A SYNTHESIS OF POVERTY LINE DEFINITIONS*

BY ALDI J. M. HAGENAARS AND BERNARD M. S. VAN PRAAG**

Leyden University, Center for Research in Public Economics

A fundamental point of discussion in poverty research is whether poverty is an absolute or a relative concept. If poverty is seen to be a situation of absolute deprivation, a poverty line will usually be defined to be independent of the general style of living in society. If poverty is considered to be a situation of relative deprivation, a poverty line will be defined in relation to the general style of living in society. The choice for one of these two approaches has important consequences for social policy, as absolute poverty may be reduced by economic growth, while relative poverty will only decrease when income inequality decreases. This paper suggests a poverty line definition that is not *a priori* meant to be either absolute or relative, but depends on the perception of poverty in society. If the poverty line is higher in countries with higher median income (as an indicator of "general style of living") the poverty line is said to be relative; if the poverty line does not vary with median income, it is said to be absolute. The poverty line definition suggested appears to be a generalization of almost all well-known poverty line definitions. Poverty lines thus defined are estimated for eight European countries on the basis of a 1979 survey. The resulting lines appear to have an elasticity with respect to median income of 0.51, and hence can be said to be halfway on the scale between absolute and relative.

1. INTRODUCTION

In the present situation of worldwide economic recession, poverty research is of increasing importance.

A basic problem encountered in poverty research is the identification of people living in poverty.

This problem is usually solved by the introduction of a poverty line, an income level that is considered to be the borderline between the poor and the non-poor. Many different poverty line definitions have been proposed, reflecting equally many different views on the nature of poverty. They vary from a certain fixed level of purchasing power to decile-definitions of poverty.

The former definition arises from an *absolute* poverty concept, where poverty is seen as a situation of insufficient command over resources, independent of the general welfare level in society. The latter definitions arise from a relative poverty concept, where poverty is seen as a situation of purely *relative* deprivation. This implies that most poverty research is based on an *a priori* notion on the nature of the phenomenon to be analyzed.

In this paper a general parameterized definition of a poverty line is proposed, which encompasses all these views for different values of the parameters. The definition is based on the perception of poverty in the population, instead of on the researcher's perception of poverty. It is seen that several well-known poverty line definitions arise as a special case of the proposed general poverty line

*This research is part of the Leyden Income Evaluation Project. We thank Huib Van de Stadt for his comments on an earlier draft. The research described here has benefited from a discussion paper by Michael O'Higgins (1980), which commented upon earlier research. The responsibility for any remaining errors is with the authors. This paper was presented at the 1983 Conference of the International Association for Research on Income and Wealth. Part of the paper was also presented at the Venice Conference on Poverty, May, 1982. (Van Praag (1982)).

**The present affiliation of the second author is with the Econometric Institute, Erasmus University, Rotterdam.

definition. The structure of this paper is as follows. In section 2 a review of some frequently used poverty line definitions is given, and it is shown that all can be seen as special cases of a general principle. In section 3 the so-called Leyden Poverty Line (LPL) is described, and it is shown that the LPL also belongs to the general class. Section 4 serves to compare the different measures. Section 5 gives an illustration of the method described, while section 6 concludes.

2. Review of Poverty Line Definitions

Poverty is the counterpart of well-being. Hence poverty may also be defined as lack of welfare.

Let welfare U be assumed to depend on some vector y describing the aspects of an individual's position thought to be relevant for the individual's welfare according to some functional relation $U(y)$. As soon as a critical level δ on the welfare scale is identified as the poverty border, then the corresponding poverty line y_δ is defined as the solution of $U(y_\delta) = \delta$. Notice that if y is a scalar and U is an increasing function, also y_δ will be a unique poverty line. If y is a vector, say, income and leisure, there will be a *poverty boundary* in (income-leisure)-space. If U depends also on personal characteristics x, for instance family size, then the solution of $U(y_\delta, x) = \delta$ will be $y_\delta = y_\delta(x)$ and we find a poverty line (boundary) differentiated with respect to x.

Thus the definition of the poverty line (boundary) depends on three elements:

(a) the choice of the relevant variable(s) y.

(b) The choice of the function U, by its very nature a proxy relationship.

(c) The critical level δ.

In this paper an *economic* definition of poverty will be used (see e.g. Watts (1968)), that is, poverty is considered as a situation where income, representing command over resources, falls below a certain level. That is we define our y to be the scalar *income*. The value y_δ will be called the *poverty line*. In the literature a host of poverty definitions have been proposed. These notions on the one extreme consider poverty to be a situation of deprivation of certain basic goods and services necessary for maintaining physical subsistence, independent of the level of well-being in society. Poverty line definitions based on this notion will be called *absolute* poverty line definitions. At the other extreme poverty is seen to be essentially a situation of *relative* deprivation: as Atkinson (1975, page 186) puts it: "It is misleading to suggest that poverty may be seen in terms of an absolute standard which may be applied to all countries and at all times, independent of the social structure and the level of development. A poverty line is necessarily defined in relation to social conventions and the contemporary living standards of a particular society."

