Review of Income and Wealth
Series 39, Number 1, March 1993

THE DISAGGREGATION OF THE GINI COEFFICIENT BY FACTOR COMPONENTS AND ITS APPLICATIONS TO AUSTRALIA

BY NRIPESH PODDER

The University of New South Wales

This paper demonstrates the proper use of the disaggregation of the Gini coefficient by factor components by deriving a formula of the elasticity of the Gini coefficient with respect to specific income components. The method is then applied to Australian household expenditure survey data to find the effects of various components of income on overall income inequality. The results are important in examining the effects of growth in specific components on the overall inequality and hence in policy decisions with respect to redistribution of income.

1. INTRODUCTION

The total income received by a person or a household can be divided into a number of components depending on the sources of income. Alternative disaggregations of total income can be made depending on the purpose of the analysis. One such disaggregation is the distinction between earned and unearned income. Irrespective of the way total income is disaggregated, one should be able to determine the exact contribution of each of the components to total income inequality. Due to its overwhelming popularity, the Gini coefficient is often used to represent the degree of inequality in the society. While some scholars maintain that the exact contribution of each component of income can be determined by using the method of decomposition of the Gini coefficient by factor components, the present paper demonstrates that incorrect interpretation of the method has lead to widespread misuse of concentration ratios. However, it is shown that with proper interpretation, the method is useful in answering important questions regarding the effects of various income components on total inequality. Thus, at times when there is a high rate of growth or inflation in the economy some components such as wages and salaries may grow relative to other components. In that case we are able to determine the change in overall inequality of income. In addition, it is possible to compute the elasticity of the Gini coefficient with respect to specific components of income. These elasticities are immensely useful in aiding policy discussions about the level of inequality in the society. The importance of the problem can be gauged by the fact that a large part of personal income comes as a cash benefit received from the government. Moreover, the government can indirectly influence the other sources of income using appropriate fiscal or monetary instruments.

Note: The author wishes to acknowledge his debt to Professor J. W. Nevile for his support and encouragement at all stages of this research. Comments from an anonymous referee were very helpful in revising the paper. The usual disclaimer applies.

The plan of the paper is as follows. In the next section we develop the method of decomposition of the Gini coefficient, derive some useful results and discuss the appropriate interpretation of the results. Also, in the same section we show how the method has been widely misused. In the third section, we present empirical results by applying the method to Australian Household Expenditure Survey of 1988–89 and discuss the implications of the results. Finally some concluding remarks are made in the fourth section.

2. The Method

Suppose X represents the total income of an income unit (person or household). If X is the sum of K components represented by $X^1, X^2, \ldots, X^K$, then

$$X = \sum_{k=1}^{K} X^k.$$

When a specific factor income is arranged in ascending order of total income and the proportions of the factor income are plotted against the proportions income units, we get the concentration curve. One minus twice the area under the concentration curve is the concentration index. Unlike the Lorenz curve, the concentration curve may lie above the 45° diagonal and in that case the concentration index will be negative.

Let G represent the Gini coefficient of total income and C_k represent the concentration ratio of the k-th component of income. Let $E(X) = \mu$ and $E(X^k) = \mu_k$. Then it is easy to show that

$$G = \sum_{k=1}^{K} \frac{\mu_k}{\mu} C_k. \tag{1}$$

This result was originally derived by Rao (1967). It is then very tempting to use $(\mu_k/\mu)C_k$ as the part of total inequality due to the k-th component of income and that is exactly what most authors have done. In fact a slight variant of this,

$$\frac{(\mu_k/\mu)C^k \cdot 100}{G}$$

has been used as the percentage share of the k-th component of income in total inequality. There are numerous references of such usage both in the theoretical and empirical literature. Kakwani (1980 and 1986), for example, advocated this usage both theoretically and in his empirical research with Australian data. That this approach is faulty and leads to a misleading result is clearly demonstrated by Podder and Tran-Nam (1991). However, it is essential that we make this point abundantly clear. Following Shorrocks (1989), we may think of four possible interpretations of the contribution of the k-th factor. These are:

1. The percentage of inequality due to source k income alone.

2. The reduction in inequality that would result if this source of income was eliminated.

3. The percentage of inequality that would be observed if this was the only source of income differences and all other incomes were allocated evenly.

