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INEQUALITY MEASUREMENT USING 'NORM INCOMES': WERE GARVY AND PAGLIN ONTO SOMETHING AFTER ALL?

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The "norm income" approach to inequality measurement is based on a comparison of the observed income distribution with a reference distribution consistent with the socially desired minimum degree of inequality (and not the equal shares distribution). Garvy and Paglin suggested such an approach, and we show that their methods, suitably modified, are closely related to the multivariate methods recently proposed by Atkinson and Bourguignon. The advantage and disadvantages of a norm income approach are analyzed in detail.

1. INTRODUCTION

In Pieland there are three families: Arthur's, Brian's and Chris's, and the current income of each, expressed as a share of total income, is as shown in Figure 1(a). Figure 1(b) shows what each would have if the economic pie were split equally between them. Suppose you are now told that Arthur is a single person, and that both Brian and Chris are married and have children. If you were to ask yourself what a *fair* division of the pie would look like, the answer is unlikely to be the "equal shares" pie. It is more probably something like (c), where the pie split takes account of differences in needs. This paper considers whether analysing the difference between a household's share in the actual distribution with its share in the fair-share distribution—generalizing the pie (a)-pie (c)-type comparison—is a useful way of examining the inequity of income distributions.

We build upon the work of Garvy (1952) and Paglin (1975) who also addressed this issue. Garvy discussed the case of comparing each household's income with what he referred to as its "norm income"—that amount each would have consistent with "the socially desirable minimum of inequality" (1952, p. 30), and he cites an earlier writer expressing a similar view: "the degree of departure

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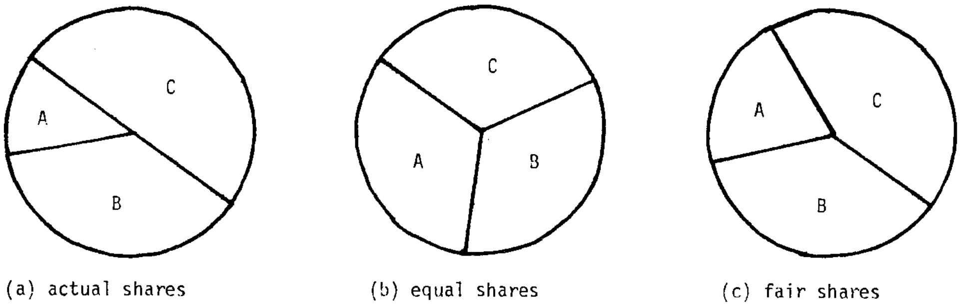

Figure 1. Income Shares in Pieland

from absolute equality, however measured or stated, must itself be referred . . . to a standard of normal or justifiable concentration" (Young, 1917, quoted by Garvy 1952, p. 30). Paglin's more recent arguments for a "basic revision" to inequality measurement apparently make the same case, for he proposed a comparison of the Lorenz curve of observed incomes with, not the usual 45° line, but "a new function generated on the basis of a more careful and explicit definition of perfect equality" (1975, p. 598). However, Paglin's article attracted a record number of adverse comments, and Garvy himself concluded that "all that can be claimed for the reference curve is its usefulness as an expository device, not as a yardstick" (1952, p. 38).

We reconsider the validity of a norm income approach to inequality measurement and show that, if Garvy's and Paglin's methods are suitably modified, there is a close link between a norm income approach and the more recent proposals by Atkinson and Bourguignon (1982, 1987) for inequality measurement using a multivariate perspective.

2. Approaches to Inequality Measurement Compared

The raw materials required for inequality measurement are the joint distribution of income and non-income equity-relevant characteristics amongst households. Three different ways of using this information are summarized in Table 1.

TABLE 1
APPROACHES TO MEASUREMENT

	For a Given Income Unit, i,	
Approach	Actual Income	Reference Income
1. "descriptive"	y_i	$\bar{y}$
2. "equivalent income"	$e_i = y_i / m_i$	$\bar{e}$
3. "norm income"	y_i	z_i

Key: y_i: observed income for income unit, i. m_i: equivalence scale rate applicable to i. e_i: equivalent income for i. z_i: norm income for i. $\bar{y}$ and $\bar{e}$: mean income and mean equivalent income.

First one might concentrate on the income differences *per se* and compare them with the equal-shares distribution, just like comparisons of pies (a) and (b) for Pieland. However, as many writers have stressed, "the mere existence of differences in income and wealth is not, of course, a sufficient basis for statements about justice or injustice" (Atkinson, 1983, p. 4). The problem is that the exercise takes no account of the information provided about differences in other characteristics.

Analysts typically take account of these differences by adjusting the income data prior to analysis: "in order to assess the implications of differences in incomes, we need first to establish that the people involved are comparable in other relevant respects" (Atkinson, 1983, p. 4). Since the observed incomes are not commeasurable, the characteristics are used to convert them into a fully comparable metric. It is then justifiable to compare pie (a) with pie (b).

The most common way of making the adjustments is to use an *equivalence scale*, and inequality is then defined as differences between households in their *equivalent incomes*. Equivalent income for a given household equals observed income divided by an equivalence scale rate which depends on its characteristics:

$$e_i = y_i / m_i. \tag{1}$$

The m_i thus encapsulates values judgements about the well-being derived per pound for each household type, relative to some reference type (for whom m is normalized at unity); e_i is a proxy for utility.[1] The reference distribution denoting minimum inequality is the one where everyone receives mean equivalent income, $\bar{e} = \Sigma_i (y_i / m_i)/n$, where n is the total number of income units.

The norm income approach uses the same raw materials, but in a different way. As when preparing pie (c) for Pieland, the social planner determines *how much each household should have from an equity point of view, consistent with the given amount of aggregate income available* and the answer is the norm income distribution, $\mathbf{z}$. Given differences in equity-relevant characteristics between households, one would expect a variation in z across the population as a whole, but none within a given group with the same characteristics. Garvy's and Paglin's propose inequality measurement be based on differences between observed incomes and norm incomes.