Definitions of this type will be called *relative* poverty line definitions. Most poverty line definitions can be placed on a scale between purely absolute and purely relative; in other words, they will have an income elasticity between zero and one (see e.g. Kilpatrick (1973)).

The choice of a place on this scale has important implications, both for the extent of poverty that will be measured and for the policy that is needed for a reduction of poverty.

A number of poverty indices have recently been proposed for measuring the extent of poverty. (See e.g. Sen (1976), Takayama (1979), Kakwani (1980), Thon (1979,1981)). In this paper we shall restrict ourselves to the simplest index possible, i.e. the percentage of people with an income lower than the poverty line. We will consider the effect of changes in the income distribution on this poverty percentage for various poverty definitions. These changes will be either changes in income inequality, to be measured by the standard deviation of log-incomes σ_y, or changes in mean-log-income μ_y, as an indicator of the average income level.

We will describe the income distribution by the distribution function of log-incomes $F(\ln y;\ \mu_y, \sigma_y)$. In the special case that the income distribution is lognormal F equals the normal distribution function with parameters μ_y and σ_y. We furthermore assume that all incomes will be incomes per equivalent adult; we will return to the effect of family size on the poverty lines in section 4.

(a) *Basic-needs Approaches*

Definitions of this type are based on the choice of a certain food basket c_0, just sufficient to stay alive. In order to transform these costs for food to an income level, which is considered to be the poverty line y_δ, a certain amount is added to account for other items like clothes and housing. This approach has been the basis for numerous poverty line definitions, starting with Booth (1892) and Rowntree (1901), up to Orshansky (1965, 1968). However, different results arise depending on the way the poverty line in terms of *total* expenditures is calculated from the expenditures on food only.

(a1) *The Basic Needs Approach According to Rowntree*

Rowntree (1901) added a fixed amount of money, in order to cover distinctly described items like fuel, food and rent. If these other costs are represented by oc_0, the poverty line is simply

$$y_\delta = c_0 + oc_0.$$

The general principle $U(y_\delta) = \delta$ now results in the choice of a linear function $U(y) = y$, and δ is chosen to be $c_0 + oc_0$. This poverty line is obviously absolute; the elasticity of this line with respect to median income is zero. The percentage of poor in the population, given this poverty line definition (which we will denote by z_1 hereafter) is

$$\Pi_1 = \int_{-\infty}^{\ln z_1} dF(\ln y;\ \mu_y, \sigma_y).$$

As an illustration we consider the situation where incomes are lognormally distributed, so that

$$F(\ln y;\ \mu_y, \sigma_y) = N(\ln y;\ \mu_y, \sigma_y).$$

We now have

$$\Pi_1 = N\left(\frac{\ln z - \mu_y}{\sigma_y}; 0, 1\right) = N\left(\frac{\ln (c_0 + oc_0) - \mu_y}{\sigma_y}; 0, 1\right).$$

It is seen that this percentage decreases if μ_y increases; it also holds that for $\ln (c_0 + oc_0) < \mu_y$ the poverty percentage decreases if income inequality decreases.

(a2) *Basis-Needs Approach According to Orshansky*

Another poverty line definition based on the costs of food is given by Orshansky (1965, 1968). Orshansky transformed food costs c_0 to an income level by multiplication by an estimate of the average income-food ratio in society. Suppose that the relationship between expenditures on food, c, and family income y may be described by a double logarithmic Engel function (see e.g. Prais and Houthakker (1955), Cramer (1973), Van Praag, Spit, Van de Stadt (1982)),

$$\ln c = \alpha_0 + \alpha_1 \ln y. \tag{1}$$

The average food-income ratio (c/y), used to transform expenditures on food into a poverty line, is chosen to be the geometric average in the population,[1] that is

$$\begin{aligned} \left(\frac{\tilde{c}}{y}\right) &= \exp E_y \left(\ln \frac{c}{y}\right) \\ &= \exp E_y (\ln c - \ln y) \\ &= \exp E(\alpha_0 + (\alpha_1 - 1) \ln y) \\ &= \exp \int_{-\infty}^{+\infty} (\alpha_0 + (\alpha_1 - 1) \ln y)\, dF(\ln y; \mu_y, \sigma_y) \\ &= \exp (\alpha_0 + (\alpha_1 - 1)\mu_y) \end{aligned} \tag{2}$$

where $E_y(\cdot)$ stands for the mathematical expectation with respect to y. The poverty line corresponding to a specific food basket c_0 is now found to be

$$y_\delta = c_0 \left[\left(\frac{\tilde{c}}{y}\right)\right]^{-1} = c_0 \exp (-\alpha_0 + (1 - \alpha_1)\mu_y). \tag{3}$$

