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A COMMENT ON CALCULATING WEALTH FROM INCOME FLOWS

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In a recent article in this Review and previous work, Wolff calculates household wealth embodied in (current and/or future) pension and social security benefits. The valuation in intended to be based on expected discounted income flows. However, the formulae Wolff uses are not the correct ones.

EXPECTED DISCOUNTED FLOW VERSUS DISCOUNTED FLOW EVALUATED AT ITS EXPECTED DURATION

Let $B(s)$ be the benefit from some source at time s. The discounted value of the benefit flow until t equals

$$DV(t) = \int_0^t B(s)\, e^{-\delta s}\, ds. \tag{1}$$

δ is the discount rate (for which Wolff uses the 10-year treasury bill rate). The *expected* discounted value is then given by

$$E\{DV(t)\} = \int_0^\infty DV(t) f(t)\, dt, \tag{2}$$

where $f(\cdot)$ is the conditional probability density function of lifetime (which can be inferred from mortality tables).

Wolff, however, calculates the wealth corresponding to $B(\cdot)$ as the discounted value evaluated at the expected duration of the flow:

$$DV\{E(t)\} = \int_0^{E(t)} B(s)\, e^{-\delta s}\, ds \tag{3}$$

(Wolff, 1990, equations (1), (2), (3) and (5)). Since in general $DV(\cdot)$ is a nonlinear function, we have—by Jensen's inequality—that $E\{DV(t)\} \neq DV\{E(t)\}$. Only in special cases we have equality, for example if the discount rate is zero and the nominal benefits are constant over time.

HOW LARGE IS THE BIAS?

To reveal the exact magnitude of the biases, recalculation of the pension and social security wealth according to (2) is necessary. However, to get some insight

Note: I thank a referee for suggesting some improvements in the presentation.

in the differences one might expect, the following numerical example may be helpful. Suppose that $B(\cdot)$ is constant over time and that $f(t)=\lambda e^{-\lambda t}$, $\lambda>0$. That is, t follows an exponential distribution, so that $E(t)=1/\lambda$. For this example, we have

$$DV\{E(t)\}=\frac{B}{\delta}(1-e^{-\delta/\lambda}) \tag{4}$$

and

$$E\{DV(t)\}=\int_0^\infty \frac{B}{\delta}(1-e^{-\delta t})\lambda e^{-\lambda t}\,dt=\frac{B}{\delta+\lambda} \tag{5}$$

so that

$$\text{Bias}=\frac{DV\{E(t)\}-E\{DV(t)\}}{E\{DV(t)\}}=(1+\lambda/\delta)(1-e^{-\delta/\lambda})-1. \tag{6}$$

Since $DV(\cdot)$ is a concave function here, we have $E\{DV(t)\}<DV\{E(t)\}$. So, in this example, Wolff's method overestimates the wealth embodied in the income flow B. For the "typical" parameter sets in Table 1, the bias appears to vary between 10 and 30 percent.

TABLE 1
MAGNITUDE OF THE BIAS (NUMERICAL EXAMPLE)

	$\lambda=0.20$ $[E(t)=5]$	$\lambda=0.10$ $[E(t)=10]$	$\lambda=0.04$ $[E(t)=25]$
$\delta=0.05$	11%	18%	28%
$\delta=0.10$	18%	26%	29%

REFERENCE

Wolff, E. N., Wealth Holdings and Poverty Status in the U.S., *The Review of Income and Wealth*, pp. 143–165, 36 (2), 1990.