Review of Income and Wealth
Series 35, Number 2, June 1989

GROSS AND NET CAPITAL, AND THE FORM OF THE SURVIVAL FUNCTION: THEORY AND SOME NORWEGIAN EVIDENCE

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In this paper we discuss the empirical measurement of capital stocks derived from data on gross investment. Two capital concepts are involved: gross capital—representing the capital's capacity dimension—and net capital—representing its wealth dimension. A brief summary of their components is presented.

The data base consists of long series of Norwegian national accounts data for gross investment at a disaggregated level of sector classification and for 1-3 capital categories within each sector. Survival functions, representing the process of retirement and decline in efficiency of capital units over time, with different curvature (concave, convex) and non-zero interest rates for the discounting of future capital service flows are considered. The effects of these parameters on the calculated gross and net capital stocks in the years 1956-82 as well as on the implied replacement and depreciation rates and rates of return are discussed.

1. INTRODUCTION

The question of how to measure real capital stocks and flows of capital services is frequently discussed in economic literature (Johansen and Sørsveen, 1967; Hall, 1968; Hicks, 1969; Jorgenson, 1974). The reason is that time series of these variables are required in important fields of economic research, such as studies of productivity and producer behaviour, analyses of profitability and national accounting. As recognized by several authors, real fixed capital has at least two "dimensions." First, it may be interpreted as a *capacity measure*, i.e. a representation of the potential volume of capital services which can be "produced" by the existing capital stock at a given point of time. Second, it is a *wealth concept*; capital has a value which is derived from its ability to produce capital services today and in the future. In Biørn (1983) a theoretical framework is introduced which distinguishes between two capital concepts: *Gross capital*, according to the definition used in this paper measures the instantaneous productive capacity of the capital objects, whereas *net capital* indicates the accumulated prospective capacity of the capital stock. Over the years, the standard representation of the retirement process for capital in the literature has been the *exponential decay* assumption. This has at least two implications which can be contested partly from a theoretical, partly from an empirical point of view. First, it implies that a constant share of the capital stock is retired each year. Second, it has the particular property that gross capital and net capital coincide numerically. An interesting research problem, not least from the point of view of national accounting, is to investigate the consequences of relaxing this rigid assumption about the survival function of the capital.

In this paper, we present and discuss, under different assumptions about the retirement pattern, series of gross and net capital stocks calculated from investment data from the Norwegian national accounts. This is a database which is particularly well suited to this kind of investigation, since it contains long series of investment (back to the nineteenth century for some categories) at a fairly detailed level of sector classification, and it also distinguishes between different kinds of capital. The estimated capital data are furthermore used to calculate derived macroeconomic variables, such as productivity and rates of return to capital. A main purpose is to investigate the sensitivity of these measures and their cyclical behaviour with respect to the form of the survival function of the capital—a problem that, to our knowledge, has not received much attention in the literature.

We have divided the paper into five sections. In the next section, the theoretical framework and the basic concepts are established. The four chosen retirement processes are presented in section 3, while in section 4 we provide an overview of the investment data used and present the empirical results. Finally, section 5 contains concluding remarks and suggestions for further research.

2. Theoretical Background

The concept of gross capital can be defined straightforwardly from a sequence of gross investment figures and assumptions of how the productive capacities of the capital objects decline over time. In the following, $J(t)$ denotes the quantity of capital invested at time t, measured in physical units or as a quantity index. The physical wear and tear of the capital units is described in a traditional way [see e.g. Johansen and Sørsveen (1967) and Jorgenson (1974)] by introducing the technical survival function $B(s)$, expressing the proportion of an investment made s periods ago which still exists as productive capital. It represents both the loss of efficiency of existing capital units and the physical retirement of old capital goods.[1] The following restrictions are imposed on this function:

$$
\begin{aligned}
&0 \leq B(s) \leq 1, \quad B'(s) \leq 0 \quad \text{for all } s \geq 0, \\
&B(0) = 1, \qquad \lim_{s \to \infty} B(s) = 0.
\end{aligned}
\tag{1}
$$

The volume of capital which is s years of age at time t is $K(t, s) = B(s)J(t-s)$. The *gross capital* stock at time t is obtained simply by aggregating

[1]Although the distinction between different capital vintages is of major importance in our theoretical model, this does by no means imply that the presented formal framework fits into any type of "vintage production model," e.g. the putty-clay model suggested by Johansen (1959). A vintage production model is characterized by the fact that there is (i) a specific technology attached to each vintage of capital goods and (ii) limited substitutability between capital goods belonging to different vintages. The concepts and assumptions in this paper are, on the contrary, implicitly related to or derived from a *neoclassical* production technology, in which (i) only the total, accumulated capital stock is specified as argument in the production function for the sector as a whole, reflecting the underlying assumption of perfect substitutability between capital vintages, and (ii) perfect markets exist for both new and old capital objects.

over capital vintages, i.e.

$$K(t)=\int_0^\infty K(t,s)\,ds=\int_0^\infty B(s)J(t-s)\,ds. \tag{2}$$

In accordance with the definition of $B(s)$, gross capital is a technical concept; $K(t)$ represents the current productive capacity of the total capital stock at time t. Thus, gross capital, or the services produced by this stock, is the relevant argument in a neoclassical production function. Its age distribution is irrelevant to the description of the technology.

Related to the gross capital stock is the volume of *retirement* at time t, $D(t)$, which is, by definition, the difference between gross investment and the increase in the (gross) capital stock. An expression for $D(t)$ can be found by differentiating (2) with respect to t, which gives

$$D(t)=J(t)-\dot{K}(t)=\int_0^\infty b(s)J(t-s)\,ds, \tag{3}$$

where $b(s)=-B'(s)$ indicates the structure of the scrapping process.

