## NOTE

# A SPECIFICATION SEPARATING FAMILY SIZE AND INDIVIDUAL AGE EFFECTS ON SUBJECTIVE EQUIVALENCE SCALES: A NOTE

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This paper analyses the Kapteyn and Van Praag method of estimating equivalence scales with respect to the way family size and age composition are incorporated into the scales according to rank and age of each family member. It becomes evident that the Kapteyn/Van Praag procedure fails to distinguish between household size and individual age effects with the result that personal weights can not be used for recomposition of household types other than wife and husband families, nor can they be interpreted as showing real age dependence of personal income equivalence. For these reasons another specification of the general approach, separating both effects, is outlined. This specification distinguishes between several consumption classes within each household. Within each class, economies of scale are attached to similar individuals while differences in individual need are obtained by comparing individuals with the same rank in the different classes.

#### 1. INTRODUCTION

Quite often, when income is analysed welfare is actually meant, but welfare depends on needs as well as on income. In order to take needs into consideration, equivalence scales are developed. Equivalence scales can be understood as relations between the incomes of households of different size and composition necessary to bring them to the same welfare level. Therefore, the field of equivalence scales is important for research in social stratification, wealth, poverty and other topics as well as for social benefits to families or households. Because of this very broad application of equivalence scales, it is necessary to carefully examine what the real welfare equivalent incomes are for families of different size and different composition, i.e. different needs.

The term "subjective" is denoted to equivalence scales that rely on subjective welfare feelings rather than expenditure or the opinion of experts.<sup>1</sup> This article critically discusses a new approach on subjective equivalence scales by Kapteyn and Van Praag (1976). In 1976 this "new approach to the construction of family equivalence scales" was outlined (Kapteyn and Van Praag, 1976) and has been applied to several countries since then (Van Praag *et al.*, 1982). Whereas research on equivalence scales as a method of determining welfare equivalent incomes between families of different size and composition has a long history, this was one of the first steps that tried to evaluate empirically the subjective welfare functions of income. Similar steps were taken by Rainwater (1974), Kilpatrick (1973), Thurow (1969) and Dunlop (1965). Nevertheless some important aspects of the approach seem to be worth further discussion.

<sup>&</sup>lt;sup>1</sup>For different approaches of analysing welfare equivalent income relations, see Klein (1986).

As has been pointed out already, much research in the fields of poverty, income distribution and redistribution, labour supply and many other areas depends on what, if any, income equivalence scale is employed. Moreover, the living standards of those living on social security benefits in low welfare areas are determined by equivalence scales underlying the official benefit regulations. Therefore, a review of the problems concerning the methods of analysing equivalence scales seems worthy of discussion even if not reconsidered empirically. In addition, the specification details are more relevant for resulting welfare equivalent income relations than the general theoretical basis of different approaches (Klein, 1986). This paper analyses the problematic mixture of family size and individual age-dependence of needs in view of the Kapteyn/Van Praag method of specifying both effects. The following article stresses the separation of family size and age effects within subjective equivalence scales.

### 2. PROBLEMS OF RANK AND AGE DEPENDENCE OF NEEDS

The Kapteyn/Van Praag method of estimating equivalence scales is based on the explanation of the so-called natural unit of income and stands for the income necessary to reach a certain welfare level (for details see Kapteyn and Van Praag, 1976, p. 318). In order to cope with family size on one side and age dependence of individual needs on the other side, a rank function and another age function is employed in the explanation of the natural unit. Denoting the rank function by  $\alpha(i)$  for the *i*-th individual within the family and denoting the age function by  $f(a_i)$  rank and age effects of an individual *i* are represented by

(1) 
$$f_i(a_i) = \alpha(i)f(a_i)$$

(Kapteyn and Van Praag, 1976, p. 322). The age function is specified as

and the rank function as

$$f(a) = \Lambda(a; \mu_2; \delta_2) + c$$

$$\alpha_i = \Lambda(i; \mu_1; \delta_1) - \Lambda(i-1; \mu_1; \delta_1)$$

where  $\mu$  stands for the natural unit and  $\delta$  for the welfare sensitivity while *c* is a constant and *i* denotes the rank number of the person in the household, beginning with the mother and continuing with father and children with decreasing age. For each household all rank-age values of equation (1) are added up to the variable  $fs = \sum_i f_i(a_i)$ , which, integrated into the estimation of the natural unit, results in the following regression analysis (see Kapteyn and Van Praag, 1976, p. 322):