This poverty line may be seen to result from the general principle with

$$U(y) = y \quad \text{and} \quad \delta = c_0 \left[\left(\frac{\tilde{c}}{y}\right)\right]^{-1}.$$

It appears that the resulting poverty line depends on mean log-income in society. The elasticity of this poverty line with respect to median income, $\exp(\mu_y)$, equals $(1 - \alpha_1)$. The percentage of poor in the population, given this poverty line definition denoted by z_2 is

$$\Pi_2 = \int_{-\infty}^{\ln z_2} dF(\ln y, \mu_y, \sigma_y).$$

[1] If another indicator of the average food-income ratio is used, (2) will become slightly more complicated. For any specific functional form of the income distribution, however, the poverty line may again be derived.

In case of a lognormal income distribution this amounts to

$$\Pi_2 = N\left(\frac{\ln z_2 - \mu_y}{\sigma_y}; 0, 1\right)$$

$$= N\left(\frac{\ln c_0 - \alpha_0 - \alpha_1 \mu_y}{\sigma_y}; 0, 1\right).$$

It may be seen that unless α_1 equals zero, this percentage decreases if μ_y increases. Furthermore if $\ln c_0 < \alpha_0 + \alpha_1 \mu_y$, in other words if the minimum food basket is smaller than the geometric mean food basket in society, the poverty percentage also decreases if income inequality decreases.

(b) *Food-Ratio Method*

One might alternatively derive a poverty line from the Engel function by setting a maximum value γ_0 for the ratio of food expenditures to total income; if someone's food-income ratio c/y is higher than γ_0, this person is called poor; if c/y is lower than γ_0, this person is called non-poor. This method has been used by e.g. Love and Oja (1975) on Canadian data.

If once again equation (1) describes the relationship between food and income in society, the poverty line corresponding to γ_0 is the solution of

$$\ln \gamma_0 = \ln\left(\frac{c}{y}\right) = \alpha_0 + (\alpha_1 - 1) \ln y_\delta \tag{5}$$

yielding

$$y_\delta = \exp\left(\frac{\alpha_0 - \ln \gamma_0}{1 - \alpha_1}\right). \tag{6}$$

In this poverty line definition the general principle holds with $U(y) = \exp\{(1-\alpha_1)\ln y - \alpha_0\}$ and $\delta = 1/\gamma_0$. If γ_0 does not change with μ_y or σ_y^2, this method thus results in an absolute poverty line that will be denoted by z_3. This assumption is not very realistic; in practice researchers or politicians will almost always relate the value of γ_0 to the income distribution in society. This has led Townsend (1979) to the observation that an absolute poverty line is not only undesirable, but almost impossible.

The corresponding poverty percentage under the assumption that γ_0 is exogenous will be

$$\Pi_3 = \int_{-\infty}^{\ln z_3} dF(\ln y; \mu_y, \sigma_y),$$

which in case of lognormal incomes equals

$$\Pi_3 = \int_{-\infty}^{\ln z_3} dN(\ln y; \mu_y, \sigma_y) = N\left(\frac{\alpha_0 - \ln \gamma_0 - (1-\alpha_1)\mu_y}{(1-\alpha_1)\sigma_y}; 0, 1\right). \tag{7}$$

If γ_0 is constant and chosen to be larger than the geometric mean food-income ratio, both a reduction in income inequality and an increase in μ_y will result in a lower poverty percentage.

(c) *Fraction of Median Income Approach*

Another well-known poverty line is defined as a certain percentage of an index of average income in society. (See e.g. OECD (1976).) If instead of a percentage of average income, the poverty line is chosen as a fraction of median income,[2] this poverty line can be denoted by

$$y_\delta = \tau_0 \exp(\mu_y) \qquad (0 \leq \tau_0 < 1). \tag{8}$$

The welfare proxy used in the derivation of this line is the ratio of actual income to median income in society, yielding $U(y) = y/\exp(\mu_y)$ and $\delta = \tau_0$. This poverty line, denoted by z_4, is obviously relative.

The corresponding poverty percentage is

$$\Pi_4 = \int_{-\infty}^{\ln z_4} dF(\ln y; \mu_y, \sigma_y),$$

or, in case of lognormal incomes:

$$\Pi_4 = \int_{-\infty}^{\ln z_4} dN(\ln y; \mu_y, \sigma_y) = N\left(\frac{\ln \tau_0}{\sigma_y}; 0, 1\right). \tag{9}$$

Hence the poverty percentage depends on income inequality only; if income inequality decreases, the poverty percentages decreases as well. Economic growth does not, however, reduce poverty.