To bring out the relationship between the distribution of equivalence scale rates and the norm income distribution, note that without loss of generality, we can write

$$\mathbf{z} = \theta \mathbf{r} + \lambda. \tag{2}$$

In the special case where the translation factor, λ, equals zero for all, the income relativity scale $\mathbf{r}$ is exactly the same as the equivalence scale $\mathbf{m}$, and θ is simply a scaling factor ensuring the equal means constraint is satisfied (to ensure $\bar{y} = \bar{z}$, $\theta = \bar{y}/\bar{r}$). In this case the Lorenz curve for the distribution of the ratios y/z would

[1]Equivalence scales have been based on econometric analysis of household budget data (the best survey is by Deaton and Muellbauer, 1980); on experts' estimates of minimum consumption needs (see e.g. Orshansky, 1965); and on aggregation of survey respondents' views (as in the Leyden approach, see e.g. Hagenaars, 1987).

be exactly the same as the Lorenz curve for the distribution of equivalent income (although the distributions have different means, the relative measure is unaffected). In this sense the norm and equivalent income approaches are exactly the same (though whether it is more appropriate to work with ratios or differences of actual and reference incomes is an issue we discuss in greater detail in Section 4). Outside the special case where $\lambda = 0$ and $\mathbf{r} = \mathbf{m}$, the close correspondence will not hold.

The norm income approach is certainly plausible, but can it be applied in practice? If not then the approach deserves no further attention, for associated with the standard approach are a set of well-known useful tools—Lorenz curves and related results about the ranking of distributions, and a wide range of summary measures. So to demonstrate the feasibility of the norm income approach we need to show that analogous tools are available. We discuss the two approaches in turn, and summarize the argument in Table 2.

TABLE 2

MEASUREMENT TOOLS: THE EQUIVALENT INCOME AND NORM INCOME APPROACHES COMPARED

Tool	Equivalent Income Approach	Norm Income Approach
Diagram	Lorenz curve for **e**	Lorenz curve for **y**; concentration curve for **z**
Partial ordering check	Compare Lorenz curves for **e**	(i) **z** not fully specified: sequence of comparisons of generalized Lorenz curves for **y**, starting with group with highest z. (ii) **z** fully specified: compare the joint cumulative distributions of **y** and **z**.
Summary indices		
Diagram based	Gini, Schutz coefficients	Relative areas and distances between Lorenz and concentration curves.
Axiomatic "distance"	Univariate Generalized Entropy family	bivariate Generalized Entropy family
"Social welfare"	Atkinson/Dalton family (using symmetric social function	Atkinson/Dalton-type (using partially symmetric social welfare function).

Measurement Using the Equivalence Scale Approach[2]

The most common-used graphical summary method is the Lorenz curve, which graphs cumulative (equivalent) income share against cumulative population share; see Figure 2. This not only provides a clear summary picture of distributions, but also provides the key elements for measures ranking pairs of distributions in terms of their inequality. Atkinson's (1970) famous result states that where the Lorenz curve for some distribution **a** is never below and somewhere above the Lorenz curve for another distribution **b**, then **a** is more equal than **b**, for all symmetric, increasing and concave additive social welfare functions, and where $\bar{a} = \bar{b}$ (or inequality comparisons are made independently of mean income). Dasgupta, Sen and Starrett (1973) weakened the concavity requirement to S-concavity, and Shorrocks (1983) has shown that where means differ, distributions

[2]For a more extensive introductory survey, see Jenkins (1989).

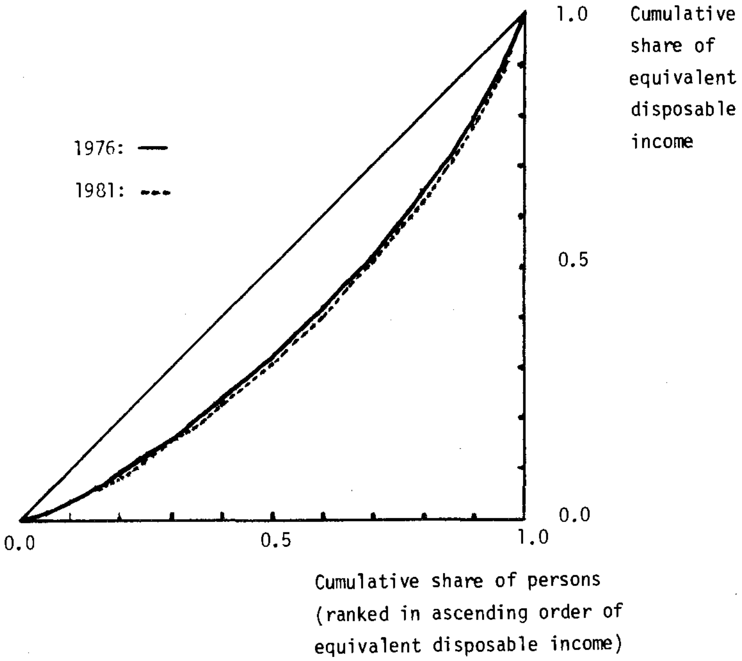

Figure 2. Lorenz Curve for Equivalent Income
Source: U.K. Family Expenditure Survey microdata.

may be unambiguously ranked where their Generalized Lorenz curves do not cross. The Generalized Lorenz curve is derived by vertically rescaling the ordinary Lorenz curve. At each point on the horizontal axis of Figure 2, the Lorenz curve ordinate is multiplied by the mean of the distribution.