Formulae for gross capital and retirement similar to (2) and (3) can be found in e.g. Jorgenson (1974) and Hulten and Wykoff (1980). Unfortunately, the terminology does not seem to be consistent in the literature. Some authors [e.g. Johansen and Sørsveen (1967) and Steele (1980)] define gross capital as the cumulated volume of past gross investment flows, without adjusting the remaining stock for physical outwear or efficiency loss. This definition is equivalent to (2) if the survival profile is of the simultaneous retirement ("one-horse-shay") type, i.e. if the productive capacity of the capital units actually remain constant (and full) over their lifetime (see below). The definition (2) is a more general and for empirical purposes a more interesting one, since it also contains other structures of capital retirement.

While gross capital expresses the current productive capacity of the capital stock, our definition of *net capital* is related to the *value* dimension of the capital. The value concepts to be introduced are implicitly based on the assumption that there exist well-organized markets for capital goods, where both new and old capital goods are traded. The market value of the capital objects will, in general, reflect the cost of producing new capital goods on the one hand, and the producers' expectations about future productivity on the other. For old capital units, it is the service flow that they are expected to produce during their remaining life-time that matters. Thus, it is reasonable to assume that capital prices are decreasing functions of the age of the capital objects.

The price of a capital unit which is s years old at time t is in the following denoted by $q(t,s)$. For new capital installed at time t the simplifying notation $q(t)=q(t,0)$ is applied. The value of capital of age s may then be written as $V(t,s)=q(t,s)K(t,s)$, and aggregating over all vintages we get the total value of the capital stock at time t, i.e.

$$V(t)=\int_0^\infty q(t,s)K(t,s)\,ds=\int_0^\infty q(t,s)B(s)J(t-s)\,ds. \tag{4}$$

Our next step is to decompose the current market value into a price and a quantity component in order to obtain a measure for the capital value that is not affected by changes in the general price level. It is then necessary to introduce specific assumptions of how capital prices vary with age, s, in order to eliminate the s index on the price variable $q(t, s)$. In this paper, following Biørn (1983), the specific assumption made is that the relative prices of capital units of different ages perfectly reflect the differences in their prospective service flows. More precisely, the price per unit of the discounted future flow of capital services is assumed to be the same for all capital vintages at each given point of time. The discounted future service flow per capital unit which is s years old is given by

$$\Phi_\rho(s) = \frac{\int_s^\infty e^{-\rho(z-s)} B(z)\, dz}{B(s)}, \tag{5}$$

where ρ is the rate of discount. Formally, our assumptions regarding relative capital prices can then be expressed as

$$\frac{q(t, s)}{\Phi_\rho(s)} = \frac{q(t)}{\Phi_\rho(0)} \quad \text{for all } t \text{ and all } s \geq 0. \tag{6}$$

This equation implies a sort of "law of indifference" to hold between different capital vintages; since the prices per unit of (discounted) prospective capital services are the same, a firm will be indifferent between investing in new and old equipment.

The common price per unit of (discounted) capital services is

$$c(t) = \frac{q(t)}{\Phi_\rho(0)} = \frac{q(t)}{\int_0^\infty e^{-\rho s} B(s)\, ds}. \tag{7}$$

This is a *general* expression for the user cost of capital in a neoclassical model of producer behaviour, in the absence of taxes.

Combining (4) and (6), the value of the capital stock may be written as

$$V(t) = q(t) \int_0^\infty \frac{\Phi_\rho(s) B(s)}{\Phi_\rho(0)} J(t-s)\, ds. \tag{8}$$

Furthermore, if we choose the current investment price, $q(t)$, as the price component of the market value, its quantity component becomes

$$K_N(t) = \frac{V(t)}{q(t)} = \int_0^\infty G_\rho(s) J(t-s)\, ds, \tag{9}$$

where

$$G_\rho(s) = \frac{\Phi_\rho(s) B(s)}{\Phi_\rho(0)}.$$

This is the variable which we shall refer to as the *net capital* stock in the following. It is seen that $K_N(t)$, like $K(t)$, is constructed by aggregating previous investment flows, but the weighting system is different; the weight assigned to investment made s years ago in $K(t)$, $G_\rho(s)$, is the share of the total discounted service flow produced by a capital unit *after it is s years old*, whereas the calculation of $K(t)$ is based on the technical survival function, $B(s)$. It is seen that $G_\rho(s)$ has the same mathematical properties as $B(s)$, cf. (1).

The conceptual difference between gross and net capital can be explained in a slightly different way: Let the elements of the net capital at time t which belongs to vintage $t-s$ be denoted as $K_\rho^*(t, s)$, i.e.

$$(10) \qquad K_\rho^*(t, s) = G_\rho(s)J(t-s) = \frac{\Phi_\rho(s)}{\Phi_\rho(0)} K(t, s).$$

While the gross capital stock is defined by simply adding (integrating over s) all $K(t, s)$, the net capital stock is calculated in a similar way after having first multiplied these vintages by the ratio $\Phi_\rho(s)/\Phi_\rho(0)$, which expresses the remaining (discounted) flow of services per unit from "old" (age s) capital vintages relative to the corresponding service flow produced by new capital. When compared with the gross capital, the net capital is thus adjusted for the fact that old capital objects generally are less productive in terms of *future* cumulated services than new ones, even if they are equivalent in terms of *instantaneous* service flows. From this interpretation, it may be concluded that estimates of net capital will normally be lower than corresponding figures for gross capital (strictly, the inequality $K_N(t) \leq K(t)$ always holds).[2]

Net capital, in contrast to gross capital, is dependent on the rate of discount, ρ. This is due to the fact that it reflects prospective capital service flows. The net capital stock will, in general, increase with increasing discounting rate.

The final concept with which we shall be concerned is depreciation. This variable has the same formal relationship to the net capital stock as retirement has to the gross capital stock, i.e. it is defined as the difference between the gross investment and the increase in the net capital stock. Formally, *depreciation* at time t can be expressed as follows:

$$(11) \qquad D_N(t) = J(t) - \dot{K}_N(t) = \int_0^\infty g_\rho(s)J(t-s)\, ds,$$

where $g_\rho(s) = -G'_\rho(s)$ indicates the structure of depreciation, in the same way as $b(s)$ represents the retirement process.