4. The reduction in inequality that would follow from eliminating differences in source k incomes.

Examining the meaning of the concentration ratio closely one would see that none of the above interpretations is valid for the concentration ratio of a component. If the k-th component of income is a constant for all incomes, its concentration ratio will be zero, leading us to conclude that the component does not make any contribution to total inequality. However, we know that an addition of a constant to all incomes decreases total inequality. Thus, Podder and Tran-Nam showed that $C_k s$ as such are not amenable to any sensible interpretation and it is not possible to determine the exact contribution of any component unambiguously. The only way equation (1) can be interpreted is by transforming the equation as

$$\sum_{k=1}^{K} \frac{\mu_k}{\mu}(C_k - G) = 0. \tag{2}$$

The sign of the quantity $C_k - G$ tells us if the k-th component has a negative or positive effect on total inequality. In other words, the sign indicates if the presence of the k-th component increases or decreases total inequality. One can intuitively understand the situation in this way. If the k-th component is proportional to total income, the component does not have any effect on total inequality. On the other hand when the component rises more than proportionately with total income then the concentration index of the coefficient will be higher than the Gini coefficient of total income and therefore $C_k - G$ will be positive and consequently, the k-th component increases total inequality. Similarly, for a negative effect the k-th component has to rise less than proportionately with total income. Another interpretation of the quantity, $C_k - G$ is that it can be shown to be the weighted sum of the deviation of elasticity (of the k-th component of income with respect to total income) from unity. In the present context the more important result is the following theorem conceived by Larman and Yitzhaki (1985). A simple proof is given below.

Theorem 1. Suppose μ_k changes in such a way that its concentration curve remains undisturbed. Then the elasticity of the Gini coefficient with respect to the k-th component of income is given by

$$\eta_k = \frac{1}{G}\left[\frac{\mu_k}{\mu}(C_k - G)\right]. \tag{3}$$

Proof. Suppose in equation (1) only μ_k changes in such a way that its concentration ratio remains undisturbed. Then the total derivative of G with respect to the mean of the k-th component of income will be

$$\frac{dG}{d\mu_k} = \frac{\partial G}{\partial \mu_k} + \frac{\partial G}{\partial \mu} \cdot \frac{d\mu}{d\mu_k}. \tag{4}$$

The derivatives are obtained as

$$\frac{\partial G}{\partial \mu_k} = \frac{1}{\mu} C_k, \qquad \frac{\partial G}{\partial \mu} = -\sum \frac{\mu_k}{\mu^2} C_k = -\frac{1}{\mu} G$$

and $d\mu/d\mu_k = 1$ because of $\mu = \sum \mu_k$.

Substituting these results in (4) we have

$$\frac{dG}{d\mu_k}=\frac{1}{\mu}(C_k-G) \tag{5}$$

and thus,

$$\eta_k=\frac{\mu_k}{G}\frac{dG}{d\mu_k}=\frac{1}{G}\left[\frac{\mu_k}{\mu}(C_k-G)\right].$$

This theorem tells us that we can compute the change in the Gini coefficient due to a proportionate change in the mean income of the k-th component as

$$\mu_k\frac{dG}{d\mu_k}=\frac{\mu_k}{\mu}(C_k-G). \tag{6}$$

It should be clear that the sum of the elasticities will always be equal to zero implying the equation

$$\sum\eta_k=\frac{1}{G}\sum\frac{\mu_k}{\mu}(C_k-G)=0. \tag{7}$$

This equation says that if there is a proportionate change in income from all sources the Gini coefficient will remain unchanged.

These results are immensely important from the point of view of redistributive policies. We are now in a position to compute the redistributive effects of any component of income. Also, the elasticities give us a clear picture of the relative importance of different income components with respect to total inequality of the society as a whole.