For many purposes a scalar summary index is required. The Gini coefficient (half the relative mean difference) equals the ratio of the area between the Lorenz curve for **y** and the leading diagonal (the Lorenz curve for $\iota\bar{e}$) to the total area under the diagonal. The Schutz coefficient (half the relative mean deviation) equals the maximum vertical distance between the diagonal and Lorenz curve for **y**, and is the proportion of total equivalent income that would need to be redistributed from high e to low e households to achieve equalization of equivalent income (Pfähler, 1983). This maximum distance occurs at the percentile with mean equivalent income and so the measure can also be thought of as a measure of distance between the actual welfare share of that household and their "fair" share. In contrast the Gini coefficient involves pairwise comparisons of differences between each unit's income and everyone else's.

The distance concept implicit in both these measures has been developed more systematically in the literature using an axiomatic approach. In terms of the two person example represented by Figure 3 we require a measure of the distance between points A and B (and of course a more sophisticated distance metric where $n > 2$). Axioms desirable for such a measure are: (i) inequality is zero when points A and B coincide; (ii) Mean Independence so that proportionate changes in (equivalent) incomes leave inequality unaffected; (iii) Symmetry, so

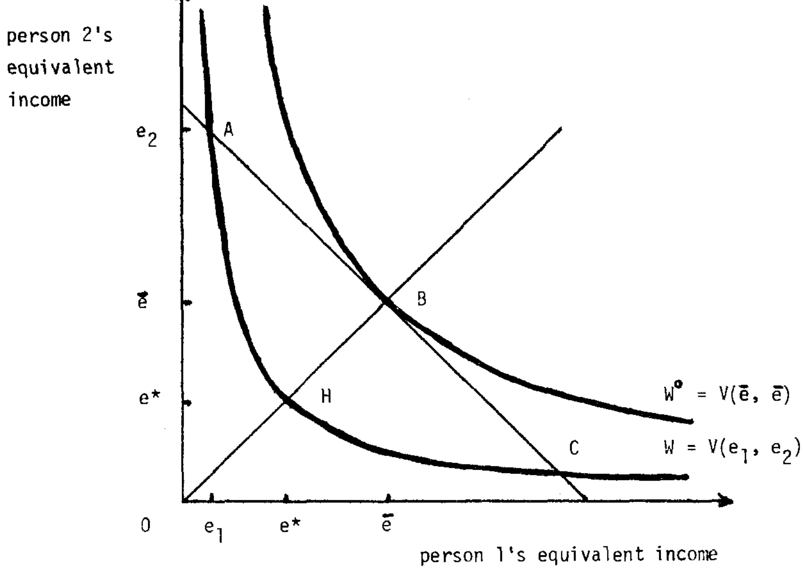

Figure 3. Social Welfare Function (Equivalent Income Approach)
Note: "Social welfare" inequality measure is (OB-OH)/OB, or $(\bar{e}-e^*)/\bar{e}$ using the "equally distributed equivalent" concept.

that inequality depends only on the information provided by the equivalent incomes per se and is not affected by any further information about the household (which simply means that the equivalence scale takes into account all relevant characteristics)—hence point C implies as much inequality as A; and finally (iv) greater distances between the distribution of observed incomes shares and the reference distribution correspond to greater inequality.[3]

If one also requires the distance measure to be additively decomposable by population subgroup[4]—this is particularly useful for empirical work—then Cowell (1985, Theorems 1-3) has shown that there is a class of distance measures satisfying these properties, the so-called Generalised Entropy (GE) family. For any two distributions **v** and **w** the class can be written in the form,

(3) $$J_\alpha(\mathbf{v},\mathbf{w}) = \frac{1}{n\alpha(1-\alpha)}\sum_{i=1}^{i=n}\left[\left(\frac{v_i}{\bar{v}}\right)^{\alpha}\left(\frac{w_i}{\bar{w}}\right)^{1-\alpha}-1\right], \qquad \alpha \neq 0, 1$$

$$J_1(\mathbf{v},\mathbf{w}) = \frac{1}{n}\sum_{i=1}^{i=n}\left(\frac{v_i}{\bar{v}}\right)\log\left(\frac{v_i}{\bar{v}}\cdot\frac{\bar{w}}{w_i}\right)$$

$$J_0(\mathbf{v},\mathbf{w}) = \frac{1}{n}\sum_{i=1}^{i=n}\left(\frac{w_i}{\bar{w}}\right)\log\left(\frac{\bar{v}}{v_i}\cdot\frac{w_i}{\bar{w}}\right)$$

[3]In formal terms this is Cowell's (1985) Monotonicity in Distance axiom. This is not the same as the Principle of Transfers because we are working with *equivalent* incomes here, and it is not satisfactory to think about transferring units of this (rather than £); see below.

[4]An index is additively decomposable if total inequality can be written as the positively weighted sum of inequality within each group, plus a between-group inequality term based upon group mean incomes, and the inequality index takes the same form in each component term (see Shorrocks, 1984).

Substituting $\mathbf{e}$ for $\mathbf{v}$ and the reference distribution $\iota\bar{e}$ for $\mathbf{w}$, the formulae reduce to

(4) $$I_\alpha(\mathbf{e}) = \frac{1}{n\alpha(1-\alpha)} \sum_{i=1}^{i=n} \left[\left(\frac{e_i}{\bar{e}}\right)^\alpha - 1\right], \qquad \alpha \neq 0, 1$$

$$I_1(\mathbf{e}) = \frac{1}{n} \sum_{i=1}^{i=n} \left(\frac{e_i}{\bar{e}}\right) \log\left(\frac{e_i}{\bar{e}}\right)$$

$$I_0(\mathbf{e}) = \frac{1}{n} \sum_{i=1}^{i=n} \log\left(\frac{\bar{e}}{e_i}\right)$$

which is the GE family of inequality measures developed by Cowell and Kuga (1981). The coefficient α summarises the sensitivity of the measure to differences in $\mathbf{e}$ in different parts of the distribution. Theil's inequality measure is I_1 and twice I_2 is the square of the coefficient of variation.