It can be shown [cf. Biørn, 1983, section 6] that the capital service price, depreciation, net capital and gross capital satisfy the following simple relationship

$$(12) \qquad q(t)D_N(t) + \rho q(t)K_N(t) = c(t)K(t),$$

regardless of the form of the survival function. Its economic interpretation is that the current "user value" of the capital stock equals the sum of the value of depreciation and a term which represents interests imposed on the capital value, and is analogous to the expression for the user value of capital found in many textbooks describing static producer behaviour. However, decomposing the user value additively, on the basis of the *gross* capital concept, is valid only in the case of an exponential retirement process. Equation (12) shows, however, that such an additive formula exists as an identity between the "value" related concepts depreciation and net capital.

[2]For the commonly applied exponentially declining survival function, $B(s) = e^{-\delta s}$, it can be easily shown that net capital equals gross capital for all parameter values. [Confer Biørn, 1983, section 7.]

The relationship (12) may be used to support the common practice applied in many countries when calculating net operating surplus as a residual, i.e. as what is left from gross factor income when wages and the value of "depreciation" is deducted. The reason for this is that national accounting calculations are commonly intended to represent the value dimension, rather than the capacity dimension of the capital stock.[3]

3. Parametric Survival Functions

In this section, we present two classes of parametric survival functions which we consider useful for empirical applications. Each class is characterized by two parameters; the first representing the maximal life time of the capital, the second indicating the "curvature" of the survival profile.

Consider first the following parametric form for $B(s)$:

$$B(s) = B^I(s;\, N, n) = \begin{cases} \left(1 - \dfrac{s}{N}\right)^n & \text{for } 0 \leq s \leq N, \\ 0 & \text{for } s > N, \end{cases} \tag{13}$$

where N is the maximal life time of the capital objects and n is a non-negative integer constant. This survival function is strictly *convex* if $n \geq 2$. The corresponding retirement function is

$$b(s) = b^I(s;\, N, n) = \begin{cases} \dfrac{n}{N}\left(1 - \dfrac{s}{N}\right)^{n-1} & \text{for } 0 \leq s \leq N, \\ 0 & \text{for } s > N. \end{cases} \tag{14}$$

General expressions and a recursive procedure for deriving numerically the weighting functions for net capital and depreciation from this class of technical survival functions are presented in Biørn (1983, section 7). An interesting result for the case when the interest rate, ρ, is zero may, however, be mentioned:

$$G_0(s) = \left(1 - \frac{s}{N}\right)^{n+1} = B^I(s;\, N, n+1),$$

$$g_0(s) = \frac{n+1}{N}\left(1 - \frac{s}{N}\right)^n = b^I(s;\, N, n+1).$$

When no discounting of future capital services is performed, there is in this case a simple way of obtaining the weighting function of the net capital from that of the gross capital: we increase n by one.

By varying the parameter n, the class of survival functions (13) generates several specifications discussed in the literature as special cases. This includes both the *simultaneous retirement* case $n = 0$, in which the capital objects are assumed to retain their full productive capacity during N periods before they are completely scrapped, and a *linearly decreasing* survival function, with $n = 1$.

[3]However, in that case an *inconsistency* of the accounting practice in Norway is that the same (linear) survival function is used both for the estimation of depreciation and the construction of capital figures presented and used as if they were gross capital stocks.

It is easily seen that when the technical survival function is of the simultaneous retirement type, the net capital stock in the zero-interest case is depreciated linearly; net capital figures estimated with this structure will thus coincide with gross capital figures calculated when the *technical* survival function is assumed to be linear. The third survival profile we shall consider is strictly convex, with n set equal to 5.

As emphasized above, the net capital stock will normally be lower than the corresponding gross capital, and the weighting function for the former will lie below that for the latter. This property is illustrated in Figure 1 for a strictly convex survival function.

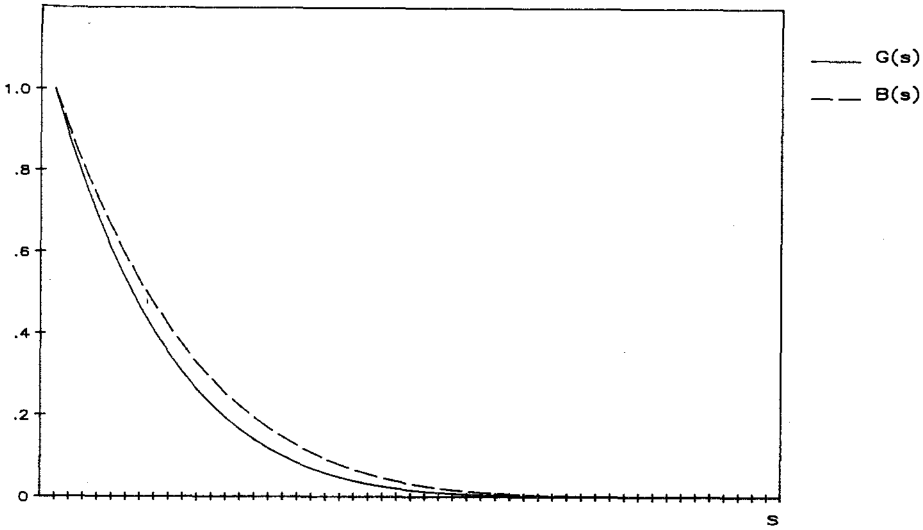

Figure 1. A Strictly Convex Survival Function and the Corresponding Weighting Function for the Net Capital

The second class of survival functions which we have applied has the following general form:

$$B(s) = B^{II}(s;\, N, m) = \begin{cases} 1-\left(\dfrac{s}{N}\right)^m & \text{for } 0 \le s \le N, \\ 0 & \text{for } s > N, \end{cases} \tag{15}$$

where m is a positive integer constant and N, as before, is the maximal life time of the capital units. This function is strictly *concave* if $m \ge 2$.