(2) 
$$\mu = \beta_1 \ln f s + \beta_2 \ln y + \beta_3.$$

Without going into further detail, the results estimated by Kapteyn/Van Praag are presented in Figure 1, where all equivalent rank-age weights of mother, father and the first five children are shown. Adding up all personal weights of two households with incomes y' and  $y^*$ , the family size values fs' and  $fs^*$  are obtained and according to

(3) 
$$\frac{y'}{y^*} = \left(\frac{fs'}{fs^*}\right)^{\beta_1/(1-\beta_2)}$$



Source: Kapteyn and Van Praag, 1976, p. 325.

a true equivalent income relation is found (Kapteyn and Van Praag 1976).<sup>2</sup>

Several problems of the equivalence scale (3) due to the Kapteyn/Van Praag rank and age specification (1) should be discussed here.<sup>3</sup> The problems quite clearly show up in Figure 1. First of all, Figure 1 shows the highest weights for the wife, lower weights for the husband and lowest ones for the children, the weights always increasing with age. This result, however, does not seem realistic as produced by the way rank and age specification are given by equation (1). Let us first look at the rank function whose sense actually is to catch economies of scale (Kapteyn and Van Praag, 1976, pp. 321ff) whereas, in Figure 1, economies of scale and individual age-dependent need differences are mixed up or even confused. As pointed out (Kapteyn and Van Praag, 1976, p. 322), economies of scale most likely increase with increasing rank. This is why individual weights, to some extent, depend on rank distribution among family members as well as on age-dependent individual differences, e.g. exchanging ranks between wife and husband would give a higher weight to the husband and ascribe all economies of the second person to the wife. This methodological artifact does not cause trouble as long as all households under consideration include both husband and

 $<sup>^{2}</sup>$ As Kapteyn and Van Praag (1976) point out, household members perceive equal welfare income equivalents which in the long run are different from true values.

<sup>&</sup>lt;sup>3</sup>The same problems arise with the perceived scale values which will not be treated here separately for space reasons.

wife, but becomes most troublesome if other household types occur. Male singleperson households for instance are underestimated when using the economiesbiased husband weights of Figure 1. Analogously, the weights of children are underestimated in single parent households with an amount of 1 rank due to smaller scale economies.

Moreover, the way economies of scale and individual need differences are confused in equation (1) not only ascribes some economies of scale to individual differences, but also hides real age dependence of individual needs. Age is allowed to influence personal weights much more, if, for instance, the first rank is considered as opposed to the second. So the wife's age by itself must have more influence on the wife's weight as shown in Figure 1, due to lower economies, whereas the husband's variability of weights is reduced by the second rank and the children's weights are levelled out almost completely by having the highest rank numbers. Yet, what is even more striking with children, is that the weight of the first child depends on age, at least to some degree, whereas the weight of the fifth child does not depend on age at all. However, no reason exists for the first child having individual need structures different from those of the second or fifth one. Obviously, Figure 1 represents a mixture of individual differences and economies of scale combined in equation (1), rather than individual age dependence of needs.

Kapteyn and Van Praag (1976, p. 324) claim that, according to several studies not considering the parent's ages, increasing needs of parents growing older are ascribed to the children. Concerning this matter, some studies do in fact investigate the age of the household head and of the children simultaneously, none of them finding the age of children less relevant than the age of parents. Table 1 surveys some results of other studies; among them the Van der Gaag/Smolensky scale and to some extent the Rowntree scale also include the age of the household head. Nevertheless, all scales in fact show a significant rise of weights as the age of the children increases, which gives evidence in favour of our thesis.

On the other hand, Table 2 looks at equivalence scales for the age of adults, some of them taking into account the age of children as well. While the age of household head does not seem to have much influence, some reversed U-shaped tendencies show up in most studies. The weights first increase and then decrease; the Rowntree scale only showing a small tendency of decrease, the Watts scale being kept out of discussion for inplausibility reasons. So, at least some evidence is given that the effects of the positive correlation between the age of parents and that of children could just as well be ascribed to the parents incorrectly, by the Kapteyn/Van Praag method.