These indices contain implicit views about the social welfare function (Sen, 1973; Blackorby and Donaldson, 1978). In contrast, the Dalton (1920)–Atkinson (1970) approach begins with explicit assumptions about the way inequality reduces social welfare. The standard axioms are those outlined when discussing inequality rankings, and are summarized succinctly for the two person case in Figure 3: they imply equal-social-welfare contour lines fanning out from the origin, symmetric about the 45° line (OB). The reference point becomes not B *per se*, but the level of social welfare associated with it, $W^\circ = V(\iota\bar{e})$, where ι is the $n \times 1$ vector of ones, and hence a natural relative measure of the extent of inequality [given the symmetry of $V(\cdot)$] is the distance $(OB - OH)/OB$, whose precise value depends on the shape of the V function. For example, letting

(5) $$V(\mathbf{e}) = (\Sigma_i e_i^{1-\varepsilon})/[n(1-\varepsilon)], \qquad \varepsilon > 0, \qquad \varepsilon \neq 1$$
$$= (\Sigma_i \log e_i)/n, \qquad \varepsilon = 1$$

implies a family of inequality indices

(6) $$A_\varepsilon = [V(\iota\bar{e}) - V(\mathbf{e})]/V(\iota\bar{e})$$
$$= 1 - (1/n)\Sigma_i(e_i/\bar{e})^{1-\varepsilon}, \qquad \varepsilon > 0, \qquad \varepsilon \neq 1$$
$$= 1 - (1/n)\Sigma_i \log(e_i)/\log(\bar{e}), \qquad \varepsilon = 1.$$

Higher values of ε correspond to higher degrees of "inequality aversion" (more rectangular contours).

Atkinson (1970) noted that this index is not invariant to linear transformations of $V(\cdot)$ and to remedy this recommended the measure $(\bar{e} - e^*)/\bar{e}$, where e^* is the "equally-distributed-equivalent income"—that amount which if given to each household would provide exactly the same level of social welfare as the observed distribution (see Figure 3). For each member of this well-known family of indices, there is an ordinally-equivalent member of the GE family and so, clearly, social welfare and distance measures are closely related (though note that neither the Atkinson class of measures, nor the Gini coefficient, are additively decomposable).

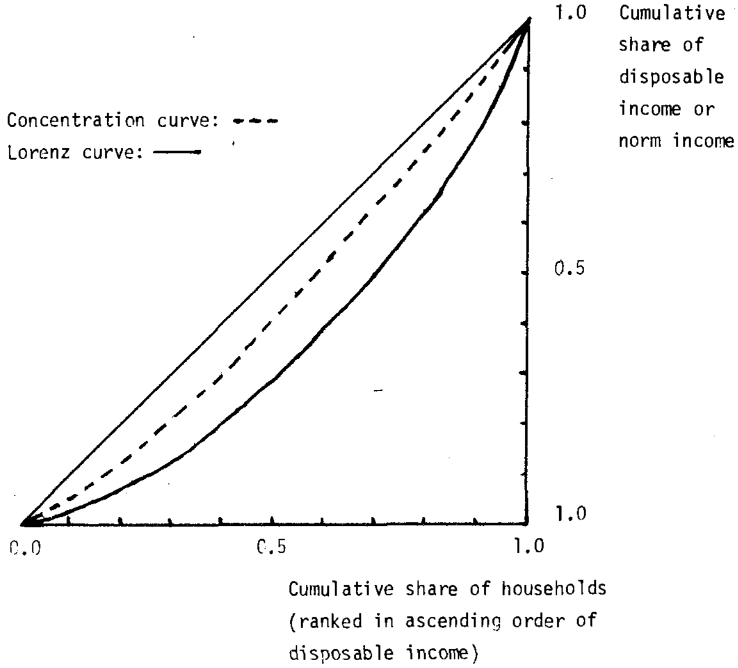

Figure 4. Lorenz Curve for Income and Concentration Curve for Norm Income
Source: 1976 U.K. Family Exenditure Survey microdata.

Measurement Using the Norm Income Approach

In the norm income framework, comparisons are done using actual income which implies that the ordering of households within each distribution is now relevant: we are concerned with income pairs $\{(y_1, z_i), (y_2, z_2), \ldots, (y_n, z_n)\}$, and symmetry must now refer to permutations of income pairs.

In Figure 4, analogous to Figure 2, the observed income distribution is represented using a Lorenz curve as before, while the other curve shows the cumulative income shares of norm income where households are ranked in ascending order of $\mathbf{y}$ to ensure that the appropriate income pairs are compared, i.e. a *concentration*, not Lorenz, curve should be drawn for $\mathbf{z}$. Perfect equality occurs when the curves coincide. The concentration curve for $\mathbf{z}$ has a non-negative slope, but potentially may lie above or below the Lorenz curve for $\mathbf{z}$, or the diagonal, or indeed wiggle around either of them.[5] As a means of summarizing distributions the diagram is quite evocative, but the range of potential shapes for the concentration curve means that basing summary indices on areas or maximum vertical distances between the curves is unsatisfactory. That description is

[5]An alternative approach would be to calculate the Relative Lorenz curve for actual incomes, which shows the cumulative proportions of y graphed against cumulative proportions of the norm income distribution (and would lie along the diagonal when there were equality). However, the Relative Lorenz curve may lie above, below or wiggle around the diagonal; we can only say for sure it has a non-negative slope. Note that microdata is essential for both exercises: with grouped data there is increased difficulty in linking up the appropriate income pairs.

more complicated now should not be surprising since we are attempting to account for population heterogeneity at the same time as income differences *per se*.

One special case is of particular interest. If households have the same *rank* in the observed distribution as the norm one, yet have the wrong income levels, then the inequality measurement problem has exactly the same structure as that commonly considered in the tax progressivity literature. In this situation the concentration curve for **z** is the same as the Lorenz curve for **z**, and so will never have wiggles in it and must lie wholly above (or below) the Lorenz curve for **y**, in which case an obvious inequality index is twice the area between the two Lorenz curves (analogous to Kakwani's (1977) tax progressivity index).[6] The maximum vertical distance between the curves is the proportion of total observed income that would need to be redistributed to obtain the norm income distribution [applying the results of Pfähler (1983)]. But we reiterate that this is a special case, and in general an alternative basis for summary measures is preferable.