Again, the "curvature" parameter m may be varied in order to generate specific survival profiles. When $m = 1$, we get the linear survival function. Furthermore, for $m \to \infty$, this function degenerates to the simultaneous retirement case, since $\lim_{m\to\infty} (s/N)^m$ is zero when $s < N$ and one when $s = N$. Our fourth survival function is obtained by setting $m = 5$.

The derived weighting function for the net capital, $G_\rho(s)$, lies below the technical survival function, $B(s)$. It should be noted that when the latter is of

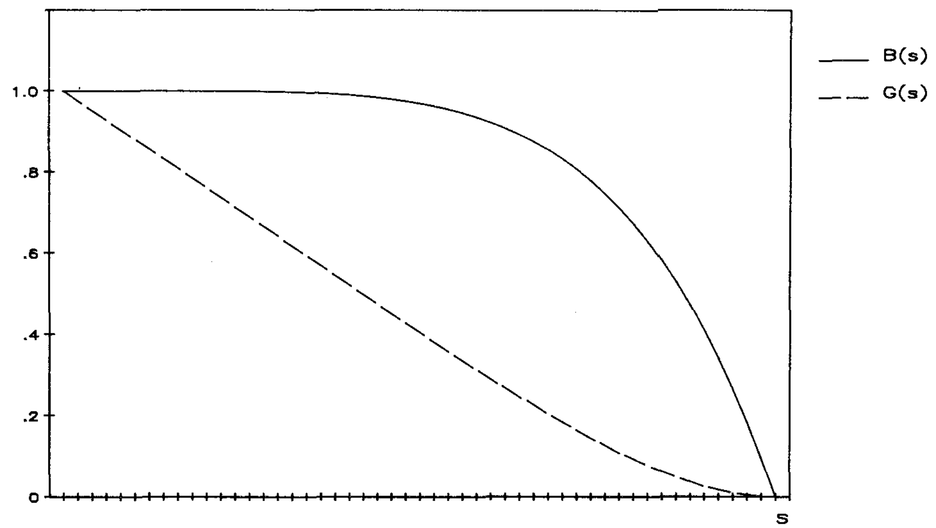

Figure 2. A Strictly Concave Survival Function and the Corresponding Weighting Function for the Net Capital

the strictly concave type, i.e. when the retirement *increases* with age, it may be the case that the decline in the net capital is represented by a convex function, i.e. that depreciation *decreases* with age.[4] An example of such a situation is given in Figure 2.

4. Data And Empirical Results

The practical procedure for constructing time series for gross and net capital stocks, as it follows from sections 2 and 3, consists in cumulating past series for gross investment at constant prices over a period of length equal to the capital's maximal life time, by application of two different weighting schemes. The weights are operationally defined once the survival function, $B(s)$, and the rate of discount, ρ, have been specified, and once an algorithm for conversion from continuous to discrete time has been constructed.

The database for the present paper is Norwegian *National Accounts Data* for gross investment at constant (1975) prices, which permits us to go further in the direction of disaggregating capital by sector and kind than is usually possible. With a few exceptions, our gross investment series go as far back in time as to permit, with the values of the maximal life time specified, the computation of capital stock series starting in 1956 (at the latest). For our purpose, we have aggregated the detailed investment data in the national accounts to a sector classification with 26 sectors and three kinds of capital:

[4]Note that for other constellations of the parameters affecting net capital, i.e. the "curvature parameter" (m) and the interest rate (ρ), the $G_\rho(s)$ function may be concave. This function increases with both m and ρ. However, with an interest rate equal to zero, the weighting function for the net capital is convex for all values of m (degenerating to a linear function in the simultaneous retirement case).

1. Buildings and structures,
2. Transport equipment,
3. Machinery and other equipment, etc.

Among the empirical results we want to focus on the following issues:

(a) the effects of the form and curvature of the survival profile on gross capital stocks under different growth profiles for gross investment;
(b) the relationship between gross and net capital stocks, and the dependence of the latter on the rate of discount assumed;
(c) the effect of the assumed survival profile on the retirement rate;
(d) the relationship between the retirement and depreciation rates; and finally
(e) the way in which the assumed rate of discount affects the estimated rate of return to capital.

The sensitivity of the gross capital stock to the form of the survival function for two selected industries is illustrated in Figures 3 and 4. We see that not only the level of the capital stock, but also its growth profile is strongly dependent on the structure of retirement. There are, however, notable differences between the production sectors in this respect. Oil production, which has been the most outstanding growth sector in Norway in the last decade, shows a sharply increasing capital stock over the entire period in the case with a simultaneous retirement pattern, whereas the capital stock attains a peak in 1977 and then decreases if a convex profile is assumed.

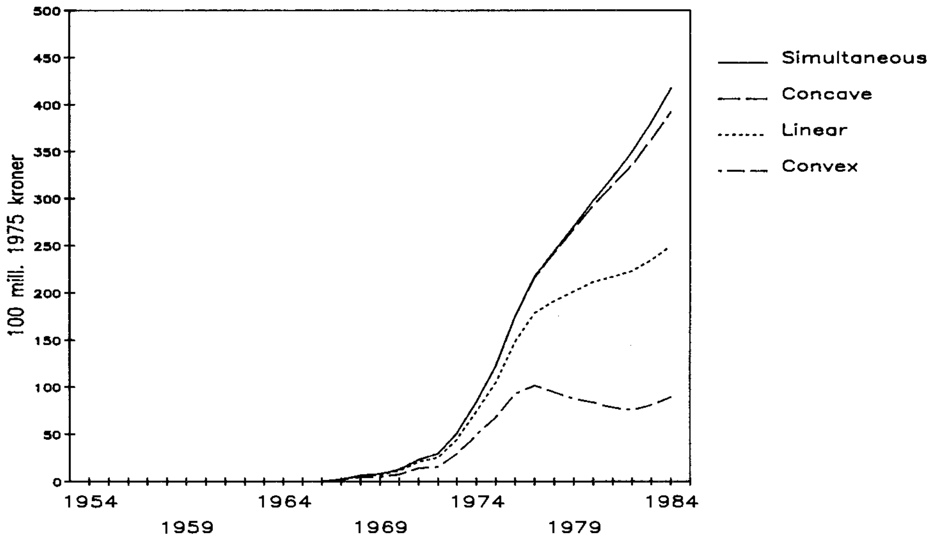

Figure 3. Gross Capital for Four Different Survival Functions. Sector: Petroleum Production Category: Buildings and Constructions

Manufacture of textiles etc., which exemplifies a sector with stagnant investment, gives a somewhat different picture; gross capital stock (machinery) is slightly increasing in the convex case, showing stronger growth in the linear case, while accelerating in the concave and simultaneous retirement case.