Before we apply the decomposition to Australian data let us consider another important problem. The impact of the government sector on the redistribution of income is analyzed with respect to various types of cash benefits received by individuals or families. There have been frequent attempts to allocate non-cash benefits to individuals such as education, medicare and subsidized housing. Whereas some non-cash benefits go to individuals in differing amounts, others go to individuals by the same amount. For example, government expenditure on defence or police force provides protection to all individuals equally. In the case of universal medicare, although the service is received by the sick people only, there is a strong argument that government expenditure in this regard should be treated as a social insurance that provides equal health protection to every individual. Treatment of such cases will lead us to allocate equal non-cash income to every individual. We know that the concentration index of any component of income that is constant will be zero. However, we also know that an equal addition to all incomes will lead to a diminution of overall inequality. Now if each unit receives an extra dollar the mean income of the community will increase by a dollar but all the concentration indices will remain unchanged. What then will be its effect on the overall Gini coefficient. This is given by the derivative of the Gini coefficient with respect to mean income as

$$\frac{\partial G}{\partial\mu}=-\sum\frac{\mu_k}{\mu^2}C_k=-\frac{G}{\mu}. \tag{8}$$

Thus if $\Delta\mu$ is the total change in the mean income, the corresponding change in the Gini coefficient will be given by the following lemma.

Lemma 1. If an equal addition of $\Delta\mu$ is made to every income then the change in the Gini coefficient will be

$$\Delta G = -\frac{G}{\mu}\Delta\mu \tag{9}$$

This equation provides us with a tool to analyse the impact of a change in any component of income that is constant for *all* individuals.

3. Applications to Australian Data

In this section we analyse the redistributive effects of various components of income on total inequality in Australia using the method described in the previous section. Our main interest lies in the redistributive effects of various government cash transfers and how the totality of government cash benefits change the inequality of the overall distribution.

The data used in this study are obtained from *Household Expenditure Survey 1988–89,* the latest of the series conducted by the Australian Bureau of Statistics. Data were made available to us as a unit record file consisting of 7,225 records. The basic unit is a household which may be a smaller or larger unit than a family. It becomes a smaller unit when we consider a single member household and a larger unit when we consider a household consisting of multiple families. However, households of the latter type are insignificant in number. The total household income is the sum of incomes received from all sources before taxes are paid. For further details of the survey the interested reader is referred to the Australian Bureau of Statistics (1989). All incomes are weekly incomes in terms of the Australian dollar. Assuming that each member of a household should be given equal weight from the point of view of economic welfare, the analysis is done in terms of income per person of a household. This means that in computing the Gini or concentration coefficients all households are arranged in ascending order of their income per person and then the households are given weights equalling to their respective sizes. It should be kept in mind that in this paper we are looking into the decomposition of gross income inequality.

Our analysis starts with Table 1 which presents an overall picture of the components of income for various deciles of households when the households are arranged in ascending order of their income per person. At this stage we consider only three broad categories of income namely, earned income, unearned income and total government cash benefits. In the table earned income consists of income from wages, salaries and own business while unearned income consists of incomes from all other sources except government. Unearned income is mainly income generated from assets. The main components of government cash benefits are various types of pensions and unemployment benefits. The age pension by far constitutes the major part of government benefits. Later in this section we will disaggregate government cash benefit further. It is quite natural that both earned and unearned income rise across the rising deciles. However, some government cash benefits should also flow to the upper deciles. Later we will investigate and comment on a more detailed breakdown of total government benefits. It is seen that the average household size of the bottom decile is the largest while that of the top decile is the lowest. The table gives us a general idea of the importance of various components of income in the income packages of different deciles. Thus we see that government cash benefits constitute a highly significant part of total income of the two lowest deciles. The existence of some unearned income in the lower deciles indicates that there are some retirees with some investment income in those deciles. The high average household size of the bottom decile clearly indicates that the poorest households consist of families with dependent children, not families mainly consisting of retired people.