(d) *Percentile of the Income Distribution Approach*

Finally, a poverty line may be defined as the borderline of a certain percentile ε_0 of the income distribution:

$$\int_{-\infty}^{\ln y_\delta} dF(\ln y; \mu_y, \sigma_y) = \varepsilon_0 \tag{10}$$

If for instance the income distribution is lognormal, the poverty line equals

$$y_\delta = \exp(N^{-1}(\varepsilon_0)\sigma_y + \mu_y). \tag{11}$$

The general principle results in $U(y) = F(\ln y; \mu_y, \sigma_y)$ and $\delta = \varepsilon_0$. In this definition the welfare proxy used is someone's relative position in the income distribution. This poverty line depends both on average income and on income inequality; for values of ε_0 smaller than 0.5, the poverty line increases if income inequality decreases.

Due to the poverty line definition denoted by z_5, the percentage of poor is

$$\Pi_5 = \varepsilon_0. \tag{12}$$

Hence neither economic growth nor changes in income inequality will reduce poverty.

[2]This choice is made in order to simplify the remainder of the discussion; if average income is chosen, formula (8) will be slightly more complicated, involving σ_y as well.

3. The Leyden Poverty Line (LPL)

An alternative poverty line definition has been introduced by Goedhart, Halberstadt, Kapteyn and Van Praag (1977). That poverty line has been called Leyden Poverty Line (LPL) after its place of origination. This definition, elaborated upon in e.g. Van Praag, Goedhart and Kapteyn (1980) and Van Praag, Hagenaars and Van Weeren (1982) is based on the relationship $U(y)$ between welfare U and income y, as derived from a specific set of attitude questions in a survey. In a similar way as before the poverty line y_δ is specified as the income level y_δ, for which holds $U(y_\delta) = \delta$.

The *cardinal* utility function chosen in that context is the Individual Welfare Function of Income (WFI), introduced by Van Praag (1968, 1971) and elaborated upon in e.g. Kapteyn (1977), Van Praag (1981), Kapteyn and Wansbeek (1982), Van Praag and Spit (1982).

In Van Praag (1968) a theoretical framework has been developed suggesting that the WFI can be approximately described by a lognormal distribution function

$$U(y) = \Lambda(y; \mu, \sigma) \equiv N(\ln y; \mu, \sigma), \tag{13}$$

where $\Lambda(\cdot; \mu, \sigma)$ and $N(\cdot; \mu, \sigma)$ are the lognormal and normal distribution function, respectively. For each individual the location and shape of this function, determined by its parameters μ and σ^2, is estimated from the following composite survey question, called the Income Evaluation Question (IEQ):

> "Please try to indicate what you consider to be an appropriate amount of money for each of the following cases? Under my (our) conditions I would call an after-tax income

per week/month/year[1] of:

about £..... very bad
about £..... bad
about £..... insufficient
about £..... sufficient
about £..... good
about £..... very good"

[1]Please encircle the appropriate period

On the assumption that people try to maximize the information given by their responses to the *stimuli*, offered by the verbal qualifications "very bad", "bad," etc. up to "very good," the answers are set equal to the means of equal quantiles of the finite interval [0, 1]. This method is comparable to the procedure used by Jasso and Rossi (1977) in a similar context. The answers of a hypothetical respondent t, denoted by $y_{1t}, \ldots$ to y_{6t} are depicted in Figure 1.

The equal quantile assumption amounts to

$$U_t(y_{it}) = \frac{i - \frac{1}{2}}{6} \qquad (i = 1, \ldots, 6).$$

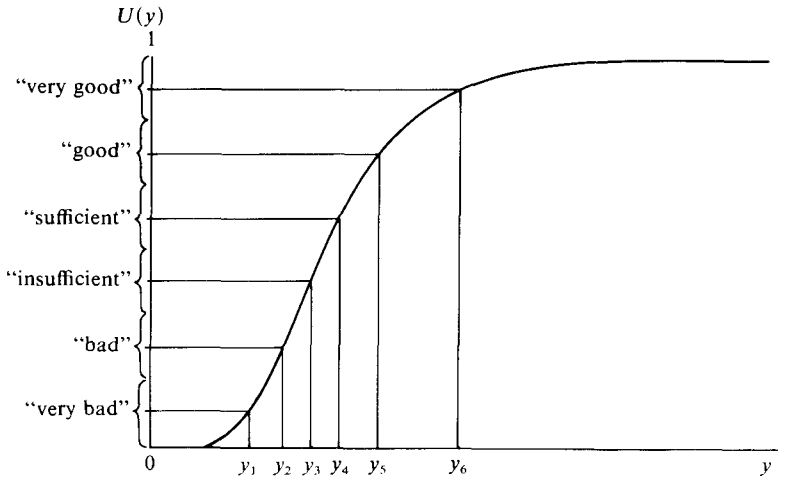

Figure 1. The Welfare Function of Income.