Consider now methods for ranking distributions analogous to those on Lorenz dominance cited above. It is here that there is a direct connection with the recent work on multivariate approaches to inequality measurement by Atkinson and Bourguignon (1982, 1987). The approach is multivariate because the distributions of income and equity-relevant characteristics (or "needs" as they refer to them) are considered jointly, and the stochastic dominance conditions modified accordingly. The social evaluator is assumed to have a set of symmetric increasing concave additive social welfare functions, with one defined over each group of households separately differentiated, and where the form of function may differ between the groups. Total social welfare is the sum across groups of the welfare of each group. Society's social welfare function is thus "partially symmetric" (Cowell, 1980a) rather than fully symmetric as before. The key assumptions characterizing the class of social welfare functions to which the conditions apply are that social welfare is increasing in both y and z, and that at any given level of y, the social marginal valuation of income for a household from a more needy group is higher than that for one from a less needy group.

The multivariate approach is particularly attractive because ranking checks may be done even if the norm distribution is not completely specified (unlike the equivalent income approach which requires exact specification of **m**).

Suppose that the social evaluator is willing to state only the norm income rankings of the groups relative to one another. Atkinson and Bourguignon derive the following ranking condition for this case (1987, Proposition 2). First compare the Generalized Lorenz curves of **y** for just the group with the greatest needs. If they do not cross, then incorporate the next most needy group into the analysis, and again compare the Generalized Lorenz curves for the expanded sample. If they do not cross (and the ranking is the same), incorporate the third most needy group and repeat the exercise, and so on for all groups. If one distribution dominates the other at each and every step, then it is socially preferred.

Specifying the norm distribution more precisely yields a simpler condition, since knowing how much more any household should have relative to another

[6]A Suits (1977)-type index would be twice the area between the Relative Lorenz curve and the diagonal.

means that the check need not proceed on a group by group basis. In this case distribution A is more equal than B if the cumulative density of the joint distribution $(\mathbf{y}, \mathbf{z})$ is everywhere greater for A then B (see Atkinson, 1981, Result A). However this is difficult to implement empirically because households will typically be clustered at certain norm income values, and these discrete distributions need not be the same in A and B. Hence the transition matrix used to summarize the joint distribution of $\mathbf{y}$ and $\mathbf{z}$ cannot be consistently defined for both A and B.

Turn now to symmary indices. Figure 5 is the diagram analogous to Figure 3 and note that actual, not equivalent, income is measured on the axes, and so a 45° ray from the origin has no special normative meaning.

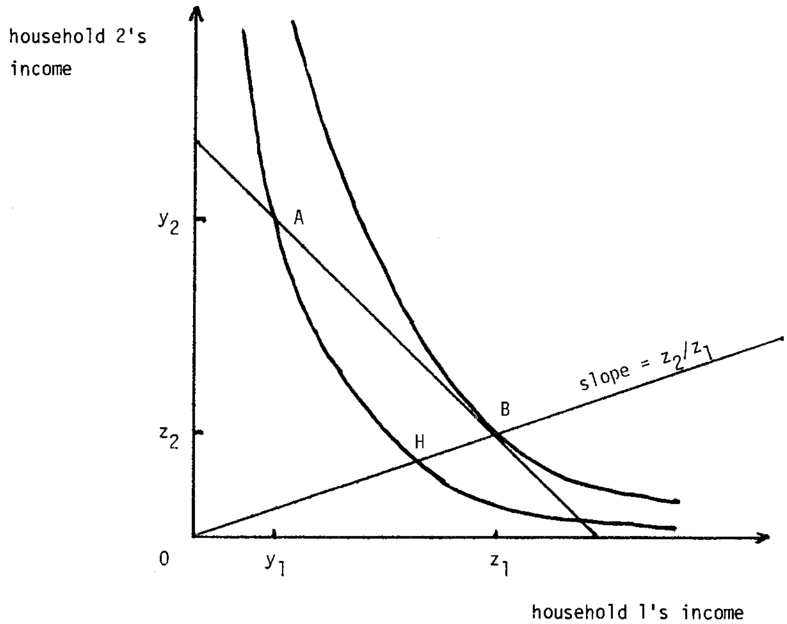

Figure 5. Partially Symmetric Social Welfare Function (Norm Income Approach)
For the SWF in (8), the contour slope is $-((z_1/y_1)/(z_2/y_2))^\varepsilon = -(z_1/z_2)^\varepsilon$ if $y_1 = y_2$.

Measurement using the distance framework is certainly feasible for straightforward application of the general formula for the GE family to this case yields, using (3) and the equal means constraint,

(7) $$J_\alpha(\mathbf{y}, \mathbf{z}) = \frac{1}{n\alpha(1-\alpha)} \sum_{i=1}^{i=n} \left[\left(\frac{(y_i)^\alpha (z_i)^{1-\alpha}}{\bar{y}} \right) - 1 \right], \qquad \alpha \neq 0, 1$$

$$J_1(\mathbf{y}, \mathbf{z}) = \frac{1}{n} \sum_{i=1}^{i=n} \left(\frac{y_i}{\bar{y}} \right) \log \left(\frac{y_i}{z_i} \right)$$

$$J_0(\mathbf{y}, \mathbf{z}) = \frac{1}{n} \sum_{i=1}^{i=n} \left(\frac{z_i}{\bar{z}} \right) \log \left(\frac{z_i}{y_i} \right)$$

To clarify the relationship between these measures and the corresponding ones based on equivalent incomes [see equation (4)], note that J_α can be rewritten when $\boldsymbol{\lambda} = (0, \ldots, 0)$ and $\mathbf{r} = \mathbf{m}$, as

$$\text{(7a)} \qquad J_\alpha = \frac{1}{n\alpha(1-\alpha)} \sum_{i=1}^{i=n} \left[\left(\frac{z_i \cdot \bar{e}^\alpha}{\bar{z} \cdot \theta^\alpha} \right) \left(\frac{e_i}{\bar{e}} \right)^\alpha - 1 \right].$$

In this special case indices I_α and J_α are very alike, yet differ in one significant way: each summarizes social welfare using a similar sum of exponentials, but in J_α each household is weighted by a term depending on norm income, rather than equally as in I_α.[7] (Of course in a homogeneous population ($m_i = 1$, all i), the two approaches would provide exactly the same result.) In the more general case where $\boldsymbol{\lambda} \neq 0$ and $\mathbf{r} \neq \mathbf{m}$, then such a close correspondence evaporates.