## 3. A METHOD OF SEPARATING FAMILY SIZE AND INDIVIDUAL AGE EFFECTS

While the theory underlying the Kapteyn and Van Praag approach sounds very reasonable, the problems discussed boil down to a specification problem in estimating the natural unit of income according to equation (2), where economy effects and age effects are combined into a single variable. Instead of incorporating the interaction effect (1) of rank and age into equation (2), family size and age effects should be treated separately in order to allow for other household types to be analysed on the one hand and to identify real age effects on the other. To do so, it seems reasonable, for instance, to start with the idea outlined by Bojer (1977) and others that several consumption groups with homogeneous needs exist within each household, say men, women and children of different age classes. Suppose the economies of scale are produced to a large extent within n consumption groups of size  $fs_i$ , the family size influence fs can be composed by

$$fs = \sum_{i=1}^{n} fs_i,$$

each group size  $fs_i$  being defined as

 $fs_i = \ln a_i$ ,

where the  $a_i$  denote the numbers of persons in the groups. By (4) the family size influence is analysed for similar individuals with the advantage that economies of scale are separated from individual age-dependent need differences. Inserting (4) into (2) the natural unit of income can be estimated by

(5) 
$$\mu = \beta_{11} \ln a_1 + \beta_{12} \ln a_2 + \ldots + \beta_{1i} \ln a_i + \ldots + \beta_{1n} \ln a_n + \beta_2 \ln y + \beta_3.$$

Within (5) *n* coefficients  $\beta_{1i}$  are separated describing the influence of household size and composition. According to (3) the true equivalent income relation of two households that only differ with respect to the consumption group *i*, is defined by

(6) 
$$\frac{y'}{y^*} = \left(\frac{\alpha_i'}{\alpha_i^*}\right)^{\beta_{1i}/(1-\beta_2)}.$$

The specific coefficient  $\beta_{1i}$  is ascribed to the specific household size difference and specifying solely the impact of this difference. By (6) only this specific coefficient is relied on. More generally, the true income relation is

(7) 
$$\frac{y'}{y^*} = \prod_{i=1}^n \left(\frac{a_1'}{a_i^*}\right)^{\beta_{1i}/(1-\beta_2)}$$

which eliminates the need to distribute ranks between family members, narrowing the analysis as described above.

If, instead of household size (i.e. the size of one group), the age of one of the household members changes, say one child growing older and moving from one age class to another, size effects in fact do describe the age dependence of equivalent incomes. Looking at the generalized model (7) the child will disappear in one age group and reappear in the next, which comes down to

$$\frac{y'}{y^*} = \left(\frac{a_1^* - 1}{a_1^*}\right)^{\beta_{11}/(1 - \beta_2)} \times \left(\frac{a_2^* + 1}{a_2^*}\right)^{\beta_{12}/(1 - \beta_2)}$$

If age-dependent needs increase with increasing age  $\beta_{12}$  is expected to be greater than  $\beta_{11}$  causing an increase of the child's weight.

Number and Age of Children (years)		Biolog	ical-Norma				Equal Budget Share Scales				
	Pow	ntraa									
	Rowntree					Adults			All and		
	Employed	Unemployed	Engel <sup>1</sup>	Klanberg	Orshansky	Cloths	Alcohol	All	Tobacco	Habib	
Couple without	children 100	100	100	100	100	100	100	100	100	100	
Couple with 1 c	hild aged										
0-1	)	}	115	)	]					)	
2			117								
0-4			118			124	105	112	116	1	
2-4			120							l l	
0-5	1		120								
0-6			120								
5-7			125								
8-10 0-18	122	126	129 130	127	123	123	123	101	125	135	
6-11	[ <sup>122</sup>	120	130	127	125	125	123	121	123	135	
10-12			132								
13-15			132	1	·						
12-17			138								
16-18			142								
18 & older				1							
22		}	148								
26 & older	J	J	150	J	J					J	

 TABLE 1

 Equivalence Scales by Age of Child

Number and Age of Children (years)	Equal Budget Share Scales (con.)						Specific Scales Based U				Utility Function Based		Subjective Scales		
	Seneca and Taussig <sup>4</sup>					tts <sup>2</sup>		h and gar <sup>1</sup>				Van der	Kapteyn and Van Praag <sup>7</sup>		
	Food	Nec.	Bojer	Food	Nec.	Rural	Urban	McClements	Lazear and Michael	Kakwani <sup>4,5</sup>	Gaag and Smolensky	Perceived	True	Rainwater <sup>6</sup>	
Couple with	ut childr 100	en 100	100	100	100	100	100	100	100	100	100	100	100	100	
Couple with	1 child as	ged													
0-1								109				108	119		
0-4 2-4 0-5						118	97	118							
0-6 5-7								121			99				
8-10								121							
0-18	130	130	124	136	120	1.47	114		120	} 120	109			113	
6-11 10-12						147	114	125			109				
13-15 12-17		1						127			112				
12-17 16-18 18 & older								136 142			112				
22 26 & older	J	]				150	150	1.2			152	105	114		

<sup>1</sup>Derived by adding up personal weights.

