

## NOTES AND COMMENTS

### DECOMPOSING THEIL'S INDEX OF INCOME INEQUALITY: A REPLY

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In a reply to Cowell's criticism of our note "Decomposing Theil's Index of Income Inequality into Between and Within Components" (Adelman and Levy, 1984), we would like to reemphasize that the purpose of our note was to show that the decomposition of Theil's index of income inequality into between and within components is misleading in the sense that it does not provide the partial contribution of each factor (schooling, ethnicity, and nativity in our example) to total inequality in an unnested manner. Therefore, this decomposition cannot tell us which factor is more dominant in determining the level of income inequality.

Regarding our assertion that total inequality varies with the order of decomposition, we are now convinced that we were in error. Using the formula

$$(1) \quad T_{ijk} = \sum_i y_i \log \frac{y_i}{x_i} + \sum_i y_i \left[ \sum_j \frac{y_{ij}}{y_i} \log \frac{y_{ij}/y_i}{x_{ij}/x_i} \right] + \sum_i \sum_j \frac{y_{ij}}{y_i} \left[ \sum_k \frac{y_{ijk}}{y_{ij}} \log \frac{y_{ijk}/y_{ij}}{x_{ijk}/x_{ij}} \right]$$

( $y$  and  $x$  are income and population shares of the respective  $i, j, k$  groups), we were not aware that  $T_{ijk}$  can be further expressed as

$$(2) \quad T_{ijk} = \bar{T}(I) + [\bar{T}(IJ) - \bar{T}(I)] + [T - \bar{T}(IJ)] = T$$

and similarly,

$$(3) \quad T_{kji} = \bar{T}(K) + [\bar{T}(JK) - \bar{T}(K)] + [T - \bar{T}(JK)] = T$$

as was shown by Cowell (1985). Our failure to reach this equality was due to a computational mistake.

However, Tables 3a and 3b in Cowell's comment reconfirm and redemonstrate our major point of criticism of the Theil Index that, "the decomposition of Theil's Index of total inequality into between and within components cannot lead to an unambiguous quantification of the causes of inequality because, in the presence of intercorrelation between the principles of decomposition, the decomposition overstates the contribution of the first cause considered and understates the contribution of the subsequent effects" (Adelman and Levy, p. 119). Information about the partial effects of variations in factors which affect individual earnings is valuable for social policy design. The decomposition of Theil's measure, which is based on the notion of entropy in information, into between and within components is not linked to individual earnings functions in a rigorous manner [see, for example, Chiswick (1968) and Levy (1985)]. Thus, although this decomposition is mathematically correct and seems intuitive and appealing for descriptive purposes, it does not provide an unambiguous basis for evaluating the partial effects of individual factors on total income inequality.

## REFERENCES

- Adelman, I. and Levy, A. (1984), Decomposing Theil's Index of Income Inequality into Between and Within Components, *Review of Income and Wealth*, 30, 119-121.
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