TABLE 1

INCOME AND ITS COMPONENTS BY DECILES

Deciles	Av. hh. Income	Av. Size Persons	Income per Person	Av. Earned Income	Unearned Income	Av. Govt. Benefit
Lowest	214.37	3.60	59.55	75.01	16.00	123.37
Second	311.70	2.96	105.30	161.21	16.17	134.33
Third	278.49	2.23	124.88	140.36	18.21	119.92
Fourth	416.46	2.83	147.16	290.58	35.99	89.88
Fifth	589.89	3.32	177.68	486.22	43.50	60.16
Sixth	680.36	3.16	215.30	583.59	49.52	47.25
Seventh	817.55	3.07	266.30	726.98	58.47	32.10
Eighth	880.30	2.26	389.51	798.98	64.00	17.32
Ninth	938.77	2.16	434.62	863.25	68.12	7.40
Top	1,270.28	1.85	686.64	1,116.78	146.88	6.62
Total	639.82	2.78	230.15	524.30	51.59	63.84

The next table presents the quintile shares in different income components as well as the population shares of the quintiles when the households are arranged in ascending order of their income per person. The last column of the table presents government cash benefits as a percentage of total household income. It is seen that the first and the third quintiles have a population share of more than 23 percent while the top quintile has a remarkably low share of 14.44 percent. We should note that the percentages of both earned income and unearned income rise more sharply across the quintiles than the percentage of total income. This phenomenon is easily explained by the highly favourable redistributive effects of government cash benefits. It is clear that the distribution of income would have been significantly more unequal in the absence of government benefits. As yet it is hard to judge the relative effects of earned and unearned income but it is abundantly clear that government benefits go a long way towards raising the income of the poorer sections of the community.

Table 3 presents the most important results of our analysis. In computing the relative contributions of different components of income to total inequality the basic ingredients needed are the numerical value of the Gini coefficient of total income, the share of each component of income, and the concentration

TABLE 2

QUINTILE SHARES OF INCOME COMPONENTS

Quintiles	Pop. Share	Total Inc. Share	Earned Inc. Share	Unearned Inc. Share	Benefit Share	Benefit/ Income
Bottom 20%	23.59	8.22	4.50	6.22	40.37	48.98
Second 20%	19.21	10.86	8.22	10.49	32.87	30.19
Third 20%	23.30	19.85	20.40	18.00	16.82	8.46
Fourth 20%	19.47	26.54	29.10	23.69	7.74	2.91
Fifth 20%	14.44	34.53	37.77	41.60	2.20	0.63
Total	100.00	100.00	100.00	100.00	100.00	

ratio of each component of income. These, in addition to the derivatives of the Gini coefficient and the elasticities are presented in Table 3. Thus we see that earned income constitutes almost 82 percent of total income while unearned income constitutes only 8 percent. On the other hand, total government income constitutes 10 percent of total income. This proportion has increased from 7 percent in the seventies due mainly to indexation of pensions and the increased number of unemployed. Later we shall examine the detailed breakdown of government benefits. The Gini coefficient of total income is estimated to be 0.36 for Australia. It should be kept in mind that this value of the Gini coefficient is estimated for income per person before income tax is paid. The values of the concentration ratio for both earned and unearned income seem to be very similar. From this and the fact that unearned income is only 8 percent of total income one can easily guess that the absence of unearned income is unlikely to make a great deal of difference to total inequality. We get a better idea by looking at the derivative of the Gini coefficient with respect to a change in any of the components. Thus, we find that a 10 percent change in earned income will increase the Gini coefficient by 0.007 whereas an increase of a 10 percent change in unearned income will increase the Gini by 0.0007. Therefore, we may conclude that unearned income has a completely negligible effect on total inequality. On the other hand, we find that government benefits have a significant effect in reducing total inequality. An increase of 10 percent in government benefits will lead to a 2 percent reduction in the Gini coefficient, a significant feat given that government benefits are only 10 percent of total income.

TABLE 3

EFFECTS OF INCOME COMPONENTS ON INEQUALITY

Components	Share	C_r	$S_r(C_r - G)$	η
Earned Income	81.94%	0.445	0.0697	0.1935
Unearned Income	8.08%	0.436	0.0067	0.0186
Government Benefits	9.98%	−0.360	−0.0720	−0.2000
Total Income	100.00%	0.36[a]	0	0

[a]This is the Gini coefficient.