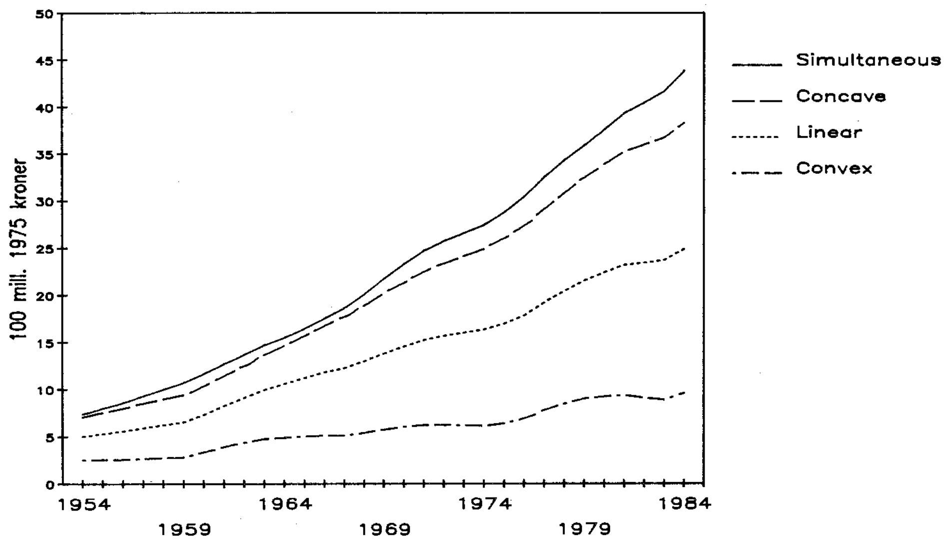

Figure 4. Gross Capital for Four Different Survival Functions. Sector: Manufacture of Textiles. Category: Machinery

To a large extent, these differences reflect the different age distribution of the capital stock implied by the four survival functions considered. The "more convex" the survival profile, the relatively larger are the weights given to investments in the current and recent vintages as compared with older vintages. Regarding the oil industry, it should be recalled that a major part of the production capacity in Norway was built up during the 1970s. A closer look at the investment data reveals that gross investment increased strongly from 1973 to 1977, but decreased thereafter until 1981, when a new peak in investment was attained. This investment path explains the development of the gross capital stock in the convex case, with a peak in 1977. When the survival profile is assumed to be of the simultaneous retirement type, the gross capital stock increases throughout the period, because this assumption implies that no "heavy" vintages have been scrapped during the present period of observation.

As mentioned in the introduction to this paper, an interesting application of the presented framework is to investigate the impact of changes in the form and curvature of the survival function on the implied measures of *capital productivity*, e.g. gross production at constant prices per unit of gross capital. The differences in productivity for an arbitrary sector/category reflect the differences, both with respect to level and growth profile, in the corresponding gross capital stock. Thus, productivity is highest in the convex case, in which the most rapid deterioration of the capital stock takes place. Calculations based on the three other survival profiles lead to higher estimates of gross capital stocks, lower average productivity, and dampened relative fluctuations.

Figures 5 and 6 serve to illustrate the difference between the gross and net capital. Here we consider a capital category with a long service life (dwellings, $N = 90$ years), which explains the smoothness of the growth curves in these

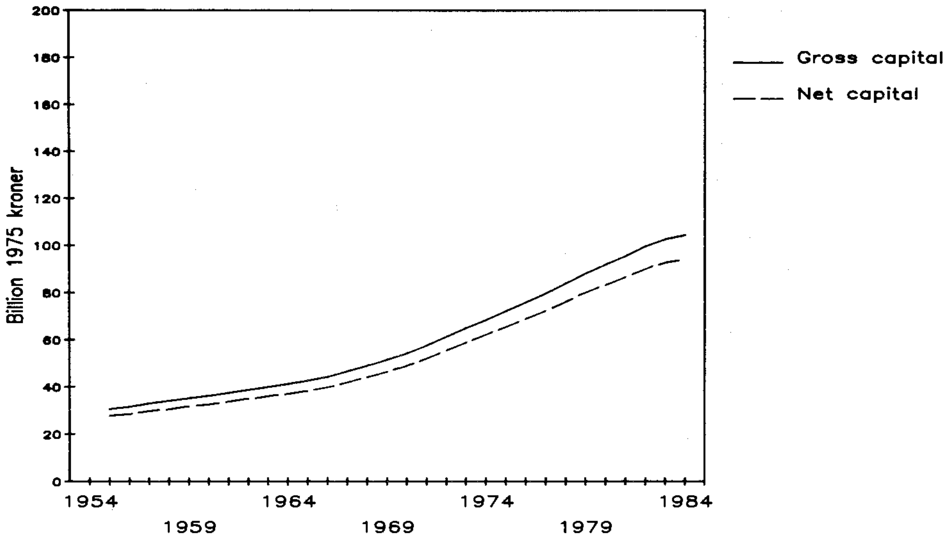

Figure 5. Gross and Net Capital Stock, Convex Survival Function. Sector: Dwellings. Category: Buildings and Constructions