Making use of the lognormal specification we find:

$$\Lambda(y_{it};\mu_t,\sigma_t)=\frac{i-\frac{1}{2}}{6} \qquad (i=1,\ldots,6) \tag{14}$$

yielding

$$\ln y_{it}=\mu_t+\sigma_t u_i \qquad (i=1,\ldots,6) \tag{15}$$

where u_i is defined as

$$N(u_i)=\frac{i-\frac{1}{2}}{6} \qquad (i=1,\ldots,6), \tag{16}$$

where $N(\cdot)$ stands for $N(\cdot;0,1)$. After adding an i.i.d. error term ε to equation (15) μ_t and σ_t can be estimated for each individual t by ordinary least squares.

Obviously the identification of the theoretical concept with the response on the IEQ is a step which one must be willing to accept. The procedure described above is tantamount to the operationalization of a theoretical notion by the definition of a measurement procedure. At many places both the philosophical and the practical value of this operationalization has been thoroughly studied (Van Praag (1971, Kapteyn (1977)). The equal quantile assumption has been tested empirically by Buyze (1982) with a positive result. Attempts to explain the variation in μ and σ, thus estimated, between individuals have been made, among others, by Van Praag (1971), Van Praag and Kapteyn (1973), Kapteyn (1977), Kapteyn, Wansbeek and Buyze (1980), Van Praag (1981) and Van Praag and Spit (1982). Their results culminate in the following hypothesis:

> "The Welfare Function of Income is approximately equal to a perceived Income Distribution Function."

A similar theory was described by Duesenberry as early as 1949 and more recently by Layard (1980) in his formulation of a Status Ranking Principle (see also Jasso (1980) for similar ideas). If the Income Distribution Function perceived by the

individual equalled the actual income distribution function, this statement would imply that an income level is evaluated by 0.6, if 60 per cent of the total population earns less. The evaluation of an income level would be solely determined by its relative ranking on the income scale. However, the perceived distribution of incomes differs from the actual distribution, for the following reasons.

First, the individual's perception of the income distribution will depend on his own relative position in this distribution, which seems adequately described by his own income. Second, the income distribution of his peers, say his "social reference group," will influence his standards. Thirdly, someone's perception of his relative position in the income distribution may depend on his income and his social reference group in the past and in the future. In what follows we will restrict ourselves to the contemporaneous factors; for a first exploration of the dynamic aspects of this theory we refer to Van de Stadt, Kapteyn and Van der Geer (1985) and Van Praag and Van Weeren (1983). If only terms pertaining to the present are used, the parameter μ_t and σ_t of the WFI are known to be fairly well explained by the equations

$$\mu_t = \beta_0 + \beta_1 \ln y_t + \beta_2 m_t \tag{17}$$

$$\sigma_t = \beta_3 s_t \tag{18}$$

where y_t stands for the income of individual t, m_t stands for the log-income in t's reference group, and s_t stands for the standard deviation of log-incomes in t's reference group (see Kapteyn, Van Praag, Van Herwaarden (1978)). If we assume that everyone in society has the same reference group, i.e. all other individuals in society, we have[3] $m_t = \mu_y$ and $s_t = \sigma_y$ yielding (cf. Van Praag, Goedhart, Kapteyn (1980))

$$\mu_t = \beta_0 + \beta_1 \ln y_t + \beta_2 \mu_y \tag{19}$$

$$\sigma_t = \beta_3 \sigma_y. \tag{20}$$

For each individual t the income level $y_{\delta,t}$ corresponding to a welfare level δ can now be derived by solving the equation

$$N((\ln y_{\delta,t} - \mu_t)/\sigma_t; 0, 1) = \delta \tag{21}$$

Yielding after substitution of (17) and (18)

$$\ln y_{\delta,t} = \beta_0 + \beta_1 \ln y_{\delta,t} + \beta_2 \mu_y + \beta_3 \sigma_y u_\delta \tag{22}$$

where u_δ is implicitly defined by $N(u_\delta) = \delta$. If $y_t > y_{\delta,t}$, we have $U(y_t) > U(y_{\delta,t}) = \delta$ and if $y_t < y_{\delta,t}$, we have $U(y_t) < U(y_{\delta,t}) = \delta$. Solution of (22) with respect to $y_{\delta,t}$ yields the corresponding definition of a national poverty line z_6:

$$\ln z_6 = \frac{1}{(1-\beta_1)}(\beta_0 + \beta_2 \mu_y + \beta_3 \sigma_y u_\delta). \tag{23}$$

[3]If cross-sectional data of one society are used, the parameter β_2 cannot be estimated, but will be included in the intercept. If however panel data are available, or international data, as we will use in section 5, β_2 may be identified.

This poverty line is partly relative; its elasticity with respect to median income $\exp(\mu_y)$ depends on the values of the coefficients β_2 and β_1. The poverty line furthermore depends on σ_y, unless the welfare level δ is chosen to be 0.5, for then u_δ would equal zero.