Let us now consider the effects of spouse's income on the inequality of total household private income. Private income is defined as income from all sources other than from the governments. Thus, total private income is now considered to be the sum of two components namely spouse's income and all other private income. Table 4 presents the results. In this table we find that the Gini coefficient of total private income is 0.46 and the concentration index of spouse's income is 0.37 and therefore immediately we can conclude that wife's income has an inequality reducing effect on total private income. However, the picture changes considerably when we add government benefits to obtain total household income.

TABLE 4

EFFECTS OF SPOUSE'S INCOME ON PRIVATE INCOME

Components	Share	C_r	$S_r(C_r - G)$	η
Spouse's Income	24.73	0.38	−0.0212	0.0461
Other Income	75.27	0.49	0.0221	−0.0480
Total Income	100	0.46	0	0

The results are presented in Table 5. This table shows that the other income is the main contributor to total household income while the existence of spouse's income has a very insignificant effect on total inequality. A percentage increase in spouses' income increases the Gini coefficient only in the fifth decimal place. Therefore, it can be safely concluded that the total inequality is almost entirely due to other income (though moderated by Government benefits.) An additional computation of the Gini coefficient after dropping spouses' income from total household income gave us virtually the same value for the Gini.

TABLE 5

EFFECTS OF SPOUSE'S INCOME IN THE PRESENCE OF GOVERNMENT BENEFITS

Components	Share	C_r	$S_r(C_r - G)$	η
Spouse's Income	22.26	0.37	0.0022	0.0061
Other Income	67.76	0.47	0.0745	0.2069
Government Benefits	9.98	−0.36	−0.0719	−0.1997
Total Income	100	0.36	0	0

Table 6 presents the detailed breakdown of the income components. Here we see that only a few components have positive effects on total inequality while the majority consisting of various types of government benefits have negative effects. As expected, wage and salary contributes most to total inequality. The second most important component in this regard is the income from own business and self employment. It should be noticed that the concentration ratio for this component is no different from that of wage and salary income. But their effects are different due to the difference of their income shares. It is remarkable that interest income has no effect on total inequality. In other words, the absence of interest income would have left the value of the Gini coefficient unchanged. Similarly, superannuation and annuity income also do not seem to have any noticeable effect on inequality. On the other hand, income from investment compared to its share has a significant positive effect on inequality. Another component worth mentioning for its positive effect on inequality is property rent. It is seen that all types of government benefits reduce total inequality. Of these the old age pension and unemployment benefits by far have the most significant effects. While all types of government benefits have inequality reducing effects, items such as supporting parents benefit have higher impact when we consider their shares. The fact that the concentration ratios are noticeably less than −1 indicate that some benefits percolate to households in the higher income brackets.

TABLE 6

EFFECTS OF COMPONENTS: DETAILED BREAKDOWN

Source	Share	C_r	$S_r(C_r - G)$	η
Wage & salary	73.32	0.44	0.0586	0.1628
Own bus. self empl.	8.63	0.44	0.0090	0.0250
Age pensioh	3.08	−0.35	−0.0219	−0.0608
Invalid pension	0.83	−0.36	−0.0060	−0.0167
Widow pension	0.49	−0.41	−0.0038	−0.0105
Unempl. benefit	1.18	−0.54	−0.0106	−0.0294
Sickness benefit	0.23	−0.47	−0.0019	−0.0053
Vet. aff. pension	1.59	−0.12	−0.0076	−0.0211
Sup. parents benefit	0.77	−0.71	−0.0082	−0.0228
Wife's pension	0.26	−0.43	−0.0021	−0.0058
Family allowance	0.80	−0.33	−0.0055	−0.0153
Govt. study assistance	0.32	−0.29	−0.0021	−0.0058
Other govt. benefit	0.43	−0.17	−0.0023	−0.0064
Interest income	2.92	0.36	0.0000	0
Superann./annuity	1.48	0.37	0.0001	0.0003
Workers compensation	0.27	0.10	−0.0007	−0.0019
Accident compensation	0.00	0.04	0.0000	0
Maint./alimony	0.14	−0.29	−0.0009	−0.0025
Invest. income	1.92	0.68	0.0061	0.0169
Property	0.75	0.64	0.0021	0.0058
Rent other income	0.53	0.26	−0.0001	−0.0003
Child earned income	0.00	0.01	0.0000	0
Child unearn. income	0.01	0.60	0.0000	0
Private scholarship	0.00	0.12	0.0000	0
Total govt. benefit	9.98	−0.36	−0.0719	0.1994
Total income	100	0.36	0	0