Turning to the social welfare based measures, we now have to specify a partially-symmetric social welfare function; one that gives contours symmetric about the 45° line only for households with the same norm income. It is admittedly difficult to think about the nature of this function, but one possibility is a function where the marginal rate of substitution between any two households—the slope of the welfare contour—depends on the ratio of their norm incomes when their actual income are equal. Such a function is

$$\text{(8)} \qquad W = [\Sigma_i (z_i)^\varepsilon (y_i)^{1-\varepsilon}]/[n(1-\varepsilon)], \qquad \varepsilon > 0, \qquad \varepsilon \neq 1.$$

and the associated family of inequality measures $[V(\mathbf{z}) - V(\mathbf{y})]/V(\mathbf{z})$—distance $(OB\text{-}OH)/OB$ in Figure 5—is

$$\text{(9)} \qquad B_\varepsilon = 1 - (1/n)\Sigma_i (z_i/\bar{z})[y_i/(z_i)]^{1-\varepsilon}, \qquad \varepsilon > 0, \qquad \varepsilon \neq 1.$$

Each member has the property that an equiproportionate change in all incomes and all norm incomes leaves the measure unchanged. The same sort of ordinal equivalence between distance and social welfare measures reported above applies here too.

In summary, one *can* develop measurement methods for the norm income approach that are analogous to those used in the equivalent approach.

3. Were Garvy and Paglin Onto Something After All?

If the norm income approach is plausible and feasible, why has it been rejected in the past? One answer is that the earlier authors used the wrong sorts of summary methods, and confusingly conflated the separate issues of inequality measurement and inequality decomposition.

[7] A further complication arises if equivalent incomes are weighted by the number of individuals in each household when aggregation is done—as in the numerical illustration—for then the indices will be based on different numbers of income units. The weighting issue is discussed by Danziger and Taussig (1979).

One of Garvy's self-criticisms was made using the following distributions and associated Lorenz curves (see Garvy, 1952, p. 38, headings added).

Family	Number of Persons	Actual income ($)	Norm income ($)
A	1	150	50
B	2	100	100
C	3	50	150

He commented that in this case the "actual and the reference curve are identical, yet the actual distribution is in drastic contrast to the criterion of equal per capita income" (1952, p. 38) and he saw this as a "serious limitation" of the approach. Clearly, following our earlier argument, he should have compared a concentration and a Lorenz curve, not two Lorenz curves.

Garvy's second criticism was that the "areas under the Lorenz curves are not additive. The area enclosed by the reference curve does not represent the portion of the total inequality depicted by the Lorenz curve that is attributable to specific factors used in constructing such a reference curve. Since the reference curve is not a demarcation line between two components of inequality the two distributions cannot be compared graphically by using the respective areas between the curve and the reference curve" (1952, p. 38). The many critics of Paglin's (1975) paper rehearsed much the same arguments. These views confuse two separate issues: inequality measurement per se, and the subsequent decomposition of measured inequality. (The probable reason for the blurring of the issues is that both exercises use information about household characteristics.)

When measuring inequality the analyst can use information about characteristics to construct either equivalence scales or norm incomes and then use a range of summary methods. For inequality decomposition by population subgroup, one should use an inequality measure that can be decomposed in a way consistent with certain desirable axioms, and this restricts one's choice to the measures based on the GE family regardless of whether one is working within the norm income or equivalence scale framework.

In short, a distinction can be made between welfare- (or norm income-) affecting characteristics and other ones. Only the former set should be used in the normative measurement of inequality, but both may be used as the basis for the decomposition of the inequality so measured.

Was Paglin onto something after all? The problem with his paper is that, although his approach was similar to a norm income one, his norm distribution was based only on differences in the age of head of household, and it is most unlikely that this is the only characteristic relevant to the derivation of **z** (though certainly relevant to controlling for lifecycle differences in income). If he had used a more plausible vector of characteristics, his inequality measure would have had some legitimacy. On the other hand, if he had wanted subsequently to decompose the inequality measured then he should have used an appropriate decomposable measure.[8]

[8]The same criticisms may be levelled at the literature which measures inequalities in health via differences between social classes. For two income distribution applications making the correct distinction between inequality measurement and decomposition, see Cowell (1984) and Mookherjee and Shorrocks (1982).

4. Strengths and Weaknesses of the Norm Income Approach

The most telling argument against the norm income approach is that it is beside the point. This view says that we should be interested in difference in *welfare* between *individuals.* In other words, given the appropriate variable of intrinsic interest and definition of the recipient unit, an equivalent income approach follows. According to this view, different definitions of equivalent income are simply different ways of measuring individual welfare, and so the emphasis in the norm income approach on "income", and the household as the unit, is misplaced. The corollary is that analysis should be directed at improving methods for measuring individual welfare, rather than changing the type of analysis. Moreover an advantage of an approach examining individual welfare is that controversial issues such as the impact on welfare of differences between individuals in their amounts of leisure time—ignored above—can be handled using standard concepts (utility functions and labour supply function estimates). Finally, it might be argued that a description of bivariate comparisons as *inequality* do not accord with the everyday usage of the term.