The percentage of poor in case of a lognormal Income Distribution can be calculated as

$$\Pi_6 = \int_{-\infty}^{\ln z_6} dN(\ln y;\, \mu_y,\, \sigma_y) = N\left(\frac{\beta_0 + (\beta_1 + \beta_2 - 1)\mu_y + \beta_3 \sigma_y u_\delta}{(1-\beta_1)\sigma_y}\right).$$

An increase in μ_y will decrease the poverty percentage if $(\beta_1 + \beta_2) < 1$. If $\beta_1 + \beta_2 = 1$ the poverty percentage depends merely on income inequality, represented by σ_y.

If in addition to a possible finding that $\beta_1 + \beta_2 = 1$ we should also have $\beta_0 = 0$, $\beta_1 = 0$ and $\beta_3 = 1$, the poverty percentage would be constant and equal to the welfare level δ, chosen to be the poverty threshold.

The conclusion of this paper so far is that any poverty line definition suggested is a specific result of a general principle and that the various specifications correspond to different choices of the function U and the welfare level δ. In the next section this will be formally summarized. In section 5 the equations underlying the Leyden Poverty Line will be empirically estimated, yielding a poverty line corresponding to the opinion of a representative sample.

4. Evaluation and Comparison of Different Definitions

The poverty lines and poverty percentages corresponding to the various definitions are summarized in Table 1. For the calculation of the poverty percentages it has been assumed that incomes are lognormally distributed. In order to distinguish

TABLE 1

Six Poverty Lines and Percentages According to Various Definitions of U and δ

Definition	Poverty Line	Poverty Percentage
(1) Basic needs, Rowntree	$\ln z_1 = \ln(c_0 + oc_0)$	$\Pi_1 = N\left(\frac{\ln(c_0 + oc_0) - \mu_y}{\sigma_y}; 0, 1\right)$
(2) Basic needs, Orshansky	$\ln z_2 = \ln c_0 - \hat{\alpha}_0 + (1 - \hat{\alpha}_1)\mu_y$	$\Pi_2 = N\left(\frac{\ln c_0 - \hat{\alpha}_0 - \hat{\alpha}_1 \mu_y}{\sigma_y}; 0, 1\right)$
(3) Food ratio	$\ln z_3 = \frac{\hat{\alpha}_0 - \ln \gamma_0}{1 - \hat{\alpha}_1}$	$\Pi_3 = N\left(\frac{\hat{\alpha}_0 - \ln \gamma_0 - (1 - \hat{\alpha}_1)\mu_y}{(1 - \hat{\alpha}_1)\sigma_y}; 0, 1\right)$
(4) Percentage median income	$\ln z_4 = \ln \tau_0 + \mu_y$	$\Pi_4 = N\left(\frac{\ln \tau_0}{\sigma_y}; 0, 1\right)$
(5) Percentile	$\ln z_5 = N^{-1}(\varepsilon_0)\sigma_y + \mu_y$	$\Pi_5 = \varepsilon_0$
(6) Leyden method	$\ln z_6 = \frac{\hat{\beta}_0 + \hat{\beta}_2 \mu_y + \hat{\beta}_3 \sigma_y u_\delta}{1 - \hat{\beta}_1}$	$\Pi_6 = N\left(\frac{\hat{\beta}_0 + (\hat{\beta}_1 + \hat{\beta}_2 - 1)\mu_y + \hat{\beta}_3 \sigma_y u_\delta}{(1 - \hat{\beta}_1)\sigma_y}; 0, 1\right)$

parameters that are chosen *a priori* and parameters that are estimated in a (budget) survey the latter are denoted with hats.

The poverty lines vary from absolute (basic needs according to Rowntree and food ratio) to merely relative (percentage of median income and percentile) methods. The other poverty line definitions (basic needs method according to Orshansky and Leyden method) may be anywhere on this scale, depending on the values of the estimated coefficients.

If $\hat{\alpha}_1 = 1$, implying that the elasticity of food with respect to income equals 1, the basic needs line according to Orshansky becomes absolute; if $\hat{\alpha}_1 = 0$, implying that expenditures on food are inelastic with respect to income, the basic needs approach yields a completely relative poverty line. Analogously the LPL becomes an absolute poverty line if $\hat{\beta}_2 = 0$, and a completely relative concept if $\hat{\beta}_2 = 1 - \hat{\beta}_1$.