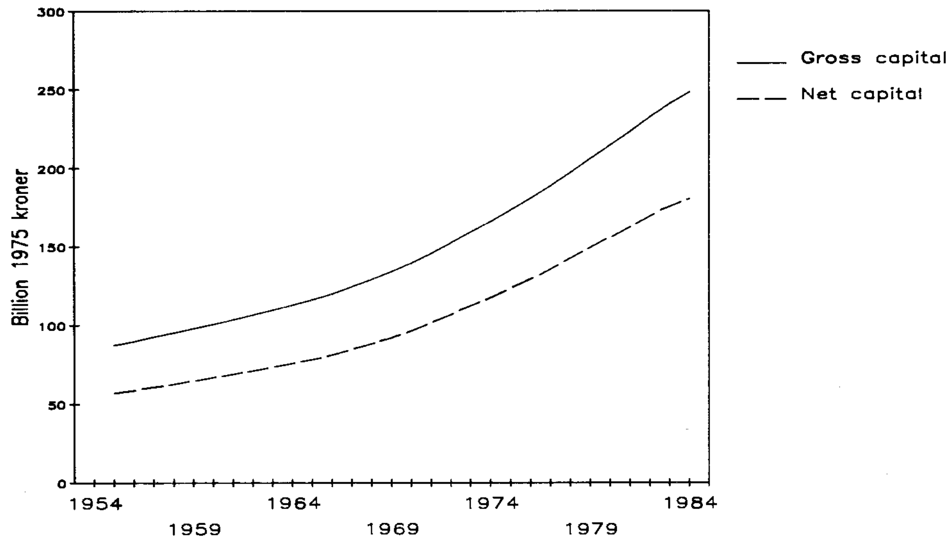

Figure 6. Gross and Net Capital Stock, Concave Survival Function. Sector: Dwellings. Category: Buildings and Constructions

figures. We see that net capital is close to gross capital if the retirement and decline in efficiency follows a convex pattern ($B(s)$ convex). As noted in section 3, the weighting function for net capital, $G(s)$, is in this case also convex. If the survival function is specified to be concave, which is probably the most realistic assumption for dwellings, we find, at least when a zero rate of discount is applied, a substantial difference between the numerical values of the two capital measures.

This reflects the basically different curvature of their weighting functions; gross capital is constructed from a concave function, net capital from a convex function (cf. Figure 2). This implies that new and old capital vintages are given widely different weights in the two capital measures.

The sensitivity of the net capital stock to the rate of discount, ρ, is illustrated in Figure 7. A change in this parameter alters the relative weights of different vintages because it affects the agents' relative evaluation of future flows of capital services under perfect market conditions. As noted in section 2, the larger the rate of discount, the closer is net capital to gross capital, and in the degenerate case when ρ goes to infinity, they coincide. An increase in ρ from 0 to 5 percent leads to a substantial rise in the net capital stock; the estimates for dwellings are for instance increased by about 25 percent. This is illustrated in Figure 7.

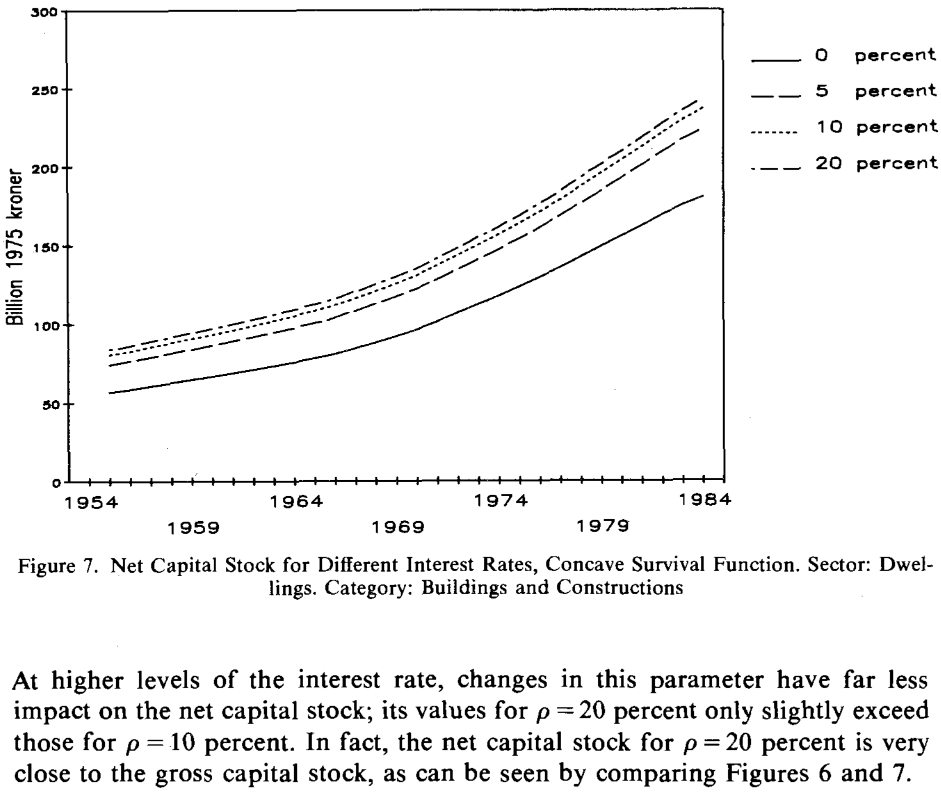

Figure 7. Net Capital Stock for Different Interest Rates, Concave Survival Function. Sector: Dwellings. Category: Buildings and Constructions

At higher levels of the interest rate, changes in this parameter have far less impact on the net capital stock; its values for $\rho = 20$ percent only slightly exceed those for $\rho = 10$ percent. In fact, the net capital stock for $\rho = 20$ percent is very close to the gross capital stock, as can be seen by comparing Figures 6 and 7.

