view of the economy corresponds exactly to Passinetti's vertically integrated "sectors." However, Passinetti, 1981, never broke down further final demand value-added by commodity into the industry space as done here to finally reaggregate it back by industry. In other words, Passinetti simply dropped the concept of industry to replace it by the concept of sector which corresponds to the vertically integrated final demand commodity production process.

The productivity gains associated with the output of a commodity can be defined as the primary resources saved through time in all industries involved in its production as shown by Passinetti. Conversely, industries share in these productivity gains according to the value of the primary inputs they have purchased and used to produce that commodity. Therefore, the contribution of a particular industry to the business sector productivity gains can be defined as the primary resources saved by that industry in the production of all final demand deliveries to which it has contributed. This is the contribution of the industry to aggregate productivity growth.

It follows that the contribution of an industry to aggregate output growth can be precisely decomposed into its contribution to primary input growth and its contribution to aggregate productivity growth. Hence, growth in the real value-added of an industry can be decomposed exactly into the growth of its primary inputs and the productivity growth associated to its direct and indirect final demand deliveries. This productivity growth is a weighted sum of the productivity growth associated with the many commodities delivered to final demand. The weights are the value-added shares of the industry into the value-added of the final demand deliveries.

Again, this concept can be formulated in mathematical terms.[9] From the preceding discussion, it appears that in order to define productivity indices on real value-added, we have to start with productivity indices defined on final demand

[9]Other aggregation formulas could have been used in Section 3 above to aggregate real value-added over commodities for each industry from the real value-added matrix $\mathbf{Y}^k$. In this section, where productivity indices are derived, it will appear simpler to work with a Divisia index number formula.

commodities or by Passinetti's vertically integrated sectors of the economy. Each commodity defining a Passinetti sector is produced directly or indirectly by using the primary inputs of the many industries of the economy. These industries and their inputs can be traced back using standard input-output relationships.[10] The equation for the vector of productivity indices, ρ on the final demand commodity vector **e** is given by:

$$\rho = \dot{\mathbf{e}} - (\Omega_L \cdot \dot{\mathbf{L}})\mathbf{i} - (\Omega_K \cdot \dot{\mathbf{K}})\mathbf{i} \tag{20}$$

where $\dot{\mathbf{e}}$ is the vector of the rates of growth of final demand commodities, Ω_L and Ω_K are the matrices respectively of direct and indirect labour and capital input shares by commodity (rows), and type of labour of capital good (columns). The dot over the symbols represents their continuous time rate of growth and the dot product is the element by element (Schurr) matrix product, so that formula (20) represents, by definition, the vector of Divisia indices of productivity growth associated with final demand commodities. Indeed, it equates productivity gains on each commodity to the difference between the rate of growth of that commodity and the weighted rate of growth of the primary inputs used in its production. Values of direct and indirect primary input requirements are obtained by the application of the usual current price impact matrix of the input-output model to the current price diagonal matrix formed with the vector of final demand. Real input requirements are obtained by deflating nominal values by input prices.

It is to be noted here that primary inputs are broken down by commodity and by type. To transpose the final demand productivity results into the industry space, it is necessary to reclassify the primary inputs so as to express them also by industry i.e. to build a three-dimensional array breaking down primary inputs by commodity, by industry and by type. This array can be constructed by first breaking down nominal gross output by industry and by commodity as follows:

$$\mathbf{G} = [\mathbf{I} - \mathbf{DB}]^{-1}\mathbf{D}\hat{\mathbf{p}}\hat{\mathbf{e}}. \tag{21}$$

Each column of **G** gives the nominal gross output vector of industries associated with the production of a given final demand commodity. Applying the primary input requirement coefficient matrices to any column of **G** gives:

$$\hat{\mathbf{w}}\mathbf{L}_i = \mathbf{H}_L \hat{\mathbf{g}}_i \tag{22}$$

$$\hat{\mathbf{r}}\mathbf{K}_i = \mathbf{H}_K \hat{\mathbf{g}}_i. \tag{23}$$

The labour and capital costs in (22) and (23) are now by type of labour and type of capital (rows), by industry (columns), and relate only to commodity **i**.