<sup>2</sup>From correlation coefficients.

<sup>3</sup>Derived as an average of different engel-functions and different total expenditure levels.

<sup>4</sup>Derived as an average of different income levels.

<sup>5</sup>Derived from *per capita* weights, multiplied by household size.

<sup>6</sup>Derived as an average of different engel-functions.

<sup>7</sup>Same age of parents supposed.

Sources: Rowntree, 1941, p. 30; Engel, 1895, p. 5; Klanberg, 1978; Orshansky, 1965, p. 75; Fiegehen *et al.*, 1977, pp. 102-104; Singh and Nagar, 1973, p. 354; Seneca and Taussig, 1971, p. 257; Bojer, 1977, pp. 192ff; Watts, 1967, p. 11; Habib, 1979, p. 296; McClements, 1978, p. 102; Kakwani, 1980, p. 363; van der Gaag and Smolensky, 1982, p. 21; Lazear and Michael, 1980, p. 102; Kapteyn and Van Praag, 1976, p. 326; Rainwater, 1974, p. 105; own computation.

		Biological-1	Normative Scal	es	А	bsolute Sper	Equal Budget Share Scales		
Age of						Fiegehen			
Household Head (years)	Rowntree <sup>8</sup>	Engel <sup>1</sup>	Klanberg	Orshansky			All and Tobacco	Habib	
27 up to 30 up to 35 30-49 40 35-54 50-64 52 57 55-64 65 & older	124	100	100	100	×	×	×	×	} 100

TABLE 2Equivalence Scales by Age of Household Head

	Equal	Budget S	Share Scal	es (con.)		Specifi	ic Scales Based	ł	Utility Fun	ction Based	Subjective Scales		
Age of Household Head (years)	Seneca and Taussig <sup>4</sup>				Singh and Nagar					Van der	Kapteyn and Van Praag <sup>7,9</sup>		
	Food	Nec.	Bojer	Watts <sup>2</sup>	Rural	Urban	 McClements	Lazear and Michael	Kakwani <sup>4,5</sup>	Gaag and Smolensky	Perceived	True	Rainwater <sup>6</sup>
27 up to 30 up to 35 30-49 40 35-54 50-64 52 57 55-64 65 & older	100	100	100	17 37 74 100	100	100	99 100 102 100	100	100	144 148 125 100	88 92 100 100	74 81 99 100	.   100

<sup>1</sup>Derived by adding up personal weights.

<sup>2</sup>From correlation coefficients of food expenditures.

<sup>3</sup>Derived as an average of different engel-functions and different total expenditure levels.

<sup>4</sup>Derived as an average of different income levels.

<sup>5</sup>Derived from *per capita* weights, multiplied by household size.

<sup>6</sup>Derived as an average of different engel-functions.

<sup>7</sup>Children.

<sup>8</sup>Derived from unemployed scale.

<sup>9</sup>For comparability reasons the man is taken for household head.

Sources: Rowntree, 1941, p. 30; Engel, 1895, p. 5; Klanberg, 1978; Orshansky, 1965, p. 75; Fiegehen et al., 1977, pp. 102-104; Singh and Nagar, 1973, p. 354; Seneca and Taussig, 1971, p. 257; Bojer, 1977, pp. 192ff; Watts, 1967, p. 1; Habib, 1979, p. 296; McClements, 1978, p. 111; Kakwani, 1980, p. 363; Van der Gaag and Smolensky, 1982, p. 21; Lazear and Michael, 1980, p. 102; Kapteyn and Van Praag, 1976, p. 326; Rainwater, 1974, p. 105; own computation.

### 4. CONCLUSION

Reviewing several very different methods of estimating equivalence scales, which is not done here, it becomes obvious that not only the more general approach, but even more the handling of the empirical problems are responsible for the results. In this case, i.e. the specification of economies of scale and individual age dependence of needs within the subjective approach of estimating equivalence scales, the handling seems especially important. Nevertheless, the effect of this new specification still has to be shown in empirical research.

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