The results presented in this section cannot be readily compared with those previously obtained by others. Although a number of works on the redistributive effects of components of income in Australia are available, the methodology used is different. For a comprehensive reference list the reader is referred to Australian Bureau of Statistics (1987) and Economic Planning Advisory Council (1989). Kakwani (1986) is the only one who has worked on similar lines using 1975-76 survey data, although he interpreted the concentration ratios in the way that has been rejected in this paper. For example, Kakwani's methodology would make us believe that the effect of family allowance to total inequality is positive whereas in fact it is negative. However, the derivatives of the Gini coefficients and the elasticities can be easily computed from his results. Although Kakwani used a different concept of income (equivalent income) in his analysis the results are compatible.

A brief comparison of the changes in the relative importance of some factors is presented in Table 7. It may be observed that while the contributions of earned income such as wages, salaries and business income have increased, the effects of various government benefits have acted in the opposite direction to reduce inequality. The level of unemployment in 1989-90 was much higher than in the previous period. As a result its (negative) contribution has increased. Due to the introduction of means tested family allowance its contribution has also changed similarly. Overall, the role of total government benefits in reducing total inequality has become more significant over the years.

TABLE 7

CHANGES IN THE CONTRIBUTIONS OF FACTOR INCOMES 1974-75 AND 1989-90 (SELECTED ITEMS)

	Elasticities 1974-75	1989-90
Wage and salary	0.137	0.163
Business income	0.016	0.025
Unemp. benefit	−0.020	−0.029
Family allowance	−0.003	−0.015
Total Govt. benefit	−0.114	−0.199

4. CONCLUSION

Whereas the main thrust of the paper is the redistributive effects of various components of income in Australia, the theoretical aspects with respect to the proper interpretation and use of concentration ratios of the components are of far reaching significance. It is argued that the disaggregation of the Gini coefficient by factor components have hitherto been mostly misunderstood and therefore misused in empirical research. It has been made clear that a positive value of a concentration ratio does not necessarily mean that the contribution of the relevant component is positive. Also, we have seen that the effect of a component on total inequality is neutral only when its concentration ratio is exactly equal to the Gini coefficient of total income. The derivation of the theoretical results and the interpretation of the results will hopefully keep the empirical researcher in this field on the right track henceforth.

On the other hand, the paper presents the most comprehensive analysis of the redistributive effects of various components of total household income in Australia using the most recent data. The results are likely to be important with respect to redistributive policies.

REFERENCES

Australian Bureau of Statistics, *1988–1989 Household Expenditure Survey, Australia: Information Paper,* Commonwealth of Australia, 1989.

Australian Bureau of Statistics, *The Effect of Government Benefits and Taxes on Household Income,* Catalogue No. 6537.0, Canberra, 1987.

Economic Planning Advisory Council, *Social Wage,* Canberra, 1989.

Kakwani, N., *Income Inequality and Poverty: Methods and Policy Applications,* Oxford University Press, New York, 1980.

Kakwani, N., *Analyzing Redistributive Policies: A Study Using Australian Data,* Cambridge University Press, Cambridge, 1986.

Larman, R. I. and Yitzhaki, S., Income Inequality Effects by Factor Sources: A New Approach and Applications to the United States, *Review of Economics and Statistics,* 67, 1985.

Podder, N. and Tran-Nam, B., Uses and Abuses of the Decomposition of Gini Index by Factor Components, The University of New South Wales, *mimeo.,* 1991.

Rao, V. M., Two Decompositions of Concentration Ratio, *Journal of the Royal Statistical Society,* Series A, 1967.

Shorrocks, A. F., Aggregation Issues in Inequality Measures, in Eichhorn, W. (ed.), *Measurement in Economics,* Physica-Verlag, 1988.