[10]In Passinetti's dynamic framework, capital goods are assimilated to intermediate inputs so that all commodities are produced using direct and indirect labour only. In the present static framework, capital goods are considered as non-produced inputs and are retained in the primary input set. This does not imply that, in a dynamic framework, we would have to admit the existence of only one primary input as Passinetti does. The dynamic framework may still include what we called elsewhere, following Rymes, the stock of waiting, which is the stock of capital measured in input units (i.e. whose magnitude is independent of technical progress as is the case for a produced input). A two primary, that is non-produced, inputs theory of economic growth still has to be constructed given that (1) the neoclassical theory is founded on labour as the sole non-produced input since capital, the second input of the theory, is a produced input and (2) the Cambridge school admits only the existence of the single inputs of labour.

Repeating the calculations over all commodities gives the elements required to build the three-dimensional arrays mentioned above. Real input requirements can be estimated by deflating the nominal input flows by the input prices.

This breakdown of inputs corresponds exactly to the cells of the real value-added matrix $\mathbf{Y}^k$. Indeed, in value terms, the coefficients of the primary input requirement coefficient matrices add up exactly to the value-added coefficients λ and the impact matrices used to compute value-added and primary input costs are otherwise identical. To each cell of the real value-added matrix corresponds capital and labour inputs by type pertaining to a given commodity-industry breakdown of net output. It is, therefore, possible to define a Divisia index of productivity growth for each cell of $\mathbf{Y}^k$. Letting γ_{ij} denote the Divisia productivity growth rate associated with each industry-commodity real value-added and Γ, the corresponding matrix of Divisia indices of productivity growth associated with $\mathbf{Y}^k$, then industries' productivity indices (denoted by the vector $\boldsymbol{\mu}$) are by definition given by weighting the row elements of Γ by the value shares of each commodity in the value-added of each industry:

$$\boldsymbol{\mu} = (\hat{\mathbf{y}}^{-1}\mathbf{Y} \cdot \Gamma)\mathbf{i}. \tag{24}$$

That is, (24) follows from the consistency in aggregation property of Divisia indices, which implies that Divisia indices of Divisia indices are themselves Divisia indices.[11] Similarly, the vector of productivity growth by commodity can be derived by adding the columns elements of Γ weighted by their value share in the total value of each commodity:

$$\rho = (\mathbf{Y}\hat{\mathbf{p}}^{-1}\hat{\mathbf{e}}^{-1} \cdot \Gamma)^{\mathrm{T}}\mathbf{i}. \tag{25}$$

By definition, the Divisia aggregate over industries of the productivity growth rates, γ, is, again from the consistency in aggregation property, given by the weighted sum of industries' productivity growth rates:

$$\begin{aligned} \gamma &= \frac{\mathbf{y}^{\mathrm{T}}}{\mathbf{y}^{\mathrm{T}}\mathbf{i}}\mu \\ &= \frac{\mathbf{y}^{\mathrm{T}}}{\mathbf{y}^{\mathrm{T}}\mathbf{i}}\hat{\mathbf{y}}^{-1}(\mathbf{Y} \cdot \Gamma)\mathbf{i} \\ &= \frac{\mathbf{i}^{\mathrm{T}}}{\mathbf{y}^{\mathrm{T}}\mathbf{i}}(\mathbf{Y} \cdot \Gamma)\mathbf{i}. \end{aligned} \tag{26}$$

This is, of course, looking at the third equality in (26), also given by the weighted average of the productivity growth rates of each cell of Γ. Aggregating over commodities yields the same result:

$$\gamma = \frac{\mathbf{i}^{\mathrm{T}}}{\mathbf{p}^{\mathrm{T}}\mathbf{e}}(\mathbf{Y}^{\mathrm{T}} \cdot \Gamma^{\mathrm{T}})\mathbf{i}. \tag{27}$$

[11]This property is also shared by the more usual Laspeyres index which is computed by adding constant price values. Adding two Laspeyres indices, that is taking the Laspeyres' index of two Laspeyres indices, gives an aggregate Laspeyres index.

The aggregate business sector productivity growth rate in (26) is the same as in (27). It is equal to the aggregate business sector productivity gain obtained by aggregating productivity gains associated with final demand output by commodity. Therefore, productivity growth on value-added by industry adds up, as required, to the aggregate business sector productivity gain with a set of weights which fulfil the condition of the Divisia aggregation in continuous time. Real value-added growth by industry can be equated to primary input growth plus a productivity residual. It can be shown that this aggregate productivity gain is identical to the one which results from aggregating productivity gains over industries that have underlying production functions defined on gross outputs and intermediate and primary inputs.