These arguments are persuasive but not entirely compelling. Although economists naturally think in terms of individual utilities, we suspect many others think about distributions in the bivariate way we have outlined (particularly since it is household distributions that are readily available), and there is a case for exploring ways to assess distributions within this perspective.

This argument is reinforced by the observation that the equivalence scale approach is not always implemented well in empirical work: often little attention is given to the derivation of the appropriate welfare proxy and definition of recipient unit, relative to summaries of income differences.[9] There is a tendency for analysts in the inequality literature to apply equivalence scales derived elsewhere and for other purposes with little discussion, with the result that the issues involved are "swept under the carpet." Certainly, differences between households are systematically taken account of in the "welfare analysis of tax reform" literature where utility measures are derived by applying sophisticated econometric techniques to large microdata sets, but problems remain. As Pollak and Wales (1979) and Fisher (1987) have emphasized, equality in estimated individual utility levels does not necessarily imply equality of well-being as judged by society. The crucial issue is whether a person's demographic or other characteristics have an independent direct effect on the social welfare function. As Fisher succinctly put it: "*De gustibus non disputandum* is not always an attractive ethical standard" (1987, p. 523).[10]

[9]This may have been implicitly encouraged by the theoretical literature since it typically considers measurement of inequality amongst a population assumed homogeneous in non-income characteristics.

[10]Pollak and Wales illustrate their argument with reference to the implications of taking family size as endogenous, arguing for example that "observed differences in the consumption patterns of two- and three-child families cannot even tell us whether the third child is regarded as a blessing or a curse" (1979, p. 216). Even if account is taken of parental benefits from children, there remains the issue of whether society's and parent's interests coincide. Fisher emphasizes the fact that tastes and hence choices depend on past incomes and experience: "As the example of race illustrates, where a particular household is correlated with past income or past social status, the taste differences in that attribute may not be ethically neutral. To treat them as if they were may simply be to build in the results of past inequities as though they no longer matter" (1987, p. 523).

Overall, then, one basic question when comparing the two approaches to inequality measurement is: which approach will best make analysts give non-income aspects more of the attention they deserve? An advantage of the norm income approach is that, by its very nature, analysts have to make explicit their views about how households with different characteristics should be treated, and this cannot but help to facilitate social policy debate. (Our arguments for the approach are therefore largely instrumental.) Moreover Atkinson and Bourguignon's results show that a distributional ranking check is possible even if the norm distribution is only partially specified.)

Numerical Illustration

To compare the two approaches in action, let us consider whether the distribution of disposable income in Britain in 1981 is more equal than that in 1976, using first an equivalent income approach and then comparing it with a norm income one.[11] To ease the analysis, comparisons of distributions are made independently of differences in means. All calculations are based on the U.K. Family Expenditure Survey micro-data tapes. We stress that they are for illustration only.

An equivalence scale that we have both used in previous work sets m equal to 0.6 for the first adult in a household, 0.4 for each additional adult, and 0.3 for each dependent child. Adjusting household incomes by the relevant scale rate, and assuming incomes are shared equally within households yields the distributions of equivalent income amongst individuals summarized by the Lorenz curves in Figure 2. As the curve for 1976 lies on or above that for 1981, we may conclude inequality rose over the period.

The exercise just undertaken is a very standard one and, like others before us, we have used an equivalence scale without considering its appropriateness. This would be much more difficult to do, or at least the basis of the assumptions would be more likely to be challenged, if a norm income approach were taken.

To see this, take the special case where the norm income distribution is a multiple of the equivalence scale [see the discussion of (2)]. The scale used above implies the norm incomes set out in Table 3. The Lorenz and concentration curves for the 1976 data are given in Figure 4; those for 1981 are almost identical and so are omitted for clarity's sake.

We suspect that if information were presented as in Table 3, there would be debate about whether these were the appropriate groups to differentiate (should account be taken of, say, disability or differences in labour force participation?) and about the scale relativities *per se.*

To implement the Atkinson and Bourguignon stochastic dominance check, the 1976 and 1981 samples were divided into four groups according to their needs as implied by their equivalence scale rate: $m \geq 1.7$; $1.2 \leq m < 1.7$; $1.0 \leq m < 1.2$;

[11]Disposable income equals market income from all sources including occupational pensions, plus all cash transfers, less direct taxes (personal income tax and employee National Insurance contributions). One household in 1976 and five in 1981 were omitted from the analysis as they had negative disposable incomes, giving sample sizes of 7,202 households (19,788 persons) for 1976, and 7,520 households (20,520 persons) for 1981.

TABLE 3

ILLUSTRATIVE NORM INCOMES

Household type		Equivalence Scale Rate	Norm Income (£ per annum)	
#adults	#children	(m)	1976	1981
1	0	0.6	1666	3326
1	1	0.9	2498	4988
1	2	1.2	3331	6651
1	3	1.5	4164	12471
2	0	1.0	2776	5543
2	1	1.3	3609	7205
2	2	1.6	4442	8868
2	3	1.9	5274	10531
2	4	2.2	6107	12194
3	0	1.4	3886	7760
3	1	1.7	4719	9423
3	2	2.0	5552	11085

Note: Norm incomes are calculated using the formula $\mathbf{z}=\mathbf{m}\cdot\bar{y}/\bar{m}=\mathbf{m}\cdot\theta$. The Family Expenditure Survey microdata tapes yield values for θ of 3381/1.22 (1976) and 6729/1.22 (1981), where y is household disposable income (defined in footnote 11).

$m<1$. (Some grouping of those with differing needs is required to ensure sufficient numbers in each group.) For the first group the 1976 Lorenz curve for $\mathbf{y}$ lay on or above that for 1981. Combining groups one and two, the same was true except for the top 5 per cent of the distributions, but the difference in cumulative shares was only 0.1 per cent, and it seems reasonable to treat this as well within the bounds of sampling error. Continuing the checks revealed that, at each stage, the Lorenz curve for 1976 was never below 1981's so, again, we may say inequality increased over the period. So it turns out that the two approaches yield the same conclusion when a basic specification of the norm distribution is used. More sophisticated analysis may well provide a different answer.