Actually, the LPL definition appears to be a generalization of all other poverty line definitions:

If $\hat{\beta}_2 = 1 - \hat{\beta}_1$ and $\hat{\beta}_0 = 0$, we have a percentile definition with

$$N^{-1}(\varepsilon_0) = \frac{\hat{\beta}_3 N^{-1}(\delta)}{1 - \hat{\beta}_1}.$$

If $\hat{\beta}_2 = 1 - \hat{\beta}_1$ and $\delta = 0.5$, we have a percentage-of-median-income-definition, with

$$\ln \tau_0 = \frac{\hat{\beta}_0}{1 - \hat{\beta}_1}.$$

If $\hat{\beta}_2 = 0$ and $\delta = 0.5$, we have a food-ratio-definition where

$$\frac{\hat{\alpha}_0 - \ln \gamma_0}{1 - \hat{\alpha}_1} = \frac{\hat{\beta}_0}{1 - \hat{\beta}_1}$$

or a basic needs definition according to Rowntree, with

$$\ln (c_0 + oc_0) = \frac{\hat{\beta}_0}{1 - \hat{\beta}_1}.$$

If $\hat{\beta}_2 < 1 - \hat{\beta}_1$ and $\delta = 0.5$, we have a basic-needs definition according to Orshansky, with

$$\ln c_0 - \hat{\alpha}_0 = \frac{\hat{\beta}_0}{1 - \hat{\beta}_1} \quad \text{and} \quad (1 - \hat{\alpha}_1) = \frac{\hat{\beta}_2}{1 - \hat{\beta}_1}.$$

This is summarized in Table 1.

The nature of the poverty line, either relative or absolute, thus arises from the estimation of the parameter vector β on survey data, instead of being implicitly imposed by the researcher by the choice of a certain poverty line.

If the parameters are estimated on data from a representative sample of the population, the resulting line may be seen as a reflection of the prevailing views on poverty in society.

The LPL definition, derived from survey data and thus based on the *vox populi*, appears therefore in a sense an improvement upon definitions that depend

merely on the researcher's opinion on the nature of poverty, like the percentile method or the fraction of median-income method. The basic-needs and food-ratio definitions, although based on an *a priori* poverty concept, depend on survey data as well, albeit of a different type, viz., budget surveys. As budget surveys require a considerable effort of the respondents, the response is usually relatively low and ridden by selection bias; the resulting data set may very well be not representative of the society as a whole. The LPL can be seen as an improvement over budget survey definitions as it is based on a few direct survey questions on the welfare-income relationship rather than the more circumstantial evidence provided by a less comprehensive welfare proxy, like expenditures on food. It may be assumed that the response under *ceteris paribus* conditions may be larger and more representative than that for a comprehensive budget survey. (See for a further comparison Van Praag, Spit and Van de Stadt (1981)).

Up to now we have assumed that all income and welfare measurements are family-size independent, e.g. by using equivalent-adult scales. The derivation and use of equivalence scales will introduce another difference between the various poverty line definitions. The poverty line definitions based on budget survey data may be easily adapted to yield a family size effect in the Engel Function (see e.g. Deaton and Muellbauer (1980) for a review of the possibilities). Both the percentage of median-income and the percentile method will need some exogenously determined equivalence scale, to be applied for the calculation of the parameters of the family-size corrected income distribution and for the differentiation of the poverty line itself.

However, most equivalence scales face a problem, mentioned among others by Abel-Smith (1982):

> "Moreover, the chosen way of life becomes very different for families with and without children. The process of having children seems to change expenditure preferences. (.....) What is really meant by "equivalence" when ways of life are so different?"

The LPL may be differentiated according to family size by the inclusion of family size in equation (19). As this equation attempts to measure welfare directly, the effects of family size on welfare may be found in a "natural" way, such as to incorporate changing preferences as well (Kapteyn and Van Praag (1976) and Van Praag, Spit and Van de Stadt (1982)). These welfare-neutral equivalence scales may also be derived for various other characteristics, like degree of urbanization, occupation, etc. (see Van Praag, Hagenaars and Van Weeren (1982)).

5. An Empirical Illustration

Once data on the welfare parameters μ and σ from a representative sample are available, equations (19) and (20) may be estimated after adding an i.i.d. disturbance term to each equation. The resulting parameters $\hat{\beta}_0$, $\hat{\beta}_1$, $\hat{\beta}_2$, and $\hat{\beta}_3$ will yield information on the extent to which poverty is perceived as a situation of relative deprivation. As mentioned in section 3, time series, panel data or international data are needed for an estimation of these equations.

In order to illustrate the method described, the parameters have been estimated using cross-sectional data of 8 European countries.[4] The data were collected in October 1979 for a research program carried out at the Center for Research in Public Economics in Leyden, with financial support of the European Communities.

For a description of the data and the results by country we refer to Van Praag, Hagenaars and Van Weeren (1982). In this paper we restrict ourselves to the estimation of the parameters β, using a transformed version of equation (21), where (19) and (20) are substituted for μ_t and σ_t. For each respondnet we have an estimate of the welfare parameter μ_t and σ_t, his actual income y_t, the log-median income of the country he is living in, μ_y, and the standard deviation of log-incomes, σ_y.