The alternative fixed base year double deflation value-added measure does not share the same properties. Value-added could still be distributed by industry and by commodity on the basis of the constant price impact matrix but the constant price cells of, say the matrix $\mathbf{Z}^k$ corresponding to $\mathbf{Y}^k$, would not have prices corresponding to the commodity prices from which the same Divisia aggregate could be built. Consequently, the Divisia indices of productivity which could be built by using the nominal value shares of $\mathbf{Y}$ (or the implicit prices obtained by dividing each cell of $\mathbf{Y}$ by the corresponding cell of $\mathbf{Z}^k$), would not aggregate to the total factor productivity of the business sector. In other words, these prices and quantities form an inconsistent set for production analysis.

More precisely this can be seen as follows. First, it can be noted that if $\mathbf{X}$ is a matrix of real industry by commodity flows to which is associated a commodity price vector $\mathbf{p}$ and $\mathbf{x}$ is the sum of $\mathbf{X}$ over its rows or industries (an aggregate commodity vector), then, the Divisia index of the matrix $\mathbf{X}$ is equal to the Divisia index of the vector $\mathbf{x}$. The proof of that proposition follows from inspection. Hence the Divisia index of $\mathbf{Y}^k$ is equal to the Divisia index of the commodity victory $\mathbf{e}$ which follows from summing over the rows of $\mathbf{Y}^k$. Since inputs are the same for both $\mathbf{Y}^k$ and $\mathbf{e}$ with same prices, this shows more formally that the aggregate productivity index associated with $\mathbf{Y}^k$ is the same as the one associated with $\mathbf{e}$. However, even though the rows of $\mathbf{Z}^k$ also add up to $\mathbf{e}$, the prices associated with $\mathbf{Z}^k$ vary from row to row and differ from the prices associated with $\mathbf{e}$. The corresponding Divisia indices of output, therefore, differ and, consequently, so do their aggregate productivity index.

5. Conclusion

This article has introduced an alternative method to compute the real value-added of industries. This alternative method was shown to give measures of industries' real value-added and their associated implicit price deflators which have better statistical properties than those obtained by the double deflation method. In particular, the implicit price deflators of value-added which result from the application of this alternative method are not likely to behave erratically as those derived from the double deflation method.

In addition, the method could be used to derive indirectly price indices for service industries which have always presented serious problems in the past. The above results could be extended to derive alternative deflators for the gross output of these industries if necessary. This is particularly interesting in the case of business services such as accounting and management services and the growing computer services industry.

No obvious physical measures of output exist for these types of services. National accountants often use last resort prices such as wage rates and other input prices to deflate their output. These prices clearly yield unsatisfactory measures of real output for productivity analysis. Indeed, deflating gross output by some average of input prices has the effect of eliminating productivity gains from the industry. This does not affect aggregate productivity growth which depends on real final demand but it reallocates the productivity gains from the service industries to the good industries. Part of the popular belief that productivity gains are larger for good industries than service industries depend on the measurement biases.

More importantly, however, the article has attempted to bring to the fore a new analytical concept of real value-added that could prove useful for economic analysis. As is well known from the modern literature on production economics, the measure of real value-added obtained from the traditional double deflation method has to be abandoned because it rests on assumptions that are too restrictive. The alternative concept presented here does not rest on these same assumptions. It possesses an interesting interpretation that we derived from the framework of vertically integrated production processes. That interpretation is that the real value-added on industries corresponds to their direct and indirect contributions to the real output of the business sector of the economy.

The measure which was associated with this concept was shown to possess interesting analytical properties. In particular, it was shown that it was possible to derive an index of productivity growth by industry such that: (1) industries' real value-added growth was equal to their primary input growth plus their productivity growth residual, (2) the value-added productivity residuals add up consistently to aggregate business sector productivity growth with weights given by the nominal value-added shares of industries. These weights, it is to be noted, sum to one.

The analytical derivations could have been pushed further to show that the productivity growth residuals of industries on value-added are equal to their productivity growth residuals associated with their gross output inflated by integration factors. These factors are industries' nominal gross output divided by their nominal value added. Combining these integration factors with the aggregation weights, that is the value-added shares, gives Domar's, (see Domar, 1961) aggregation weights for productivity indices defined on gross output. The interpretation of Domar's rule, therefore, becomes quite clear: To aggregate traditional industry multifactor productivity gains to the business sector, one has first to integrate the results, that is to express industries' productivity gains in terms of their use of the economy's primary inputs. The latter gains are thereafter aggregated with the nominal value-added shares of industries which, contrary to Domar's weights, sum to one.

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