In Figure 8, we illustrate the effect of changing the form of the survival profile on the implied *retirement rate*, defined as the ratio between retirement and gross capital stock, i.e.

$$\delta = \frac{D}{K}. \tag{16}$$

We find, not surprisingly, that its level strongly depends on the curvature of the survival profile, taking its lowest average value in the simultaneous retirement case, and its highest value in the convex case. It can be shown [cf. Biørn, 1983,

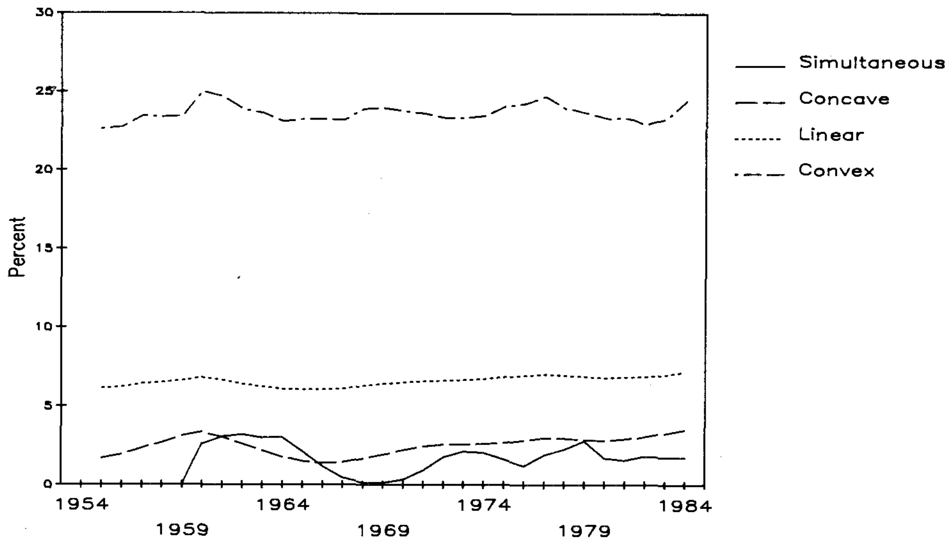

Figure 8. Retirement Rates for Four Different Survival Functions. Sector: Manufacture of Textiles. Category: Machinery

section 7] that in a situation with constant gross investment, we get the following expressions for the retirement rates in the alternatives considered:

(17)
simultaneous retirement:	$\delta = 1/N$,
concave:	$\delta = 1.2/N$,
linear:	$\delta = 2/N$,
convex:	$\delta = 6/N$.

The departure of the actual retirement rates from these "theoretical" values reflects the growth and cyclical variations in gross investment and the resulting variations in the age distribution of the capital stock over the period of observation.

The relative fluctuations of the retirement rates are widely different in the four cases. Statistically interpreted, the retirement rates are ratios of two moving average processes in gross investment, their length and weighting system reflecting the maximal life time and the form of the survival function [cf. (2) and (3)]. Both these lag distributions imply a high degree of smoothing of the investment profiles in the linear and convex case, which explains the smoothness of their retirement rates. In the simultaneous retirement case, however, retirement coincides with gross investment lagged a number of years equal to the (constant) life-time N, i.e. the moving average process in the numerator of (16) degenerates to a process with a constant lag. Its denominator is simply the cumulated flow of investment effectuated during the past N years. This explains the volatility of the retirement rate in this case.

The *depreciation rate* is the ratio between depreciation and net capital stock, i.e.

$$\delta_N = \frac{D_N}{K_N}. \tag{18}$$

An illustration of the difference between the retirement and depreciation rates is given in Figure 9. Here a simultaneous retirement survival profile is assumed, and the calculations are performed with a zero discount rate. With these assumptions it was shown in section 3 that *depreciation* follows a *linear* function. Combining this fact with (17), it follows that if investment were constant, the retirement rate would be half the depreciation rate. From Figure 9, it is confirmed that the retirement rate is far smaller than the depreciation rate. The latter is fairly stable around 2.5 percent, while the former shows considerable fluctuations, reflecting primarily the cyclical variations in gross investment.

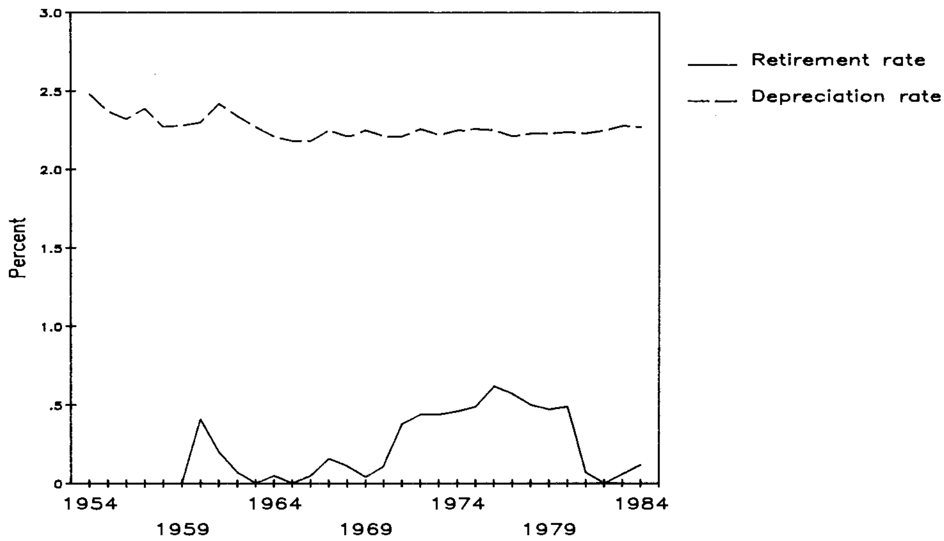

Figure 9. Retirement Rate and Depreciation Rate, Simultaneous Retirement. Sector: Manufacture of Textiles. Category: Buildings and Constructions

Finally, let us examine how our choice of capital measure affects the implied *rates of return to capital.* The formula used for calculating this variable is

$$r = \frac{E - qD_N}{qK_N}, \tag{19}$$

where E is the gross capital income, i.e. the gross operating surplus (excluding remuneration to self-employed persons) as recorded in the national accounts, before deduction of depreciation in value terms. Since the numerical values of D_N and K_N depend on the assumed survival function $B(s)$ and the rate of discount ρ, the value of r will also depend on these parameters. We will not discuss and interpret these relationships in depth here. We only select a single survival profile to illustrate the numerical relationship between r and ρ for this profile. The survival profile selected is the concave one with $m = 5$. Numerical results for the sector Manufacture of metals, with all capital types aggregated, are given in Figure 10.