These data enable us to calculate for each individual the value $y_{\delta,t}$, the income level that is evaluated by δ, and to use this as the dependent variable in the equation

$$(21') \qquad \ln y_{\delta,t} = \beta_0 + \beta_1 \ln y_t + \beta_2 \mu_y + \beta_3' \sigma_y + \beta_4 \ln fs + \varepsilon_t,$$

where $\beta_3' = \beta_3 u_\delta$ and where ln *fs* stands for log (family size), in order to allow for family-size differentiated poverty lines. Moreover, the data for different welfare levels have been pooled, assuming that the values of the parameters do not change over welfare levels, and allowing for a direct effect of σ_y on $\ln y_\delta$:

$$(21'') \qquad \ln y_\delta = \underset{(0.102)}{1.410} + \underset{(0.003)}{0.574} \ln y + \underset{(0.010)}{0.224}\, \mu_y + \underset{(0.026)}{0.712}\, \sigma_y + \underset{(0.013)}{0.807}\, \sigma_y u_\delta + \underset{(0.003)}{0.085} \ln fs$$

$$\bar{R}^2 = 0.626$$

$$N = 39{,}396.[5]$$

It is seen that $\hat{\beta}_1$ is about 0.57 and $\hat{\beta}_2$ equals 0.22, implying an elasticity of the poverty line with respect to median income of $\hat{\beta}_2/(1-\hat{\beta}_1) = 0.51$. This may be interpreted as the extent to which the poverty line is relative. It is furthermore seen that σ_y has a significant positive effect on the poverty line. The level of inequality in a country has apparently *per se* an increasing influence on the poverty line of a country, whatever the welfare level considered.

This result may be explained theoretically by the fact that the *preference drift* β_1, the parameter that reflects the change in evaluation when family income changes, is probably related to the income inequality in a country. This can be seen from the estimates of an alternative specification:

$$(28) \qquad \ln y_\delta = \underset{(0.093)}{1.763} + \underset{(0.003)}{0.536} \ln y + \underset{(0.003)}{0.072}\, \sigma_y \ln y + \underset{(0.010)}{0.226}\, \mu_y + \underset{(0.013)}{0.807}\, \sigma_y u_\delta + \underset{(0.003)}{0.085} \ln fs$$

$$\bar{R}^2 = 0.626$$

$$N = 39{,}396.$$

The coefficient of ln *y* equals $0.536 + 0.072\, \sigma_y$, reflecting the fact that changes in income will give rise to larger changes in needs, if the income inequality in a population is larger. This relationship follows indeed from Van Praag and Spit

[4]These countries are Belgium, Denmark, France, Germany, Ireland, Italy, The Netherlands and The United Kingdom.

[5]This number is composed of 13.132 observations, for each of which three welfare levels, $\partial = 0.4$ 0.5 and 0.6, are used.

(1982) and Van Praag (1981), where the variables explaining the Individual Welfare Function of Income are derived theoretically according to social filter theory. In future research this relationship will be further explored.

This implies that an increase in the income inequality increases the number of poor for two reasons: firstly, because more people will have an income below the poverty line as defined before the change in income inequality, and secondly because *the poverty line itself will shift upwards*, causing an additional increase in the number of poor.

All coefficients are significantly different from zero, and the $\bar{R}^2$'s are very high for cross-sectional individual data. However, these figures should be interpreted with some caution: it is of course possible that the perception of poverty and welfare differs over countries for more reasons than the ones mentioned here, for instance, because of differences in income history, climatological differences and cultural differences. The use of this method might be better illustrated if panel data were available, in order that some of these factors may be accounted for.

Nevertheless, the illustration given above shows that the method described in this paper, which is based on direct measurement of welfare, may yield family size differentiated poverty lines which reflect the perception of poverty in the society, and hence are related to median income and income inequality in society. Their specific form, however, is not *a priori* postulated, but estimated from survey data. For the calculation of poverty lines proper more detailed data are required. For these figures we refer to Van Praag, Hagenaars and Van Weeren (1982).

6. Conclusion

It has been shown that most well-known poverty line definitions are based on an assumption about the nature of poverty, viz. whether poverty is absolute, relative or somewhere in between. The Leyden Poverty Line has the advantage that this choice is not made by the researcher but as a result of the perception of poverty by a representative sample of the population. All poverty line definitions considered may be seen as special cases of the Leyden definition, depending on the value of the estimated parameters, and hence on the perceived nature of poverty. The Leyden Poverty Line may be differentiated for family size and other characteristics in a "welfare-neutral" way.

An empirical illustration showed that the Leyden Poverty Line is dependent on both median income and income inequality in society: the elasticity of the poverty line with respect to median income appeared to be about 0.51. The effect of increasing income inequality is to shift the poverty line upwards.

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