Additional Properties of the Norm Income Approach

The first is that it provides a very natural way of thinking about the overall *inequity* of a distribution, subsuming separate concerns about inequality and poverty within it. (The complaint above about the use of the term "inequality" is thus met if relabelling is done.) Lewis and Ulph (1988) have analyzed the question of how inequality and poverty interrelate and contribute to overall social welfare and the norm income approach provides a different perspective on the same issue. For example, we suspect that many people's specification of the norm distribution would be "raise the poor to the poverty line, and reduce the incomes of the rich by enough to finance this." This view is easily incorporated.

Suppose that there is some agreed poverty line $\mathbf{q}$ (varying across income units of different types), and that using this the population is divided into the "poor" and the "rich," with income distributions $\mathbf{y}^p$ and $\mathbf{y}^r$ respectively. Now consider the norm income distribution within which all the incomes of the poor are raised to the poverty line, while the incomes of the rich are each reduced by some tax which varies across units according to their characteristics and which ensures that the means of observed and norm income distributions are equal. The social welfare measure of Figure 4 is thus $[V(\mathbf{q}; \mathbf{y}^r - \mathbf{t}) - V(\mathbf{y}^p; \mathbf{y}^r)]/V(\mathbf{q}; \mathbf{y}^r - \mathbf{t})$, which corresponds to the "net relative welfare poverty index" of Vaughn (1987).[12] Inequality aspects can be incorporated by not setting $\mathbf{y}^r - \mathbf{t}$ equal to $\mathbf{q}$ when specifying the index in more detail.

Second, the norm income approach allows a clearcut distinction between horizontal and vertical equity aspects of inequality to be made. There is now some agreement that horizontal inequity refers to a divergence between the rankings of households in the observed income distribution and those in some reference, "initial," distribution, but there is disagreement about how the latter should be defined (see Plotnick, 1982). In contrast, using the norm income approach there is no such problem. The appropriate reference distribution is clearly $\mathbf{z}$.[13] In the special case discussed above where the Lorenz and concentration curves for $\mathbf{z}$ coincide, the distance between $\mathbf{y}$ and $\mathbf{z}$ consists of vertical inequity elements only.

A third notable feature of the norm income approach is that all the income variables are denominated in the same units (£ or \$, say). This is particularly relevant when discussing the effects of income redistribution on inequality: a transfer of £x has a consistent meaning for all income units, and does not first have to be converted to commeasurable terms using a range of "exchange rates" implied by an equivalence scale. In other words, the Principle of Transfers becomes ambiguous in a heterogeneous population.[14] Moreover the government's main redistributive instrument, the tax and transfer system, is denominated in £ not "equivalent £." Of course, the norm income approach does not remove the need to consider the value of £1 to different individuals—but one must now consider it explicitly.

To illustrate this, consider the implications for index J from equation (7) of a transfer of an infinitesimal amount of income, δ, from some richer income unit k to a poorer one j. The "transfer effect" on J is given by

$$\lim_{\delta \to 0} \Delta J/\delta = 1/[n(1-\alpha)]\bar{y}^{-\alpha}\bar{z}^{\alpha-1}[(y_j/z_j)^{1-\alpha} - (y_k/z_k)^{1-\alpha}] \tag{10}$$

It follows that inequality measured by J will fall (LHS $(10) < 0$) if

$$y_j/y_k < z_j/z_k. \tag{11}$$

[12]Vaughn actually considers the situation where $m_i = 1$, all i, so that all households have the same poverty line.

[13]This accords with Plotnick's *obiter dicta*: "suppose that the fairness of the initial ranking is questioned. Let the analyst specify the fair initial ranking. This can be compared with the actual final ranking to assess horizontal inequities" (1982, p. 376*n*).

[14]This point was also made by Cowell (1980a). A similar problem arises in the tax progressivity literature. Several authors have derived formulae relating changes in inequality, as measured by the Gini coefficient for pretax income minus that for post-tax income, to Kakwani's tax progressivity measure (for a critique, see Jenkins, 1988). To have any normative meaning their inequality measures should be based on *equivalent* income, while the tax system is summarized in terms of actual income. Hence the reported relationship between changes in inequality and progressivity only holds when $m_i = 1$, all i.

Thus whether norm-income-based inequality increases or decreases depends on whether the ratio of the unit's actual incomes is larger than the ratios of their norm incomes. If the norm income for the poorer income unit is greater than that for the richer one then inequality definitely falls, for the left hand side of (11) is less than one by definition. (The size of the effect depends on α.) Nevertheless there are situations where a transfer from a richer to a poorer unit may increase measured inequality. This is not as stange as it seems. Some income units may have a high y and a high z, so transferring income away from them would conflict with society's view that they ought to have more.

5. Concluding Remarks

A key difference between the equivalent income and norm income approaches is in the way in which the views about non-income heterogeneity are incorporated into measures. In the equivalence scale approach this is done by making prior adjustments to incomes, and subsequent analysis is eased because the population can then be treated as homogeneous. In the norm income approach, income and non-income differences are assessed at the same stage. Each approach uses different definitions of the income unit.

The norm income approach appears to require relatively more complicated measurement methods, but in part this is because insufficient attention has been given to the treatment of non-income characteristics in analyses to date. An advantage of the norm income approach is that by its very nature the assumptions made are very explicit, and should facilitate debates about specifications of relative needs, and results. On the other hand, the equivalent income approach does have benefits of its own.

We recommend combining both approaches. A particular advantage of the norm income approach is that inequality ranking checks can be carried out without completely specifying how households of different types should be treated. However, if you are confident you can derive a satisfactory measure of individual welfare, explain it and then proceed with the equivalent income approach, taking advantage of its less-complicated univariate measurement methods.

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