It should be admitted that the interpretation of rates of return calculated in this way is not obvious. The resulting figures may be characterized as *ex post* rates of return, being ratios between *observed capital revenues* and the computed *market value* of the capital stock. This may be a natural procedure for calculating rates of return for national accounting purposes. It may, however, be alleged that there is a theoretical inconsistency between this approach and the neoclassical theory of producers' market behaviour. This basically stems from the fact that we take ρ as an *exogenous and time invariant* parameter and estimate *sector specific rates of return as time functions* conditional on this value. *In principle,* ρ and r should have been treated as *jointly determined,* as market equilibrium interest rates, within the framework of a multi-sectoral model of market behaviour.

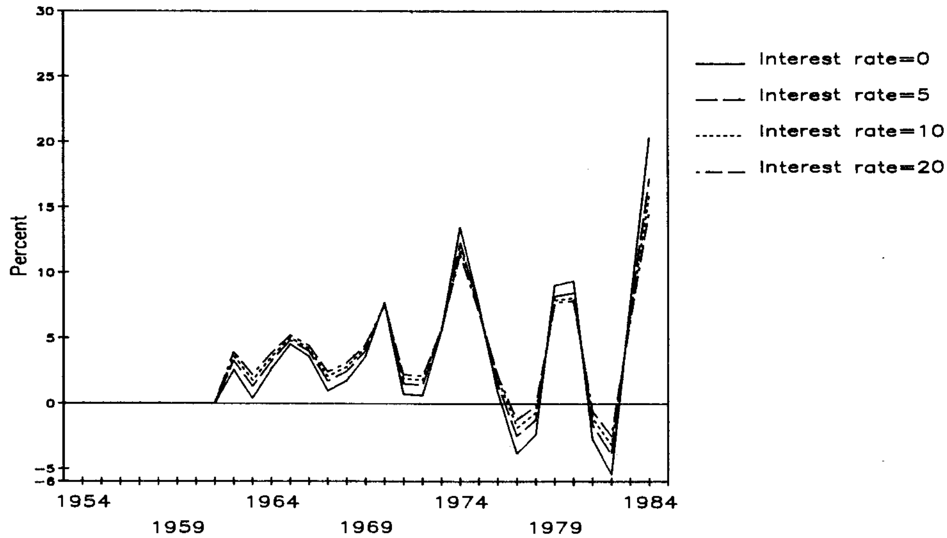

Figure 10. Rate of Return to Capital, Concave Survival Function. Sector: Manufacture of Metals. Category: Total Capital Stock

The nature of this problem is, perhaps, best understood by considering the following simple case: If all production functions were linearly homogeneous, if perfect market conditions with no uncertainty prevailed in all markets (including the credit market), and if all errors of measurement in our data on E and q could be disregarded, then the theory would predict equality between the ex post rate of return and the interest rate applied by market agents when discounting prospective incomes and costs "in the long run," i.e. we would have $r=\rho$. If we had chosen to stick entirely to these neoclassical assumptions, then we should have estimated net capital, depreciation and the rate of return simultaneously by setting $r=\rho$ in the equations from the outset for each sector under consideration. A partial *sector specific* "equilibrium value" for the rate of return, ρ^*, could then, in principle, have been obtained from

$$E=\rho^* qK_N+qD_N, \tag{20}$$

with all variables interpreted as time functions, when we recall that K_N and D_N are functions of ρ^*.[5]

Figure 10 shows the variations in the rate of return by successively choosing 0, 5, 10, and 20 percent as discounting rates. In all cases, the rates of return show strong fluctuations. This is caused mainly by the variations in the observed operating surplus. Concerning the question of how variations in the discounting rate influence the rate of return, we can make the following observations from this figure:

—It seems to exist a level at which the rate of return is independent of variations in the interest rate used in discounting future capital services. All graphs intersect at this value. Furthermore, this critical level seems to be stable over time.

—The cyclical movements of the rate of return around this "intersection level" are dampened when the discounting rate increases.

For an attempt to explain these results, we refer to Biørn, Holmøy and Olsen (1985, section 5).

5. Concluding Remarks

From the empirical results presented above, two main conclusions emerge. First, the distinction between the capacity dimension and the wealth dimension of the capital stock, i.e. between the gross and the net capital, is not only of theoretical interest; it may be empirically very important. How important it is, depends on the form of the survival profile. The difference between the two capital measures is larger for strongly concave profiles than for strongly convex ones, and is larger the smaller is the interest rate at which the future flow of capital services is discounted when constructing the net capital stock. Second, the chosen form of the survival profile may strongly influence measures of macroeconomic variables like retirement rates, depreciation rates, and capital productivity. This is true not only for the level of these variables; their cyclical behaviour may be strongly affected as well. These results have obvious implications for the interpretation of macroeconomic performance and for the specification of capital accumulation in macroeconometric models.

The four survival functions used as illustrations throughout this paper represent ways in which we could imagine the retirement of capital units or the decline in technical efficiency to take place. The simultaneous retirement profile and the convex profile are probably extreme cases from this point of view. Needless to say, we strongly need empirical evidence on survival profiles from which we could further constrain the set of specifications relevant to empirical work. Such information could be obtained in two ways: by observing the *actual age distribution* of the capital stock and the firms' *actual scrapping behaviour*, or by observing the development of *vintage prices* for sufficiently homogeneous capital units and exploiting the assumed law of indifference between vintages, (6), which underlies the construction of the net capital stock. A closer examination of the econometric implications of these research strategies is outside the scope of the present paper.

[5]Solving (20) for ρ^* is basically the same procedure for calculating rates of return to capital as the method applied in Holmøy and Olsen (1986).

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